Seakeeping and Manoeuvring

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Module No. # 01
Lecture No. # 10
Coupled Motions

So far, we spoke about single degree freedom equations of motion right.

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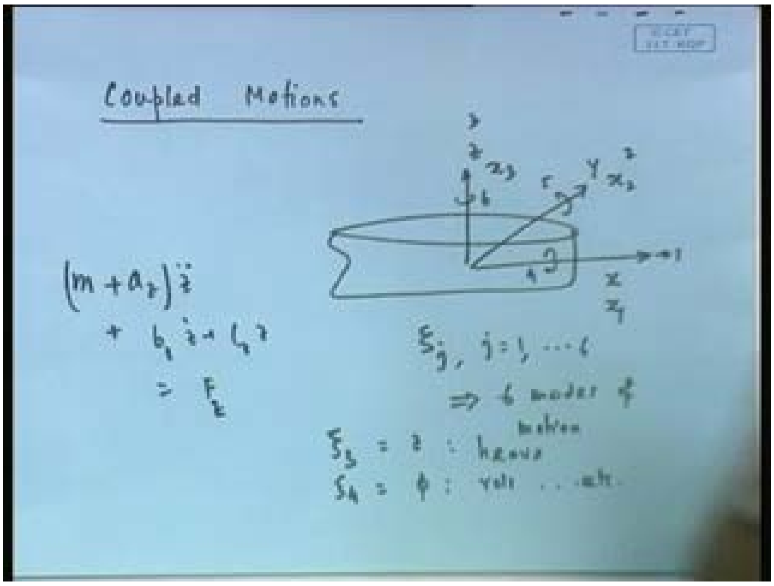

Today, I will like to talk about Coupled Motions. See, what we said earlier is that one mode of motion have no influence on the other mode of motion, so we have an equation which are all single degree. For example, let me first of all call this for convenience, I am going to like you know use index notation, or we can call this also x 1, x 2, x 3 direction, like this is direction 1, 2, 3, in x 3 means z, x 2 mean y, etcetera, as well as this motions, rotational motions; we will call by index 4, 5, and 6.

In other words, let us call this xi j 1 to 6; they are the 6 modes of motion. For example, xi 3 equal to z heave, 4 is phi etcetera. This is for convenience, because earlier we used you know like z as xi 3, phi as xi 4 (()), theta as xi 5, now let us use that.

So, what we had, an equation earlier is something like this, see m m plus say heave equation, if I write a z into z dot dot plus b z into z dot C z into z equal to F z, we have this equation. That means, heave is as if uncoupled, no effect of heave on any other mode of motion, etcetera. Now, suppose I presume that all 6 modes of motions are connected to each other, coupled to each other.

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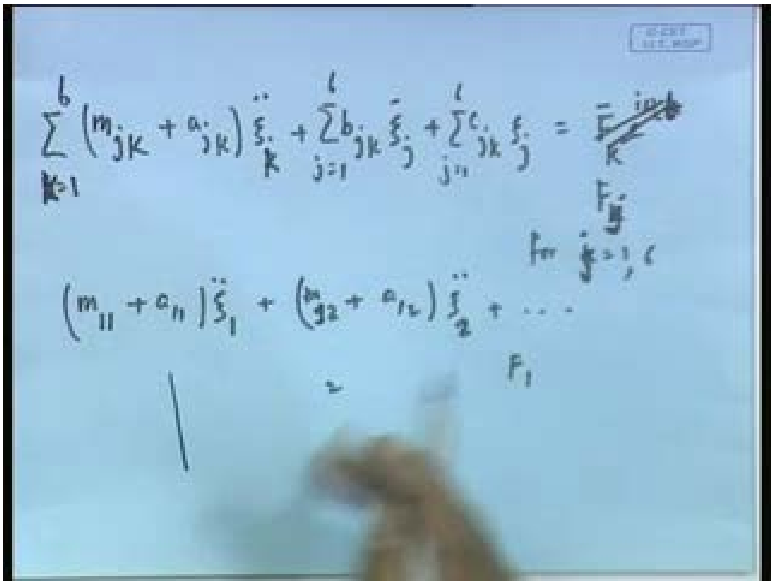

How is the equation of motion look like, see it would then appear as, let me write j k (No audio from 2:56 to 3:28) this is going to be ok, or maybe I may, let me put in this way F bar I k no into sorry this is t, let me just write this as right now right F k, exciting force k.

See with what I mean is that (No audio from 4:14 to 4:20), in other words what happen see, it is like a matrix form, you end up something getting like m 1 1 plus a 1 1, 1 dot dot plus m 1 2 sorry this is j you know j equal to, anyhow I think this index we will write i j, no we will make it this way, other way K equal to 1 to 6, and this we make it this as j sorry we will do it other round, it does not really matter, this is k equal to 1, this will be k.

Anyhow, I think that this is becoming basically m 1 1 plus a 1 1 xi 1 dot plus m 1 2 a 1 2 xi 2 and this is k only dot dot plus, like that equal to F 1, like that we will have 6 motion equation. In fact, it will be easier to see while write in a matrix form.

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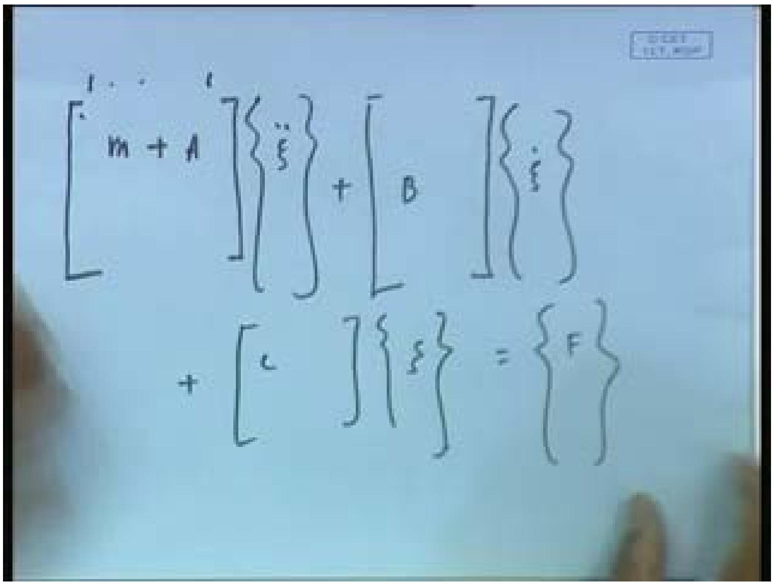

What would happen, we will end up getting a matrix m plus A, added mass (No audio from 5:58 to 6:21). What happen see, if I write 6 by 6 equation of motion presuming that all 6 modes of motion are coupled, means one influences the other, I end up getting an equation something like that m 1 1 plus a 1 1 xi 1 dot dot plus m 1 2 plus a 1 2 xi 2 dot dot, etcetera, etcetera. The question is that, what is the physical meaning that is what you globally want to know now, because this is a kind of a, just a mathematical expression in matrix form saying that each 6 modes of motion influences the other 6 modes of motion in some sense.

