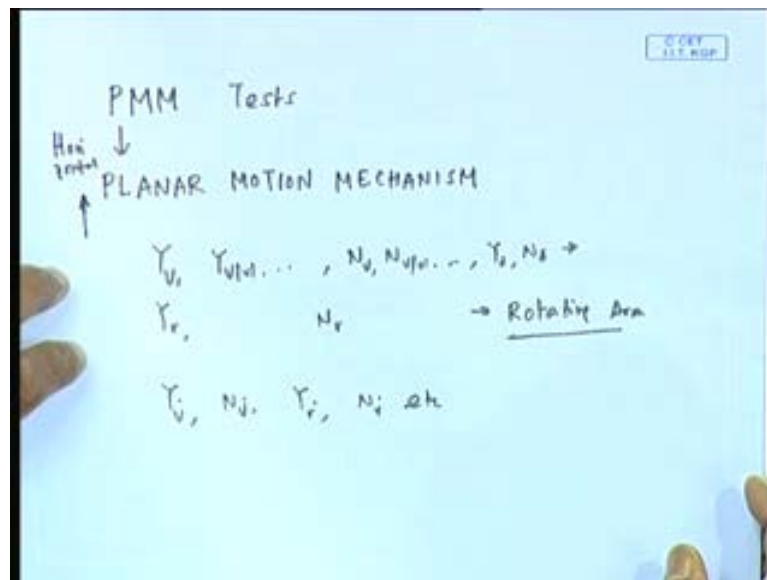


Seakeeping and Manoeuvring
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Module No. # 01
Lecture No. # 35
PMM Tests - I

See, today we are going to talk about **PMM Tests** (No audio from 00:22 to 00:30).

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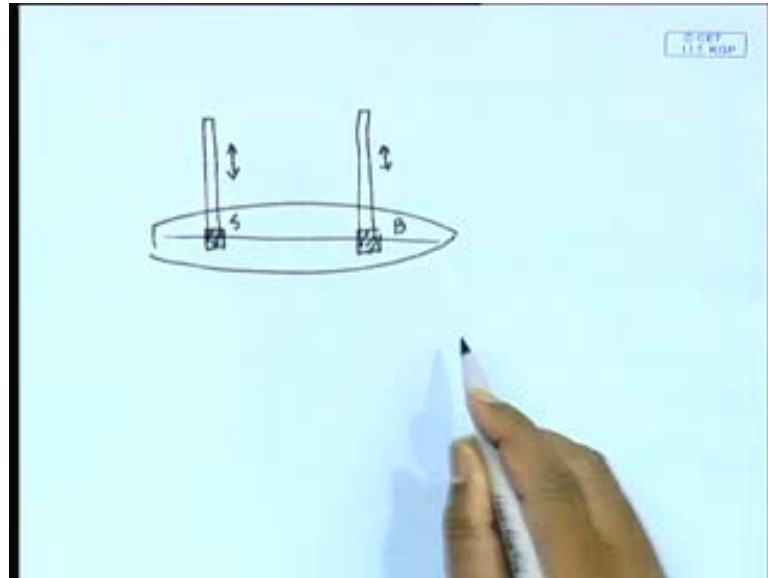


Actually this is **Planar Motion Mechanism** (No audio from 00:32 to 00:50). See, in the last class, we were talking about briefly just mention this test. Remember we are talking about experimentally determining all the hydrodynamic coefficients. Now, what we found out? We found out that among the all derivatives Y_v , etcetera, N_v , Y_δ , N_δ , this is what we can do using straight line test, then Y_r , N_r , in addition to this we can do using rotating arm test.

Of course, this test required a large facility, etcetera etcetera. Now, however, we still have to determine acceleration derivatives like **$Y_{\dot{v}}$, $N_{\dot{v}}$, $Y_{\dot{r}}$, $N_{\dot{r}}$** in addition to this. So, for that this new technique. Basically it is an equipment system, you can say is a system is an equipment as well as a measurement system came up.

In fact, for ships we can for the mechanism, we used for ships, surface ships, one can add a word here, horizontal horizontal planar motion mechanism, HPMM. What is this system, and what is the beauty of this system? Let us talk about it in a minute.

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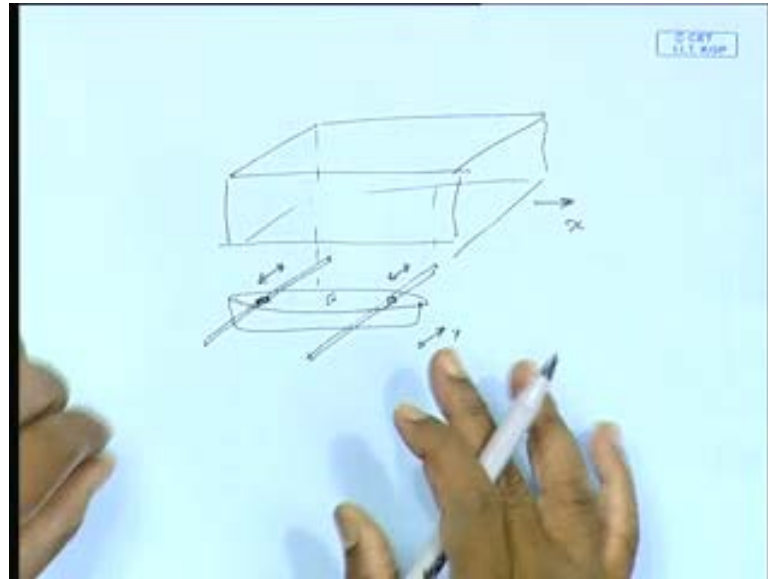


See, this system consist of two is it, is a system where we have got a mechanism where the ships can be held at two locations.

