

INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
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NPTEL ONLINE CERTIFICATION COURSE
Structural Analysis of nanomaterial's
Lecture – 12
Diffraction Methods and Directions of XRD
With
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In our previous lecture, we have discussed about the, basics of XRD or may be the X-ray diffractions and the equipment, so today we are going to discussed about diffractions methods and the directions of XRD, so first we will discussed about, what are the different diffractions methods.

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Diffraction Methods:

- Diffraction can occur whenever the Bragg's Law is satisfied.
- With monochromatic radiation, an arbitrary setting of a single crystal in a beam of x-rays will not in general produce any diffracted beams.
- Some way of satisfying the Bragg's law must be devised, and this can be done by continuously varying either λ (wavelength) or θ (Bragg angle) during the experiment.
- The ways in which these quantities are varied distinguish the three main diffraction methods:

Method	Wavelength	Angle	Specimen
Laue Method	Variable	Fixed	Single Crystal
Rotating Crystal Method	Fixed	Variable (in part)	Single Crystal
Powder Method	Fixed	Variable	Powder

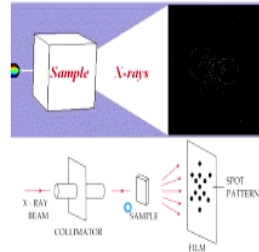


So diffractions can occur whenever the Bragg's law is satisfied, this we have already gone through, in our pervious lecture, with monochromatic radiations, an arbitrary setting of a single crystal, in a beam of X-rays, will not in general produce any diffracted beams, some way of satisfying the Bragg's law must be devised and this can be done by continuously, varying either, the wavelength λ , or may the Braggs angle, which is nothing but the θ , during the experiment, the ways in which these quantities are varied distinguish, the three main diffractions methods, so first is known as the Laue method where you, we are wearing the, variables is the wavelength angle is fixed and generally, we are doing it, for the single crystal, if it talk about the second one that is Rotating crystals methods, where the wavelength is also is fixed, angle variables in some part and that is also, we are doing for the single crystals and when we are talking about the powder XRD or may be the powder method, so the generally the wavelength is fixed here the Bragg's angle is the variable one and generally, we are doing it for the powder sample, so first we are going to discussed about the Laue method.

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1. Laue Method:

- The Laue method was the first diffraction method ever used, and it reproduces von Laue's original experiment.
 - It is mainly used to determine the orientation of large single crystals while white radiation is reflected from, or transmitted through a fixed single crystal.
 - The Bragg angle θ is fixed for every set of planes in the crystal, and each set picks out and diffracts that particular wavelength which satisfies the Bragg law for the particular values of d and θ involved.
 - Each diffracted beam has a different wavelength, selected out of the incident beam of white radiation by the d spacing and θ value of the crystal planes producing the reflection.
 - The position of any Laue spot can be altered if the wavelength of the diffracted beam changed otherwise it will be unaltered.
- ✓ It follows that two crystals of the same orientation and crystal structure, but of different lattice parameter, will produce identical Laue patterns.



So the Laue method was the first diffraction methods, because it has been developed fast, ever used till today and it reproduces von Laue's original experiment, it is mainly used to determine the orientations of large single crystals, so generally this is for the large single crystals samples, while white radiations is reflected from, or transmitted through a fixed single crystal, the Bragg's angle that is fixed in this particular case, for every set of planes, in the crystal, in the crystal and each set picks out and diffracts that particular wavelength which satisfies the Bragg's law for the particular values of D which is nothing but the inter-planer distances and θ involved, so from this particular image, you can see that we are having the sample which is nothing but the single crystals, so the X-rays are coming from these way that incident beam and then through this, the X-ray are maybe the diffracted beam has been generated, in this particular region and then it is directly falling on to the particular film and we are getting some kinds of white spots over there, so that is nothing but the different diffraction picks, or maybe the spots, each diffracted beam has a different wavelength selected out of the incident beam of white radiations by the D spacing and θ value of the crystal planes producing the reflections, the positions of any Laue spot can be alter early, if the wavelength of the diffracted beam changed otherwise it is not, it follows that two crystal of the same orientations, suppose we are having the two crystals having the same orientations and same crystal structure, but of different lattice parameter, will produced the same identical Laue patterns.

So whatever I have discussed into the upper image the same thing is replicated over here, we are having that incident beam then we are having collimators, then it is going through the samples then rays has been diffracted and then on the film it is giving some kind of spot pattern.

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Laue Description of Diffraction:

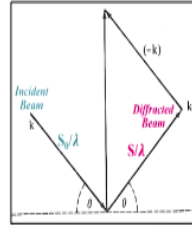
➤ Elastic Scattering:

$$|k| = |k'|$$

➤ Laue's Condition for Diffraction:

Laue's theory states that, for diffraction, the difference in the two wave vectors must be equal to a reciprocal lattice vector, i.e.

$$\vec{k}' - \vec{k} = \sigma_{hkl}$$



$\vec{k} = S_0/\lambda =$ Incident wave vector being absorbed
 $\vec{k}' = S/\lambda =$ Re-emitted as an outgoing wave vector

$\vec{S}_0 \rightarrow$ incoming X-ray beam
 $\vec{S} \rightarrow$ Scattered X-ray beam

Ewald Sphere:

- The condition of diffraction can also be shown in a pictorial way by a construction which is known as the Ewald construction.
- A geometrical construction that provides the relationship between the orientation of a crystal and the direction of the beams diffracted by it.
- If the origin of reciprocal space is placed at the tip of incident beam then diffraction will occur only for those reciprocal lattice points that lie on the surface of the Ewald sphere.



Now what is the Laue descriptions of diffractions, so now we are going to discuss from this particular graphs, so in this particular case, first we have to know what is the Elastic Scattering, it is nothing, but the total K, I means magnitude, magnitude of the incident beam and the magnitude of the diffracted beam which is nothing but the K and the K prime, now is the Laue's on this on spot diffractions, so Laue's theory states that for diffractions the differences in the two wave vectors must be equal to a reciprocal lattices vector, what is that, that is K prime vector minus K vector is equal to sigma prime HKL, so what is K prime, so K prime is nothing but the S not by the λ which is nothing but the incident wave vector break it being absorbed, so here the D is the S not by λ and K prime vector, which is nothing but the A by λ Re-emitted as an outgoing vector, which is nothing but the S λ , so what is S's not prime incoming X-ray beam and S prime is the scattered X-ray beam.

