Performance of Marine Vehicles At Sea

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Lecture No. # 19

Propeller Theories Part - 1

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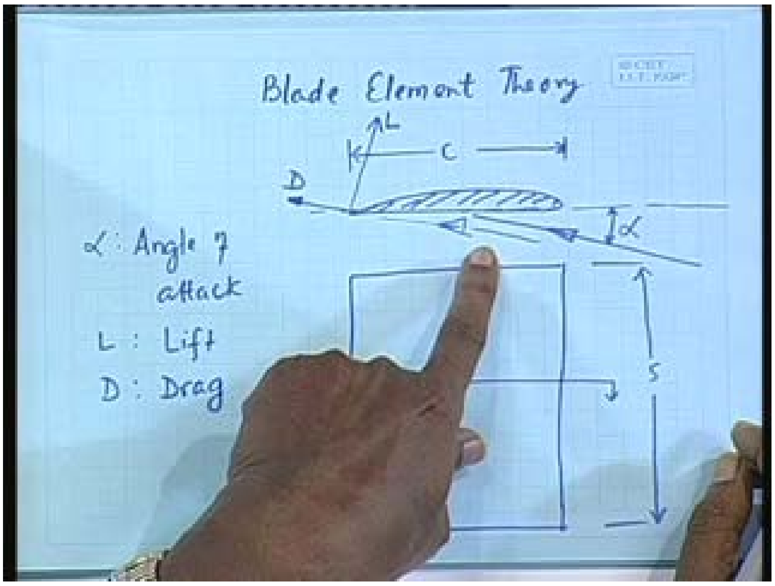

Good afternoon. We have seen the working of a propeller as a simple actuated disc and we have discussed about the axial momentum theory and the impulse theory. Today we look at a more advanced theory commonly known as blade element theory, where we will try to find out how actually the propeller develops thrust by absorbing a certain amount of power or torque and how this depends on the shape of the propeller blade.

To do this, let us assume a propeller blade consists of a number of blade elements rather than the considering the whole blade as one, we can consider a small element of the blade and will try to see what is the contribution of that element to generate, generate, thrust and how much torque it absorbs, and then, if we integrate it over the all the elements of the blade and saying the blade to consist of a number of elements, we will get the total thrust generated by the total blade.

And if there are n blades or set number of blades, then the total thrust will be thrust generated due to one blade multiplied by the number of blades z. Now, when the element moves in water, it generates an element of, generates or it is subject to an axial velocity of the fluid and also a tangential velocity of fluid. The axial velocity of the fluid and the tangential velocity of fluid together generate force on the blade element. That force can now be composed in the two directions - the axial direction and the tangential direction.

The axial direction force would be nothing but thrust; it will be in the longitudinal direction of the ship. So, that is the thrust, and the tangential direction force that is generated for the propeller rotating in this thing. If there is a tangential directional force, then that causes some moment around the propeller rotational axis and that is the torque. You understood? This basic thrust and torque for each element we can calculate like that. If we calculate all the axial forces and sum them up, then will get the total axial force, and if we get the tangential force, take the moment about the rotational axis and sum all the moments, then you will get the total torque absorbed by the propeller, is that clear?

So, this is the basis on which we will try to find out how thrust and torque are generated by an, by a small blade element. So, now, before doing that, lets consider a flat, consider, let us consider a wing consisting of an aerofoil section. You have heard of aerofoil section I am sure, yes. So, if we consider a wing of a aerofoil section having a rectangular shape, how will it look? This is aerofoil section, and if the wing has a plan like this, the section of this wing and aerofoil are shown above. Then this wing, if it is subject to a flow at an angle, then it will generate a lift, is not it?

Yes

Right

So, we can call this as chord c chord of the aerofoil that is this and this is the span of the wing. Now, if I have let me call this the base line aerofoil section, if I now have a flow coming on to this aerofoil at an angle, this is my angle of attack alpha, yes, what will happen? There will be a force generated which will be perpendicular to the direction of flow.

What is the direction of flow at an angle? What is alpha called? Angle of attack. This will generate a force which is this straight at the end, its convenient, perpendicular to direction of flow will we will have lift l, and in the same direction, opposing the flow will be the drag d. So, l will be lift. You know this know.

So, the, if, if, flow is at angle alpha, then you will generate a flow lift force l perpendicular to direction of flow and a force opposing the motion which will be the drag force. Now, this drag, remember, we are talking about inviscid fluid, that is, no viscosity. This drag would be there due to action of the flow itself without considering viscous. This is called induced drag. This will be further increased due to viscosity, but at this moment, we are not talking of viscosity. We are ignoring viscosity. So, we have this for a span.