I will mention this later on, it can be held at two points, let us call it point B and point S, some some two points and independently the model can be made to oscillate this way. So, what it means is that, see it is something like that you look at this. So, this is the kind of a two struts there, model is tied here and you can independently move this, say this one and this one. That means, I am attaching the model here, attaching the model here, these are the two struts (Refer Slide Time: 3:40).

And there is a force dynamometer here, this point and this point, and I can make the ship move this point up and down as well as this point up and down. For example, if I did together, it will be like that, but if I did separately I can have a motion like that, do you understand the point now? See, here this system again I will tell you, if if I were to look in a in a carriage form in a just very rough way.

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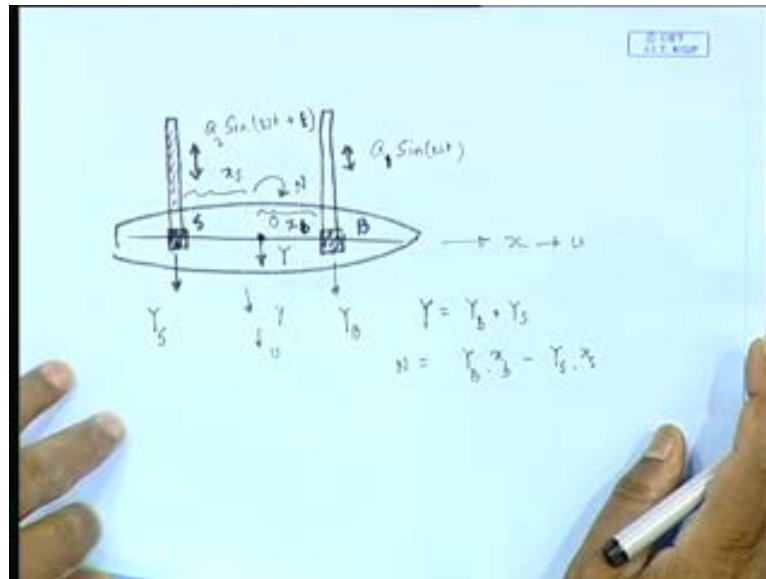


I would have had a, I will come back to this then, a say a carriage here, some kind of a carriage here. Below that I have say two struts, the system. At some point of here, I am attaching my model, this attachment point can be fixed, this full thing is attached to that by some means, along this line I can make the model to go up to and fro horizontally.

This is a horizontal plane, see this is a x, this is a this thing x direction, **this is the y you know** this is a y direction say, this is the for and of course, you make the model move together. That means, there is a towing carriage here, full thing is moving like a towing carriage, what we are doing in a **in a in a** resistance dynamometer, we actually have a point attachment here, you attach with a equipment here and tow the full model.

Here the attachment is not that, it is **(O)**, there is a system, it is about two rods, as I said two rods like that in which you are connecting your model. This points where you are measuring the model can be made to move independent like **like** this can be made to move like that, this can be made to move like that, independently which means going back to this system is schematic (Refer Slide Time 5:32).

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I can give this one a motion a 0, say sin or cos omega t and here I can give a 0 or another one, see let me call it a 1, a 2 sin epsilon independently, these two are independent mechanism. So, independently I can oscillate them in a horizontal plane, remember this my x axis u, this is my y axis small v, I will come to this full thing in a in a minute, but we should understand this system here. So, the PMM system essentially consist of an equipment where the model is attached, this full equipment system as I showed in the previous one, this this equipment system is what is called PMM mechanism.

Two struts there on which you attach a model, there are dynamometers here where you can measure the forces coming in this y direction, that is I can measure the forces here, I can measure the forces here, and the control is that I can independently make the attachment point move of an given amplitude a 1 or a 2 in a sinusoidally, say this is sin omega t (Refer Slide Time: 6:48). So, let us say, I call it sin omega t plus epsilon.

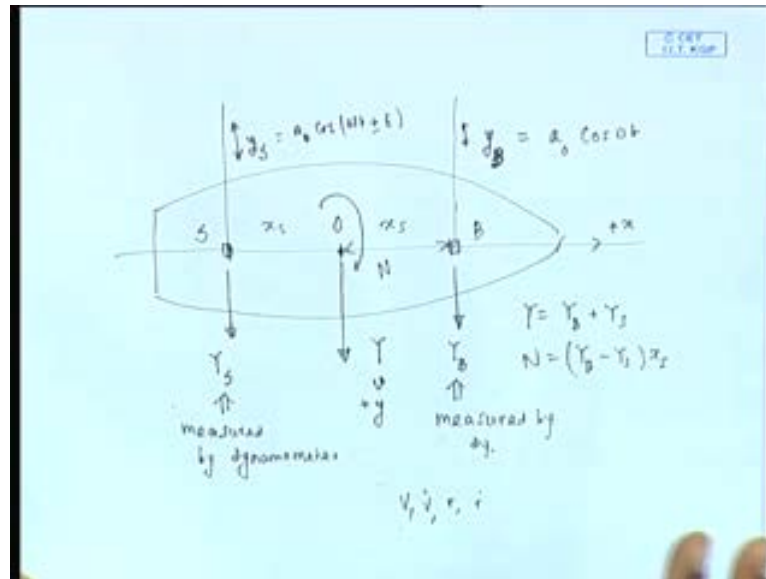
Because there can be a phase gap independent, that is why I write epsilon. Now, you see that this is the mechanism; we must understand this mechanism first. And of course, what we are measuring here, we are measuring here, Y stand point, Y bow point, because there is an equipment here, you can also measure x. So, suppose this point is O here, the origin, then I end up getting here Y, Y will be therefore Y B plus Y S and I end up getting here N, if I call this to be x s x b let us say, if I call this to be x s, then my N become remember Y B into x b minus Y s into x s.

Because this net force **know**, so what is happening, I am having this system here, I will **i**
will draw another one with **with** this diagram better, dynamometer here, dynamometer
here, measuring force in the Y direction, the attachment point. Obviously, the attachment
point is where the, because this is the rod here on which I am giving a motion, here I can
give a motion. That means, the model therefore, you see something like this model, I can
tow the model with this way.

