Seakeeping and Manoeuvring
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Lecture No. # 12
Description of Irregular Waves by Spectrum

See in the last class, we were talking about describing irregular waves.

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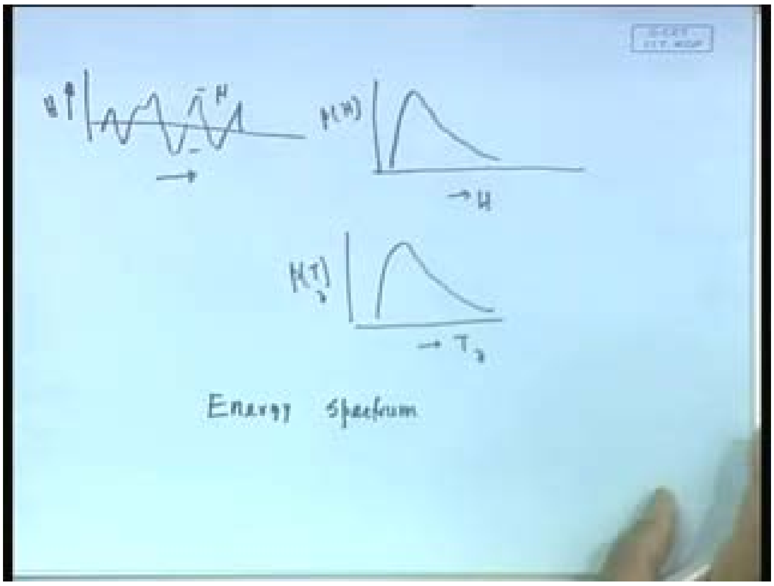

What we mentioned is that you could take wave heights and you can plot a histogram, which looks like this. We also mentioned that you can take instead of height, you can take say period and you can plot p of T z versus T z or so. You can get all statistics about height – one-third significant height, etcetera. Probability of how much height occurring where; what is the chance of a given height to be exceeded, etcetera. However, the problem in this is that, although we are getting this information of height and period, there is no correlation between the two. Remember, height is a measure in vertical direction here. And, period, length, frequency, all are measured in the horizontal coordinate. And, these two are independent as such, because for the same height, you can have same frequency (( )) different heights. But, when I want to go to a sea, we actually require information of a description, where both are coupled. Now, how this is done is the interesting concept. It is done usually using the concept of energy spectrum.

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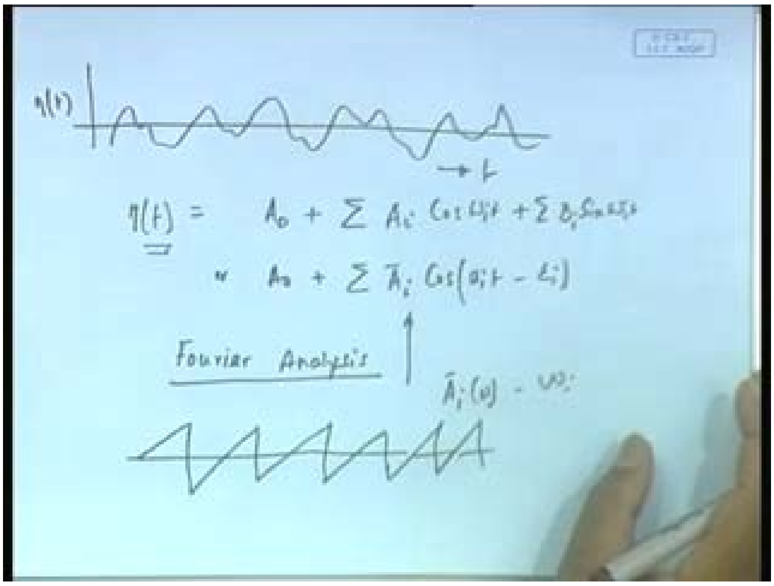

Now, we come to this interesting thing. See take some irregular signal t versus eta. Now, you see I can represent an irregular signal by eta of t equals A naught plus sigma A i into cosine omega i t plus B i into sine omega i t or A naught plus sigma A i bar into cosine of omega i t minus epsilon i. See what I wrote here. You know everybody has heard of this Fourier analysis. Any irregular signal can be expressed as if it is sum of number of sine frequency and cos frequency or number of cos frequency with the phase angle. This is only a generic, may not be exact. The point is that in signal processing, you would have seen that any signal, which is of this nature, you can express them by means of a long series, sine series.

Let us say look at that. Now, what this (Refer Slide Time: 03:43) represent? This represents here that as if I can conversely get an irregular spectrum, I can break it down to these components, that is, different so-called A i bar omega versus omega; or, conversely, if I add number of sine waves, I end up getting an irregular wave. Whichever way, because essentially, what is happening here, there is a way to get… See this is time series – time to frequency, frequency to time. Now, the question is like this – any signal, maybe the signal is also like that; you would have seen in mathematical physics book, any signal – I can always break it down to such Fourier analysis. That is from signal processing. The question is that is it possible for us? Can we really do that for seawave description? The answer is very interesting.

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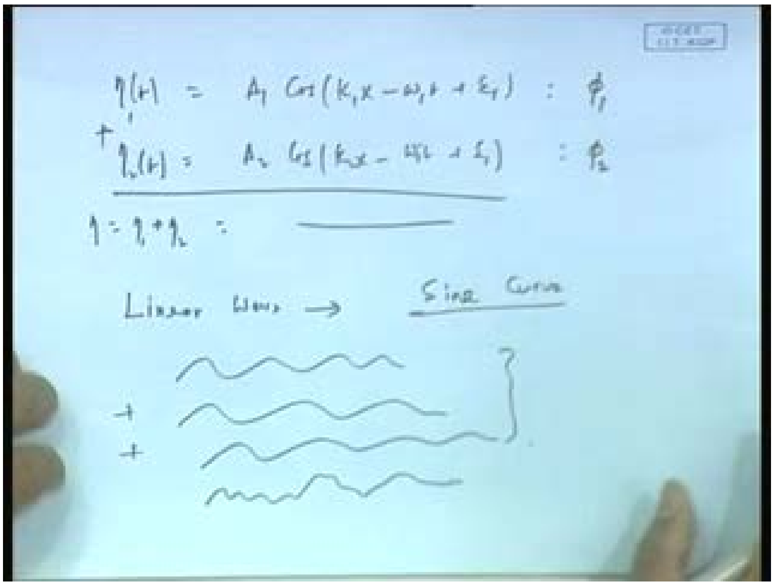

What we have seen in linear wave theory is that according to the linear waves, the individual waves eta t 1 is basically is a cos curve – A 1 cosine of K 1 x minus omega i t plus epsilon i; something like that. Now, eta 2 T – take another way, which is another cos curve – A 2 cosine of K 2 x minus omega I t plus epsilon i. Now, the point is that we have seen according to linear theory, sum of these two waves will simply be given by this sum of the two. Why? Because linear theory allows me to superpose waves. You see if for example, this is a solution the for problem phi 1, this is a solution for the problem phi 2, then we find out that basically, phi is a linear, the system is linear as well as… Therefore, phi 1 plus phi 2 can be added, superpose.

In other words, what happened, if I add these two up (Refer Slide Time: 05:58), what I get becomes actually as per the linear theory of the wave, which is sum of the two. This is very important, because what it means that it allows me to superpose waves, which therefore, tells me that supposing I can break it down to this (Refer Slide Time: 06:21)? Yes, the every one of them individually would represent a typical regular wave. In other words, I can think therefore, that an irregular wave of this signal is nothing but a sum of regular waves.