Again looking at that, if I have to lo first term, see it going to be m 1 1, see this is 1 to 6, 1 like that 6, m 1 1 plus a 1 1 into xi 1 dot dot, it has 6 modes.

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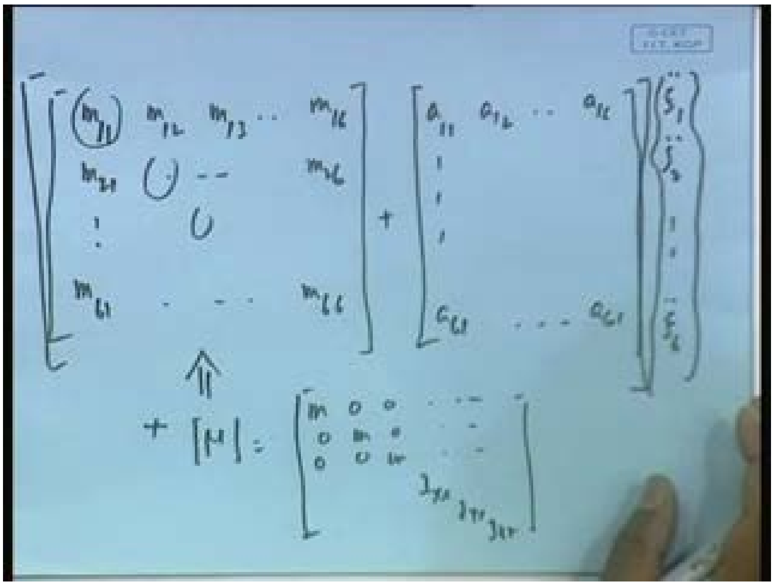

Let me write one one expression, just one, see rather we can write separately m 6 1 (No audio from 7:24 to 7:36), this we can break it down in two parts, plus let me write small hand, this full thing (No audio from 8:04 to 8:15), like that this is this is my first term. What it shows here, m 1 1 plus a 1 1 xi 1 dot dot plus a 1 2 a 1 2 xi 2 dot dot etcetera.

In fact, this matrix of course, we will all know that this matrix is very simple, this one will turn out to be simply the diagonal terms m m m and the three i terms, I 1 1 I 2 2 I 3 3 or I itself, because we are taking the c g at the origin of the c g. Therefore, all other terms becomes 0, this is there is no debate on that much, because this is a rigid body moment of inertia. In other words, this this this matrix M matrix is m 0 0, etcetera, 0 m 0, 0 0 m and here you end up getting I X X I y y I z z.

So, we do not have to worry to much on that, this I think we all know it, there is not much coupling what is important is to recognize this one, what is meant by a i j.

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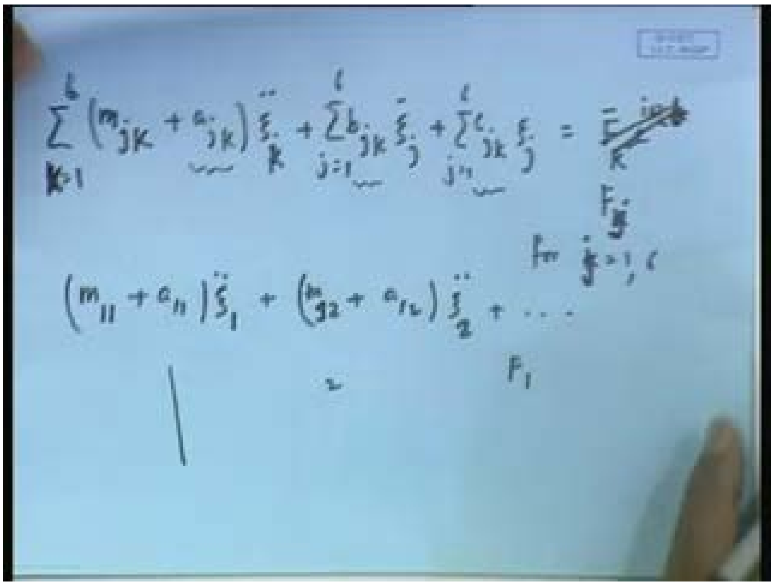

Let us now understand this, what is meant by this, see if I go back to this or j k, we are having this kind of expression b j k, a j k, c j k, this is my radiation force, say a j k and b j k, what is a j k, what does it mean, this is what we want to know.

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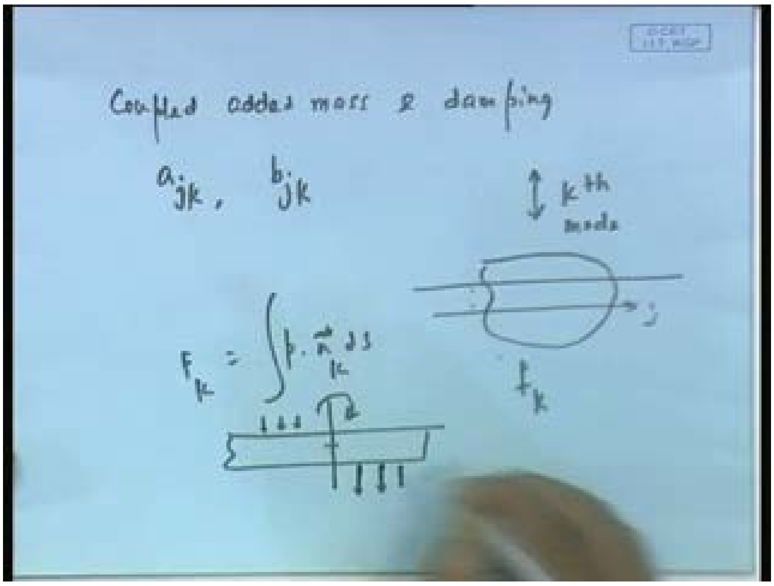

Now, let us let us look at this, expression of a j k and b j k. See, (No audio from 9:50 to 10:07), now remember this one thing, what is added mass, what we said physically, if I took a body, if I oscillate at in some mode of motion, let us say I oscillate it in j th mode or in this case well let say j th or k th mode of motion, say k th mode of motion, I what that get, I get f force in the k direction, say f k, you oscillate in this direction and I get a force in this same direction, then this was my the, or let me put the other way along.

Let us say that I put, I took a body in oscillated that in the k th mode of motion. Now, obviously what happen, there a pressure field created throughout, now this pressure if I integrate them, I get force this, but force is a vector.

So, supposing I integrate them and get a force in direction number k, then I get f k, but I also can get a force in direction number j. So, what would happen that, I can get a force in direction j, because of motion of direction in k. Because see, it is very very, I will explain this in a physical sense, I am oscillating this; if I oscillate that, there is pressure around the body; if I oscillate this, there is a pressure around the body.

Now, I integrate the pressure, obviously when I integrate the pressure, I do this p n d s to get F. Actually, if I put k here, I get k, if I put n k here. In other words, depending on which direction I take normal, I may get that particular that that particular direction of the force in that direction, I think I need to clarify this even more.