So, this one, this tow lines, I can independently oscillate and I model, I can go like that.
And of course, I am measuring at this two attachment point, the Y forces, this is the
mechanism, let us understand the equipment may differ in detail, normally you are going
to attach this mechanism below a carriage, obviously there is a carriage here, below that
the mechanism is existing on which you attach the model, and the full carriage is towed
forward.

So, that means you are towing the model like this forward. Now, I will give some
specifications, what we do now, I will draw now one more diagram to show. Normally
what you do, let us draw a bigger diagram here, see this is my point O, normally what
you do, you attach these two struts at equal distance. See, here I of course mention here,
that it can be any distance **right** x s x b whatever. But normally what you would do, you
would keep this to equidistance, you can always place the model, this **this** distance is
more or less fixed, but you can also, the equipment can also separate out slightly, but you
give that.

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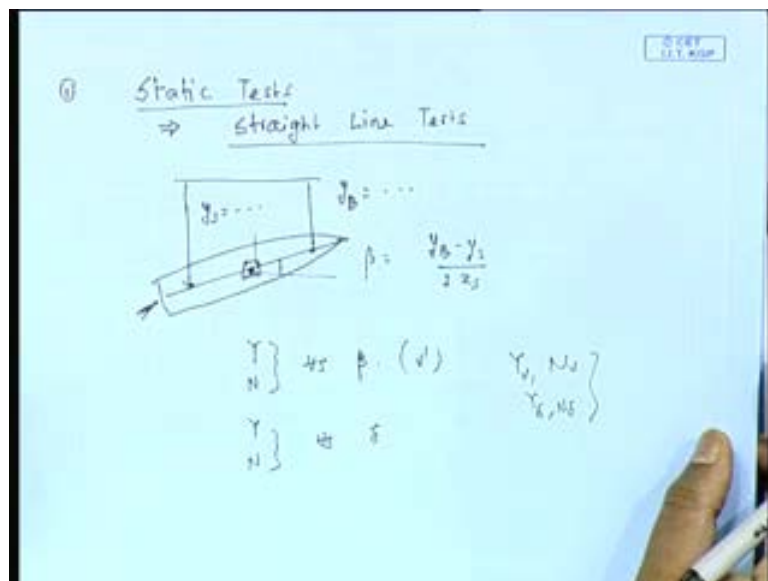
So, let us say this is I call this here, let me call this to be a bow point, point B attachment point as we said, S point and we have here x_s , now you remember this side, this is my Y, this is my N, this is also my direction v , this also plus y plus x , this flow side the one that I am measuring **measuring** is Y Star Y_B , this what we are measuring, remember this we measured (No audio from 10:43 to 10:59) (Refer Slide Time:10:00). Same is this (No audio from 11:01 to 11:08), this two are you are measuring, you are giving an independent motion here, let me call it this **y stern sorry** this is y_B , this is y_S , both of them normally you give a , we can call this **well**, let this I let me now call it \cos (No audio from 11:36 to 11:46), some what I am saying is that in the actual test, although you can have this amplitude different, normally you are going to keep the amplitude same I am, equipment allows you to fix it anywhere.

This can be different than this, this can be different, independent control, but what happen, we this is the equipment, now we have to do this test, let me call this to be as if this is $a_0 \cos \omega t$. I am calling $\cos \omega t$ for **you know** is of plotting, this is my plus y and I call it $x_0 \cos \omega t$ plus minus ϵ , some phase gap obviously. Now, you see, of course now, remember I am measuring this quantity and this quantity. Of course, as I said earlier, then my Y, this Y is Y_B plus Y_S and N equal to Y_B minus Y_S into x_s , this measured. So, I am measuring this, remember in any experiment, you are having system of control and system on measurement, this is in short the description of a

horizontal planar motion mechanism. Because in the horizontal plane, **in the horizontal plane** I am able to give a planar motion in the plane, of course I tow along with that.

Let us see, how what we can do with this. So, this once we understand that, see remember what we told in the last class, I want to ensure that I can selectively make one of the four $v \cdot r$, non zero and rest are zero, something like that, because I need to measure a force when all are 0, except in one, this is my objective basically. Now, there are you see, this is a system, so there are different kind of test that can be done.

(Refer Slide Time: 13:52)



Number 1 **number 1** is that, now I will refer back to these all the time. Number 1 is that see here, we call (No audio from 13:52 to 14:00), this is this becomes actually same as straight line test, what we do, suppose I keep **you know** one more thing I can always tell, that y , what is this y ? y is nothing but y_S plus y_B , y_S plus y_B **right, no** basically average of that.

Depending on how much, you this is y here is y_S plus y_B **(O) alright**. Now, what we do, you see, interesting point, suppose I do not change it, I make ω to be equal to infinity, means I do not change it I just keep a fixed. So, I keep y_B something and y_S something.

So, how the model is oriented then, I just keep y_B as some value, constant value; y_S as another constant value. What would happen, this angle (No audio from 15:25 to 15:33), I