Now from this particular we are going to discuss about the Ewald sphere, so Ewald sphere generally, it is a one kind of conditions of diffractions can also be shown in pictorial way by a constructions which is known as the Ewald constructions, so this is known as the Ewald constructions, so here the incoming beam is coming is nothing but the S0, then through single crystal it has be diffracted and it is going to the S and it has falling on to the film in this particular point.

So generally the Ewald sphere, if we make the thing that a virtual, or maybe the imaginary sphere that sphere is known as the Ewald sphere, whose radius is one by λ , I means the one by the wavelength, now this actually if is put the perpendicular, this is known as the one by DHK, so geometrical constructions that provides the relationship between the orientations of a crystal and the directions of the beams diffracted by it, if the origin of reciprocal space is placed at the tip of incident beam then diffractions will occur only for those reciprocal lattice points that lie on the surface of the Ewald sphere.

So in this particular case also we can see that the incoming X-ray beam is coming through is this one and then one this is for the, if it is passed diffracted and one is through transmitted, so if it is transmitted it is also coming into this particular point and here also the same thing Ewald sphere

the radius is one by λ , now actually we are going to make some relations or may be co-relations in between the Laue descriptions and Ewald sphere with the Bragg's law.
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Bragg's Law from Laue Description and Ewald sphere:

$$\left| \frac{S}{\lambda} - \frac{S_0}{\lambda} \right| = YZ + XZ$$

$$\Rightarrow \left| \frac{S - S_0}{\lambda} \right| = CY \sin \theta + CX \cos \theta = \frac{1}{\lambda} \sin \theta + \frac{1}{\lambda} \sin \theta = \frac{1}{\lambda} (2 \sin \theta)$$

$$\Rightarrow |S - S_0| = 2 \sin \theta \dots \dots (1)$$

From Laue's condition;

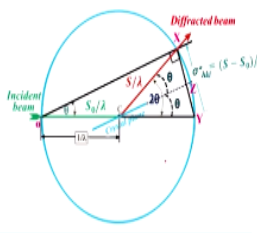
$$\bar{k}' - \bar{k} = \sigma'_{hkl}$$

$$\Rightarrow |\bar{k}' - \bar{k}| = \left| \frac{S - S_0}{\lambda} \right| = |\sigma'_{hkl}| = \frac{1}{d_{hkl}}$$

$$\Rightarrow |S - S_0| = \frac{\lambda}{d_{hkl}} \dots \dots (2)$$

From equation (1) and (2),

$$2 \sin \theta = \frac{\lambda}{d_{hkl}}$$

$$\Rightarrow \lambda = 2d_{hkl} \sin \theta, \text{ which is Bragg's law.}$$


Laue's Equation:

$$a \cdot \frac{(S - S_0)}{\lambda} = a \cdot (ha' + kb' + lc') = h$$

$$b \cdot \frac{(S - S_0)}{\lambda} = k$$

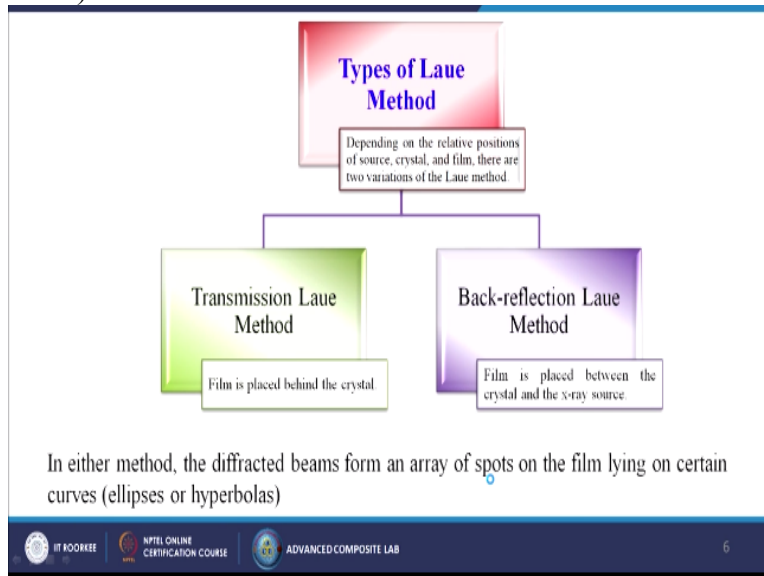
$$c \cdot \frac{(S - S_0)}{\lambda} = l$$

So in this particular image may that figure you can understand that the incident beam is coming and then you are having your samples are maybe the single crystals over here and it has been diffracted beam and it falling on to the film, in this particular point which is nothing but known as the X, so now if this angle is two θ it is nothing but the diffracted angle, so this is the total two θ value over there so here in this particular cases so your C is going if we make a lines so this angle is θ and this angle is θ , so whole parts is known as the two θ value.

So now this value is θ by λ which we have already gone through and now this one diffracted beam is also by the $S \lambda$ so now we are going to make a perpendicular on to this point on X and we are going to make a perpendicular from O to X also, so what is total XY, XY is nothing but the XZ+ZY, which is nothing but the sigma prime HL, which is nothing but the S-S0 by λ , so now come to here and here this is the radius here R is equal; to one by λ so now if we write like this that S by λ - S0 by λ which is nothing but the YX+ XZ, so here S0 λ and S λ so S λ - S0 λ is equal to YZ, YZ + XZ which is nothing but the S -S0 by λ is equal to CY, CY mean this C and Y, so CY $\sin \theta$ + CX $\cos \theta$ so this one is $\cos \theta$ + CY $\sin \theta$ which is nothing but the one by $\lambda \sin \theta$ +one by $\lambda \sin \theta$ that means if we it common one by λ so one by λ two $\sin \theta$ so that means S- S0 is equal to two $\sin \theta$ this is the number one equations.