See, take this way a longest body, this will very clearly explain to you, I oscillate that this way, I oscillate it this way, what happen, a pressure get created for an up side (Refer Slide Time 12:21). So, there is a, when I oscillate that, let us say this is coming down. So, I have got acceleration coming and here acceleration is coming down; after all when I do that, when I press that, this fluid is going to give a resistant force or you can say other way round, this particle particles are getting accelerated this side.

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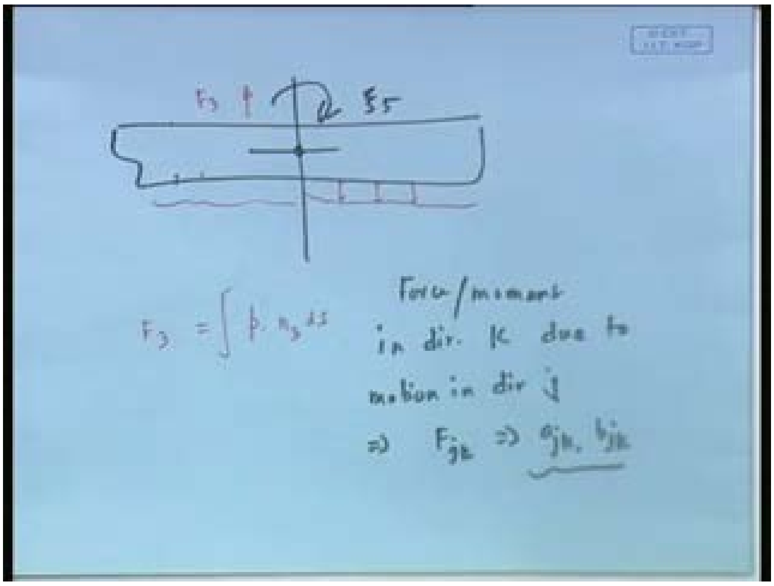

I mean take another piece of paper, this I am giving it, let us say also this is pitch motion which is actually xi 5. I have inducing xi 5, I am giving a pitch motion about its origin right. When I induce that, what happens, see what happen to this particle here, this particle is getting accelerated down, this is getting accelerated down, accelerated down, whereas here the particles are, because this is going up, it is going to I mean motion inward right. So, there is a field created here, there is a field created here.

Now, if I have to integral the pressures, if I do this p and if I do n 3 d s, that means if I want to find out what is my force in this direction, force in the vertical heave direction, because I have given a motion. It may not be 0, why it may not be 0, because this and this side may not cancel each other, because this you see, there is a net pressure for this part, net pressure for this part, but they are not symmetric or enough side.

So, since it is not symmetric, they will not cancel out each other. So, I will not end up getting it a net vertical flows; in terms of acceleration, if I if you remember we are talking about as if there is a mass being attached, what would happen, if I take each particle and see acceleration, these particles are accelerating down, but this is up, these two when I sum them up do not become 0, there is a net acceleration in the vertical direction.

Some small value, but non 0 value, this is what we can call the force created in direction 3 because of motion in direction 5. That means, this is what is my radiation force in direction 3 for motion in 5 and that give rise to my added mass a 3 5 and damping b 3 5, the part of these, because after all added mass damping up two parts which means, because it is not symmetric about all planes, I can always have force in direction. In other words, I can say that I can have force or moment in direction K due to whichever way you can interchange, motion in direction j that is my some kind of F j k which gives rise to my a j k and b j k. Because after all, this two terms as I said are nothing but expression of the forces.

What we do added mass damping; we say that that part of the force which is in face with acceleration is called added mass, out of phase in acceleration or in phase of velocity called damping. So, these two are nothing but the force, we are we are always saying that added mass and damping are nothing but basically the two part of the same force. But here the important point is that, you can understand that, if I have to keep it a pitch motion, I may have a heave force, so pitch influences heave.

Now, you see when I want to explain the total force, what is happen is that, just these two examples if I give. Now, let us (()) looking at pitch motion or heave motion, what would happen, obviously the body is simultaneously pitching.

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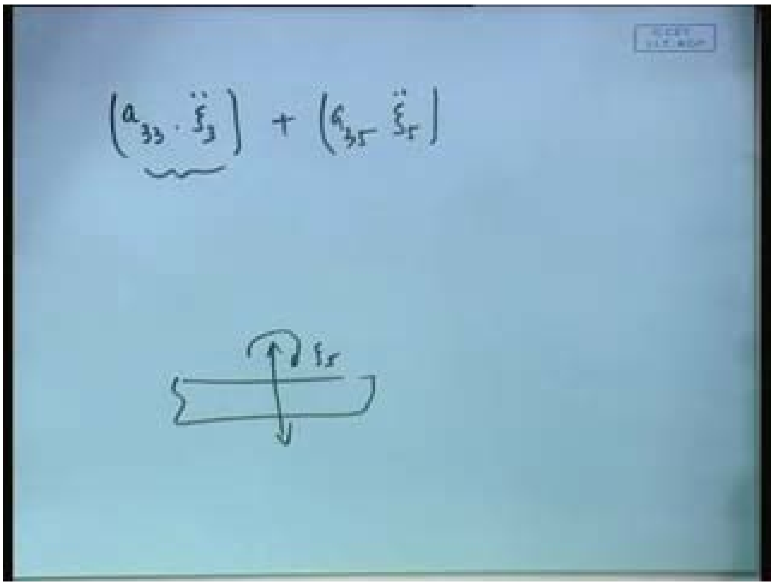

So, I have got one part of the, say radiation force, I have got one part of the force is a 3 3 into xi 3 dot dot, this is my added mass force in heave direction; see here, I am heaving it, I want to find out what is the force in this direction, net.

So, one part of the force is this, but remember the simultaneously that what is having xi 5, simultaneously at the same time. Because of this, there is additional force of a 3 5 into xi 5 dot dot coming, arising, so at this is exactly what is happening. If I if I look that at the matrix equation, you see here, I am looking at this 3 this 3. So, I have m 3 3 xi 3 dot dot plus here a 3 3 that, but a 3 5 into xi 5 dot dot, that comes in if I look back.

So, essentially what would happen, if I have to look back at that at that in a, in terms of j k, if I have to looking back to the first one that it it essentially means that, I am looking at the all the forces in directions say j. But what is happening is that, j k k dot, j k k dot, k double dot, there is force coming in direction j, because of motions in direction in direction k, this is what is called coupling. And this of course, exists as you have seen in heave and pick is strongly coupled very much. So, we will see that another example of roll, hydrostatic force itself, we will see that this roll part.