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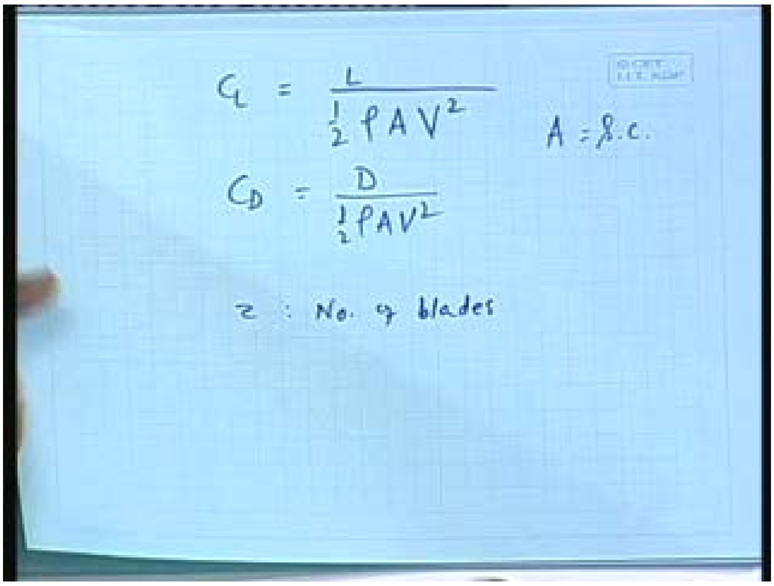

Now, let us take a propeller or lets define the lift coefficient and drag coefficient. Lift coefficient is half rho A V square. I have set V here and the area is s into c. Am I right? And drag coefficient. So, if we have a wing section and it is at an angle to the direction of flow, then it will generate a lift perpendicular to its direction of flow and a drag opposing the motion. This we understand. Now, let us consider a propeller with z blades, z number of blades let us say. We will consider first a single blade and an element inside it.

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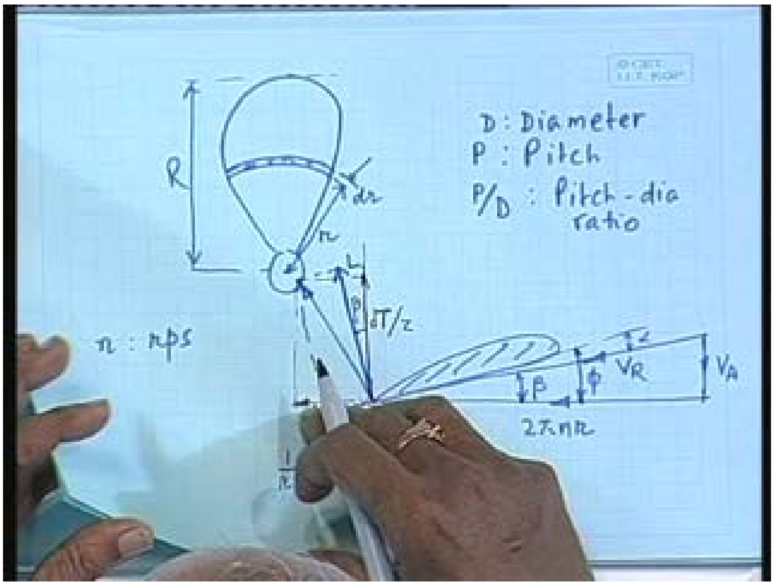

Let us say this is the propeller; this is a rotational axis. We will consider an element which is the best way to consider an element is between radii r and r plus D r from the centre; that means we will consider an element like this. This is r and this is D r and the propeller overall radius this we can call it capital R to distinguish it between an element and the maximum radius. We will consider this element and we will see the thrust and torque generated due to this element.

Now, if I draw this aerofoil diagram, first let me draw the diagram, then I will explain. Now, let us see. This propeller is rotating. See when the propeller is rotating like this, we can consider the water flowing in the opposite direction as we had decided earlier. Now, that tangential velocity is how much? How much is the tangential velocity of water which is equal to the tangential velocity of the blade in the opposite direction? Is not it? So, how much is that?

(( )).

Omega into r that is 2 pi n r. Omega is what? If n is the r P s rotation per second, then 2 pi n is the omega, is not it? Omega is a angle, angle, of this thing; 2 pi is the angle for one rotation, n rotations per second to 2 pi n into r is the tangential velocity, that is, water is impinging on to the propeller. If the propeller is rotating like this, water is impinging on to the propeller blade each section like this.

Now, the propeller what are the characteristics, I have mentioned you about characteristics of the propeller, that is, D is the diameter. What are the other characteristics of propeller? P is the pitch and P by D is the pitch-diameter ratio. Now, the propeller is twisted. Imagine that small element we have considered. Suppose this a propeller disc. I have got a propeller here; a small element is there.

I have taken this small element. This has a pitch; that means, it is not in this plane, but at an angle. You understood? A pitch is at an angle to the vertical plane. That angle we have shown here as this phi. Phi is the pitch angle; I have drawn the diagram here. Can you see that? This is the pitch angle. Now, the propeller is rotating in that disc, this plane, you understand, but the element is twisted by an angle phi. So, the rotational velocity is at an angle phi 2 pi n f. Do you get my point? Can you understand what I am saying?

