Performance of Marine Vehicles at Sea

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Lecture No. # 21

Regular Sea Waves - I

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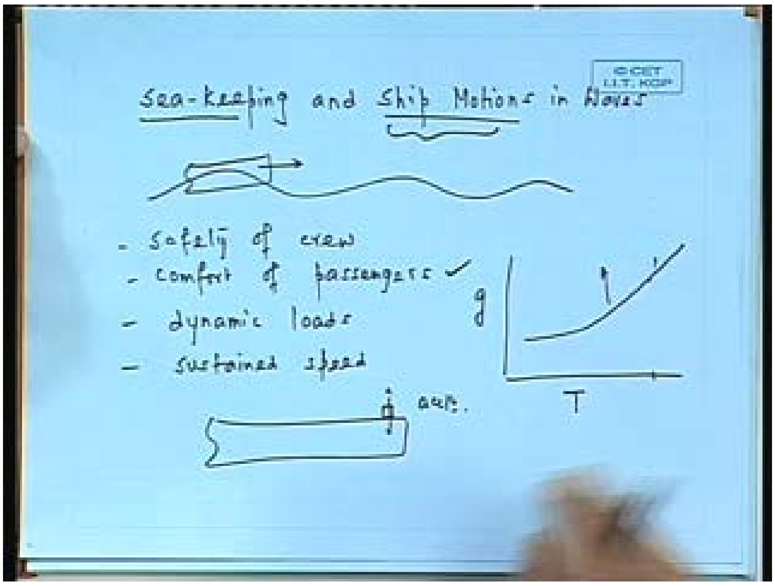

Good morning. I am going to start a new part of this course, basically connected to what we call sea keeping and ship motion in waves. We will be talking about sea keeping and ship motion in waves. So, these first two lectures I have termed as regular ocean waves, but before I go to that, I would like to give a brief introduction to what part of this performance of marine vehicles that we are going to talk. See, this, what sea keeping and ship motion essentially implies, if there is a surface wave and the ship moves, how it behaves in waves and all the attendant factors.

For example, safety of crew is associated with this. See if there is a large wave, the ship might capsize. Comfort of passengers, this I need not tell this audience much because you have been on ship basically. Then there is dynamic loads, then as you all know sustained speed, free rough weather, you have speed reducing.

In short, this part of the course is dealing with how a ship behaves in real ocean which has waves, all kinds of waves and the obviously its attendant function. See what happen is that, we call it ship motion in wave. Essentially, if there is a wave there, the ship would undergo some kind of motion, oscillation, heave, pitch roll etcetera. By itself, they may not be effecting us, but they have all kind of consequence. Let us take for example, human comfort, this part comfort of passengers.

It turns out that if you subject your body at certain frequency, like an oscillatory frequency, and subject it to certain acceleration, there is a threshold limit it goes normally like that beyond which, if you are here, that means if you have a subjected body at some period and more than this acceleration, you would feel uncomfortable. In fact, tend to vomit. It is actually connected to your some time the resonance of internal organs. This is why you vomit in a bus in a hilly region, which has such kind of T and G combination. This is a simple example why people sort of have sea sickness.

Then, you see this ship, there is some point you have a gun mount in navy ship. Now, as the ship moves it has a very large acceleration here. And if there is a large acceleration, there is going to be a large load coming on there. So, it must be designed for that load. Again the acceleration is a direct result of how the ship has behaved in a wave because it is moving up and down, as a result it has got this acceleration.

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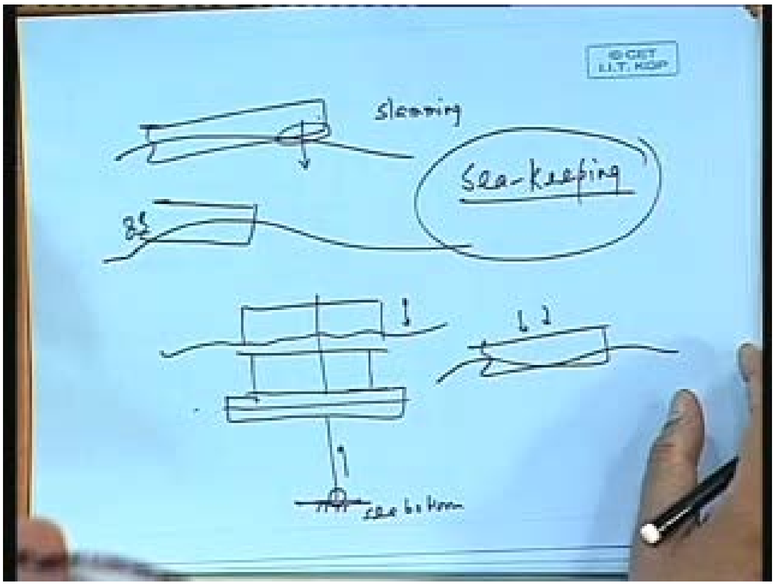

Let us see the case of what is called slamming. It may happen that the ship actually some time comes out of that water and then falls. The forward hull goes up and comes down, as it enters you have a large impacting force here coming, this is what is called slamming. Or you may have the other extreme where the propeller may have come out or a partly come out. All these are what we can call a consequence of the ships behavior in waves. And all of them can be sort of defined by means of the rigid body motion of the ship; that means, it is basically heave, pitch roll.