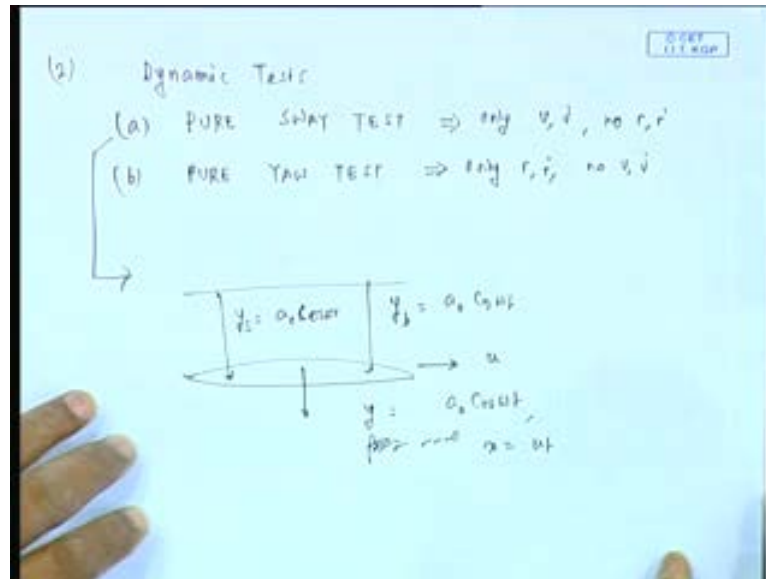
am just not here becoming correct dimensionally, because it can be **you know** like if there is plus B here, plus B here, then the theta is on this side, etcetera **etcetera** never mind that. But **but** the drift angle is like this, if my y B is fixed, not a function of time, y is not a function of time, beta is this. So, what I can do, of course I tow it in this fashion, what we have achieved, exactly what we have done in a straight line test.

In a straight line test, what we did, we simply took an angle and towed the model and what we did, we measured Y and N versus **versus versus** beta which is actually v dash, we also measured Y and N versus delta which is rudder angle. So, you see here, one default option of this mechanism is that I can in fact, I can use as a resistance test, supposing I keep both of them exactly same and tow it and measure in this the x values, x forces in the dynamometer, then I can find resistance.

The way we do that **you know**, the x force if I keep this two same, exactly same, that means beta is 0, then I tow it and measure x force that is my resistance force. So, if I keep beta equal to some beta as I showed here and tow it with of course rudder to be 0, then I get Y versus N against this thing, or if I tow a straight line **with a beta with with with a sorry** with a delta, a rudder angle I will get the y delta.

So, what happen, this is by default version, reducing the mechanism to do straight line tests and therefore, I can do exactly whatever I could do here, Y v, N v, Y delta, N delta, I can measure from what is called static test. Of course, I can also measure the non-linear terms, no V delta, N delta, etcetera whatever we have talked earlier. So, static test is nothing but reviews from a straight line test. Of course for that purpose, you would not have come with a new mechanism, because you could always do straight line test.

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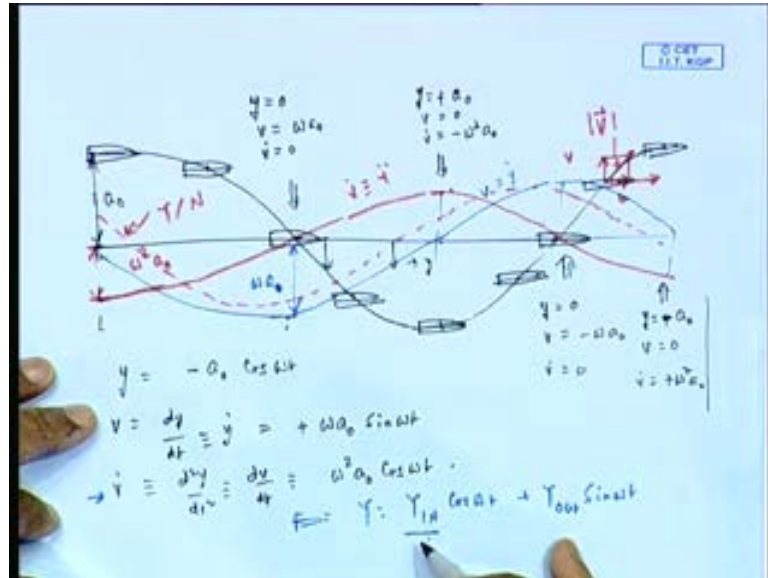
So, the next, the more important part for this mechanism of course is dynamic test, what we call number 2, where we can measure $v \dot{r}$, all the term. So, the next system is dynamic test, then there are two versions of that, one is what we call pure sway test, I will come to that.

We will **we will** today's class we will discuss the first one probably. In pure sway what we want to do, we want to make sure the model is purely swaying, there is no see, means I want to see here only $v \dot{v}$, no $r \dot{r}$, here we will come to that. So, there are two kind of tests, so we will talk of this pure sway test first. Now, you see here, once again this system here, now suppose I keep this two same, a $0 \cos \omega t$, a $0 \cos \omega t$, what is my delta? delta will be 0, because after all remember the delta is the difference between the two.

So, if I now were to keep the model here and here same, these distances that is y_B I call it a $0 \cos \omega t$, y_S also a **0 sorry** $0 \cos \omega t$, then what is y , all a zero $0 \cos \omega t$ **right**. How does y look like or what is beta, 0 if beta is 0, angle of attack going, not angle of attack, not actually **well** you can say that beta is in a sense, but I would not want to call it beta 0, we will just leave it like that, because it depends in the forward speed part little bit. So, what is happening, we are actually moving this way and also, but however, we are actually beta is on this 0, because we are having a forward speed u . So, what

happened, if you look at that this model is going to have y equal to a $0 \cos \omega t$, why? But x of course is $u t$ forward, that is why β would not be a 0 x and y .

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So, how does this look like, now this is the most interesting part, the model will therefore go, if I call y equal to a 0 , this side my plus y . So, the model is let me call it in this diagram minus of course (No audio from 21:04 to 21:31) **right**, look at this any **any** point here (No audio from 21:38 to 22:04).

See, I will have forward flow motion, this I will come to that how this v comes in **you know**, see what is happening, why is this, actually my diagram is of course this become minus this, because I am calling this to be plus y , **yes** it is a question on nomenclature, I am calling this side plus y . So, of course, this is happening is that is minus, the graph I plotted here is minus cos curve, because this is my plus side, this would have been a cos curve.