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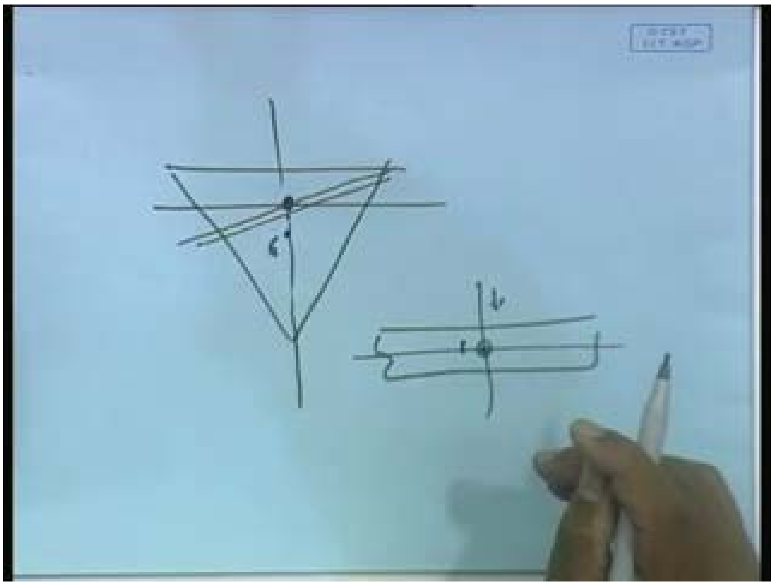

See, I have this say, I have this body, this is my center of gravity G. Now, remember that let me just take this point; if I have a heel, say I let me take this to be the reference point, easier to explain the concept. Suppose, I give a heel does it, does the line go through this, if the heel is large? No, it goes to somewhere else, you know that in large angle heel it will go through that.

What does it mean; it means that if I have to heel the body about this point, it will actually undergo some kind of a upward motion, a heave motion. That means, roll necessarily causing a heave, same thing is pitch. See, or other you see pitch and heave, that is better to see that, now you see here; if I have to make it go down, heave about this point, remember tell me will it trim? The answer is yes, it will trim, because the LCF is somewhere here. So, what happen if I were to cause a heave, then it automatically also trims; that means, heave is always causing a trim here, or rather in this case, parallel sink is causing a trim. So, heave and trim are coupled, moment you introduce that, that means when you want to push down the body, it also undergo the trim, this is what is called coupling.

So, what we are finding out in a general sense that, when I write in a general sense, the coupling is looking like that, every I am allowing every mode of motion to be coupled with every other mode of motion, that is of course how I will start writing it. But now, you will see from practically, is it so, because of symmetry.

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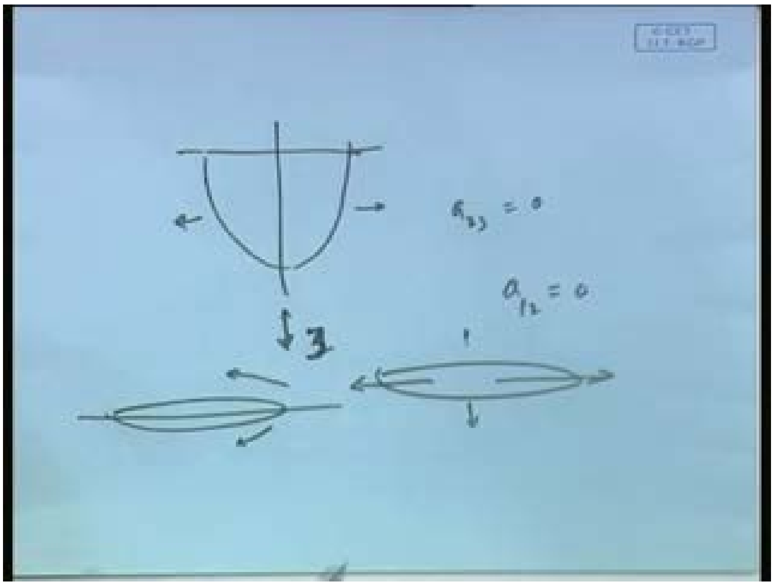

For example, added mass I said you see, let us lets take an example of this added mass only again, now most ships are symmetric about this center plane. Now, if I oscillate this side, say direction 1, suppose I take a body and oscillate in the sorry this is direction 3, will it cause a force in this direction, why because pressure exactly symmetric, so what would happen a 2 3 becomes 0.

Similarly, say (()) if I have to move in this direction, oscillate, will it cause any force in this direction? No, so a 1 2 is 0. So, therefore, because of symmetry we would know some modes of motions is 0, some are not 0, there is a strong coupling. Now, understand that almost all ships are symmetric about center plane. In fact, all vehicular systems are usually geometrically similar, external geometry similar about the center plane, whether you take a automobile or aircraft or so, because you would always wanted to that, since the fluid around this side and that side you want to be same, otherwise it is going to stir on side.

See you are making a body to move, obviously you would want to be symmetric about this line, otherwise it is time to go on other side right. So, therefore, they are mostly symmetric about center plane, but not about other planes, there is always one plane symmetry, so we that causes many terms to equal 0.

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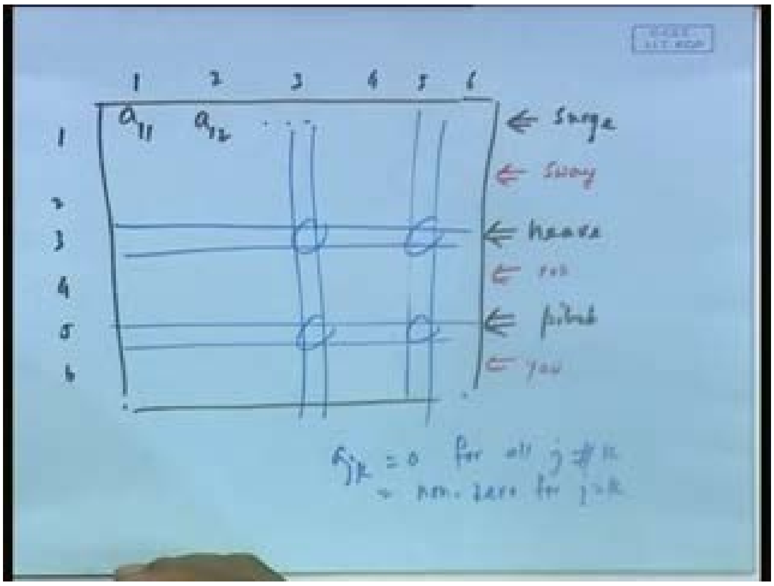

So, what happen if I have to look at added mass matrix, you now see this again, I write a 1 1 a 1 2, etcetera, actually let me write this 1, 2, 3, 4, 5, 6, and here also 1, 2 (Refer Slide Time 22:15). So, this is a matrix here right, it turns out that 1 that is surge, three that is well no heave and pitch, this three are strongly coupled. And similarly, you will find out I mean I am I am stating that without proof, basically what it happen many of them see what would happen supposing a 1 2, etcetera.

Some of the terms are 0, what would happen if you write it down, there will be no influence of this mode of motion with the first mode of motion, this one sway, roll. It turns out that essentially the strong coupling exists between surge, heave, pitch, and sway, roll, yaw. In other words, see this is 1, 3, 5 index wise, this is 2, 4, 6. So, it is any term which is this and this, or this and this, that black and red are 0.