Let us take an offshore structure. So, many things where this comes in, say an offshore structure is gadding oil. Obviously, it will have some kind of performance based on heave motion. Suppose, it moves up and down 5 meter, then this oil riser will not be able to work because you cannot have this point going of 5 meter up and down, because you are actually this is the sea bottom and you are pumping oil. So, there is a restriction. You cannot actually have too much of motion. Like that there are numerous examples, where ship behavior in or ship or offshore structure or a floating body behavior in waves becomes important. This entire study is what you sometime call sea keeping.

How the vessel keeps a sea? You use the term sea kindly ship not so sea kindly ship etcetera, things like that. A passenger would not want to be on a passenger vessel which heavily rolls for example, or has a very low period. You do not want to travel on a ship which is very uncomfortable. This is why some ships are known to be very uncomfortable, you will not get passengers. Some ships are known to be very safe, but not comfortable, but again they can handle their half weather very much like that.

So, all this parts comes under this general generic term of sea-keeping and ship motion which itself is a very large subject by itself. Including the fact that load say ship is going like that it has got this bending moment, it must not break into pieces, it should be able to withstand their load.

Earlier days we use to calculate the load based on simple formulas, quasi static formulas, but as the knowledge is going up. This full thing is dynamic, there is a large scale motions and some other terminate like, I will show you some videos which shows really how rough weather can become.

So, what I mean is that, if we go back to this first slide again, this is ship motion in waves. So, first lecture today and tomorrow would be to understand the waves because you see it is the waves, that is the environment.

You have waves in the ocean which exists, we have to we have to understand how the waves are defined. Because if we do not define the waves, we cannot go to the next step up, when we put a ship in the waves how is the ship behaves. So, first of all we have to understand how typically wave is defined. This is what we will be doing the rest of the class today. That is we are studying what is called regular sea waves. I will explain the term why we say regular sea wave later on.

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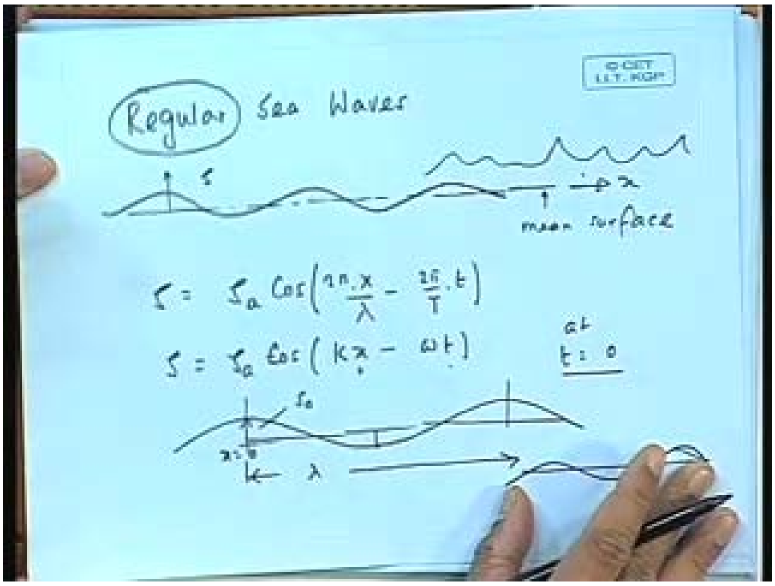

Now, again if you go to an open ocean, you will find the sea surface to be going like that. It does not same to be very regular, it is very random. Now, in order to go to that, we have to first go to the case of a what is called a single regular wave, which looks very nice and like this. This is what is called a regular wave or a single wave.

Now, if you look at this diagram what happens is that, if I call this height say, we have to keep on introducing some mathematical terms now, and if I call to be the mean surface, this is my mean surface, this writing is too small, is it? Then what happen, that xi can be expressed as a cos function, cos of… I explained to this terms later on. See if you take a function xi as xi a cos k x minus omega t, this is just a sine function, a sinusoidal function, when x represents the x axis, this is my x and t of course is time.