What is v ? $d y$ by $d t$, how much is this, this is going to be plus ωa_0 , you believe in that, and what is v dot? (no audio from 22:58 to 23:13). So, if I were to plot here the two graphs, v is going to look like sin, plus sin, this is plus **(O)**, that means it will look like (No audio from 23:28 to 23:38), this is **this is** here, maximum is 0 here. So, this is going to be v dot and how does v dot look like? v dot will look like a cos curve plus **(O)**, this is actually minus cos, so it is going to look like this way (No audio from 24:05 to 24:28).

What is the amplitudes? (No audio from 24:32 to 24:50), and this of course (No audio from 24:53 to 25:02) **alright**, that is what you are getting, now what happened, you are matching the obviously, so what we have achieved here, remember what we have achieved here **what you have achieved here** what is my r , r is 0 because there is no term it is rotating. So, this test where I am purely swaying with keeping these two amplitudes same, so what I am doing is going like that.

So, I am only imposing v , but no v and v dot, but no r , because for r , I need to have rotation, there is no rotation there **right**, for r I need to go like that, but I am only going this way. See, when I go this way, there is no rotation, what is the rotation about this point, about the ships, about this point, the rotation is actually rotation about this point, no rotation any point, just translating. So, this is what we call pure sway test, you are achieving a pure sway.

But the interesting point is that, you see that v and v dot, both are present throughout the time, but there are instances at this point For example, I have only v dot, but v is 0, this point, this point, this point, but look at this point here, where I have only v , but v dot is 0, I can call this to be in phase and **out of** call this out of phase (Refer Slide Time :26:28).

Now, what we are doing, we do of course is that I will **I will** maybe we can write it down here with this time, let me write it down here at this point. See, here my y is 0, my v is ωa 0, my v dot is 0 **right**, at this point my y is **minus no** plus a 0 here, **plus a 0** my v is 0, my v dot is this side, this is actually negative side. Now, if you look at this point when a y equal to 0, v equal to here I say plus minus and if you look at this point you will find **y equal to plus no** y equal to minus a 0, y has gone this side, v equal to 0, v dot equal to plus $\omega^2 a$ 0. So, you see depending on this periodic time, you have this **this** thing, now what, I will draw another diagram later on to illustrate, what we are doing we measuring a force.

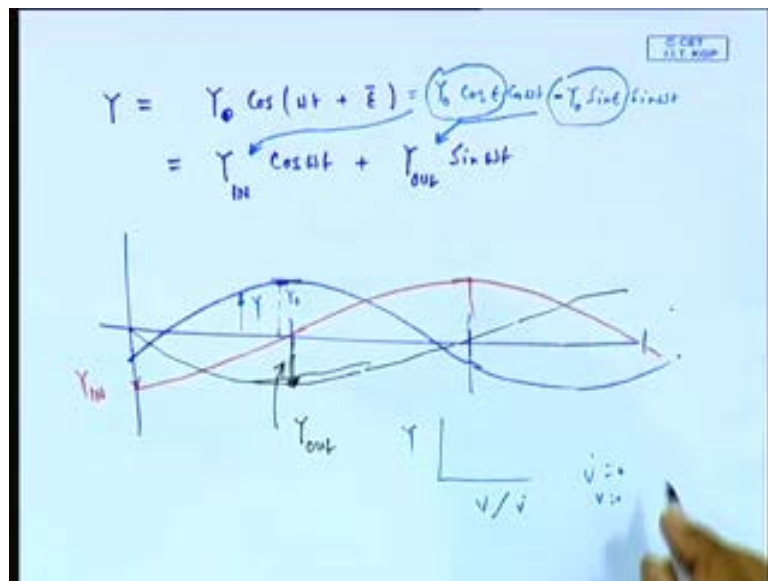
How will the force, you are only measuring the force **right**, the force would look like something like maybe **maybe** this is \cos maybe something like this, this maybe my Y , you are measuring that remember. I am measuring y b, and of course N also maybe like that, Y and N . Let **let** me call it, then what is happening **you know**, therefore you are able to find out, I will come to the maths part in a minute, you have to find out this point. See, this much of y , is the y arising only because of this v dot, v is 0, because remember this y

is $y \cdot v \cdot v$ plus $y \cdot v \cdot v$ plus maybe an inertial force, here $v \cdot \dot{v}$ is 0. So, this much of force is arising only because of this much of a 0, here this much of y force is arising only because of this much of v . So, you see what is happening, I measure the force, I am just talking the principal, we will talk about the maths in **in** a minute.

So, what happens, I have measured the force history, now there are fixed times, if I call it t_0 , at t_0 then after this is a full period t , then t equal to **you know** like in **in** this thing if I have this a period, then t equal to 0, t equal to **you know** like this is t by 2, t equal to like I mean basically half a period **you know** $n \cdot t$ by 2, n by 2 into t , for $n = 1, 2, 3, 4$. I am getting purely acceleration, no velocity, and if I just freeze it by half, n by half t by 2, you will end up getting v .

So, what is happening, therefore you end up getting cases and you have no r there. So, I have a only v and $v \cdot \dot{v}$, so now I can find out, all I have to do is to find out this force and that force, now I will come to the force part in a minute.

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You see, now let me put it this way, let me call this force Y , the what I measured this Y this dotted like this, dotted line. I can call this to be $Y_0 \cos \omega t$ plus some epsilon or it can also be called as Y , I will tell you here, this I we call it sin and cos **know** Y_{in} or Y_1 , we can call it, we are calling Y_{in} . You remember that this is very simple, now that you can always make, remember that you are measuring this forces. See, once again I will see here, what I measured is, see this is my period, 1, 2, 3, 4. I have a signal which

may look like if I were to start from here (No audio from 31:37 to 31:45) something like that which of course is with a phase gap.