No, see the propeller is if I had all the blades flat and the propeller rotating in the same disc and the other the tangential velocity would have been 2 pi n r in the same disc, that is, this blade would have been flat here, you got it? If there was no pitch, but I have got A pitch; that means the element is at an angle to that vertical plane on which the propeller blade is rotating, you got it? And what is that angle, is the pitch angle of that element that is phi.

So, that is that phi I have shown here and the rotational velocity is 2 pi n r in the opposite direction. Have you understood? Along with it, either the propeller is moving or the water is flowing fast. That is an axial velocity which is just perpendicular to the tangential velocity. That is what is shown here. You understood? 2 pi n r and V A. Have you understood everybody clearly?

Yes sir

Then the resultant of these two velocities is V r which is now impinging on the propeller blade. If you take the rotational velocity and axial velocity both into account, then the resultant velocity is not at an angle phi A pitch angle, but at a angle smaller than that, and the angle of attack alpha is equal to phi minus beta where beta is tan inverse of V A by 2 pi n r. Have you understood?

That means if we know the axial velocity and if we know the tangential velocity, then we can calculate beta and we can also see what is the geometric angle of attack, is that clear?. So, now lift and drag will depend on this angle of attack. So, lift will be here and the drag will be like this and the resultant force therefore will be here. There will be some resultant force between lift and drag acting on the propeller blade, is that clear? Now, this if I now compose, I resolve in the axial direction and tangential direction, I will get thrust and torque.

So, what is my axial direction? Is this direction? This is my axial direction, and this axial direction, what is the force? This is the lift part plus or rather minus because it is the other way the drag part which will be somewhere here. This is my thrust that is this. Is that right? And my drag will be, sorry, the tangential force will be this. Now, I will just change this. This thrust I wrote is an elemental thrust we have considered an element. So, first I write D t. This D t would have been the thrust for all the blade elements. Suppose z elements, then contribution by each element into z.

So, the elemental thrust of an element of one blade will be D t by z, you understand this per blade, element per blade D t by z because the lift is for that element only. Similarly, here, the drag will be how much? If we write in terms of torque, this is a distance r. So, torque is tangential force into r. So, elemental torque will be this force will be, if d q was the torque due to all the elements, then one divide by z D q is the torque due to each element, and if I divide that by r, I get the tangential force. Remember I am doing a force diagram, I am not doing a moment diagram; torque is a moment.

So, I must convert it to tangential force. Which is a tangential direction? This is tangential direction, is that clear? So, now you understand, I have got a blade which is twisted slightly and it is rotating in this plane with an angle phi. So, when it is rotating, the tangential velocity in the opposite direction equal to two pi n r and there is a axial velocity flow past the propeller because the propeller is moving forward we are considering the flow in the other direction as V A that is here. Thus a resultant flow instead of being like this, it is at a small angle, angle of attack. That is giving a lift which is predominantly in the axial direction but not exactly in the axial direction. There is A small correction here. What is this angle beta?

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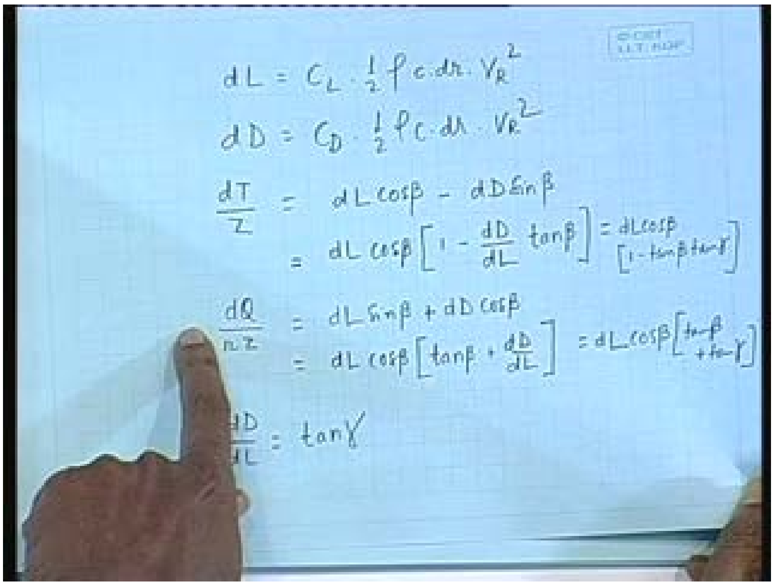

So then, we can write the relationship between thrust and lift. Delta lift will be c l into half rho c into D r into V r square. We have defined lift coefficient before. If we get the lift coefficient by some means for a section of that ship as a function of angle of attack, then I can calculate this c l if i get, I can calculate this knowing the angle of attack. Can you note that is in this diagram it is very easy for me to calculate the angle of attack. If I know the V A, I know the tangential velocity. So, if I draw this, geometrically I can find out what is the angle of attack.