What it means is that, if you look at this function how it behaves at a given time t, let us say at t 0 over the time, you will find that it looks like a cos curve. It will look something like this, where the amplitude is going to be equal to xi a and this distance is going to be what is called lambda.

See if I plot that at T equal to some constant, say 0. See if I again I want to go slowly. You take this function, suppose we take this function; obviously, it is a sinusoidal function changing with X as well as with T. Omega is some number, I will explain that K is some another number I will explain that. Let us say K is we can call it two pi value. Let me take this expression rather, say this expression, we have two pi by lambda X minus two pi by T. Now, what happen if you plot that, if you plot this at T equal to 0, then this formula becomes xi equal to xi a cos two pi x by lambda.

So, that if you plot against X, this is my X axis. So, at X equal to 0, you will see that it is equal to xi a. At X equal to lambda by 2, you will see it will be xi a into cos pi, it will become minus xi a. So, it will become this, like that it will look like this, if you plot then it will look like a wave form. This distance is going to be lambda because exactly at when X equal to lambda, it repeats itself. So, it will look something like that. So, this will represent a wave form at a given time, like a sine curve with a distance between crest to crest as equal to lambda. On the other hand, I will just draw it here only for clarity, once more. Supposing, I now take this then, anyhow let me take the next curve.

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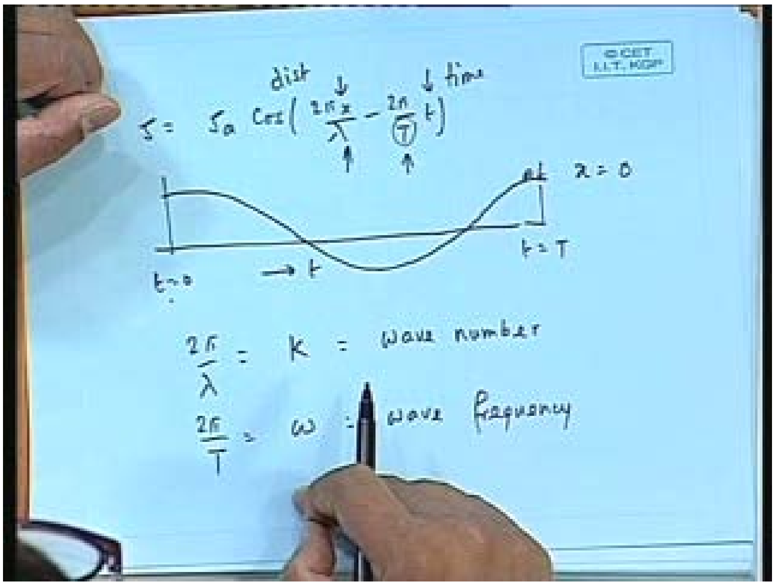

So, we have again, I have to write this. Now, I am doing it at a fixed at a given X at X equal to 0. Then what happen, you obviously plot that, that means as if you are putting a stick on the open ocean and trying to find out with time how much the height is going. See at a given location you are putting a stick and trying to measure how much the water has moved up and down with time.

So, at t equal to 0, you will find that this becomes xi a cos 2 pi T by t, basically xi a here. Again you will find that it goes like that. And this time is T equal to 0 and this is T equal to T. In other words, in a period of this t it has repeated itself. So, when you write them as two pi by lambda X minus 2 pi by capital T into T. This is time, this is distance, it turns out that this curve represents a sinusoidal wave form which has a wave length, which repeats itself at every lambda and also at every t. At every time t it repeats and every distance lambda it repeats.

So, it is basically a wave form both in time T as well as in distance X. In fact, by definition 2 pi by lambda is called K. This is known as a, this is by definition, one calls it as a wave number, this is by definition. You define 2 pi by lambda equal to K. In other words, instead of writing 2 pi by lambda, you write it as K. What is K? K is actually inverse of the wave length, in a sense 2 pi by lambda, 2 pi is a constant. So, this is an inverse of a wave length. So, this wave number is essentially an inverse of a wave length. In other words, it may be a measure of how many wave lengths are there in a given length. And 2 pi by T is of course, very common is called omega, that is wave frequency. This of course everybody knows that, wave frequency is the inverse of 2 pi by T.