Now from Laue conditions already we have gone through in our last slides that K prime vector - K vector is equal to sigma prime HKL where the magnitude of K prime vector- K prime is equal to S-S0 by λ which is nothing but the sigma prime HL is equal to one by DHKL so from this particular point we are getting this value so that means S-S0 s equal to λ by DHKL now from equations if we combining the both so now two $\sin \theta$ here two $\sin \theta$ and in teams of S- S0 we are replacing it by the λ by DHKL so two $\sin \theta$ is equal to λ by DHKL that means λ is equal to two DHKL $\sin \theta$ which is nothing but the Bragg's law so what is the Laue conditions so Laue condition is that A multiplications with S-S0 by λ is equal to A into HA prime + KB prime +LC

prime is nothing but the H so B prime into $S-S_0$ by λ is equal to K and C prime $S-S_0$ by λ is equal to L so this is the Laue equations so from this particular case we have made some co-relations with the Bragg's law from the Laue descriptions and the Ewald sphere.
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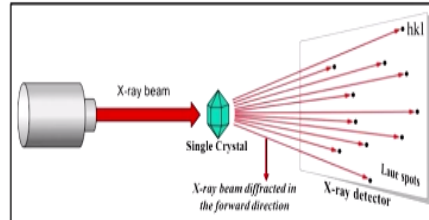
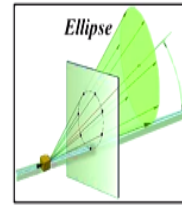
Now we are going to discuss about the different types of Laue methods so already what is Laue method so it is depending on the relative positions of the sources crystal and film there are at two variations of the Laue method one is called the Transmission Laue method another one is called the Back-reflections Laue method what is transmission Laue method that is film is placed behind the crystal so that means you are having that source of X-ray you are having are having that crystals and you are putting film the that means so that means the ray is transmitting through crystal and falling on to you are film and what is back reflections Laue method the film is placed between the crystal and the x-ray source that means you are having that source you are having the film and you're having that crystal.

So direct incident beam is going on into the crystals then it is back scattered, or maybe back reflected and then it is falling on to the film itself in either method the diffracted beam from an array of spots on the film lying on certain curves maybe whatever the points it is falling on to the film of you see it carefully we can find it either the shape of that point is maybe the elliptical, or some kind of hyperbolas, okay

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Transmission Laue Method:

- In the transmission Laue method, the film is placed behind the crystal to record beams which are transmitted through the crystal.
- One side of the cone of Laue reflections is defined by the transmitted beam.
- The film intersects the cone, with the diffraction spots generally lying on an ellipse.



Specimen Preparation of Transmission Method:

Thinner specimen, low absorption, Not too thin or the diffracted intensity will be too low, since

$$\text{Intensity of a diffracted beam} \propto \text{Volume of diffracting material}$$

Now we are going to discuss about the Transmission Laue method, so in the transmission Laue method the film is placed behind the crystal to record the beam which are transmitted through the crystal itself, so in this case this is our crystal films so one side of the cone of Laue reflections is defined by the transmitted beam, so this side is the transmitted beam, so the film intersects the cone with that diffractions spots generally lying on the ellipse, so you can see this in this particular image. Now specimen preparations how we are going to prepare the specimen for transmission maker, thinner specimen, low absorption not too thin or the diffracted intensity will be too low, since intensity of the diffracted beam is directly proportional to the volume of diffracting materials, so in this beam you can see the X-ray beam is coming from this particular side then we are having that single crystal and through that the beam is transmitted and it is falling on the X-ray detectors, these spots are known as the Laue spots and which we can determine by the HKL value.

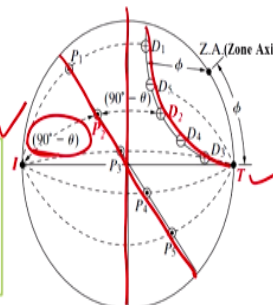
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Stereographic Projection of Transmission Laue Method:

- The crystal is at the center of the reference sphere, the incident beam I enters at the left, and the transmitted beam T leaves at the right.
- P_1, P_2, P_3, P_4 and P_5 five planes belonging to a zone.
- D_1, D_2, D_3, D_4 and D_5 are the respective diffraction directions required.

To find D_2 :

1. I, P_2, D_2 and T are coplanar. ✓
2. Angle between I and P_2 is $(90^\circ - \theta)$, and D_2 must lie at an equal angular distance on the other side of P_2 .



The diffracted beams so found, D_1 to D_5 , are seen to lie on a small circle, the intersection with the reference sphere of a cone whose axis is the zone axis.

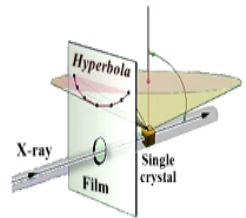
So in the next slides we are going to discuss about the stereographic projections of transmission Laue methods, so the crystal is at the center of the references sphere the incident beam is I, so this is the I enters at the left, so this is the left side and the transmitted beam T leaves at the right side, now this P1, P2, P3, P4, P5 this five planes belonging to a Zone, so this is known as the zone D1,D2,D3,D4,and D5 are the respective diffractions directions required.

So this one is the diffractions directions, now suppose we are going to find the point D2, so what we have to do we have to know that I P2,D2 and T are the coplanar, so this I, this P2, then D2 and the T are coplanar, now angle P2 and I and P2 is $90 - \theta$ and D2 must lie at an equal angular distances on the other side of P2, so if you take this one as a mirror so just D2 the is the mirror image of P2 now the diffracted beams so found D1 to D5 are seen to lie on this small circle the intersections with the references sphere of a cone whose axis is the zone axis.

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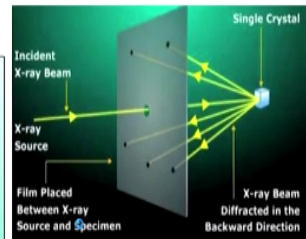
Back Reflection Laue Method:

- In the back-reflection method, the film is placed between the x-ray source and the crystal. The beams which are diffracted in a backward direction are recorded.
- One side of the cone of Laue reflections is defined by the transmitted beam.
- The film intersects the cone, with the diffraction spots generally lying on a hyperbola.



Specimen Preparation of Back Reflection Method:

No restriction on the specimen thickness and quite massive specimens may be examined, since the diffracted beams originate in only a thin surface layer of the specimen.



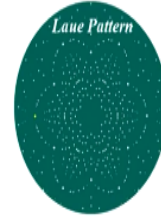
Now we are going to discussed about the back reflections Laue method so in this back reflections method the film is placed between the X-rays sources and the crystal the beams which are diffracted in a backward directions are recorded see so this is our film and this is our single crystal, so in this case the incident beam is going through this particular hole and directly it is falling on to the single crystals and then the back reflections is talking place and then those diffracted beam is falling on this particular film and they are making the shape in a hyperbola manner.