But remember that this signal can be written as part of two signal, one can be a cos curve (No audio from 31: 57 to 32:05) going like that, and one can be a sin curve (No audio from 32:09 to 32:38). See, this is my may be this **this** peak **peak** wave height happen this peak is Y_0 , see understand this, what is happening that I had this signal, this force which is coming, I have no control on that, it is coming something like that which is amplitude, why? And it has a phase, so that is why it is not 0 at 0, when t is 0, it is not 0, some kind of phase is there.

But the same signal can be written in terms of Y_{in} $\cos \omega t$ plus Y_{out} $\sin \omega t$, because you can if you, it is very simple to see, if I expand that $Y_0 \cos \omega t \cos \epsilon - Y_0 \sin \omega t \sin \epsilon$ **is it not**. If I expand that, so this will become something like, see if I expand that what is this $Y_0 \cos a \cos b$ **know** $\cos \epsilon \cos \omega t$, $\cos a \cos b$ is what, $\cos a \cos b - \sin a \sin b$ **right**. So, this will be minus Y_0 , so **you know** that this minus simple as that, you can get this two, now what is happening look back at this one. Remember that I have this force now, which is I will write the force here again or Y force, Y_{in} in what did we write $Y_{in} \cos \omega t$. So, this part is in phase with this, because here it is $\omega^2 a \cos \omega t$ and I am getting $Y_{in} \cos \omega t$.

And here v is $\sin \omega t$, I am getting $\sin \omega t$, what it means is that is why I am calling in phase and out phase force. So, if you measure this what is happening, this force arises because of this much acceleration, and this will arise because of this much of velocity, simple as that, because why, because remember that essentially what is happening at the, see what is Y_{in} , amplitude **no**, when remember when $\sin \omega t$ is 0 $\cos \omega t$ equal to plus or minus 1. So, when this goes off, I have only Y_{in} in the force and this Y_{in} force will come because of this much velocity. Once again just understand this part, when instances when $\sin \omega t$ is 0, what would happen v is 0, \dot{v} is $\omega^2 a$, what is Y force, this is 0, Y force is Y_{in} .

So, therefore, I am getting for \dot{v} equal to $\omega^2 a$ is 0, Y force of Y_{in} in opposite, if I want to get the opposite I **I** take $\cos \omega t$ is 0 at the instant when $\sin \omega t$ is plus or minus 1, because you are out of phase. So, if I you can see easily that

is basically even if I have acceleration 0, my omega a 0 velocity I get Y out. So, what is happening d v y at Y equal to Y out, my v is equal to omega a 0; at Y equal to Y in, my v dot equal to omega square a 0. So, d y by d v, I can easily find out, remember what we are trying to do, I want to find out V versus Y, v dot versus Y, when I have **no** v, see I want to find out V versus Y when v dot is 0. And I want to find out opposite v dot versus Y when v is 0 **right**. So, what I find out when v is equal to omega a 0, y is equal to Y out, when v is equal to omega square a 0 y equal to Y in.

(Refer Slide Time: 37:04)

The image shows handwritten equations on a blue background. The equations are:

$$Y_v = \frac{|Y_{out}|}{\omega a_0} = \frac{(Y_B)_{out} + (Y_S)_{out}}{\omega a_0}$$

$$N_v = \frac{x_s [(Y_B)_{out} - (Y_S)_{out}]}{\omega a_0} \leftarrow N_{out}$$

$$(Y_i - m) = \left| \frac{(Y_B)_{in} + (Y_S)_{in}}{\omega^2 a_0} \right|$$

$$(N_i - m x_i) = \left| \frac{[(Y_B)_{in} - (Y_S)_{in}] x_s}{\omega^2 a_0} \right|$$

So, what is my Y v, Y v is going to be therefore, basically Y out divided by I am just not taking this sign properly, that sign one has to take, Y out divided by omega a 0, what is N v then, similarly **well** actually this we can write, this we can write basically Y B **right** Y B out plus Y S out by omega is 0, because Y out is basically this two sum **no**, see as I mentioned here Y out or Y any Y force.

Actually Y force is sum of this two, what we do is separately actually the processing is done, because **you know** remember you are not measuring this, you are measuring this and this. So, basically we have this Y S is also as Y S in and Y S out, Y B as Y B in and Y B out. So, **you know** that way if you do, you end up getting separately like this. Similarly, N v becomes in fact, it **it** becomes because for this only, we need that it becomes x s Y B (No audio from 38:35 to 38:47).

Because this part is your basically, this is N out, N out is essentially Y B out minus Y S out into x s. So, you end up getting this value, but for **for for** acceleration, now there is a you have to take into account of this **this part you know** Y v dot minus m, I will tell you why (No audio from 39:20 to 39:30), I am just putting this bar because why I am putting this bar **you know**, because this sign you have to be careful, which side you are taking Y, which side you are taking omega 0, automatically sign will come out.

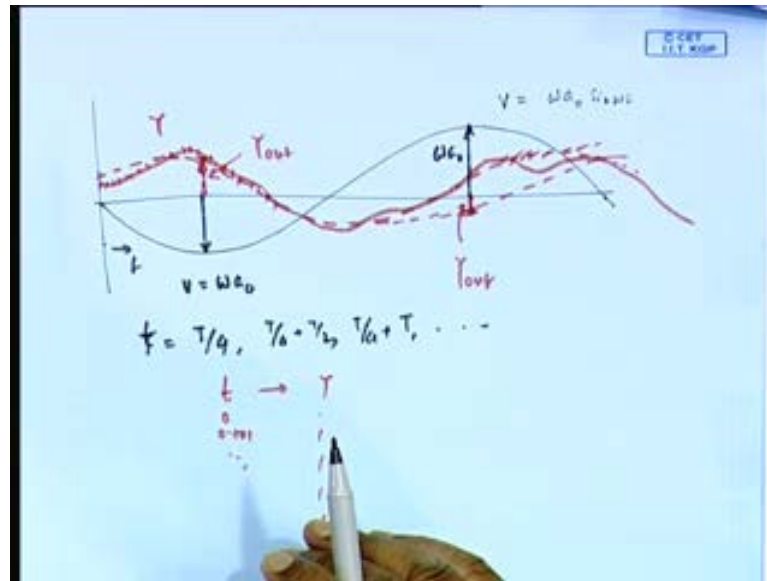
See, here I will tell you why this minus m comes in in a minute (No audio from 39:46 to 40:00), this in Y B out minus Y S in, we are just writing this way only. I can put this thing, actually this as one second I am telling you this bar I put may not be the actual bar, this I am just leaving to you, because what happens there will be a typical sign. See, you have to take a correct sign for this and this, that means the amplitude whether this plus or minus, whether this is you are taking plus or minus, etcetera, the sign will automatically come.