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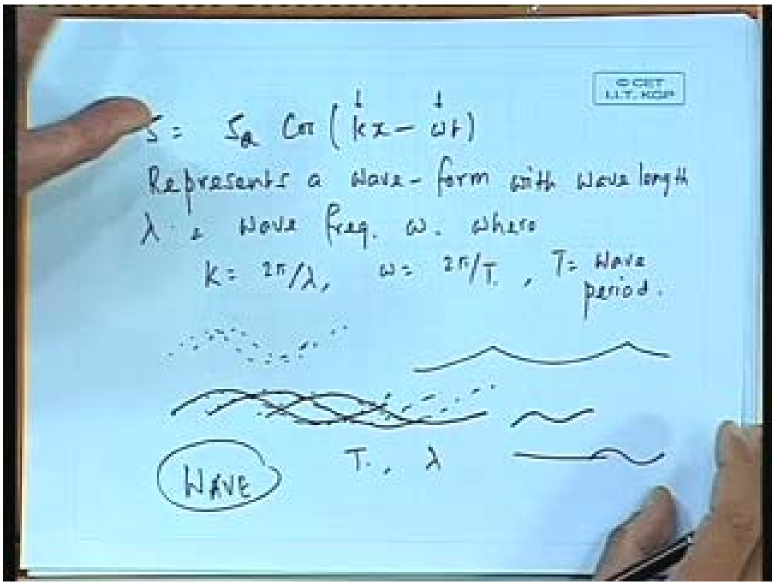

So, this is basically a sine curve. So in other words, I will write in next slide that if you have here a curve xi equal to xi a cos K X minus omega T, this represents a wave form with frequency omega, where K equal to 2 pi by lambda and omega equal to 2 pi by T, T equal to wave period. This is by definition. Whenever you find a curve, whenever you want to plot something cos K X minus omega T, it always means that it is a wave of wave length K and frequency omega, that is all. You take a computer program and try to plot that for a given T for a given X a number of points, you will see that they will all look like that.

Suppose, I am plotting it at different T, you will find that it will look like at equal to something, next T it will look like that, next T it will look like that. If you keep plotting at C at different T, say a set of point at number of X, I am finding out what is xi. So, I get this curve. Then I find out after little T equal to, say some other time, you will find that it has come here, it will look like that. It automatically means as if the form is moving forward and you will find that it would move forward exactly at the time of, there is a celerity part that in the period T, in time T the form would have moved a distance of lambda because after T this. See you started with T equal to 0 seconds, let us say that capital T is 10 second, after 10 second exactly, this crest has come here. So, this was here, then as if it has moved to at this point.

Right now, we are not going to say it is a wave form, I will come to that surface wave. See the word wave is a very common word. We are going to talk in this class about sea waves, but so far what I talked is just a wave, any wave form. Then we will say why sea wave is of this type, that is I will come to that little later. What we started is that, if you go to sea you it looks like a wavy form, anything wave means there is some change with space and change with time. Like what you see here now, next second it is not there, it is something else, it is oscillating, again space it is oscillating.

Now, this curve actually represents a typical sine curve oscillating in a space and time, this curve. Whether this curve represents our ocean waves, I will come to that progressively or should it represent because you are prone to see all cartoon waves looks like that, but this is does not look like that, this look like a sine curve. But this is a very part of the standard sinusoidal wave form, not surface wave as on now.

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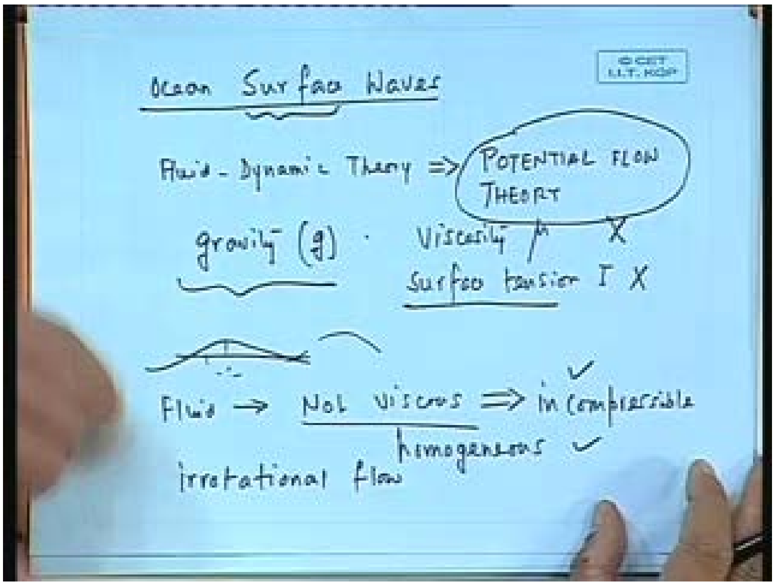

See this we can call ocean or sea surface wave. Let us talk about this now. This is a ocean surface waves, it is very important term because the general word waves is a very wide term. If you go to internet and give wave, you will probably find all waves, electromagnetic wave, acoustic wave, every kind of wave excepting surface water waves. What we are talking of is water wave or water surface waves.