Here in other words, what would happen, if I want to find out this or the surge, heave, etcetera, I simply have to delete those term, I will show that to you in a minute. So, because of symmetry, it turns out that one can show this. In fact, what we will talk about is most of heave and pitch coupling, because that is the most important coupling; what would happen, if I want to only show heave and pitch coupling, just neglect that.

Let us say that I only presume as if I have only heave, and I have only pitch, and nothing else is there. What would happen, I have only this line, the terms in this line, and then terms in this line, and here again terms in this line, terms in this line. So, I only will I consider these two and this two, everything else 0, then I end up getting what is known as coupled heap and pitch equation, because what I am saying is that that I have only heave and pitch, couple, nothing else is there.

Now, remember see, what is meant by uncouple? Suppose all modes are uncouple, there is no coupling, what will happen, everything every a j k would be 0, if j is not equal to k; supposing I say single degree freedom equation of motion, from here how do I derive well I say no coupling, what does it mean, a j k is 0 if j is not equal to k which means no modes of motion influence, other modes of motion except its own modes means if I heave, it only give a heave force, nothing else.

So, when I say once again, this should be understood if a j k is 0 for all j not equal to k, equal to nonzero for only j equal to k, this is imply that all the motions are uncoupled. If I say that some of the j and k are influenced, I will say that j is coupled with that k. So, if I want to look at heave and pitch coupling, I will only have to have a 3 3 and a 3 5 as non 0 or a 5 3 a 5 5 as non 0, everything else is 0.

So, I can construct from this now, you see these four equations means, I allowed all these to be coupled, symmetry tells me that if you go ahead that essentially these three modes are coupled these 3 mode are coupled. Now, let this surge, heave, and pitch, surge actually is of no interest to us, because surge is which motion, forward motion. See, a ship is moving at 20 meter, 10 meter per second, how does it matter it is going to make some oscillation about 10 meter, you understand this; suppose it is going like 10 meter per second, it is going forward and on that mean position making some oscillation, not really interested much.

So, interest become heave and pitch, and similarly roll and yawn, and sway you will find, but we look, let us look at heave and pitch mostly, so let us say heave and pitch.

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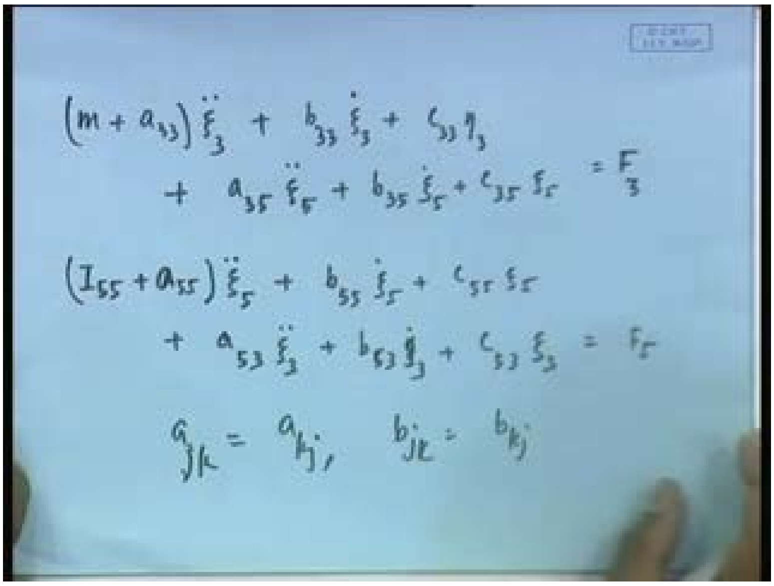

Now, if I want to write the heave and pitch coupled equation, how it look like, you see the full form, it is now look like that, m plus a 3 3, this we need to write this plus (No audio from 27:02 to 27:26) plus (No audio from 27:27 to 27:44), I may call it this F, similarly we are going to have (No audio from 27:53 to 28:18) actually this is (()) we are writing sorry (No audio from 28:24 to 28:43) (Refer Slide Time:27:02).

Anyhow this way, basically what you find out is that 3 3 and 3 5 terms are there a 3 3 xi 3 dot dot, this is actually the heave part, this is the part this is come for couple for pitch. That means, this part represent the forces in the heave direction arising from pitch motion, this is my pitch moment, and this is my pitch moment arising out of heave motion. So, this terms a 3 5, b 3 5, etcetera or a j k are the coupled dump, this is how we are getting basically equation of motion.

Now, one interesting point that I need to tell you is about, this added mass is, because we now end up getting large number of added masses right, because it now not only those diagonal added masses, we also need to have many more added masses. It turns out that added mass actually symmetric, that means a j k is equal to a k j, b j k equal to b k j, same is with the c terms; also c the restoring force is symmetric a j k, I fact restoring force 3 5 will not be existing. So, we do not talk about it, but high the time it exists here.

So, what is happening is that, you end up having these kind of added masses, we call this say a 3 5 will be called heave pitch coupled added mass. It has a unit remember, this is what is the unit mass is ton, moment of inertia if I would be ton meter square, this is going to be ton meter. So, this is have this is, so you cannot say it is either mass or moment. So, you can say it is couple added mass, because it has unit of ton into meter, why, because remember it is a part of force.

See, always you should remember this into this should give you a force unit, remember this into this should give you a force unit, this coupled term, this has a what is the unit of that? 1 by second square, so this is going to be ton meter into 1 by 7. So, you will be able to check that it is actually gives a unit of moment, a force sorry not more this thing a force.

So, like that you will be able to find out that essentially it has unit like that, see a 3 3 into xi 3 dot dot is mass into acceleration, here it is remember it is xi 5 dot dot with a 3 5. So, it has got a different unit, so this will have unit of same as this and you will be able to find out that how it works So, moment of inertia is ton meter square mass into length square, because it is mass moment of inertia we are talking, whereas this is mass only in ton, so this is in between.

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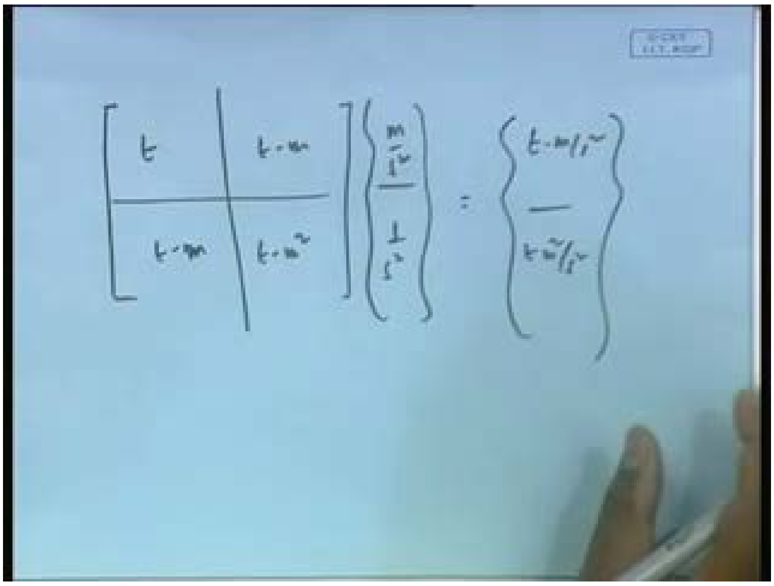

So, anyhow in other words what is happening is that, if I have to draw this, may be I do not draw this matrix here, or rather if I have to draw this matrix of a, you have this as ton, this is ton into meter square, this has ton meter, this has ton meter, I mean you know that is obvious right, because the way it is goes symmetric. If I have to look at the look at the, see here this is actually acceleration, linear acceleration, this is rotation acceleration.