Now, if I got this aerofoil data from some tests may be in a wind tunnel, then I know what will be the lift for the flow at a particular velocity at angle of angle of attack as alpha. So, I can calculate this c l and c d. So, once I know the c l and c D, I can calculate the lift. Remember this c D r is the elemental area; I can calculate. Similarly, elemental drag I can calculate. Then what will my thrust, elemental thrust and torque? I can now write D t by z is equal to, please see, see, the diagram which was drawn in the last page, which I can write as (Refer Slide Time: 21:35) Can you accept that?

And similarly, we have done this force diagram. From here, I have taken. You can see that D t by z will be this D l cos beta, D l coos beta minus D D sin beta. That is what I have written here and I have just simplified it in a particular manner. Now, if i say drag to lift ratio is a constant say tan gamma, then I can write this as D l coos beta into 1 minus tan beta tan gamma and this is equal to D l coos beta tan beta plus tan gamma. So, if I now calculate the total thrust and torque, I will multiply this with z this quantity. I will get my total elemental thrust of all the blades and this by r z to get the total torque.

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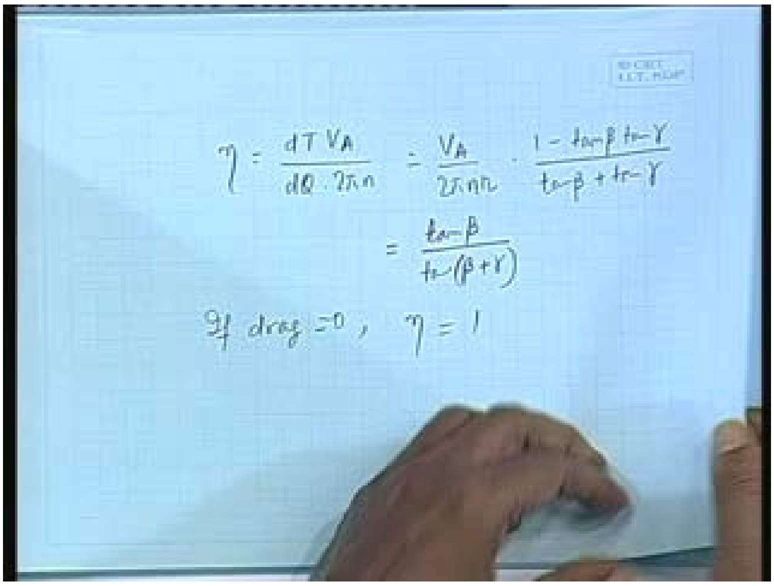

Efficiency, what will be the efficiency? It is very simple now really D t into V A divided by D q into 2 pi n which is the omega of the disc is rotating at n rpm. Now, this you will find to be 2pi n r into 1 minus tan beta tan gamma divided by tan beta plus tan gamma. You can just work it out; it will come out to be this. That is equal to tan beta divide by tan beta plus gamma, am i right? What is tan beta? This quantity, this is 1 by tan beta plus tan gamma, yes. So, now, we can express efficiency as this. What is it tell us?

If we now assume the drag is 0, suppose we assume drag as 0, then what we get efficiency as, as, as, as, 1. So that means, if drag is 0, efficiency equal to 1, which is quite in contravention to what we have seen in our previous lecture. We had said that efficiency will never be equal to 1, it will always be less than one, but here is a case when due to whatever action if drag is 0, induced drag is very small and there is fluid is totally non-viscous, and if induced drag also is 0, then we will get efficiency as 1 which is quite in contravention to what we have seen in the momentum theory.

So, what could be the reason for this? Is this wrong or is what we talked about earlier is wrong? The thing is in the earlier theories, we did very simplistic assumptions and we found efficiency is less than 1. That cannot be contradicted in a sense that no system has efficiency more than one any way or even equal to 1. So, this something we have missing here in this theory which is not showing us an efficiency less than 1, and what could that be? Think of what we did in the previous lecture compare this.

That is we had said in the momentum theory there is an axial in flow factor and a rotational in flow factor.

That means there was a change in velocity in both axial direction and in the rotational direction. If you look here, what we have done so far? There is no change in velocity. We have taken V A as the free stream velocity and 2 pi n r as the disc velocity. So, there is no change in fluid velocity. You understood? The diagram that you did this diagram if you look at. We have taken the free stream velocity here and the tangential velocity here which is equal to the disc velocity. So, there is no change in velocity we have imposed on this analysis so far. Therefore, we are getting result what we have got.