Now, what I mean is that, when we talk of ocean or water surface wave, it turns out that the theory by which it has been developed is based on a fluid dynamic theory. So, the fluid dynamic theory, it is based on what we call. I will have to be very brief, but I have to still mention this. It is called a potential flow theory. What does it mean? Basically it mean that surface waves are dominated by gravity. And viscosity, seems to have practically no effect on formation of a waves neither that surface tension. These two do not seem to have much effect on formation of waves at a large scale. See surface tension seems to be affecting only at the scale of this pen kneed at 2 millimeter, 3 millimeter. But in ocean we are looking at 100 meter. There is no effect of surface tension that coming. Even if you take a beaker, if you take just a glass of water, only at the edge 2 millimeter, there is a slight surface tension, water going up. You take a rain water droplet coming from a leaf, there is surface tension in that. Anything bigger there is no surface tension, that is at only 5 centimeters.

So, we are looking at here 100 meters. So, no question of surface tension. Viscosity, it turns out that viscosity has a very low effect on surface waves, it has been found. As it is, water is actually a very low viscous fluid not much viscosities there.

So, it turns out that the main thing that effect water surface waves is gravity force. You are pulling up the water some from some point and no this is not balance. So, this will tend to go down, as it goes down this side water goes up etcetera. So, it is essentially gravity. So, if you actually go to moon, you will find waves have completely different height because the gravity is different.

Now, if it is gravity and not viscosity, it turns out that you can use a theory in fluid dynamics called potential flow theory. Potential of flow means ideal fluid, that means you can study the fluid properties assuming there is no viscosity. Now, it turns out that, that is not the only thing. First of all fluid mechanics, fluid is not viscous, but that does not mean potential flow. After that in water, you can say water is unlike here, can be treated as incompressible. This is quite acceptable because if you take a water and try to push water, water does not get pushed, you can say it has compressibility. But at such a scale that is really not coming in the picture. Unless you give very large pressure, it will not be compressed. So, you can always call it non compressible.

Similarly, we can call it homogeneous. At least in the scale of it, ship scale. You see when you have a ship 100 meter, 200 meter long, we are looking at best the water about half a kilometer around it. Half a kilometer square or 2 kilometer square kilometer ocean surface is nothing compared to a 1000 by 1000 meter ocean. So, in a 1000 by 1000 meter ocean, you may have density change slightly, that too very slightly. But in 500 kilometer type scale, there is no density change. You can safely assume that our rho is constant. It turns out if it is this and this and there was another thing that occurs which is difficult to explain, there is something called irrotationality of flow. This I will explain little in a next slide.

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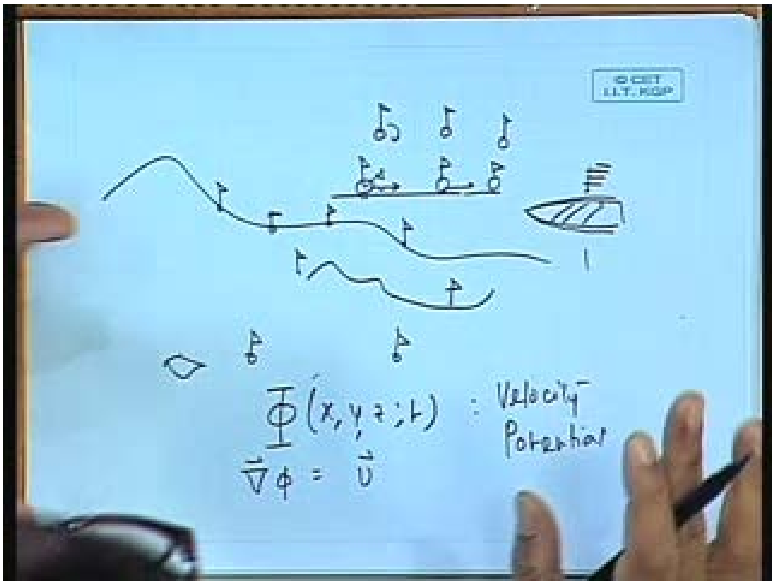

If there is an irrotationality of the flow, it turns out we can use potential theory. Now, what is irrotationality? You take a fluid particle here, something like that, now let us say I have flag here, now rotation means if this particle can rotate, now what happen, if there is a surface here, and there is a particle here, as it tries to go, if they are stuck here which would happen if they are a viscous. Supposing, they are viscous, they get stuck here then this ball cannot go, they will just rotate. So, it will just try to rotate because it is stuck here. But if there is no viscosity which I have assumed, then this ball can simply go like that, slight passed it. Now, it turns out that there is a fluid mechanics theory is that, if there is a flow which was not rotational at some point of time, it will never become rotational.