Conversely, I can add regular waves together to get irregular waves. Possible, because remember, regular gives the sine curves and also linear. See the two things are important; regular waves are not only sine curves, we have seen it is also linear. In other words, I should say other way round – as per linear theory goes, regular waves are sinusoidal curves. And, because it is linear theory, these curves can be added together to get a sum wave. So, super position is possible. So, you see there is physics involved and maths involved. Maths, because any signal can be broken down to this (Refer Slide Time: 07:16). I can always break it down, but the breaking down would have been meaningless provided this sum does not really represent a realistic physical wave. But, we find out that according to the linear wave theory; linear wave theory tells me a sine curve and this can be added together, superposed together; that means I can have plus. If I add it up, I will get a wave, whatever sum wave. This is actually also a possible wave. So, I can think now a converse process that I have begun with this wave, I can think I have this (Refer Slide Time: 08:01). So, I can break it down and I can think that it is nothing but sum of these regular waves. So, this is the concept behind that.

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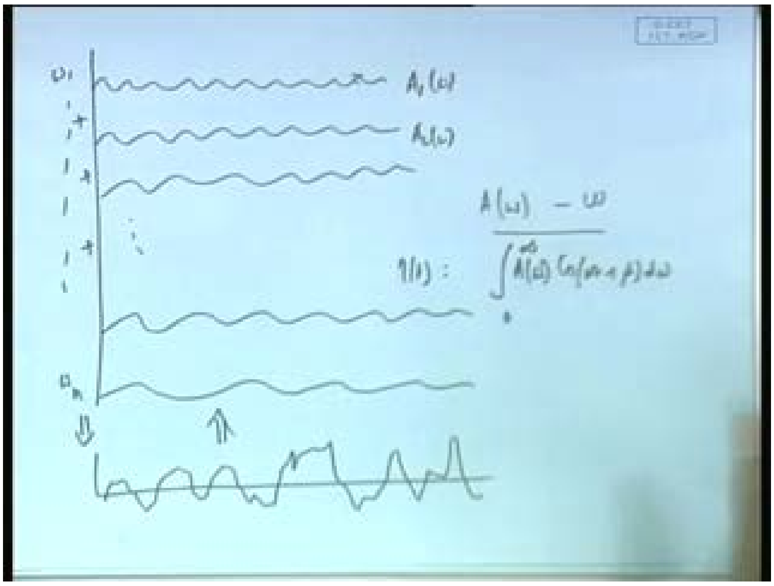

In other words, what is happening now, we will just now go to this picture part; it will be easier. What is happening, then, I can say that I have say in this all sine waves. Say this is omega 1; like that say omega n. This when I add it all up, I get an irregular wave. So, you see of course, here we have assumed that all the waves are travelling in the same direction, all of them are same direction and I am adding up. What is happened in reality now that what I find out, if I add linear waves, I get an irregular signal. Therefore… And, this irregular signal also satisfies the linear boundary value problem and they represent a realistic wave. Now, think opposite. I have this wave to start with; I have got a signal that the wave is this. So, what I do?

I now, break it down, do a Fourier analysis; break it down and find out from… As per this equation, both are possible; that I will tell afterwards. I have this (Refer Slide Time: 09:46) signal; I can break it down and determine this A i omegas, so-called amplitudes of different frequencies and find out individual A i's (Refer Slide Time: 09:58) for different omega i's. See this has got A – basically, A 1 omega; this is A 2 omega, etcetera; that is, this amplitude.

In other words, I started with that and I can always go back in the opposite direction. So, I have this wave; I can say look I have an irregular signal. Let me find out this signal (Refer Slide Time: 10:21) consists of how many sine waves of what type? With which I can do that.

Sir, there (( )) possibility (( )) we are doing it (Refer Slide Time: 10:28).

Absolutely, we will come to those answers separately later on regarding various possibilities. The question is that I can break it up; number 1 is that if you have this signal, the possibilities are not infinite at all; you will find out if I do an FFT, you will end up getting A omega versus omega, so-called… If I break it down like (Refer Slide Time: 10:54), so-called time domain to frequency domain, what I will find? See if I… Let me write it down (Refer Slide Time: 10:59), eta t – I can write it actually this way – A omega into cosine of omega t plus beta d omega integral 0 to infinity. Instead of discrete form, I can write it in continuous form. Rather, let me put another signal.

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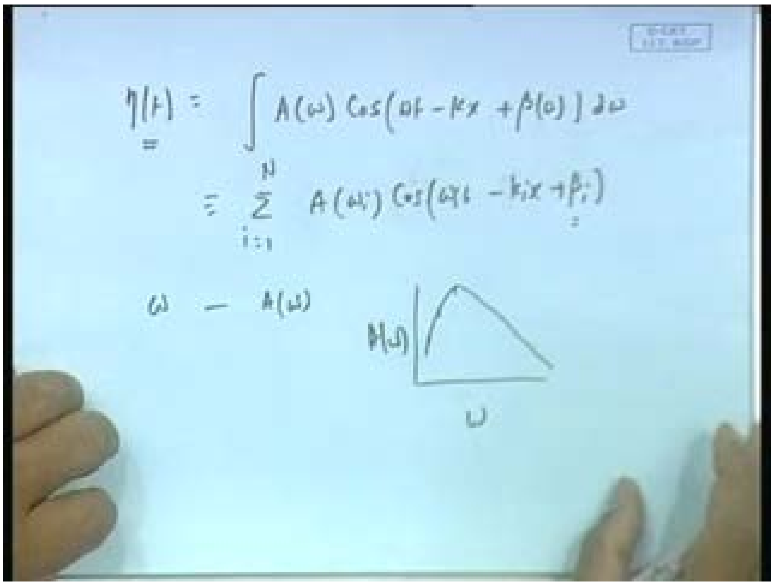

See I can write eta t as integration of A omega into cos omega t minus – let me also write k x plus see this beta omega – let me write it this way – d omega. I can write it this way, which is nothing but same as sigma of A omega i into cosine of omega i t minus k i x plus beta i; i equals 1 to N. These two are same thing. The question is that given a signal, if I break it down, I will get always omega versus A omega fixed; I am not going to get different; I may be getting… The question only is that if I take N – 100, N – 200, N – 500, I get different numbers. But, that is only a question of discretization. This one if I plot the graph, if I plot it omega versus A omega, I will get some value; no matter how. Whether I get this point or the other point, I get the same graph. So, it is not non-unique. The reverse is of course different.

Now, what happens, if I have a signal (Refer Slide Time: 12:36) here, I can break it down to this. But, if I have these A omegas, if I add them all up, because there is a phase involved, this beta i value (Refer Slide Time: 12:44) – depending on what beta i I take, I end up getting different kind of signals. This I will come later on. These are phase parts. See if I sum it all up depending on where… See this phase means where I start it from; I could start it from 0 here; I could start it from slightly here, etcetera. I can shift it and add it all up. If I do that, I will end up getting different signals. So, there is non-uniqueness, comes from that. But, if I do have a random signal, if I break it down, I will get a unique omega versus A omega. That is unique; that means, I can tell, this is composed of how many or in what way sine curves of amplitude; that is, omega versus A omega. That is this graph (Refer Slide Time: 13:33).