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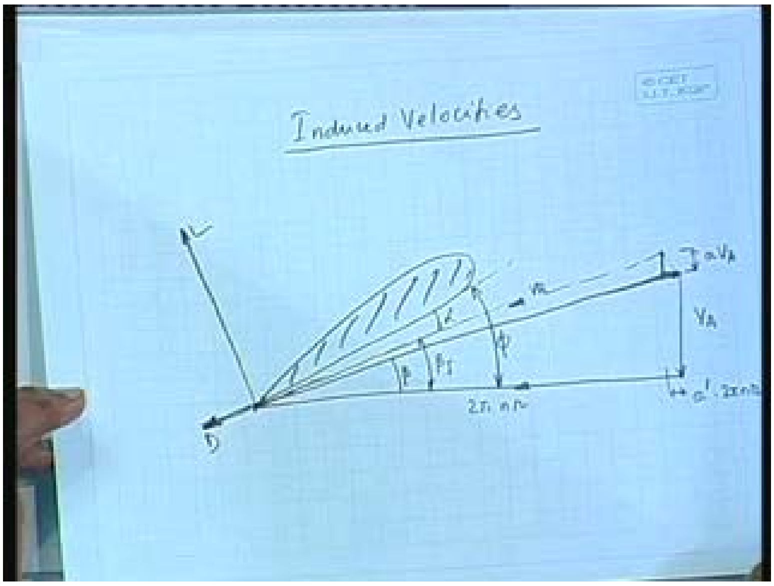

So, now, we will look at the modification of this theory by introducing what is known as induced velocities, that is, both the axial velocity and the rotational or tangential velocities undergo change due to propeller action. Due to propeller action, the flow is pulled a little bit and reduced a little bit in the rotational direction, that is, if the propeller is moving in this direction, the rotational tangential velocity is not equal to 2 pi n r, but slightly less than that and the axial velocity on the fluid is accelerated. Therefore, the axial velocity is slightly more than the free stream velocity.

These increments and decrements in velocity are called induced velocity. Velocities induced due to the propeller rotating. If the propeller did not rotate, then these velocities would not have been there, but because of the action of the propeller, there are induced velocities changing the axial velocity slightly in an incremental manner and changing the rotational velocity slightly in a detrimental manner.

So, if we redraw this diagram, let us redraw this diagram. This is the propeller blade. We have the 2 pi n r here, V A here. This is the induced velocity in the, this was the, I made a mistake. This is the line we had previously. Now, because of this induced velocities, this line shifts to, yeah, this amount of increment in velocity in the axial direction by our previous nomenclature A into V A. A is the axial inflow factor, and similarly, this one is A dash into 2 pi n r.

So, you see what is the effect? The V r is changed slightly; angle of attack further reduces. Can you see that? This was the initial geometric pitch angle beta as defined earlier, but now, the pitch angle, the effective pitch angle or the hydrodynamic pitch angle changes to we will call it beta I, not pitch angle. I mean pitch angle is this; pitch angle is this phi. The angle due to the rotational and axial velocities which was beta earlier changes to beta I. Have you understood this? Now, thrust and torque of course are same. I am not drawing thrust and trough or lift and drag. Lift now will be perpendicular to this. This is my velocity line.

So, lift will be perpendicular to this. This is lift and the drag will be this to this new V r line, and you will have the thrust and torque in the same vertical and horizontal directions. In fact, I should write D l and D D say elemental thing.

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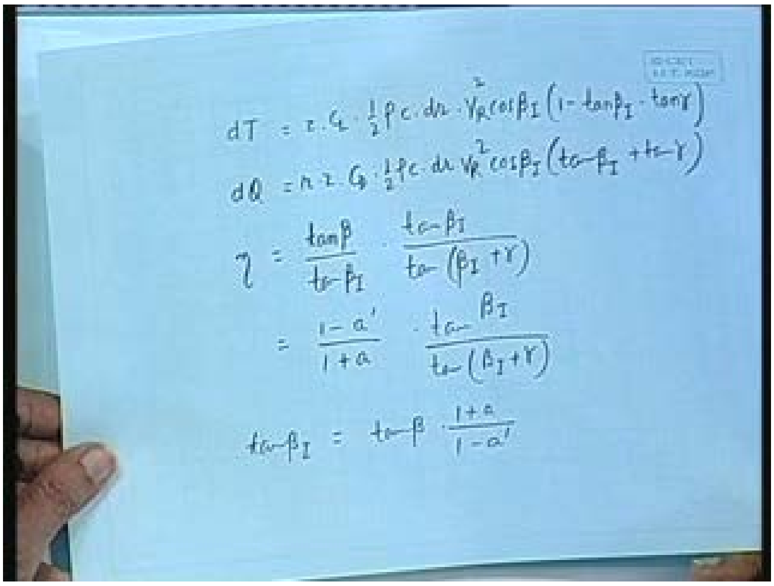

Now, we can go ahead and calculate the D t and D q. D t will be z into c l into half rho c D r into V r square coos bet I into 1 minus, you see this is same as before except that beta is replaced by beta I and D q similarly will be r z c t half rho c D r V r square coos beta I into tan beta i plus tan gamma. So, efficiency we can now write in terms of D t and D q as we have written previously and it can shown to be tan beta by tan beta I into tan beta I divide by tan beta i plus gamma, sorry.

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We can, but we will not. Now, tan beta, what is tan beta?