So the beams which are diffracted in the backward directions are recorded on this particular film one side of the cone of the Laue diffractions, is define by the transmitted beam the film intersects the cone with the diffractions spots generally lying on a hyperbola, so how we are going to prepare the specimen for this back reflections methods, no restrictions on the specimen thickness and quite massive specimen maybe examining, since the diffracted beam originate in only a thin surface layer of the specimen.

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Crystal Structure Determination by the Laue Method:

- ❖ The positions of the spots on the film, for both the transmission and the back-reflection method, depend on the orientation of the crystal relative to the incident beam.
- ❖ The spots themselves become distorted and smeared out if the crystal has been bent or twisted in any way.
- ❖ Although the Laue method can also be used to determine the crystal structure, several **wavelength can reflect in different orders from the set of planes**, with the different order reflections superimposed on the same spot in the film. This makes **crystal structure determination by spot intensity is difficult**.
- ❖ The rotating crystal method overcomes this problem.



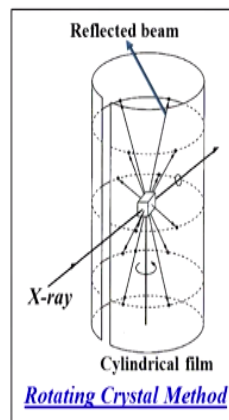
Main Uses of the Laue Methods:

1. Determination of crystal orientation.
2. Assessment of crystal quality.

Now we are going to discuss about the Crystal Structure Determinations by the Laue method, the positions on the spot on the film for both the transmission and the back-reflections method depend on the orientations of the crystal relative to the incident beam, of course from this particular image you can see that we are having the Laue pattern and through that you are getting some spots which is nothing but the Laue spots and it is giving you the picks are may be the information's about the crystals, the spot themselves they became distorted and smeared out. If the crystal has been bent or twisted in any way, otherwise it will be a form spot on it, although the Laue method also can be used to determine the crystal structure, several wavelengths can reflect in different order from the set of planes is of course with the different order reflections superimposed on the same spot in the film this makes crystal structure determinations by spot intensity is very, very difficult because there is a chance of overlapping of different spots each other, the rotating crystal methods overcome this problems, what is the main use of the Laue methods determinations of the crystal orientations and the assessment of then crystal quality. (Refer Slide Time: 18:10)

2. Rotating Crystal Method:

- ❖ In the rotating-crystal method, a **single crystal** is mounted an axis **normal (perpendicular)** to a monochromatic **x-ray beam**.
- ❖ A **cylindrical film** is placed around it and the **crystal** is **rotated** about the **chosen direction**, the axis of the film coinciding with the axis of rotation of the crystal.
- ❖ As the crystal rotates, sets of lattice planes will at some point make the correct Bragg angle for reflection of the monochromatic incident beam, and at that instant a reflected beam will be formed.
- ❖ The reflected beams are located on the surface of imaginary cones. When the film is laid out flat, the diffraction spots lie on horizontal lines.

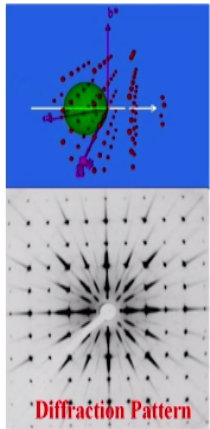


Now we are going to discuss about the second type which is nothing but the known as the Rotating crystal method, so in the rotating crystal method a single crystal is mounted on an axis normal to a monochromatic X-ray beam, so in this particular case we are putting the crystal in this particular position a cylindrical film is placed around it and the crystal is rotated about the chosen directions, the axis of the film coinciding with the axis of rotations of the crystal also. So in this particular case what we are doing that we are putting a single crystal and then single crystal is also rotating in a particular direction and that crystal is surrounded by some films and that film is also rotating into the same direction with the crystal itself so as the crystal rotates the set of lattice planes will at some point make the current Bragg's angle for the reflections of the monochromatic incident beam and at that instant a reflected beam will be formed, the reflected beams are located on the surfaces of imaginary cones, when the film is laid out flat, the diffraction spots lie on the horizontal lines, so you can see that you are getting that horizontal spots on to that from the diffraction beam.
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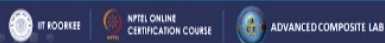
Crystal Structure Determination by the rotating crystal Method:

- By recording the diffraction patterns (both angle and intensities) for various crystal orientations, the shape and size of the unit cell as well as arrangement of atoms inside the cell can be determined.
- For given wavelength λ if angle θ at which a reflection occurs is known then interspacing d is determined by using Bragg's law.

$$n\lambda = 2d \sin \theta \quad \&$$

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad (\text{for cubic crystal structure})$$


Diffraction Pattern



12

So how to get the Crystal Structure Determinations by the Rotating Crystal Method, so by recording the diffraction patterns both the angle and the intensities, intensities here you can that how we are getting the different points on to the film itself, so for various crystal orientations the shape and the size of the unit cell as well as the arrangement of atoms inside the cell can be determined for given wavelength λ , if Bragg angle θ at which a reflection occurs is known then interspacing D is determined by using the Bragg's law which is nothing but the $L \lambda$ is equal to $2D \sin \theta$ and D is equal to A by root over $H^2 + k^2 + l^2$ generally it is for the cubic crystal structure.
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3. Powder Method:

- ❖ In the powder method λ is fixed but θ is variable (the sample consists of crystallites in various orientations).
- ❖ The crystal to be examined is reduced to a very fine powder and placed in a beam of monochromatic x-rays.
- ❖ Each particle of the powder is a tiny crystal, or assemblage of smaller crystals, oriented at random with respect to the incident beam.
- ❖ Some of the crystals will be correctly oriented so that their (100) planes, for example, can reflect the incident beam. Other crystals will be correctly oriented for (110) reflections, and so on.
- ❖ The result is that every set of lattice planes will be capable of reflection.
- ❖ The mass of powder is equivalent, in fact, to a single crystal rotated, not about one axis, but about all possible axes.

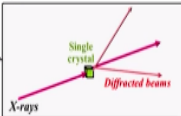
Now we are going to discuss about the Powder method, so in the powder method as I already told you initially that here the wavelength λ is fixed and the Bragg's angle or maybe the θ is variable the sample consists of crystallites in various orientations, the crystal to be examined is reduced to a very fine powder, so first we are making the material in a powder form and then placed in a beam of monochromatic x-rays, each particle of the powder is a tiny crystal is a small, small crystals or assemble of smaller crystals, oriented at random with respect to the incident beam, some of the crystal will be correctly oriented so that their 100 planes, for example, can reflect the incident beam, other crystal will be correctly oriented for 110 reflections and so on the result.