Now the question is of course… Now, another question is of physics. So, having said that, our answer is like this – once again, I have this (Refer Slide Time: 13:45) to start with; I break it down here. What do I plot? I can actually plot this (Refer Slide Time: 13:52) A omega versus omega. But, sometime, I can also plot energy of that – this side. Now, there is a reason for that. When I do an FFT of a signal, I can actually plot amplitude; this is the amplitude here; or, I can plot here so-called energy, which is square of amplitude. Why this comes?

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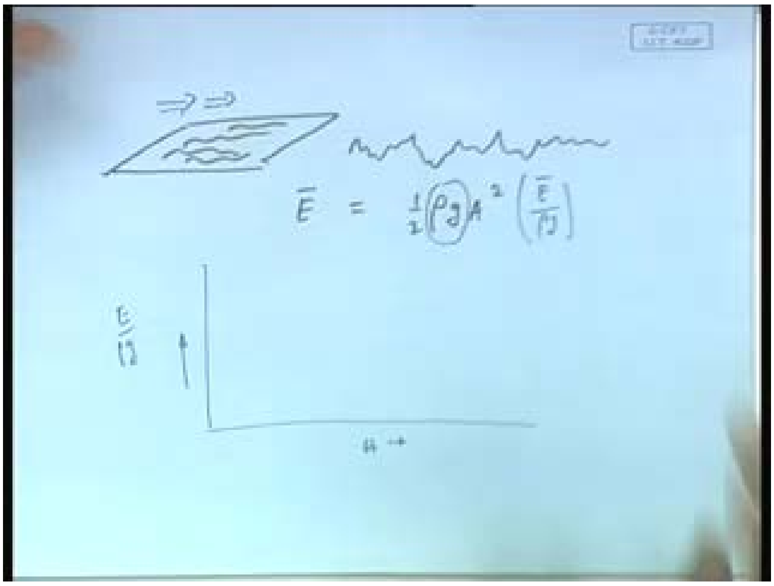

Now, look at this here. Let us take an area on the ocean surface, some area. The waves are of various types; I mean, there is a wave picture; it is changing. You have a stick here measuring; it is changing continuously. It goes like that whatever, continuously. But, the question is like this – how did the wave get generated? Before that, let me tell you this aspect – the energy average of the wave per unit square area – what is this? It is half rho g A square; it depends on amplitude square.

Now, the second part, wind is blowing over this (Refer Slide Time: 15:00). Normally, these are because of wind generation; wind is blowing over this. For a long time, the wind is transferring certain energy to the surface. Now, the wind has been blown for such a long time that whatever it could transfer has been transferred, which is what we call fully developed sea. So, what happens, for a give wind speed, whatever energy that could be developed has got developed; picture remains different, but its energy content… you expect from energy conservation to be constant, because for a given location – see if I have taken a given location and I am measuring the wave height, this record from time t 0 till say 1 hour, 2 hours, 3 hours, 4 hours, 5 hours. Now, if I take any segment for first 1 hour signal or second 1 hour signal or third 1 hour signal, because wind was going for a sufficiently long time; I would expect the energy content of that signal remains same, because no further energy can go in. Yes, maybe after 1 year, different; that is a different thing. But, for over a time span, for a given signal, you would expect energy to be same. So, it is therefore, logical to actually plot. Instead of A w, plot energy.

Now, energy means this (Refer Slide Time: 16:19). But, actually, rho g is a constant. So, what we can do, we can plot E by rho g. But, E by rho g if I plot here, it is not going to give me a proper… See there is omega here. Now, here comes again the question. Supposing I break it down, this part to 100 components. This n is (Refer Slide Time: 16:53) 100. So, what happens, I get 100 points; half a square. But, if I have 200, I will get 200 points. If I add them all up, answer is different. So, what happens, what we do is that… Remember that last time we say about energy band; now, I take this frequency band.

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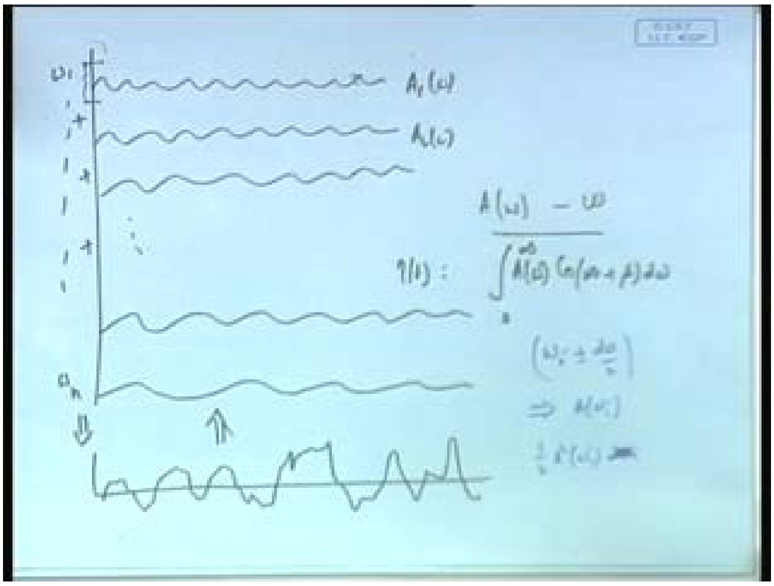

Now, I say all waves between this to this band; that means, omega 1 plus minus d omega by 2. Energy of all the waves is represented by only one frequency – this much; that means, energy of all the waves between omega i plus minus d omega by 2. This is represented by A omega I; this amplitude. So, what is energy of that? It is half A square omega i.

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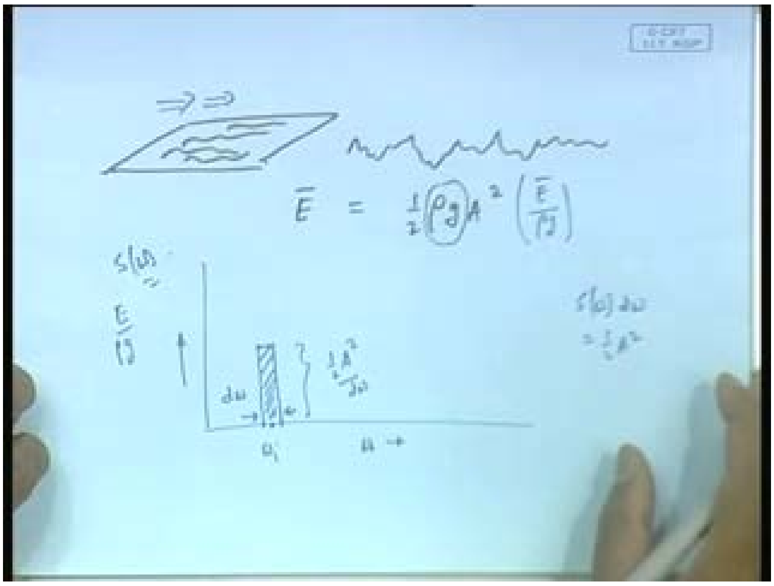

Therefore, what happens, what I want to plot here this side; see this. If I call this to be s omega, what I want to plot is that s omega d omega must be equal to half A square. In other words, see all the waves… Let us say this is omega i. This area should represent energy of all the waves, which means this area should represent half A square. Then, how much is this value? This is of course…

A i omega

This is what we plot; this is what is called an energy spectrum. So, when I plot this… that means, on the vertical axis when I plot half A omega square by d omega; that is very important to have by d omega, because if you did not do, then if you add it all up; if I take 200, then if I add all the A square and if I take 300, then you will find the answer do not match. Remember that total area; see the total energy must be constant. Once again, I will give this demonstration here differently.