Now, it turns out that in non-viscous fluid, if there was no rotation, there is nothing to cause rotation. So, in water surface wherever you see, you take a fluid particle, particles move, but if I am putting the flag to see orientation, it will always move wherever it goes, flag remains up. So, it goes like that. Non viscous is rotational motion. But it goes like this. First of all, we say that viscosity has practically no effect on your wave formation, which is very true. It has actually an effect, if you look at a ship resistance you have done, there is a ship hull. It has only effect over a small boundary layer. Beyond that there is nothing, and how much is the width, few centimeter. Beyond that there is nothing, the waters are just going passed it.

Similarly, a wave also there is no structure in that yet. Even if there is a structure, it is only small thin layer, rest of the part the fluid particle, there is the particle. Suppose like a fish there is a particle here. This particle, wherever it goes it just stays like that because there is nothing to cause it rotation. See, there is nothing to turn it. So, this is quite acceptable in practice. In fact, if you throw something on the surface, you will find that it always stays, an orientation stays, wherever particle goes it actually stays like that. So, this is called irrotational flow. It turns out that if the fluid is irrotational, then you can have something called a function phi, again this is hull very basic, which is the function of X Y, it depends on X Y Z and T. You can have a function pi, it is known as velocity potential so that if you take the gradient of that, that means if you take Del of pi, you get the velocity something like that. So, there is a some kind of concept of velocity potential. Let me not go into this detail, but it turns out that if you systematically follow fluid mechanic theory, then the water surface wave.

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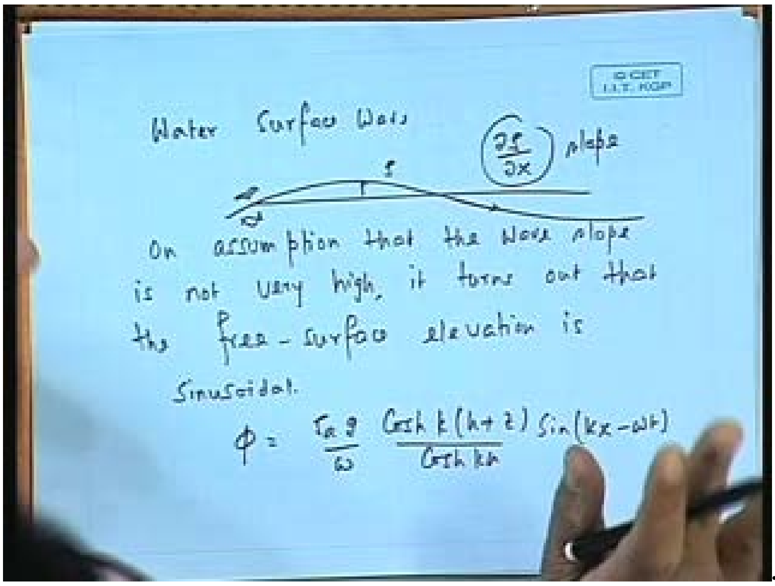

The water surface wave, if you assume the slope is not very high. See this is the slope, this is my xi and this is my slope. Any time this d xi by d x is a slope, if you assume the slope is not very much, not very high, if you assume. It turns out that the solution of the fluid mechanics problem tells us that the surface of the wave will become sinusoidal. We call it free surface elevation, you can call it free surface elevation. See, this surface is free, there is a fish here, I mean this is the water here in surface, it is called a free surface, it is free to move. This surface elevation is sinusoidal. In fact, what happens is that, the theory says that the velocity potential phi, actually phi turns out to be xi a g by omega into cos h, I just write from here k h plus z by cos h k h into sin K X minus omega T.

It turns out that you can think in other way round. If you use a fluid mechanic theory, based on the fact that the water is not compressible, water is homogeneous and of course, it is non viscous and it started at some time when it was stationary. So, there was no rotation, then which is all of them are absolutely justifiable, then it turns out that your result will show that the free surface elevation, if the elevation is assumed to be of having a small slope, then it will be sinusoidal.

In fact, the solution will show that the fluid particle, any point the particle, this function will become also sinusoidal. Why this function is important? Because if you take gradient of the function, you get velocity. There are theories which says, if you get time gradient of the function, you get pressure. Actually this function tells us everything of the fluid property, including free surface elevation. Just to tell you the theory tells us that if the slope is small, then the surface elevation is sin. that is all.