Since, there maybe confusion, I am just putting a bar, but in reality you will have to actually take care of what should be the thing, the bar may not be there, for example this you will find out that if I were taking consistently, if I measure Y on this side, see in this case Y positive this side, then I must take the capital Y and force on Y this side, then I must take **you know** like omega a 0, this one as plus and this one as minus depending on the time, that means I must take v to be v an amplitude positive on this side, etcetera, etcetera **you know**, so that you have to take care.

Because you will find out that when the amplitude in fact, what would happen, when the amplitude is positive here, this is **(())** done when I actually do this way, this force would have been dotted line would show on the other side typically, the dotted line would have shown this way, that is if I give plus omega a 0 as v, force would be measured forces would have come on the other side. I mean the take the measured force should actually have looked something like this.

So, this is a question of what the output comes and how you measured it. So, this is in principal, the way we are doing the pure sway test. Now, once again I want to tell you that see that now the data processing part comes in, let me draw this once, once more just this velocity part, then we will know so, velocity comes here as sin omega t **know** for v part. So, I have this say, so the v is written as sin omega t sin is this.

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So, see this is actually v , say ω what happens is that see here, this is an **an** interesting point here, my force would have come, acceleration is let me just see so that I want **I want** to make sure that I am getting that right sort of a this thing, a your $v \dot{\cos} \omega t$. So, $v \dot{\cos}$ would have been this side plus. So, my means I get a force would have been here, may be I would get a force from something like (No audio from 42:58 to 43:07), this is like my Y , I am just presuming it, I am just drawing it.

Now, what is happens remember that at this instant, this v at this instant actually this would have this **this** line would be still down **sorry**. So, this is not the one, something like that, you have got and this is my Y , this is my Y , what you expect this same as this, same as this, etcetera, **is it not**? See, at this instant the point I am saying that this **this** value, if you look out, this will be actually same as Y out in my nomenclature.

What **what** I am just understand this, this is important for me to convey to you, this red line is what you measured, what the instrument has measured, this red line is what the instrument that you have placed it here as measured, this here I have put an equipment, they are measuring the Y forces. So, I measure this Y S Y B and I sum them up and divide them, I get Y , I have measured, I have no control on that, this is as measured that is what the red line looks like.

Technically or theoretically, this should be a sinusoidal curve, theoretically it should be sin curve, therefore what happens here, that is this value should be this value, should be

this value, if you do 10 oscillation, the 10 values or each oscillation you get two points, so 20 **you know** points should be same value.

So, what I can do, I can measure this, that means the t equal to, in **in** this case here, T by 4 **right**. If I make it **no sorry** t this is just t , this T by 4 here, because from here to here is 1 T . So, then T by 4 plus T by 2 like that **you know** T by 4 plus T like that, these instances. I have this Y out, what I can do, I can average this out and basically measure my Y out and take that to be my this Y out this **Y out**.

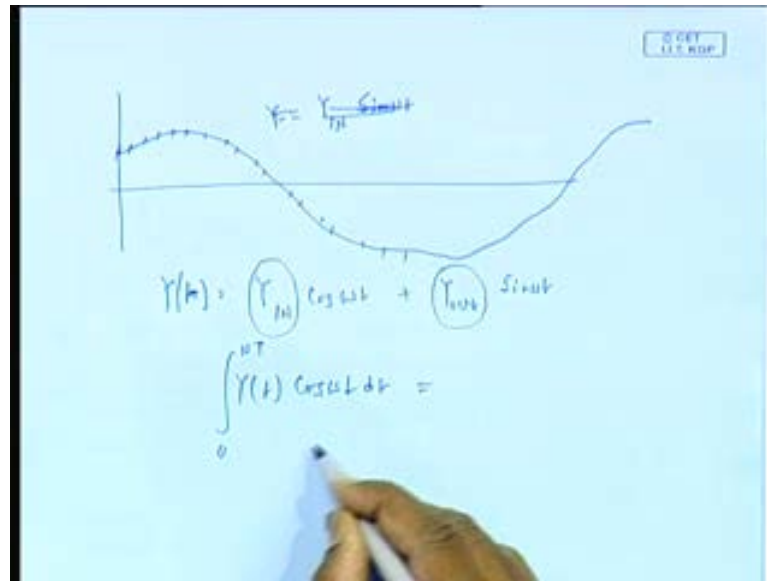
But of course, when you want to do data processing, this value may not look like \sin , it may look something like this, what would you do then, that is the question that comes in, because you have no choice. So, **you know** so of course, one thing is that you will try to tell that I will take average above this forces, but remember one thing, that is one the most interesting point that is what I want to tell here.

When I do that, what did I do, I measured remember all these points **you know** like in a digitized form, I have got in my record all the points, my computer has actually given t versus y , **you know 0.0, 0.001** like that all this values. Suppose, I am taking forty points per oscillation or hundred points say what we call hundred hertz **you know** like sampling weight hundred per oscillation, ten oscillation at thousand points, but each out of which hundred point only two points two measurements what I am using for Y out.