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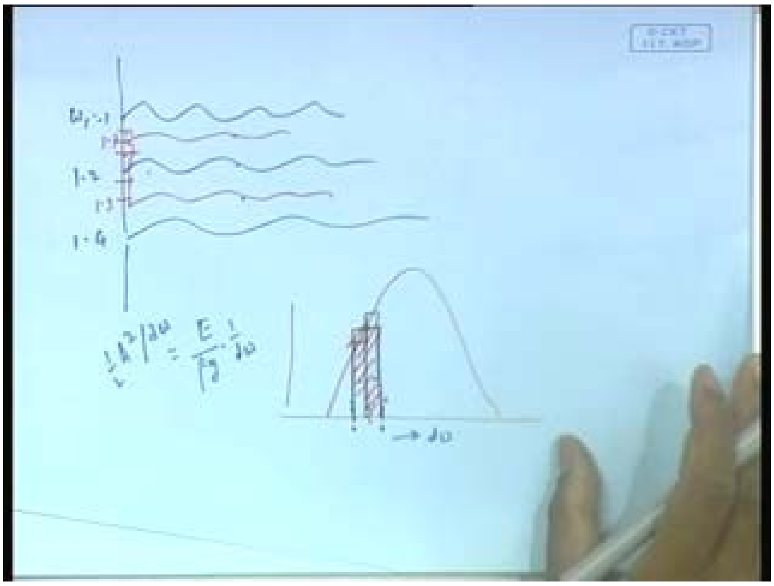

See take these two cases. You have taken this one – some value say omega 1 equal to say 1 unit and some next one is 1.2 units; next is 1.4 units, like that. What it means? I have broken it all like that. Now, there can be wave, which is at 1.1 also in between. But, when I do that, what I did, I say that all the waves… Let me take this one. All the waves with the frequency between 1.1 to 1.3 – this much; energy of all the waves is equal to half of this A square, because I have only one wave; I broke it down.

Now, supposing somebody else have another wave here (Refer Slide Time: 19:51) and another one here. Now, if I were plotting only half A square, here what is happening? Remember, here all the energy of this wave would have been half A square; and, this wave – half A square. In other words, in this graph here, what I am plotting here, you would have plotted rather other way round. This value with one value – somebody else would have plotted. Basically, you just check that two – one here and one here. It is the same thing.

See the question is that if I rate (Refer Slide Time: 20:32) this, then I would have got rather two or three; I would have got these two. If I took that one, I would have got this one. Both of them have… because the question is that the energy contained of… See what is the red line? All the waves between this to this frequency have so much energy. What is the blue one? All the way between this to this and this to this have this blue energy. They must be constant; they must remain same. So, what is happening, what I am plotting here is therefore, E by rho g into 1 by d omega; or, rather half A square by d omega, so that when I integrate or when I take a d omega bandwidth, this area (Refer Slide Time: 21:17) represents me the energy of the wave. And, when I integrate over the entire thing, this graph tells me the total energy of the wave. See this much would be energy of the wave between this frequency band; next one is energy of the wave between the next frequency band. So, this over all area would represent the total energy and this energy remains constant for a particular sea, for a particular wind speed, whatever; for everything else remaining constant. So, that is the reason why…

In that entire width as d omega.

No, not in that. That depends on the signal, depending on how many you have broken it down. See how many I have broken it down; then, of course, I will take this frequency bandwidth (Refer Slide Time: 22:00); then, I will plot that part.

So, in the entire spectrum… [Not audible] (Refer Slide Time: 22:03)

Of course, you will get that. So, you will get piecewise when you add… Let me show that now in a different graph form.

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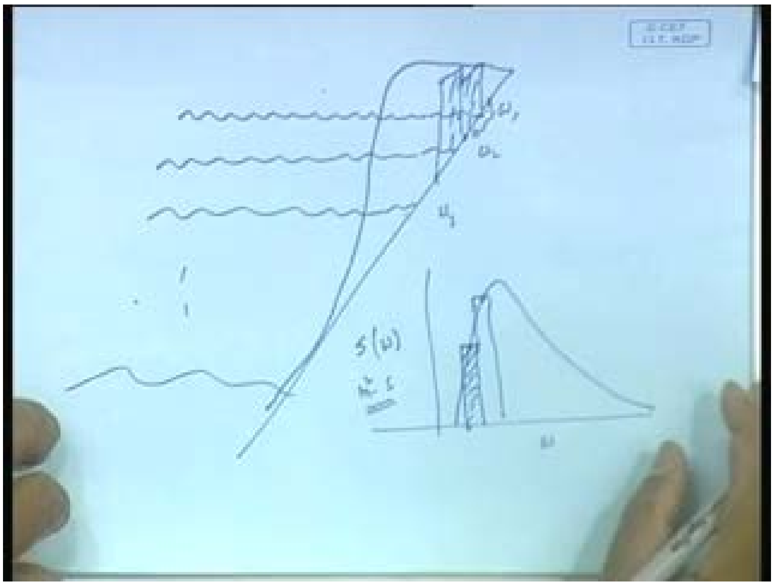

Let us put it this way. So, what you do here, see here; this is omega 1, this is omega 2, etcetera. Now, between this to this, this energy part you plot it here; between this to this, you plot it here. So, if you do that, you end up getting a graph like that in this axis; or, rather if I were to plot in this axis, you end up getting a graph, which will be looking like that. What you have done? You have plotted all the waves of this here; next one here, etcetera. Now, your question is that supposing I put more line inside? I will still end up getting the same graph, because what happens, I will end up breaking it in two parts. So, I end up getting the same graph, because it is only discrete form.

See this (Refer Slide Time: 23:19) – when I am having omega 1, omega 2, omega 3. What I am saying is that all the waves between this frequency, is essentially represented by one wave; that means, energy of all the waves between this to this is equal to half A square. So, what is the energy then? Half A square for this. So, this side axis should be half A square by d omega, so that this into this (Refer Slide Time: 23:42) that is, this area gives me the energy; that is the idea.

Now, you are saying that… Let me make it smaller. You make it smaller; what would happen? You would automatically end up getting a smaller amplitude at closer frequencies if I break down a signal, if I were to do Fourier analysis, if I were to do this (Refer Slide Time: 24:02). See it is very simple. If I take N to be 200, I will get smaller value of A over smaller omegas. But, if I take N to be say just 5, I will get larger value of A over larger chunks. So, I have end up getting the same graph. So, I will end up getting this graph and this is what is called energy spectrum; we call it by s omega. And, what is the unit of that? You will see it is meter square second, because it is A square by d omega and this is omega. So, this is… Now, what is the uniqueness of this representation?