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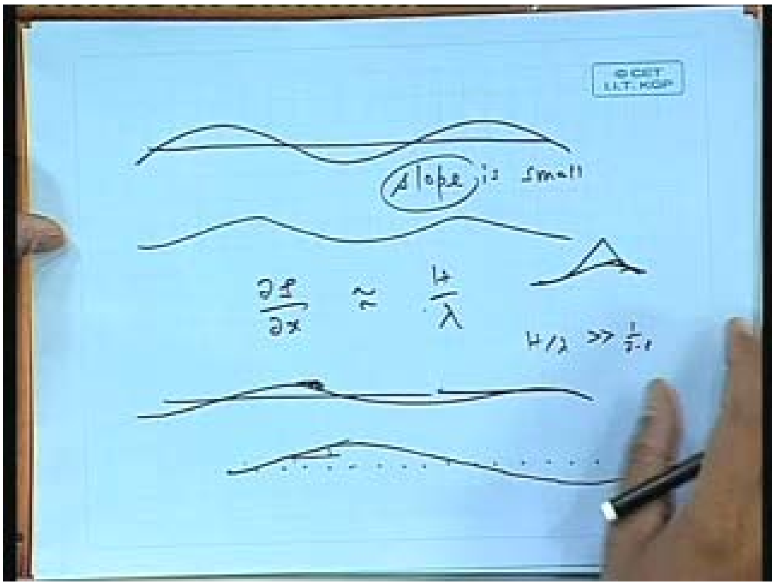

So, for us we should only know this, that the fact that we use a sin curve is not, see real ocean curve may be like this, not sin. But the theory will tell me that if I make the slope to be small and if you make the slope to be not small, then you get kind of more and more advanced theories, becomes more complicated. But at the first approximation, the first level of approximation, if you assume the slope to be small, then the free service elevation is basically sin.

Now, what it is not the height. What is slope? Slope is actually, what is slope now? Slope is D xi by D x it was proportional to h by lambda, if h is the height. It is proportional to height. When you say 30 meter wave, please understand it is 300 meter long. By theory, you see that this I want to ask you this. Suppose, there is a wave here, you tell me at which height the wave begins to break.

See, you will find in Open Ocean the wave breaking, white cap forming, what is the height? If you see that you will find that this height is not very high. In fact, this height is by length. The theory says that if h by length is exceeding more than actually one by seven or eight. Actually one by seven is theory, but in practice it is one by eight or one by nine.

So that means, it is about small slope, that 30 meter height 300 meter long. So, if you draw in scale, let me see here the scale, if this can be scale here, the scale will look like this, something like that. 1 2 3 4 5, it is exactly scale will look like that, this is the my maximum height I can get. This is I have just drawn in this more or less scale. So, it is not having high slope. The slope of that maximum slope that you get actually is only in order of point one. Anything beyond that cannot sustain. This is actually, we have a lot of, in a perception of waves when we see some kind of very high at some localized phenomena we think it is very stiff, but if you think in terms of the length, you will see that the slope is not high. You cannot have a wave like that, it just does not occur. It will be like that before and then it will only it will break, it cannot sustain itself.

So, the slope is not very high. In fact, a typical slopes will be only one by twenty. What you see everyday. Go to ocean, when you are sailing you find waves which is what order, may be a 4 meter height, 3 meter height, typical order of waves are being in that order, 3, 4, 5 meter means 10, 15 feet, isn’t that?

Say sea is 20 feet, let us say 6 meter. Now, this 20 feet wave what will be the length of it? Think of it in terms of ship, your ship on which you are going is 100 meter long, it is 300 feet. So, this length is incomparable to, the wave length is comparable to the ship length. So, it is may be half a ship length, which is still 150 feet. So, 20 feet by hundred 50 feet is not very high. We only see normally the height, we do not see the length part of it. So, it is always a reasonably small slope. No x is xi is this height. H is the basically this, peak to peak. Xi is not the mean height, xi is just at any point the height of that above the free surface. See, xi is a function of x and t, means xi is actually height at any point. When xi becomes maximum, it becomes equal to xi a.

See, when you say a wave which is 5 meter high, actually the point is not always at 5 meter, no when this point is a 5 meter, this is actually some other height. See, if you look at a point, it goes from minus 5 meter up to 5 meter and then close down and goes up etcetera, that is what it does. The form moves up and down from minus 5 meter to 5 meter. Xi is actually for any point free surface height. So, xi becomes maximum as equal to xi a and height is twice the xi a because it is minus xi. This I will come later on, this is the kind of an example, try to have to prove that. So, it turns out that the fact that we use, that the wave height is sinusoidal, the wave is sinusoidal form of it is not really without foundation.