Is that every set of lattice planes will be capable of reflections, the mass of powder is equivalent, in fact to a single crystal rotated, not about one axis, but about all possible axes.
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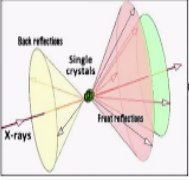
The Powder Method:

The powder method is used to determine the value of the lattice parameters accurately. Lattice parameters are the magnitudes of the unit vectors a , b and c which define the unit cell for the crystal.

- If a monochromatic x-ray beam is directed at a single crystal, then only one or two diffracted beams may result.

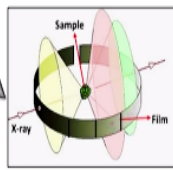


➤ If the sample consists of some tens of randomly orientated single crystals, the diffracted beams are seen to lie on the surface of several cones. The cones may emerge in all directions, forwards and backwards.



➤ A sample of some hundreds of crystals (a powdered sample) show that the diffracted beams form continuous cones.

➤ A circle of film is used to record the diffraction pattern. Each cone intersects the film giving diffraction lines. The lines are seen as arcs on the film.



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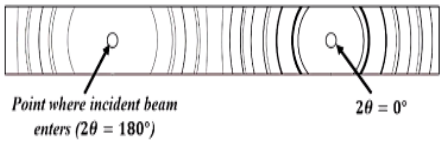
So how we are going to do the powder method, the powder is used to determine the value of the lattice parameters accurately, lattice parameters are the magnitude of the unit vectors A,B and C

which define the unit cell for the crystal, so here simple in a monochromatic X-ray beam is directed at a single crystal then only one or two diffracted beams may result, so from this picture you can see that X-ray when it is coming, it is falling on to the single crystals may be one or two diffracted beam has been fall it, but when you are talking about the powder crystals. Simple when the incident beam is coming on to that powder materials then there are, so many different diffracted beam has be generated and it is falling into the film itself, so if the sample consist of some tens of randomly oriented single crystal the diffracted beams are seen to lie on the surface of several cones, the cones, may emerge in all directions forwards and backwards. So you can see that plane falling, there are so many single crystals over there that is some kind of powder materials, so tiny single crystals are there so when the X-ray beams are coming then after reflections it is creating so many cones for different crystal materials, so a sample of hundred of crystals a powder sample show that the diffracted beam from continuous cones, a circle of film is used to record the diffractions pattern each cone intersect the film is giving diffractions lines, the lines are seen as the arcs on the films , so this lines actually it is known as the arcs on to the film itself.

(Refer Slide Time: 23:19)

Powder Diffraction Film:

- The cones of diffracted radiation intersect the cylindrical strip of film in lines or arcs and, when the strip is unrolled and laid out flat.
- Each diffraction line is made up of a large number of small spots, each from a separate crystal particle, the spots lying so close together that they appear as a continuous line.
- The lines are generally curved, unless they occur exactly at $2\theta = 90^\circ$ when they will be straight.
- From the measured position of a given diffraction line on the film, θ can be determined, and, knowing λ , we can calculate the spacing d of the reflecting lattice planes which produced the line.



Appearance of film when laid out flat

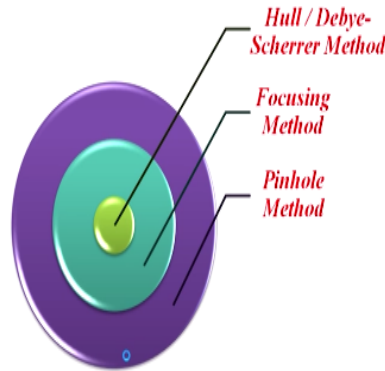
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Now how it looks, Powder Diffractions Films the cones of diffracted radiations intersect the cylindrical strip of film in lines or arcs and when the strip is unrolled and laid out flat, so after that we are making the film in a flat and we are getting the lines in such a manner some lines are strip some lines are in a circular shape, or may be the round shapes, so each diffractions lines is made up of a large number of small spots each from a separate crystal particle the spots lying. So close together that they appears as a continuous line, it looks like a bar code the lines are generally curved unless they occur exactly at 2θ equal to 90 degree when they will be straight in this particular case you can see that we are getting the lines almost straight, so from the measured position of a given diffractions line on the film, θ can be determined and knowing the λ value we can calculate the spacing D of the reflecting lattice planes which produced the lines, so from this particular case we are getting the θ value is 0 and in this particular line here the two θ is 90 degree, and in this particular case the 2θ is equal to 180 degree.

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Types of Powder Method:

There are three main photographic powder methods in use, differentiated by the relative position of the specimen and film:

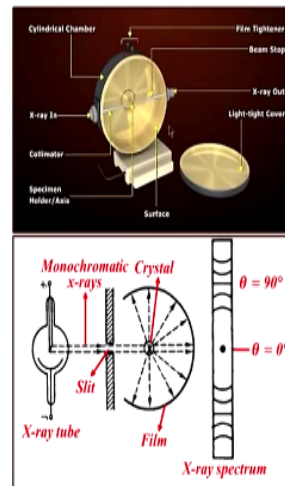


Now what are the types of the Powder methods so generally three types methods are available, so there are three main photographic powder methods in use, differentiated by the relative positions of the specimen and the films, so first one is known as the Hull Debye- Scherrer methods next one it is called the focusing methods and the third one is known as the Pine hole method.

(Refer Slide Time: 25:04)

I. Debye-Scherrer Powder Method:

- The Debye-Scherrer and other variations of the powder method are very widely used, especially in metallurgy.
- In this method, a narrow strip of film is curved into a short cylinder with the specimen placed on its axis and the incident beam directed at right angles to this axis.
- The method is especially suited for
 - ✓ determining lattice parameters with high precision
 - ✓ the identification of phases, whether they occur alone or in mixtures such as polyphaser alloys, corrosion products, refractories, and rocks.