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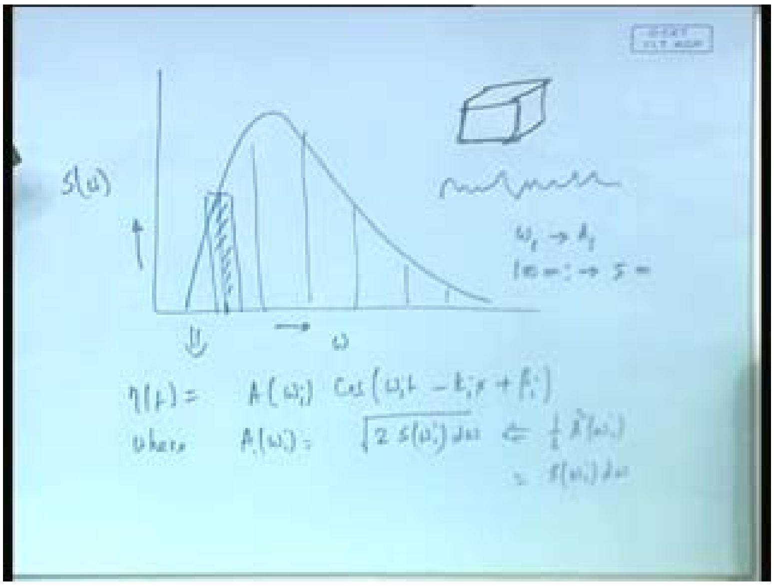

Let me again go back to this spectrum. Supposing I could plot this. What I can tell, I can tell from the plot immediately that look, this signal consists of these frequencies and for each frequency, the energy is this much. In other words, this signal represents this eta t equal to A omega i into cosine of omega i t minus k i x plus beta i; where, A omega i is equal to square root of 2 s omega i square d omega, because see this area – this comes from where? This comes from the fact that half A square omega i is equal to s omega i d omega. That is what we have used. So, what happens, when I have this, suppose I took a signal, see now I could represent in this form as I have shown.

In other words, I have the signal; I broke it down to amplitude against the frequency. So, I will know that this consist of so much amplitude for this frequency (Refer Slide Time: 26:07). Remember, they are connected now. Amplitude means height. So, I will know this particular wave signal that this wave signal consists of one – 100 meters long wave of height 2 meters plus one – 150 meters long wave of height 3.5 meters plus etcetera, because when I say 100 meters long way, what I am saying, one way of frequency omega 1. Remember, length, frequency, k are connected. So, I am saying 100 meters long wave means one wave of frequency omega 1 of height A 1 – 3 meters plus another wave of frequency omega 2 of height so much meter. I know that. So, this signal – therefore, I am able to breakdown, where I can correlate the length with the height. It means it consists of so much waves of so much length of so much height.

Length means period means frequency. So, you see, I could get frequency this side (Refer Slide Time: 27:08) against you can say measure of height this side for the same signal. So, it is correlating both the things. Earlier, what happened, I only said I have so many heights occurring so many times, etcetera. But, here these signals – I can now break it down to say how many frequencies exist of what height or what is the combination; that means, this irregular spectrum consists of what regular waves. Remember one thing – if I say there is a wave of omega 1 of frequency A 1, like of amplitude A 1; let us say this is omega 1 means 100 meters long wave; of amplitude A 1 means 5 meters. What I am saying effectively is that I have in this signal one component of 100 meters long wave of height 5 meters or five 1 meter waves, because five 1 meter wave also gives me 5 meter wave for the same length. So, here we are doing that.

Remember, why I am saying this five 1 meter wave? Because earlier, what happened, let us say in a signal I broke it down 5 meter occurred once, twice, three times, etcetera. So, I say 5 meter occurred four times; 3 meter occurred one time, etcetera. But, here the time part is not taken. If 5 meter occurred four times, then all get embedded as one number, lump number. So, this is what the concept behind sea spectrum is. This of course, any signal as I mentioned before, at the very beginning, can be Fourier analyzed and you can always plot this A i square (Refer Slide Time: 28:44). If you go to any computer program say matlab or anywhere, there is an FFT subroutine. You feed a signal; there will be output file.

How do you want to do FFT results? Do you want amplitude with omega or energy with omega? It is called energy spectrum. You can always find that energy. So, energy spectrum of any signal is very common in any electrical engineering or any engineering, because it is known as energy is always in proportion to square of amplitude. So, you are plotting square of amplitude. The question is that in our case, the second question is I know that, I have an irregular signal, but can I break it down to sine waves? Does it make any physical sense? Can I say that regular waves can be added? It so beautifully happened that because linear wave theory gave me sine waves, I could add the linear waves.

It is something like I have a black box; maybe (Refer Slide Time: 29:34) I should use the black color for that black box here; something like I add wave number 1, wave number 2, wave number 3, etcetera; churn it out; outcome is the regular waves. Converse – I have irregular waves there; I have the mechanism; I just shake it; separates out all the regular waves. So, you see the idea of irregular wave therefore, is obtained by simply summing regular waves; and conversely, I can sum regular waves randomly to get irregular waves.

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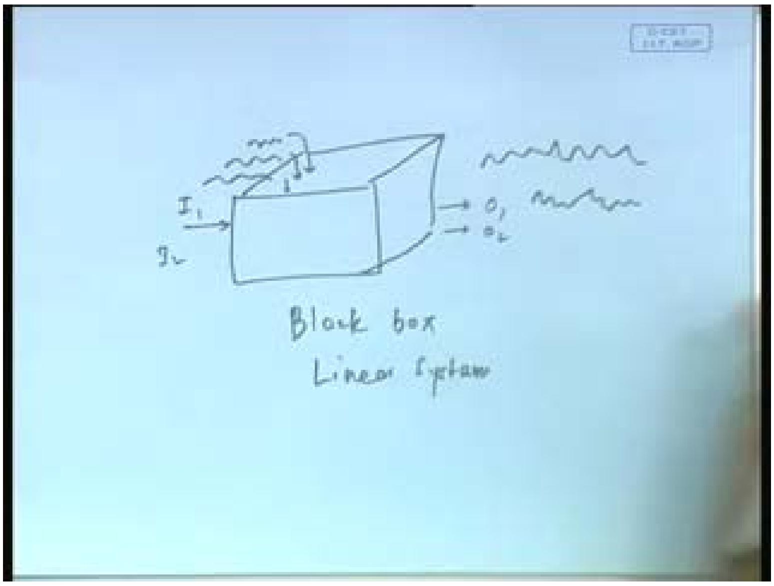

Now the question is… See this black box thing I want to tell you once more. Suppose I have this black box. Once again, I put it this way. This is the black box, which is a linear system. And, linear system – I told you that if I have an input 1, gives me an output 1; input 2 gives me output 2; then, input 1 plus 2; output 1 plus 2. That is the definition of linear system. Now, the waves are linear. So, I am putting waves number 1, which is a linear one of omega 1; sum this one; I put it back. Then, another one I put it here. I keep putting this inside this box. I do not know when I put it; maybe the time I put is a start time. So, I put it now; next one after 2 seconds; next one maybe after one and a half second. Randomly, I keep putting. So, the addition is random up to some point.

But, energy remains same. Remember, energy will remain same because energy has nothing to do with the time at which you are putting; you are putting this wave (Refer Slide Time: 31:10) having some energy; he puts this wave after maybe he is strolling 2 minutes later and puts it, but his energy is same. So, when you add it all up, the energy remains same, but the signal that I get up when churn like this. But, I shake it differently. After a while, a get another signal. The point is that whether I get this or this signal, if I break it down, I will get this component, same component. So, you see the point remains therefore, that in a space if I have a fully developed sea, wind is blowing for a while, my instantaneous signal completely defers. But, if I break it down, then I would expect that they will filter out to same frequency components, because that is what is logical that you would expect that at least for a same place. In other words, I took a 10 hour signal of the same place; whether you take a 1 hour churn, fifth hour to sixth hour or first hour to second hour, you would expect it to be same. But, the signal will not repeat; it is random. You can check that.