V by 2 (( ))

V A by 2 pi n f, and tan beta i is V A plus V A into one plus A divided by 2 pi n r into 1 minus A dash. So, if we put that, it can be shown that this is equal to 1 minus A dash by 1 minus A into tan beta i divide by tan beta i plus gamma, mistake again. What is it tell us? Just see previously we said that the efficiency could be 1. Now, you put the same condition, that is, what is that condition that there is no drag, that is, gamma is 0. If you put that, then you get efficiency same as what we had found in the impulse theory momentum theory with rotation.

So, now, this theory and the previous theory are compactable. Do you understand? Yeah. So, also we can write now tan beta I is equal to tan beta 1 plus A divided by on minus A dash. This, this, is the derivation we can get from the blade element theory. We can calculate the efficiency of an element. Now, if we assume that all the elements of the blade produce the same efficiency that this will be the efficiency of the overall propeller.

If we design, if we can design all the elements such that the efficiency of elements is same as this, then my overall propeller efficiency is also equal to this thing, is not it? What is the propeller efficiency? Total thrust, if we, if we, if we want to calculate the overall propeller efficiency, how would you go about it? We have got the elemental thrust and elemental torque; we have to integrate it over the radius entire radius 0 to capital r.

I can get the total thrust. I can get the total torque. Then efficiency will be same as we have written the formula before. D will vanish; we have calculated the total thrust and total torque. So, t into V A divided by two pi n q will be the efficiency of the propeller. What I am saying is that you can go ahead and do this there is no problem there, but suppose we had design our elements in such A manner that all the elements are this efficiency eta, then I need not go through this my efficiency of the propeller will be same as this efficiency. You got it?

So, this is what is the blade element theory, and one can design a propeller based on this theory by designing each single blade separately. See the beauty of this theory is that each element you can design for the prevailing V A. Do you understand? Previously we had assumed V A to be constant across the disc complete disc. Now, we have a option of if the V A varied also radically. Then I can design my propeller for that V A. Do you understand? We have talk about wake behind a ship.

We have seen that the axial velocity is not constant in the propeller disc it varies. So, it is now possible for me to design each element separately for the particular axial velocity at that blade radius. There also there is a assumption that the along the circumference the axial velocity is constant which may not be so, but at least there is a possibility that I can design my blade element for that particular velocity prevalent at that radius which was not possible for me on other theories. So, as you can see there is an improvement, but still there is a lot of lacuna in this, because when we design the propeller, when we measure the thrust, we measure the torque they do not tally. So, what is the next? If this theory is not correct or not accurate enough, there must be a different mechanism which are missing out all together.

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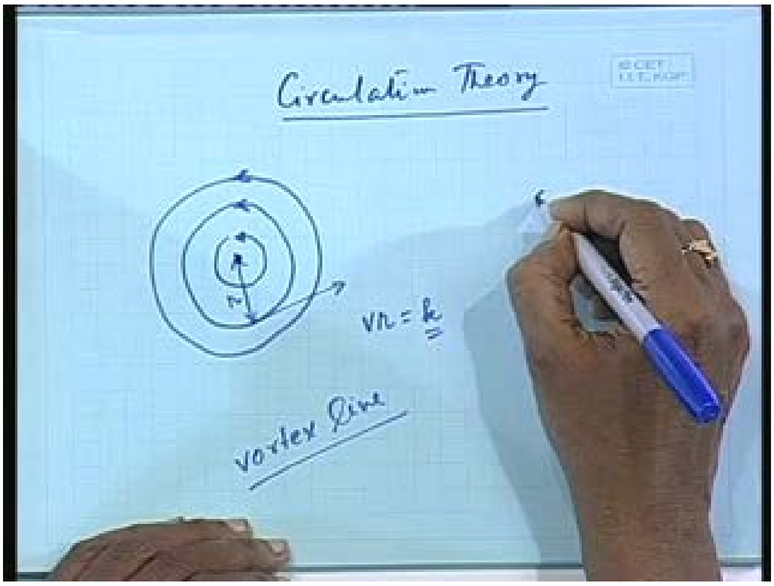

So, the most, more recent theory is the so called circulation theory. In this theory, we assume the fluid as a circulation around the propeller blade, that is, we superimpose a vortex on a linear flow around a propeller. If we superimpose a vortex, a vortex in hydrodynamics is represented by what is called circulation, that is, fluid circulates in a rotational manner. If we impose a circulation, on a streamline flow, on a completely translational flow, then the flow pattern changes and this circulation generates a lift around the section.

This is the most recent theory of wings as well as in propeller blade sections. As you have seen propeller blade section made of aerofoil sections, and therefore, the wing theory applies to propeller blade sections also. Have you heard about this? Yes. Now, let us see how this takes place. We have a vortex flow is simply represented by a circular flow around a centre, that is, flow is circulatory in nature around A centre.

Now, what is the characteristics of this flow? If a flow like this exists, the tangential velocity is inversely proportional to the radius, that is, one we know, that is, the velocity here will depend on this distance r. The further we go away, the velocities will be less. In fact, yes, velocity will be inversely proportional or directly proportional V into r is equal to k constant. So, it is inversely proportional, that is, near the circle, the velocity, tangential velocity is faster as you go away it reduces.