So what is the Debye-Scherrer Powder so the Debye-Scherrer and other variations of the powder method are very widely used especially in metallurgy that means for the metallurgical samples, so in this methods, a narrows strip of film is curved into a short cylinder with the specimen placed on it is axis and the incident beam directed at right angles to the axis itself this methods is especially suited for determining the lattice parameters with high precision, the identifications of phases whether they occur alone, or in mixture such as polyphaser alloys corrosion products refractories and rocks.

So in this particular case we are having the X-ray tubes, X-ray beam incident is going we are having that slit and then we are having the film and here are we are going to put our crystal structure and we are getting g the x-ray spectrum in this particular manner this is X θ is equal to 0 degree and in this particular line the θ is equal to 90 degree.
(Refer slide Time: 26:10)

Debye-Scherrer Powder Method- Resolving Power:

Resolving power is the ability to separate diffraction lines from sets of planes of very nearly the same spacing.

$$S = 2\theta R$$

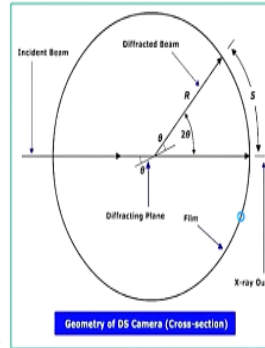
$$\Delta S = 2R\Delta\theta \Rightarrow d\theta = \frac{dS}{2R}$$

From Bragg's Law, $\lambda = 2d \sin \theta$ ($\because n = 1$)

$$\frac{d\theta}{d} = \frac{-1}{d} \tan \theta \Rightarrow \frac{dS}{d} = \frac{-2R}{d} \tan \theta$$

Thus, **Resolving Power** = $\frac{d}{\Delta d} = \frac{-2R}{\Delta S} \tan \theta$

where, d is the mean spacing of the two sets of planes, Δd difference in their spacing, and ΔS the separation of two diffraction lines which appear just resolved on the film

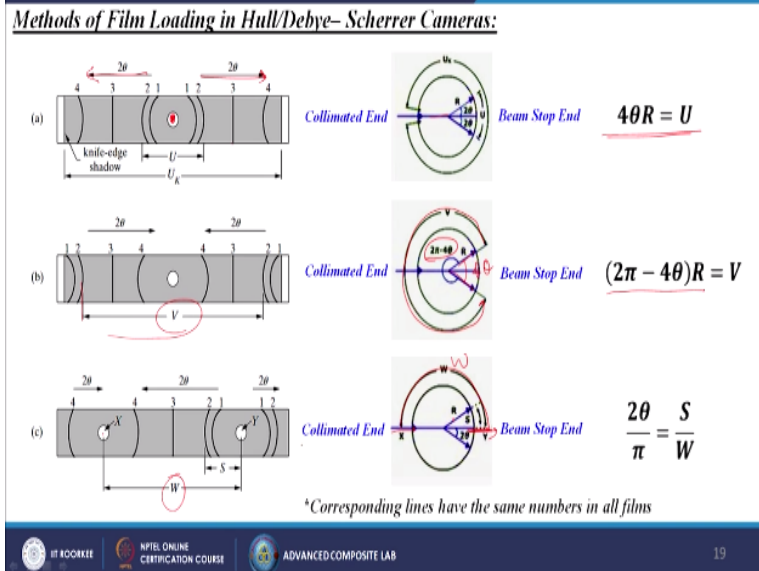


Now what we are getting from the Debye-Scherrer Powder Methods Resolving power, if you are going to calculate the resolving power, so resolving power is the ability to separate the diffractions lines from sets of planes of very nearly the same spacing, so here in this particular case what we are going to do we are putting the incident beam and then it is falling on to the samples and the reflected beam is falling on to the film in this particular point.

So this is the two θ which is the nothing but the diffraction angle and this is arc and here the incident beam angle is θ which is nothing but the diffracting plane, so this is the film and if we go, make a straight line from this point to this point which is nothing but the known as the S , so here the S is equal to two θ into R , so what is the Debye so if we do the diffractions is equal to two R which is constant in $d\theta$ which is nothing but the $D\theta$ is equal to DS by QR .

So from Bragg's law what we know, that λ is equal to two $D \sin \theta$ where N is equal to 1, so $D\theta$ by DD is equal to -1 by $D \tan \theta$ is equal to DS by DD is equal to $-2R$ by $D \tan \theta$, so thus the resolving power D upon $d\Delta d$ is equal to $-2R$ by $d\Delta d$ is $\tan \theta$, so where D is the mean spacing of the two sets of planes $d\Delta d$ is the differences in their spacing so in the last slides you have seen that so many lines are coming so what is the distances in between those spacing which is nothing but the know as the $d\Delta d$ and $d\Delta S$ is the separations of two diffractions lines which appear just resolved on the film itself.

(Refer Slide time: 28:09)



Now there are so many methods of film loading on the Hull Debye-Scherrer Cameras so first when we are talking about these say suppose we are having that here the θ is equal to 0 degree here the θ is 90 degree, two θ is increasing in this way, two θ is increasing in this way from 0 both the sides so this is known as the knife-edge shadow, so these distances is known as the capital U.

So one we are talking about the collimated end, so this side is the collimated end over here and then when the incident beam is going and then it is reflecting and it is making an angle of four θ , so here the four θR is the U, so from here to here, this distances to this distances when we are talking about the capital V means here how to calculating the spacing's in between the lines so if we want to calculate the spacing's in between these two points, number two to this number two points.

So in this particular case which is nothing but the V, so how to calculate the V so V is what the V is the total length of these one so this is the R and this the 4 θ so now rest of the thing is two $\pi - 4\theta$ so $2\pi - 4\theta$ into R is equal to capital V, now if we are measuring the distances in between this centers and this centers which is nothing but we are denoting as a capital W so now capital W means from this point to this point which is nothing known as the capital as the W, so incident, we was going and then we are thinking that it is going through fully transmitted, so now this angle is totally again it is 4 θ so 2θ by π is equal to capital S by W, corresponding lines have the same numbers in all films.

(Refer slide Time: 30:19)

II. Focusing Method- (SEEMANN-BOHLIN CAMERA):

- All converging rays from the target enter the slit and after passing it they diverge to the specimen.
- For a particular hkl reflection, each ray get diffracted by the same angle 2θ and converge to a focus at F.
- A number of incident beams contribute to each reflection, and a diffraction line is formed by the intersection of a number of cones with the film.