This part is linear acceleration means what, meter by second square, rotational acceleration is 1 by second square. See, ton meter by second square, ton meter by second square everywhere the same unit will come, this is ton meter square by second square, ton meter by second square you know like if you look at that, see here this side this is ton. So, you can see that, what I am trying to say is very easy to see the units, you can always see that the units kind of match right, the units will be like that. Now, let us look at another interesting point of solving this equation of motion, I want to go back to the first equation of motion and I try to see how you can do that.

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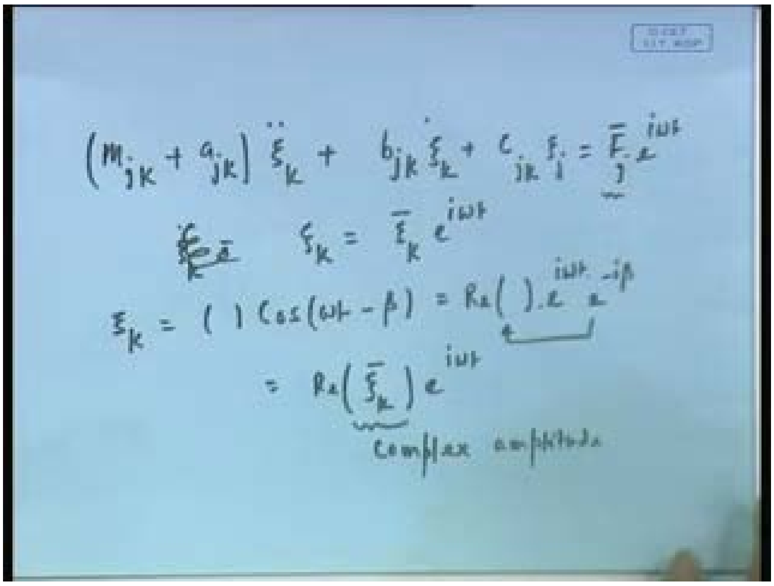

See, I had this m j k plus a j k that is why I wrote k and j, that is why this confusion came you know. I am omitting this sigma term, because I presume that sigma term exist I I will tell you about this or may be we can put them (No audio from 33:41 to 33:56).

Let me put this now this way, this we can (()) this term a little bit or I will put this way (()). See, what I wanted to say, I need to solve this, there are six equations, how do I solve this. So, we have this or other sorry, this is my motion, remember that this motion is a sinusoidal motion with an amplitude and this, this is what I call complex amplitude, why complex amplitude because actually it should be actually it should be something into cos omega t plus beta. In other words, I think I have mentioned that before if I want to lo at this again, see xi k equal to something into cos omega t minus or plus beta we can call it, this is actually real part of something into e of i omega t minus i beta.

I can bring this here, so I can call it is real part of, so this is what we will call (No audio from 39:40 to 39:48), this is also actually, this is also amplitude into e of i omega t, this is also a complex number I can call, force also a sinusoidal function, the beauty is that all are sinusoidal function. So, I can call it in this way with a complex number. The reason of doing that is because what would happen you know is that, now if I look at this part xi double dot k k, I will just go to the next one.

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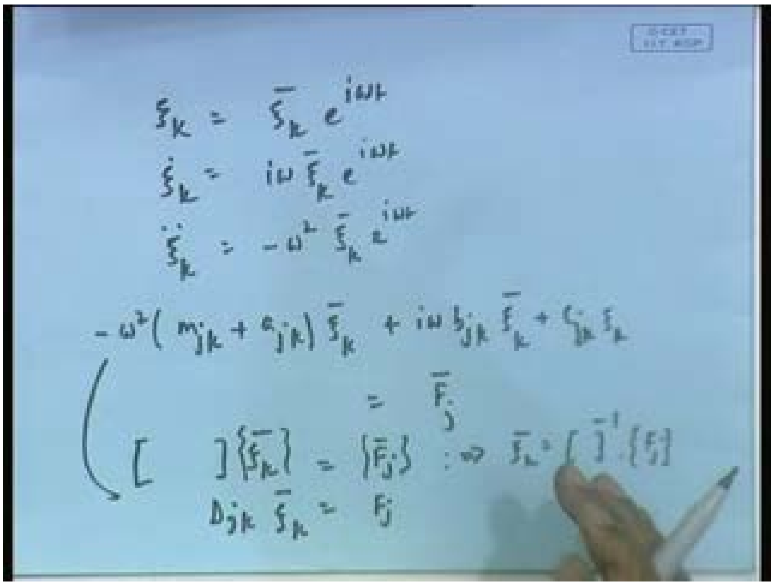

So, you see if I have xi k equal to xi k bar e of i omega t, then xi k dot equal to i omega (Refer Slide Time: 36:24). Now, I put it back to that equation what do I get, I will get minus omega square m j k plus a j k e i omega t all will be common, then we end up getting plus i omega b j k bar plus c j k equal to F j.

You understand that this will, it will lo like that with a sigma of course, there. So, this entire thing, therefore it becomes a matrix into xi k equal to F j. So, what is this, this is therefore, this gives me I write straightway. You know if you want to manipulate that, what what I am trying to say, I could call this to be this whole thing is a complex equation. So, it becomes something like or one can write this equation to be, if I if you want it you can write it as a complex equation D j k into sigma k bar equal to F j.

This is the complex matrix, complex number, complex number 6 by 6 equation, this is inverse of this into this, one line solution (Refer Slide Time 38:05). So, end up getting all those six modes of motion, what you get complex numbers, you end up getting this number which is complex number, next line the real amplitude is equal to you know the absolute value of this F is equal to tan inverse of imaginary real part. My point is therefore you know is that, the main part that I am saying is that, these equations the solution is trivial, absolutely trivial. In terms of solution if I know the component, because if I assume the equation has a nature of sinusoid. So, if I say that it is like an sinusoid, then this 6 by 6 such long term you know, if I look at that, basically there will be 6 term 6 term. So, you have got you know like 6 plus 6 plus 6 plus 6 very long expression, entire thing becomes one number, something into xi k, some D j k into xi k equal to this thing.

So, what happen that, solutions become trivial, in a computer especially all that you have to do just keep on adding the terms, you end up getting this all complex number. So, there is nothing to worry about solving this, when we look at for example, our this you know like this couple couple sort of equations that we are talking, same thing will happen as I said this couple heave and pitch equations that we are talking about, where we have done that coupled pitch and heave equations, yeah this one. If I am looking at this part, all that I have to do is to say this is equal to something into cos omega t, or something into e i omega t, then you end up getting all the one number.