Free vertex

Free vortex, it is free vortex, so far we are talking of free vortex. We will talk of bound vortices later. Now, this centre is called a vortex line. You imagine this I have draw in two dimensions. In three dimensions, the flow will be moving like this and the centre, the axis around which the flow rotates would be called a vortex line, this point. Now, the vortex lines are not created nor are they destroyed. Therefore, if a vortex exist in a flow, it will go to infinity in either directions unless it meets a boundary or it is an unending loop, that is, it is bound. Vortex line can be enclosed or it has to go to infinity till it meets the boundary. That is the vortex theory without viscosity. Now, let us see flow around a circular cylinder.

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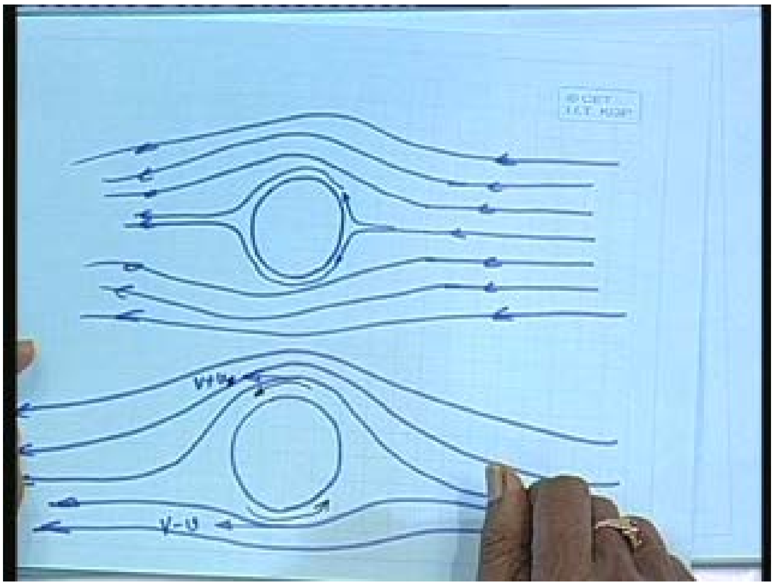

Let me draw in another paper. If i have a circular cylinder and a uniform stream of flow is coming on to it. This flow is coming on to the cylinder. How will the flow change? Flow will, there will be stagnation point and flow will separate into two stream lines like this so on and so forth. So, there is no circulation here as you can see and the flow here will be parallel. There will be no force on the cylinder because the pressure distribution here and here will cancel each other. Therefore, no resultant force or here and here also will not, will cancel each other. So, there will be no resultant force on the cylinder.

Now, if we put A vortex flow on top of it, what happens? This is the cylinder, and if there is a vortex flow super imposed on the stream line flow, flow will go like this and flow here will go like this, that is, I have imposed a vortex flow on top of this stream line flow.

A vortex which is moving in this direction. So, it has got a tendency to pull the force that side and the stream lines will be more oriented like this and this side stream lines will be flattened. Yes. So, what will happen? Here, the velocity will be higher V plus V 1 and here it will be lower V minus some amount v. So, following Bernoulli's equation, you will have low pressure here and high pressure here. Therefore, there is a pressure difference and a vertical force will emerge, is that clear?

So, if i impose a vortex, then why in the free stream flow there was no force. Now, there will be force imposed on the cylinder. So, this is the principle of vortex flow that, if there is a vortex super imposed on a free stream flow, it will impose a vertical force to the direction of free stream flow on the object. This is similar to what we call lift force.

That means now, if you have a aerofoil section, you can consider that when it is at an angle, angle of attack is there, instead of doing the analysis that we did before, we could also represent it as if there is a free stream flow and a circulation around the aerofoil which is causing a lift as we have seen.

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I have not this is just (( )) forget about the, the, one part of it this.

(( )).

Because the vortex is there; the vortex flow is in that direction.

(( ))

Yeah, it is in that direction. So, there is an addition this way and subtraction this way. That is how the extra this thing tangential.

(( )).

If the vortex was this way, it would be the reverse. It depends on which way? You see here, I have, I should have done that I have imposed a vortex flow like this. I have got a vortex flow like this. So, there is a addition here and a subtraction here. This is what I have shown.

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How it generates? We will see that, we will see that. On aerofoil, it is very easy to see how the circulation is generated.