Focusing Method- Resolving Power:

To calculate value of θ : $4\theta R = U + \text{arc } SABN$

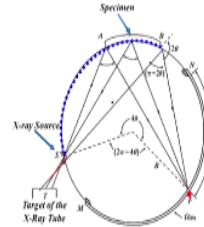
Differentiating we get,

$$d\theta = \frac{dU}{4R}$$

From Bragg's Law, $\lambda = 2d \sin \theta \Rightarrow \frac{d\theta}{dd} = \frac{-1}{d} \tan \theta$

Combining above bold equations,

$$\frac{dU}{dd} = -\frac{4R}{d} \tan \theta \Rightarrow \text{Resolving Power} = \frac{d}{\Delta d} = \frac{-4R}{\Delta U} \tan \theta$$



Now we are going to discuss about the Focusing methods which is nothing but the SEEMANN-BOHLIN CAMERA it is also known as, all converging rays from the target enter the slit and after passing it they diverge to the specimen, from HKL reflections each ray get diffracted by the same angle 2θ and converge to you are focus at F, a number of incident beam contribute to each reflections and diffractions line is formed by the intersections of a numbers of cones with film. So in this particular case you can see that target of the X-ray tube is this one then it is passing through to the materials then again it is diverging and then after that diverging it is falling on to the specimen and then again it is converging on to the film itself on a particular point, so here from that last slides lectures PPT's we have already gone through that here the S and R so here it is known as the $2\pi - 4\theta$ because this is known as the 4θ so here to calculate the value of θ , $4\theta R$, $4\theta R$ so $4\theta R$ into R which is nothing but the $U + \text{arc } SABN$. So the total this area so differentiating we get $D\theta$ is equal to DU by $4R$ from Braggs law λ is equal to $2D \sin \theta$ which is $D\theta$ by DD is equal to $- \text{one by } D \tan \theta$ combining our two equations this is number one equations and this is number two equations so DU by DD is equal to $-4R$ by $D \tan \theta$ so the resolving power D upon dl D is equal to $-4R$ by dl $U \tan \theta$. (Refer Slide time: 32:21)

Back Reflection Focusing:

- The most precise measurement of lattice parameter is made in the back-reflection region.
- The film straddles the slit and the specimen is placed diametrically opposite the slit.

Back Reflection Focusing- Resolving Power:

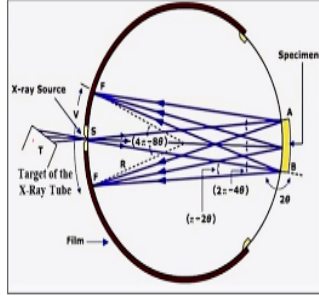
To calculate value of θ : $(4\pi - 8\theta)R = V$

Differentiating, we get $\Delta\theta = \frac{-1}{4R} \Delta\left(\frac{V}{2}\right) \dots \dots (1)$

From Bragg's Law, $\lambda = 2d \sin \theta \quad (\because n = 1)$

Differentiating, we get $\frac{d\theta}{d\lambda} = \frac{-1}{d} \tan \theta \dots \dots (2)$

Combining equation (1) and (2),

$$\text{Resolving Power} = \frac{d}{\Delta d} = \frac{4R}{\Delta(V/2)} \tan \theta$$


Here, d is the mean spacing of the two sets of planes, Δd difference in their spacing, and $\Delta(V/2)$ is the separation on the film of two reflections differing in Bragg angle by $\Delta\theta$.

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Now we are going to discuss about the Back Reflections Focusing so in that back reflections focusing what happens the simple thing the target is here the X-ray is coming then it is falling on to the specimen then back reflections is taking place and through that all the beams are falling into these two points into the slit on to the film itself so the precise measurement of lattice parameters is made in the back-reflections region, the film straddles the slit and the specimen is placed diametrically opposite the slit.

So in the back reflections focusing resolving power to calculate the θ value, so in this case what is that this angle is $4\pi - 8\theta$ so $4\pi - 8\theta$ into R so we are talking about this one, so $4\pi - 8\theta$ into R is equal to capital V so differentiating we get $d\theta$ is equal to -1 by $4R$ into dV by 2 so from Bragg's law λ is equal to $2D \sin \theta$ where N is equal to one so differentiating we get $D \theta$ by $D \lambda$ is equal to -1 by $D \tan \theta$ so combining equations 1 and 2 resolving power D up an dD is equal to $4R$ by dV by $2 \tan \theta$ so here D is the mean spacing of the two sets of planes $1/d$ D is the differentiations in their spacing and dV by 2 is the separation on the film of two reflections differing in Bragg's angle by $d\theta$.

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III. Pinhole Method:

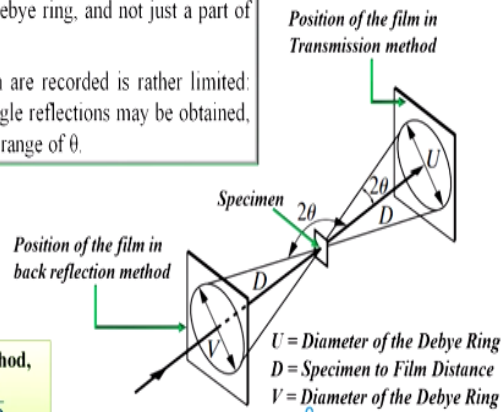
- In this method, an entire Debye ring, and not just a part of it, is recorded on the film.
- The range of values which are recorded is rather limited: either low-angle or high-angle reflections may be obtained, but not those in the median range of θ .

For transmission method,

$$\tan 2\theta = \frac{U}{2D}$$

For back reflection method,

$$\tan(\pi - 2\theta) = \frac{V}{2D}$$



So now we are going to discuss about the Pinhole methods so in these methods, an entire Debye ring and not just a part of it, is recorded on the film, the ranges of values which are recorded is rather limited either low-angle or high angle reflections may be obtained but those in the median range of theta so in this particular case we are putting one spacing over here, and now some rays maybe it has transmitted through samples or maybe it has been reflected through samples.

So both the diffracted beams are recording in this particular case so here whatever the shape are forming that these total distances is known as the capital U and here it is known as the capital V so U is the diameter of the Debye Ring D is the specimen to film distance which is nothing but the this one and in this case is this one and capital V is the diameters of the Debye Ring into the back reflections methods so here this is the film for the back reflections methods and this is the film for the transmission methods

So for the transmission $\tan 2\theta$ is equal to capital U by 2D this is number one and for back reflections methods $\tan \pi - 2\theta$, so this angle is totally 2 theta so 2 theta is angle over here diffracted beam so here $\pi - 2\theta$ is equal to capital V by 2D so this is now as for the Pinhole method.