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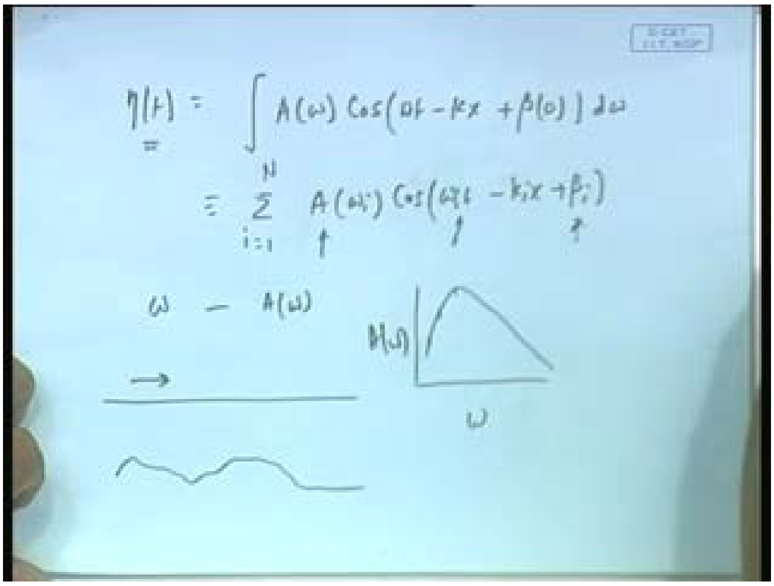

If I were to use this signal in a computer program and if I generate beta i to be random – you just use beta i to be random and use same values of A i and omega i's, you will see that the signal will not repeat itself, because there is a random phase involved here; it will keep on… But, it will have a same statistical property. If I filter it down and plot an energy spectrum, I will end up getting the same energy spectrum. And, that makes sense, because we are thinking that in a given location with a bathymetry and all that, for a given wind speed, only a given kind of energy can get transferred and given type of waves get excited.

For example, if in a location with some bottom – say there is a location here, the sea bottom is going like that. This is the sea. You would expect that it will always produce more number of 100 meters long waves and much less number of 200 meters long waves whether it is today, tomorrow, day after if the wind was 30 knots, because nobody knows it yet. But, at least you would think it will depend on the bathymetry. So, this is the reason; this is the logic of introducing this wave spectrum. So, this idea is really interesting. This takes some idea. And, you end up getting this shape, which is nothing but representing the irregular waves in a frequency domain.

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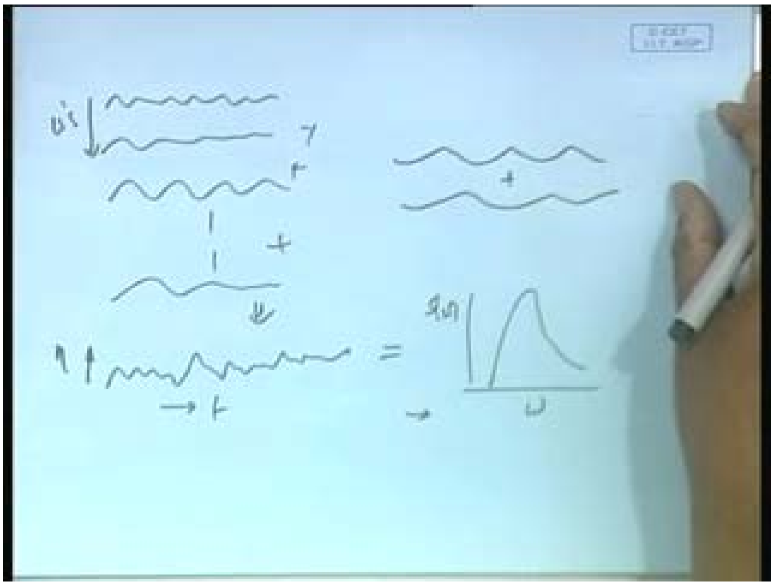

Once again, I may always say now one further sum that I have something – if I were to plot again this, many times we are plotting it. So, you think these sine waves; this sum, sum, sum gives me… which I am representing as this way. So, here the omegas. So, this is the time. What I did? Eta versus time is represented in terms of s omega versus omega. This is what is called time domain signal represented in a frequency domain; that is all. A time domain signal represents frequency domain and it is permissible for us, because this process is permissible as per linear wave theory. If the wave was not linear, suppose you use trochoidal wave in ship strength, you cannot add; two trochoids cannot be added.

If you have done marine (( )) suppose you use stocks second order wave, looking like that. You cannot add them; you cannot add these to get a wave. Remember, these are not additive. Supposing I take a second order profile looking sine, non-sine – another second order profile. If I add them, the profile I get is not a feasible wave, because that does not satisfy the boundary value problem, whose solution should be a wave. But, if I take 2 sine or 3 sine or 5 sine, the resulting wave also satisfies the same linear boundary value problem. So, it is theoretically a feasible wave. This is something that has to be understood. So, this thing is possible only for a linear wave system. If I had a very rough, very steep wave, I cannot add two steep waves period. So, linearity of wave theory really allowed me this. And, that is very… I am repeatedly stressing that to you.

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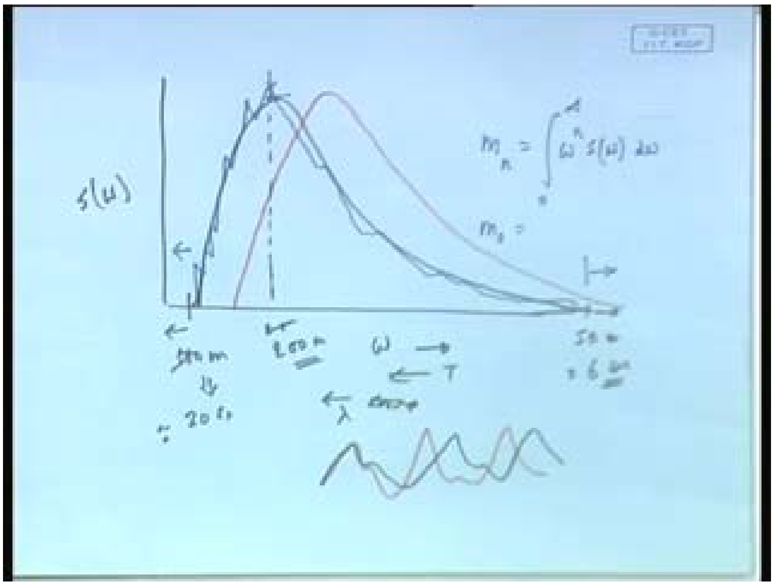

Having said that, now, lets us go back to some other interesting parts of this property of that. Number 1 – if I see this spectrum, suppose I take a signal; it will look something like that typically. Now, you see omega is reverse of T; and, that means, this side is omega, this side is T; that means, this side is also lambda in proportion, because remember, higher frequency means lower period means lower wavelength. So, in a sense, it is like that.

Most of the signals that has been analyzed turns out… When people do that, is having a shape like that. Once again, it is a non-Gaussian, but Rayleigh shape. Same as what we discussed earlier. Obviously, what people have done, oceanographers – they went around the oceans for ages, for 100 years in their oceanographic ships; people have kept on measuring these signals and analyze that. So, you would have done it. You will end up finding some spectrum; it will probably look something like that. Energy – it will… (Refer Slide Time: 37:03). Obviously, it will not be a nice smooth curve. Mostly, people found out that it follows this shape. There is a reason for that.