So, actually you end up getting only a 2 by 2 matrix, you know something absolutely you will end up getting this full thing into equal to something like this.

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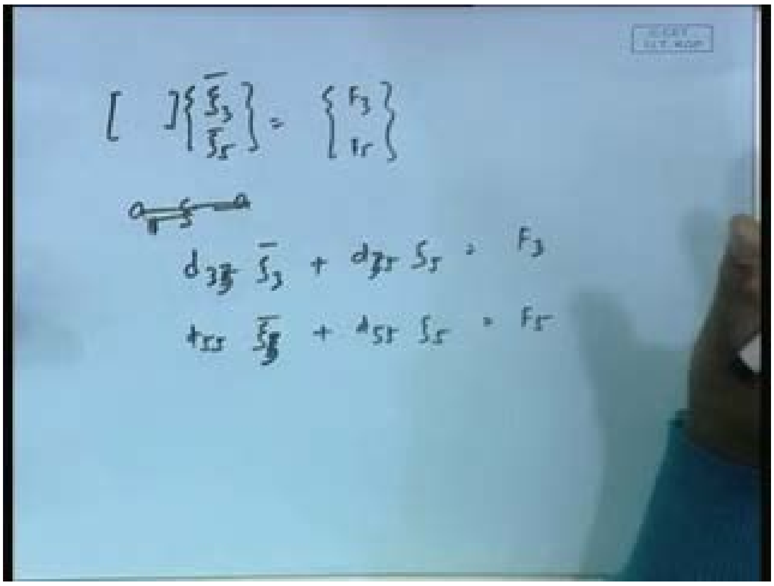

This expression you will end up getting something into or rather bar; in other words something like you know a I am writing this in terms of matrix a 1 1 xi rather rather this is sorry no no no this is 3 3, this is 3 5 something like that (Refer Slide Time: 40:40).

What I am trying to say you know, I think I think this say I think this something into xi 3 plus something into xi 5 is F 3, something into xi 5 is F 3 something into xi 5 is F 3 is equal to sorry not xi 5, it is 3 5 equal to this thing. In other words what what happen is that, it is just a 2 by 2 matrix it is one line solution, so solution is not important.

Now, what is important is that, how do I get the estimate for that just like what I said earlier; in earlier I could get away by taking only diagonal added masses, for coupled one all I have to do is add those couple, they added masses that is all. So, there is really no difficulty in the solution part of it, what is important is that you have to account for that, because both are influencing each other.

The important point is that, you must account for the two if I want a couple mode of motion, but having obtained the hydro times if the coupling, means couple added masses couples damping, etcetera, solution is trivial. So, the main point once again is that, get this number right, it is only a one line job to get the solution in that case, that is the point I am repeatedly trying to get in your you know in your mind that, the solution becomes always not very important, it is invariant, it is the terms that go in that is what is important, the terms that go in this one, this one, this one, these are more important. If you get the numbers, you can get the solutions and you find out in computation, it is only the numbers that is more important.

Now, the other thing is that, why we always could afford to study uncoupled motions, one of the reason is because normally these terms are small in number, although it may have some influence. Because they are small in number, we could sometime ignore it, of course from also study point of view obviously, the uncouple modes will be the much larger, coupling my influence there, but might may be few percentage depending on situation, although heave and pitch coupling is somewhat important. Now, let me just put one more important thing about combining the motions now, see we have we have let say we have obtained all the couple motions we we want to.

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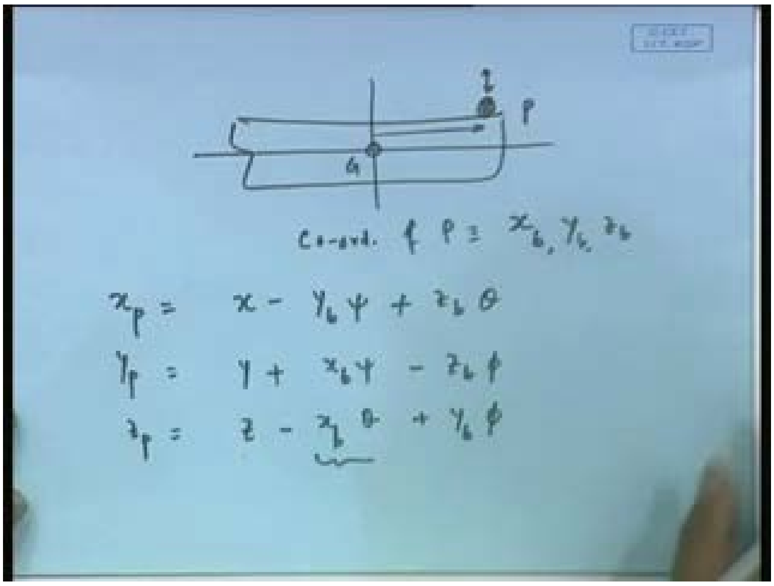

So, look at this point here, I have the ship here, there is a point here, somewhere. I want to find out the motion of this in the vertical direction, some point, remember here, now this phase thing will come in picture that is very practical. I have got a let us say gun mound on a naval vessel, then I need to find out the acceleration of this point, so I need to find out vertical oscillation.

So, this point is let us say a point p and its location is let us say distance is I mean the coordinate of p x b y b z b, let us say you know it is located at some point, 50 meter forward of the coordinate system that is x based 50 meter, 5 meter (()) and 2 meter high, some location and it is what is its motion at this point then, now I can always get for small amplitude motion by combining the two, it becomes x minus y b into psi plus z b into theta y p becomes (No audio from 45:12 to 45:44) (Refer Slide Time: 45:12). See, this part what we have done here, remember that actually if this was my G, I was knowing the modes of motion of this rigid hull, how much this is moving that is my x here, how much it is moving that is my y here, similarly theta xi and phi are the rotations.

Now, if I want to know this, it is a simple kind of coordinate transformation. See here, this is going to be say say vertical direction, if I want to look that, vertical is going to be z, but now because of this pitch angle is going to be having contribution of x b this thing, because you know that this is coming down so much and also for roll. So, what is happening you know that, this is a very simple addition to find out the vertical mode of motion combining them what do I explain that mode.