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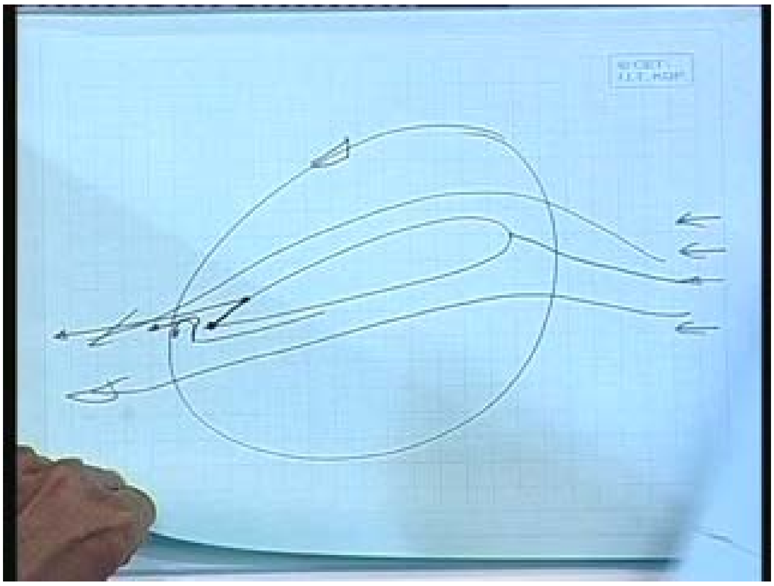

Now, let us see this is the aerofoil section and this is at an angle as you can see. I have already drawn it at an angle to free stream velocity which is horizontal in this direction.

Now, I have, I have the free stream velocity coming here. How will this flow go around this propeller, this section? Let us consider it a constant called and having finite span. So, how will this, this, thing go? It will go like this. This will be the stagnation point, and after which, this will move like this.

Now, there will be high pressure here, sorry, there will be stagnation point here also a stagnation point this side. Where will the stagnation point be? If you see the pressure distribution, the stagnation point b will be somewhere here; that means this flow will come go like this, and from here, it will go like this. What about this flow? This flow will come like this and go like this and go like this. Do you understand? Now, this as you can see is unsteady flow, the flow cannot circulate go and such a thing, such a large curvature and follow this. This is unsteady flow.

So, the steadiness of the flow will come if the stagnation point moves down here; that means this flow will go like this and that flow will go like this. You can see here what is happening. As the flow starts because of the pressure distribution, there will be a stagnation point here which is unsteady, and as soon as the flow starts, there is a rotational velocity being imposed.

Can you see that and this is unsteady. So, this vortex that is been shed is going away from the trailing edge. It will pass and slowly the unsteady flow will become steady and the stagnation point will move. So, this vortex that is shed is called the starting vortex. It is not continuously being shed. It shed only at the starting of flow because of the initial pressure distribution causing unsteady flow.

So, this vortex is shed and gone. Now, this vortex, stagnation point has come here. So, what is happening? This flow is going like this and this flow is going like this. You can see it is as if there is a axial flow and a there is tangential velocity also. So, this can be represented by an axial flow like this and a circulative flow which is like this. Do you understand? So, therefore, now you get a flow around aerofoil as shown here, can you see this? So, this is how the circulation is being imposed on the aerofoil.

Now, I will go A little quickly because I do not have much time and I will show you these diagrams. This is where the flow started. I have drawn this diagram to you. This is where the flow started and there is a circulation being imposed at the trailing edge a trailing edge is shedding a vortex starting vortex. Later stage, you can see there is circulation and the starting vortex has been shed. So, now, if we look at aerofoil section, you can see that there is a circulation around the section all along the section and there is a starting vortex which has been shed from the trailing edge, and since no vortex can start or finish in flow by itself, there must be two vortex lines here coming up which are called trailing vortices, which are because of the endings, they have been shed because of the endings.

Now, we have seen that the vortex flow generates a force on the body. Remember, this trailing vortices are free vortices with no body around it. So, there will be no force due to staring vortex or trailing vortices. The force on the propeller blade will come only due to the bound vortex here which is around the propeller blade shown here is a constant strength circulation.

Now, this can be extended to the propeller. If we extend it to the propeller because of the section blade section changing, the circulation on the span of the propeller blade changes from 0 at the ends to a maximum at the middle. There is a change. It is not a constant strength of circulation. Circulation changes from one end to the other end and you have this trailing vortices shed because of change in circulation strength artificial endings are created because of the change in circulation.

So, instead of one trailing vortex at the end and one trailing vortex at the other end, you have as if a vortex sheet, a trailing vortex sheet is produced and there is starting vortex. Now, as the propeller a move forward, the staring vortex is left behind we can ignore it. What we have is a trailing vortex sheet which is created due to a distribution of circulation along the propeller blade and this circulation creates the lift. We will not go further than this. My purpose of giving you this theory is only to introduce you how circulation is generated around a propeller blade.

Now, if you have to do a propeller design, you can now see that we can represent the propeller blade by a line with a varying circulation and that line will shed trailing vortex sheet and we can do the calculation by representing the propeller blade by a line which is a lifting line because this will generate lift. So, this is called the lifting line theory. This is the most modern propeller design theory on which propeller design is based is called lifting line theory.

Now, there is an advance theory more advance than that where we think that the propeller is circulation also changes along each element across is, that is, chord wise. That is called the lifting surface theory, but those are more advanced and we will not go into that. If you have to do propeller design, then you have to do. Most commonly used propeller design theory is the lifting line theory based on the principle I have just explained. Thank you.