Again. you see what is it; not uniformly distributed; you have got more number of smaller waves, less number of broader waves, which is true, which is how it happened. Not only that, people will find out that for a given energy, normally, there is no waves beyond some long wave; and, no wave beyond this thing. Practically, no wave. There will be maximum number of waves or most dominant waves at some length. This is what has been always found out from observation and that makes sense. Example – take a location; wind is going at 20 knots. You will find out that there are… Let us put numbers here. This is say 500 meters long wave; this is say 50 meters long wave. This 500 meters long wave is going to be approximately 20 seconds or so; maybe, 20 seconds. This may be 6 seconds (Refer Slide Time: 38:14) approximately. Remember, we need to have this feel of this number, which we will see from spectrum eventually. So, what happens, you will find out that if 20 knots wind is blowing, practically, no wave waves if I break down the signal; longer than 500 meters or shorter than 50 meters. Most of the waves are within that; of which maximum waves might have occurred at this length, which might be equal to say 200 meters.

Now, take another location; same wind; maybe its shape is different. Same location, more wind; maybe, it may be different. But, point remains is that in a given location, given bathymetry, given wind speed, the kind of energy transfer that would excite waves, you expect them to be having a similar bandwidth, similar nature. So, oceanographic people would have actually gone round and they would have found out this. And, they found out that the shape of that for a given location remains more or less similar; and, that is very rational, very logical.

Let us look at some property of this wave (Refer Slide Time: 39:28). When I have this signal this way, I can immediately find out several things. I can find out that there are no waves shorter than this length; there are no waves practically longer than this. The maximum wave occurs here. And, I can also find out statistically what is the total energy content. What is it? It is the area under them. You see now, let me define this thing m n – nth moment as omega to the power n s omega d omega integral 0 to infinity. This represent nth moment.

What is 0th moment? What is m 0? m 0 is area under that. So, m 0 therefore, becomes a measure of wave height square, because it is an area under that. see area under that is of course, in proportional to in some sense, height square. Remember, I can have this; I can shift, but now, there is an interesting question. I take another spectrum – just shifted it; just parallelly shifted it; area remains same. So, m 0 will remain same in both cases. So, of course, that make sense. The height part is not connected to the length part. But, what does this show? See this (Refer Slide Time: 40:56) would be showing that there are more number of shorter waves.

In other words, if the black one was looking… I will just show one signal like that. The red one is suppose to be similar height say, but like that shorter, because it is number of shorter waves there. Now, I need to know that also. How do I know that? That is given by the center of this, which is basically, moment of this graph about this point. So, if I take m 1, what is m 1? m 1 (Refer Slide Time: 41:30) tells me the center of this graph, that is, this distance. m 2 will be second moment of inertia. So, m 1, m 2, etcetera would represent the characteristics of the time period of the waves; whereas, m 0 gives me characteristics of the height of the waves.

In fact, now, interesting point comes. Statisticians have found out that this signal, this (Refer Slide Time: 42:06) shape as I said, follows Rayleigh distribution, a particular theoretical distribution. Moment you know and moment I can fit it a theoretical distribution, it turns out I can just like waves, I can express this in terms of some parameter m 0. I can actually write this graph s omega as a function of a parameter.

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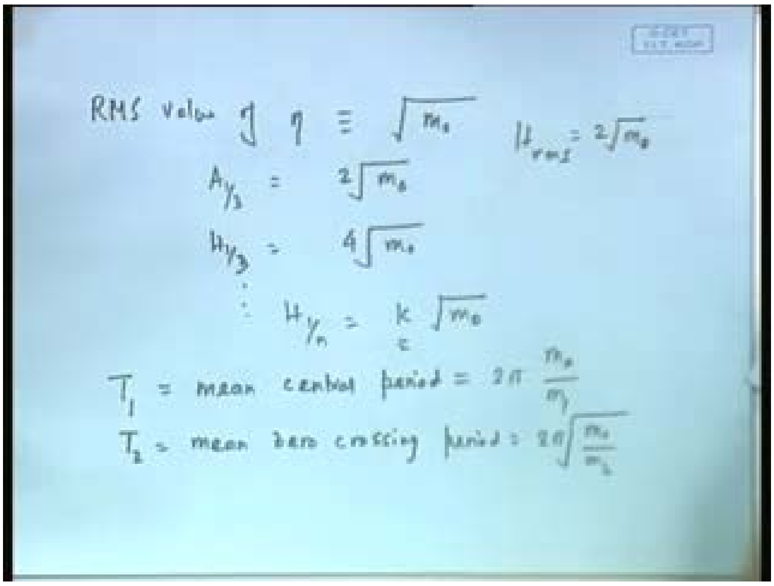

It also turns out that all these values like RMS value of the signal, that RMS value of this wave height – this is given by root over of m 0. Then, amplitude one-third – this is amplitude one-third, which is given by 2 root m 0. Then, of course, H one-third will be therefore, 4 root m 0. Then, also, H 1 by n would be k root m 0. What I am trying to say is something like that; that if you were to take this shape to follow a Rayleigh distribution, theoretically, an expression, in that case, all parameters of importance becomes no in terms of either area or H one-third. And, you can express relationship between the two. For example, the RMS value of the signal becomes square root of that. In other words, RMS value of height would become 2 m 0 if I take other way round, H RMS, because remember, H is always twice that if I take one-third value of the wave height. Remember why this 4 comes? Because I plot it A square; whereas, H is twice A; remember.

Sir, it is dimensionally same.

Of course, it is dimensionally same, because this side is (Refer Slide Time: 44:17) meter square second; this is 1 by second. What is m 0? It is meter square; square root of m 0 is meter.

Area is the energy (( )) (Refer Slide Time: 44:24)

No, rho g we have taken down. See that this is the interesting part. We said energy; actually, it is not energy; it is energy by rho g. So, we took energy out by taking rho g. So, it does like… What I say it here is it is proportional to energy essentially. Why bother with a constant rho g. So, that is why people take out rho g. So, then, it makes sense. You see what mean by k is that (Refer Slide Time: 44:52) I do not know the value of k, but for any 1 by n, theoretically, you know; like 1 by 10 k may be 5.53; 1 by 100 k may be something else, etcetera; it is all known.

Now, similarly, mean central period T 1 – mean central period means basically the mean period. It is 2 pi – I will tell you this makes perfect sense – m 0 by m 1 see m 0 by m 1; m 1 by m 0 should be omega. See what is this? See (Refer Slide Time: 45:34) if I take m 1 by m 0, what do I get? I get the c g of that – this distance is centroid of that; first moment about that divided by the area. You can think if this centroid of this graph from this side, that is equal to omega 1. T 1 therefore is 2 pi by omega 1. So, 2 pi into m 0 by m 1. That is what is called the mean period, the mean central period. Similarly, one can find out that you can also get things like mean zero crossing period (Refer Slide Time: 46:09) (( )) In fact, all of them are possible. This is given by theoretically, 2 pi square root of m 0 by m 2. There is a square root comes, because we have m 2 coming here – second part.

Now, not only this relation, actually, every possible relations with respect to statistical quantities becomes known once I assume this shape (Refer Slide Time: 46:45) to follow a particular statistical formula, which goes like this – we have done number of measurements; the oceanographers have done, analyzed it. And, by doing that, it is found out that they closely follow a shape like that, which can be fitted with a given formula, a Rayleigh distribution formula. Having said that, now, you say I will fit a Rayleigh formula. Moment you fit a Rayleigh formula, the formula is expressible in terms of m 0. m 0 is expressible in terms of H one-third or H average or H RMS. So, you can also write the formula in terms of H one-third; also, of course, in terms of T in order to know the position of that; that is, m 2 or so.