If I use only those two points, do you understand that this important to understand you are measuring one hundred points of which only the two points are the one where my v equal to 0 and $v \dot{=} 0$ **right**. So, essentially I used only two points and the out of hundred points, and the other two points when $v \dot{=} 0$ equal to something v equal to 0 , that means I am only using four points if I were to use only this selected points to measure my Y out and Y in.

What about the rest **you know** ninety six points, not getting used up, wasted, do you want it, you do not want it. So, what we should do, this is a point, what we should do is that we would like to ensure and that is a part of data processing, you would want to ensure that I want to measure this Y in and Y out taking all the points, all the measurement points, now this is interesting.

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So, here how do I do that once again I have a signal, it may not be sin, this is my Y signal, all the points have been measured, this I am writing **I am writing** remember as Y in $\sin \omega t$ that is what we write **know no** $\cos \omega t$ **sorry**. Let me write I am writing this Y equal to Y in that is what I want to write, remember and what is my objective, find out this and find out this, because this is the one which are those values remember in this case this is nothing but Y out and this is nothing but Y in, this is nothing but Y in, this is nothing but Y out **right**.

That is what I want to find out, but here I find Y out for observation 1, Y out for observation 2, all different and not only that, I end up using only fixed points. So, what shall we do, this I will leave it to you, there is a very interesting way of doing thing, what we can do **you know** I **I** have this y, this is a function of time what I should do you take this, multiply with $\cos \omega t$ dt integrate 0 to some **some** say period say mT .

Now, I will want to leave it to you that, when I do that what **what** happens, I am doing up to some point forget this NT part, now you see here this what is YT , let it write it this **this** way here.

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$$\begin{aligned}
 & \int_0^{NT} (Y_{in} \cos \omega t + Y_{out} \sin \omega t) \cos \omega t \, dt \\
 &= \int_0^{NT} \left(Y_{in} \cos^2 \omega t + \frac{1}{2} Y_{out} \sin 2\omega t \right) dt \\
 &= \int_0^{NT} Y_{in} \cos^2 \omega t \, dt \\
 & \quad \left[\frac{1 - \sin 2\omega t}{2} \left(\frac{1 + \cos 2\omega t}{2} \right) \right] \\
 &= \int_0^{NT} \left(\frac{Y_{in}}{2} + \frac{Y_{in}}{2} \cos 2\omega t \right) dt = \frac{1}{2} Y_{in} NT
 \end{aligned}$$

Y in **y in** is cos omega t plus Y out this I am multiplying with cos omega t d t integrating over 0 to some fixed period, because I know the oscillation period T, 10 seconds. So, I will **I will** start from some point and do for say 40 second or 50 second or 60 not 42 or some second that what I mean N T, some N, it can be any N.

Now, you see this value, what happened to this **this** is Y in cos square omega t plus Y out sin omega t cos omega t is how much, sin 2 omega t by 2 **right**. Now, tell me integration of this is what 0, so what you end up getting is therefore, 0 to N T Y in, but what is cos square omega t, 1 minus sin 2 by 2, 1 plus cos 2 omega t by 2. So, this becomes Y in by 2 plus Y in by 2 cos 2 omega t, integration d t **you agree with that no**, so again this term goes to 0.

So, what you end up getting what that is my point, I am leaving it to you for exercise, what you will end up getting is if you do that, you will end up getting an expression of something into Y in, that something you work it out. What is that, that means if I take the signal, if I took the signal all the points of the signal multiply by cos omega t, integrate it, then I will end up an expression that is going to be basically something into Y in, what we have achieved there, I have used all the points, all the data points remember all the data points I have used and ended up getting an integrate, basically if you do that you are talking about integrate area **you know** in a some sense integrate area.

Because you are multiplying this with $\cos \omega t$ and integrating the area, that becomes something like $Y \sin$. So, if I did that I am ending up getting this $Y \sin$ in a so much better way. See, I wanted to find out this and this, so I first I take the full signal, multiply it with $\cos \omega t$, integrate that, I end up getting $Y \sin$, take next time multiply with $\sin \omega t$ and integrate that, you will get $Y \cos$.

I will that to you to work it out, but this process becomes so much better, because what is happen here that even if the signal is actually not \sin , it is looking something like it may happen that this point you want to use abject point there was some kind of a this thing, **you know** some kind of a spike. So, if you were to use this, how much error you would have had, just this point, because just that point some vibration came, let us say in an equipment, because out of hundred points, you are using only that point.

And just at that instant or along that instance somebody jumped on the carriage of there was a vibration. So, you end up getting this wrong result, but if you did that way, this integrate Fourier sort of method analysis, you end up using all the points and this is exactly what is done. So, that means even though we are saying, see now the I will just sum it up and we will do it tomorrow for the other one, even though I am saying I am doing this test, then I am saying that there are instances.

Remember, I am saying instances where there are v zero $v \dot{\text{some value}}$ and $v \dot{0}$, v some value, even then the force that I measured, I end up using all the points of getting measurement to determine my $Y V$, $Y V \dot{\text{N V}}$, $N V$, $N V \dot{\text{N V}}$. That means, to find out that part of the force which is arising only because of v and that part of the force arising only because of $v \dot{\text{I used entire point}}$. So, you see how nice it has become, of course the principal if you see, you **you** will find out that what we have done is we have presumed this broken line is nothing but a sin curve.

We actually what you have done is that is broken line, this black line I actually presumed to be a nice fit of sin curve so that the area under that curve into $\cos \omega t$ remains constant, we have used the area concept. And then found **found** out the in phase and out phase component, that is what has been done and this is exactly what is done normally. So, this is my PMM, pure sway test, I end it here.

We will do tomorrow pure yaw test which is little more complicated in terms of phasing. Because now I have to make sure, I have only r and no $v v \dot{\text{which means my body}}$

must go tangential to the **you know** the **the** path **path** line should be tangential to the **you know** or velocity vector should be tangential to the path line, this we will discuss tomorrow's class. So, with that, I will end today's class