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Directions of Diffracted Beams:

What determines the possible directions, i.e., the possible angles 2θ , in which a given crystal can diffract a beam of monochromatic x-rays?

- We need a general relation to predict the diffraction angle for any set of planes.

For example, if the crystal is cubic,

$$\lambda = 2d \sin \theta$$

$$\frac{1}{d^2} = \frac{(h^2 + k^2 + l^2)}{a^2}$$

Combining these equations, we have

$$\sin^2 \theta = \frac{\lambda^2}{4a^2} (h^2 + k^2 + l^2)$$

This equation predicts, for a particular incident wavelength λ and a particular cubic crystal of unit cell size a , all the possible Bragg angles at which diffraction can occur from the planes (hkl) .

So now we are going to discussed about the Directions of the Diffracted Beams so what determines the possible directions that is the possible angle 2θ , in which a given crystal can diffracted a beam of monochromatic X-rays we need a general relations to predict the diffractions angle for any set for planes so for example if the crystal is cubic so generally λ is $2D \sin \theta$ so one by D^2 is equal to $H^2+k^2+L^2$ by A^2 , combining this equations we have $\sin^2 \theta$ is equal to λ^2 by $4A^2$ into $H^2+K^2+L^2$ the equations predicts for a particular incident wavelength λ and a particular cubic crystal of nits cell size A all the possible Braggs angle at which diffractions can occur from the planes which is nothing but the HKL lengths.
(Refer Slide Time: 36:36)

- If the crystal is tetragonal, with axes a and c , then the corresponding general equation is

$$\sin^2 \theta = \frac{\lambda^2}{4} \left(\frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2} \right)$$
- Diffraction directions are determined solely by the shape and size of the unit cell
- Conversely, all we can possibly determine about an unknown crystal by measurements of the directions of diffracted beams are the shape and size of its unit cell

Here, d is inter-planar distance;
 V is the volume of unit cell;
 a, b, c and α, β, γ are the lattice parameters of unit cell and angle between them respectively.

Formulas for Inter-planar spacing (d) calculation of different crystal structures

Crystal structure	Formula to find " d "
Cubic ($a=b=c$ & $\alpha=\beta=\gamma=90^\circ$)	$\frac{1}{d^2} = \frac{h^2 + k^2 + l^2}{a^2}$
Tetragonal ($a=b \neq c$ & $\alpha=\beta=\gamma=90^\circ$)	$\frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$
Orthorhombic ($a \neq b \neq c$ & $\alpha=\beta=\gamma=90^\circ$)	$\frac{1}{d^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}$
Hexagonal ($a=b \neq c$ & $\alpha=\beta=90^\circ, \gamma=120^\circ$)	$\frac{1}{d^2} = \frac{4}{3} \left(\frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2}$
Monoclinic ($a \neq b \neq c$ & $\alpha=\beta=90^\circ \neq \gamma$)	$\frac{1}{d^2} = \frac{1}{\sin^2 \beta} \left(\frac{h^2}{a^2} + \frac{k^2 \sin^2 \beta}{b^2} + \frac{l^2}{c^2} + \frac{2hk \cos \beta}{ac} \right)$
Triclinic ($a \neq b \neq c$ & $\alpha \neq \beta \neq \gamma \neq 90^\circ$)	$\frac{1}{d^2} = \frac{1}{V^2} \left((h^2 b^2 c^2 \sin^2 \alpha + k^2 a^2 c^2 \sin^2 \beta + l^2 a^2 b^2 \sin^2 \gamma) + 2hkb^2 c^2 (\cos \alpha \cos \beta - \cos \gamma) + 2kla^2 bc (\cos \beta \cos \gamma - \cos \alpha) + 2hlab^2 (\cos \alpha \cos \gamma - \cos \beta) \right)$

Now if the crystal is tetragonal with axes A and C, then the corresponding general equations is $\sin^2 \theta$ is equal to λ^2 by 4 into $H^2 + K^2$ by $A^2 + L^2$ by C^2 diffractions directions are determined solely by the shape and of the unit cell, conversely all we possible determine about an unknown crystal by the measurement of the directions of the directions diffracted beams are the shape and size of it is unit cell so here in this particular case we having formulas for inter-planers spacing D calculations of different crystal structure.

So for Cubic's so generally where A is equal to B is equal to C and alpha, beta, Gama all are 90 degree so what is the D formula on by D^2 is equal to $H^2 + K^2 + L^2$ by A^2 so like this way we can calculate for cubic, tetragonal, oribarbambic, hexagonal, monoclinic, or may be the triclinic structure and here all are the their assumptions and right hand side is the how to find the D or may be the calculations of the D value, so here D is known as the inter-planer distances capital V is the volume of unit cell small a,b,c and alpha, beta gama are the lattice parameters of unit cell and angle between them respectively.

(Refer slide time: 38:00)

Advantages of XRD:

- X-rays are the least expensive, the most convenient & the most widely used method to determine crystal structures.
- X-rays are not absorbed very much by air, so the sample need not be in an evacuated chamber.

Disadvantage of XRD:

X-rays do not interact very strongly with lighter elements.

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So now we are going to discussed about the advantage of the XRD, so X-rays are the least expensive, the most convenient & the most widely used method to determine the crystal structure X-ray are not absorbed very much by air, so the sample need not be in a evacuated chamber, but of course there are been certain disadvantages also so X-ray do not interact very strongly with lighter elements.

(Refer Slide time: 38:29)

Summary:

- ❑ There are different types of diffraction methods depending upon the wavelength and types of specimen analyzed.
- ❑ All methods (e.g. Laue, Rotating Crystal and Powder methods) and their application to analyze of different crystal structures of single and polycrystalline materials are discussed.

So now we are going to the whole study of this particular lecture, so there are different types of diffractions methods already we have discussed depending upon the wavelengths and the types of specimen analyzed say for single crystal or may be for the powder samples all methods example like Laue methods Rotating Crystal, and Powder Methods and their applications to analyzed of different crystal structure of single and for the polycrystalline because we have discussed about the powder samples which we are thinking that is a combinations of different so many types of single crystals materials has been discussed in this particular lecture, Thank you.

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