Having said that, once I have that formula in place, say I have an H one-third and T 1 given; then, I can figure out all other statistical quantities of importance from that formula. For example, if I want to find out what is the chance that my wave height is going to be (Refer Slide Time: 47:50) more than so and so, I can find it out. What are the number of times in 24 hours a particular wave height will occur? I can find out. What is the mean period? I can find out. All quantities of statistical importance can be found out theoretically from the ship, which we will not discuss here, because that is a part of wave statistics and irregular wave theories. But, it is possible to find out all.

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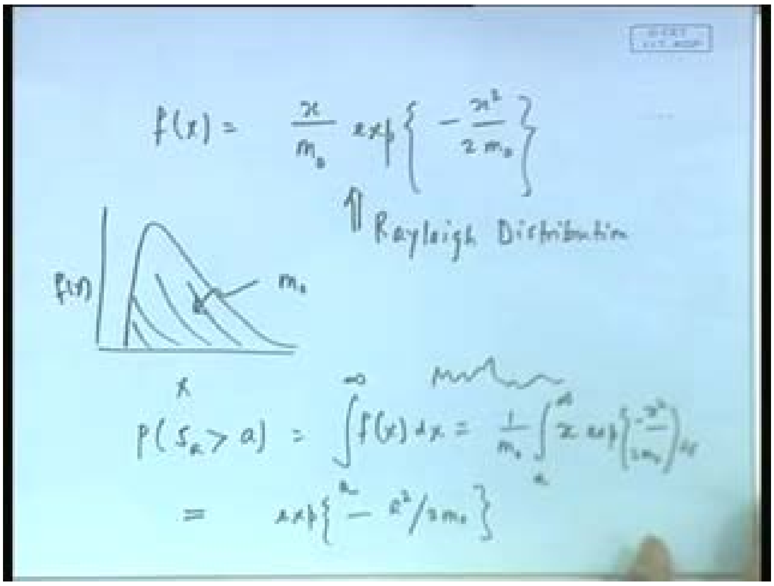

An example is this – see a Rayleigh distribution is given by f x given by x by m 0 – area under that – exponential minus x square by 2 m 0. See here this is the graph; that means, this shape. This is f x; this is x; area is m 0. You see this with respect to m 0; this is the formula for a Rayleigh distribution. Rayleigh distribution formula is like that in terms of m 0; unknown is m 0. Now, if I therefore know m 0, if I knew the area, I know the shape. Conversely, I got the shape; I found the m 0 and I fit this formula. So, I will end up getting this shape.

See what happened, I took this signal (Refer Slide Time: 49:23); I broke it down; I found out total like this signal; I found m 0; then, I fit this graph. I end up getting this smooth curve. If I get this smooth curve, for example, I can find out things like probability of say xi a more than a. This will turn out to be integration of a to infinity f x dx. And, if you work it out, it will turn out to be 1 by m 0 integration of a to infinity x exponential into minus x square by 2 m 0 into dx. You can work it out. One can get this. Actually, this will turn out to be – if I work it out, exponential minus a square by 2 m 0.

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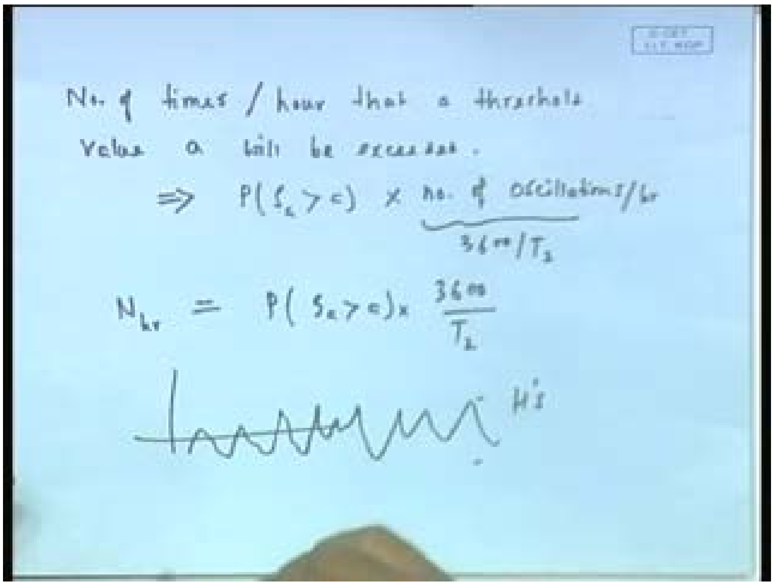

Similarly, for example – I will give an example – number of times per hour that a threshold value – a particular value – a will be exceeded. This becomes probability of xi a more than a into number of oscillations by 6x, which is 3600 by T 2. N hr is equal to probability of xi a more than a 3600 by T 2. These all that can be worked out. What I am trying to say is see instead of thinking, a probability of a particular threshold. Suppose you want to find out how many number of times in an hour a particular value a will be exceeded. For example, you want to find out in 1 hour, how many times the wave height will be more than 5 meter; two times, three times, how many times? You can find it out, because this is probability multiplied by number of oscillations per hour. So, the point I am making here is that every quantity of statistical importance can be determined once I presume the graph follows a Rayleigh distribution; and, the Rayleigh distribution becomes knowable provided I know area under that m 0 or H one-third or one parameter. So, what therefore, means is that see – it is very interesting from this point of view – I have a signal; I went to ocean and I measured the signal; I broke it down; I got a graph; I found out m 0 – area under the graph or total energy content.

Now, I assume that with that energy content, the graph will be Rayleigh distribution. So, I put the Rayleigh distribution back. Then, from there, I can find out all the property I want. This is a uniqueness of spectral representation. See if I were to use with a actual observed data, probably, I could not get all this part. But, if I in fact, went to an observation and fitted that to a Rayleigh distribution, I will end up getting this. And, interesting part is that we will be saying that – suppose I take a long signal; I found out all these H's, then I analyze in one hand how many times, what H occurred, etcetera. I found H one-third; kept it aside. Then, I took the same signal with FFT; fitted the signal and fitted a graph; found out the spectrum. Again, area under that is 4 root m 0. I will see that will be same as this. They are all theoretically consistent. It is only stating the same thing from other way around. In other words, if I have this signal, if I had broken it down, if I had certain wave statistics, same statistics is obtained by fitting it to a Rayleigh spectrum with of course, the input parameter given by H one-third. So, this is what is happening.

Now, we will end it today this description part. What happens is that obviously, for my description of open ocean, remember that I have to design a ship that has to go to ship tomorrow, not yesterday. I knew yesterday's data; I analyze to a spectrum. But, I need to design a ship that goes tomorrow. So, I have to have a mean subpredicting or describing ocean for tomorrow. You understand what I am saying; because what I am designing must be in an environment that is not occurred yet. So, I have to have some formula that will describe the open ocean. So, these are given by certain closed-form formulas, which is called theoretical wave spectrum, which we will discuss in next class. That is what spectrum I should use to find out how the ship would behave when it is going from Calcutta to Port Blair for next 20 years. With that, I will end here. Tomorrow, we will discuss about what is called theoretical wave spectrum.

Thank you.