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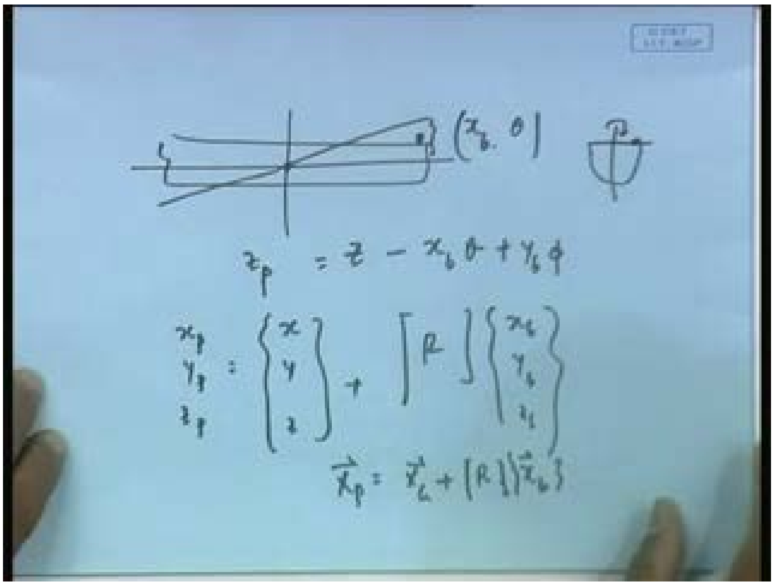

Let us look at this in a this way, see the heap part, if I have to find out a point here, if the ship is trimming, then you see amount of this that moves out up is going to be x b into theta, this is the amount of its z displacement, because of theta this thing, of course our theta is positive down. So, therefore, what happen if I combine heave which is z and the vertical displacement, because of trim that is this much and also if I check rotation, remember this was this point is located somewhere here. So, if I take phi, another contribution comes that will become y b into phi, this is my net z point p.

In other words, the displacement at any point is very easily obtained by the modes of motion that I have already by rotation, and by you know by rotation by linear displacement. Actually it is something like this, now x y z at a point p is x y z plus in transformation matrix into x b y b z b, this is the general formula for any point; in other words, x at a point vector b x of the central gravity plus R into x of the point b, this is the kind of a expression that we know.

This is a, this has nothing to do with hydro dynamics, it is purely a kinematic equation, its coordinate transformation, now looking back at this now I go back to this expression.

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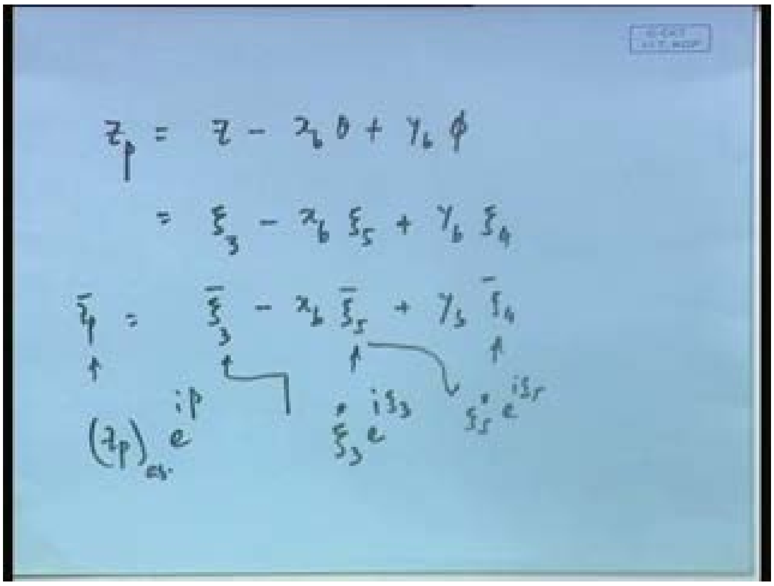

What, why I am coming back is that see, we end up getting, therefore z at the point p is equal to z minus x b theta plus y b phi. Now, if I want to write in terms of xi, it is basically xi 3 minus x b into xi 5 plus y b into xi 4. in terms of now this is the actual modes of motion, in terms of amplitude, etcetera, this becomes an amplitude function into i omega t say complex amplitude or other, let me put it this way if I want to find out this t (Refer Slide time :50:00).

See, now what happen, this expression I know, if I add it up remember all are sinusoidal function all are sinusoidal function we will do this later on again. So, if I add this up, it will going to be sinusoidal function cos omega t plus something. So, the entire thing is going to be again another sinusoid. So, the question is that, this is going to be again something into e i omega t. So, let us let us call this amplitude, because you can always call that as an amplitude into e i omega t, if I am writing this in terms of z p bar as a complex amplitude into e i omega t, each of them I am writing bar into e i omega t, but bar represents complex amplitude complex amplitude.

So, what would happen if I combine them, I get a complex amplitude of this, now remember this is the complex number, this is the complex number, this is the complex number (Refer Slide Time 50:54). So, this is very easy to do in a complex case, because I end up getting a complex number, this becomes this and this number is nothing but z p bar which is actually z p absolute you know, I can say this absolute into e of some i beta (Refer Slide Time: 51:00).

So, I can get both of the information from here. So, this is exactly how is one of the way how we can combine and now here, there is the most important point here, remember that this one, what is this one, this one is xi 3 into e of i xi 3, it has a phase angle, remember because this is a complex number with a phase angle, this has also a phase angle, now this let me call is star as the absolute value.

The point is that, I am adding the complex numbers which has a phase, so therefore the phase of these, and these, and these are very important when I add this up. So, in other words, I cannot simply, I cannot say that the ship is heaving 2 meter and pitching say 3 degree. So, I cannot add 2 meter plus 3 degree into the distance to get this at all, this is what I am trying to tell, suppose my xi my rather heave, absolute heave was 2 meter. I am looking at a point 50 meter in front of at the bow point which is 50 meter, x b is 50 meter, 50 meter ahead of mid ship, or the center of gravity, and my maximum trim is say 3 degree.

I cannot say my z p is going to be 2 meter plus 50 into 3 degree. I have to add with a phase in that and this is exactly where the phase information comes in, which are very very important. So, this particular expression you have to operate in a complex domain, if you do not want to operate in a complex domain, you have to operate in a the algebra will become more complicated like, if I if I want to write them in terms of cos and sin, what would happen, remember this will have something into cos omega t plus beta z, something into cos omega t plus beta theta, something into cos omega t beta phi, add them all up, entire thing you manipulate will be something into cos omega t plus beta z, much more difficult, normally for operation point of view.

But normally that is the reason why we always use complex algebra, but I will end today's you know the couple thing, because we will not (()) repeat that to just by saying that, if I were to consider couple modes of equation, the form of equation remain exactly same. It is just that I must consider those additional terms that you know coupling term, so called these terms.

In other words, I have to have all the j k terms when j is not equal to k, just add them all up, practically it turns out not all j and k are coupled, it is only 1 3 5 and 2 4 6 of couple, primarily 3 and 5 are most coupled, solution is trivial, all I want to know, have to know is that, the terms what is well earlier I need it to know a 3 3 a 5 5, now I need it to know I 3 5 I 5 3 in addition to a 3 3 a 5 5, once I know the solution is trivial.

Once I get this coupled solution, then also I can also join them to get displacement at any other points which becomes more practically important like this, because I am more interested to find out ultimately practical things like, what is my velocity here, acceleration, etcetera. And I tell you this, I will just end it by saying that once I know this kind of expression, I can find that velocity in acceleration by just one line, I just have to put a dot here, you know i omega into this double dot here minus omega into this.

So, once I know one, the other two automatically comes out. Why I am saying, because you may be interested to find what is acceleration here, because that is the load coming in the gun mound let us say, not the displacement find the displacement, then take double dot that is it. And you know that is where I will end today, next class onwards we will be doing some some other topic from tomorrow, thank you.