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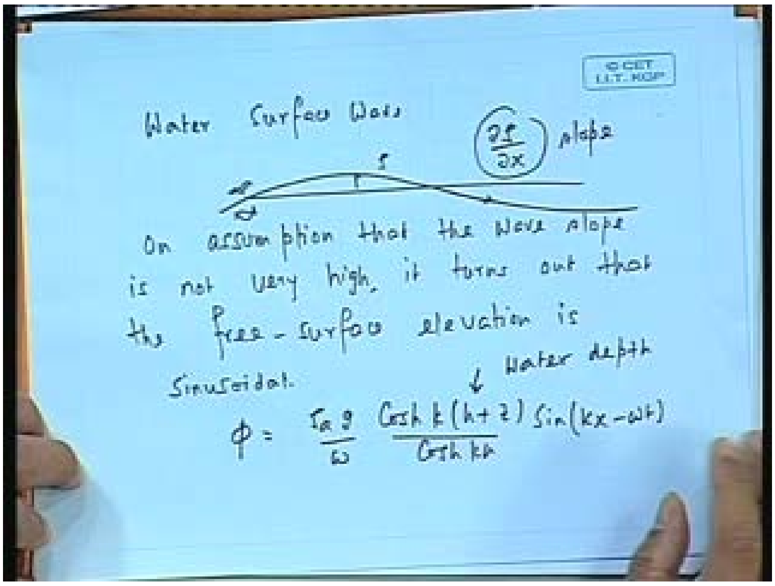

It is true. Let me write this phi wave again, it turns out that it can also be expressed as… This is a deep water wave form, this is called velocity potential for a deep water wave. See the previous formula. If you see that I wrote earlier one was this cos h and sine h. Now, see in this formula I wrote cos h k sine h k and this h was water depth.

But what happens is that, if you assume the water depth is very high. Actually in reality if the depth is more than half the wave length, may be see a typical wave length is 100 meter. So, if the water depth is more than 50 meter, then it turns out that this factor gets equal to an exponential function, like maybe I should show it here.. This function and this function, this cos can you see this? This and this becomes same or rather this reduces to this function.

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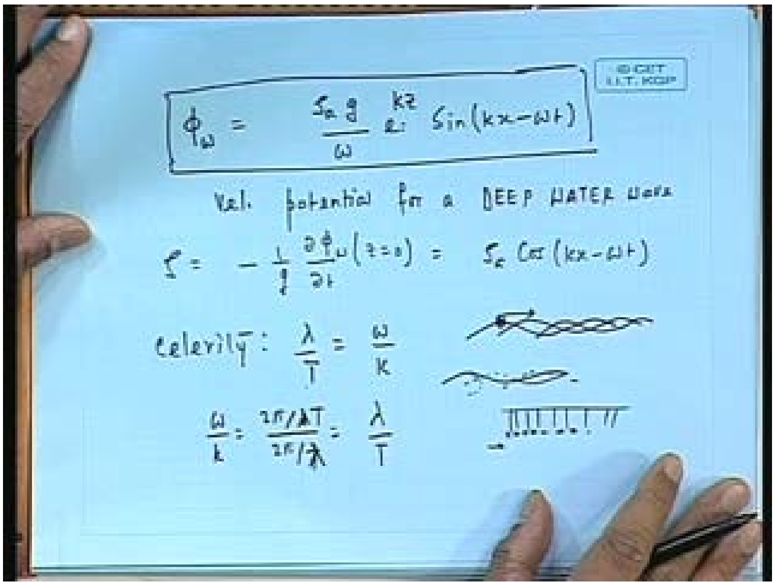

So, what happened, for our class, for our naval architecture subject, normally our water depths can be taken deep because mostly the ship is flying in water depth of few 100 meters. So, we will be concerned henceforth with this. Mostly in open ocean, the water depth is always more than half. See, if you go, except continental shelf, moment you go outside it is seen order of few 100 meter or more. Only coastal water becomes actually shallow water. But that is of no sort of issue because if it was, if you knew the height you can always use this formula, no problem on that.

But this makes it somewhat simpler. Now, this is actually what we call is this the potential and it turns out that every parameter including wave height xi or a function of this. See, xi can be, for example, given by what is meant as one by g of D phi by D T taken at Z equal to 0. If you do that, you will see that it will turn out to be xi a cos K X minus omega T. What I mean is that every parameter of water wave can be expressed in terms of this. One thing I will tell afterwards.

Let us say now celerity c, actually we do not call velocity this wave form, we call the what celerity, how fast the form is moving. Now, this is very important concept. See, the form was here today, then it looked like this, then it looked like this, then it looked like this. So, this form is the wave form, the surface shape is moving, but you take a particle here, does it mean the particle is moving? No the particle does not move, it is only the shape that moves. That is why we use the what celerity as opposed to velocity mostly.

See, it is the form that moves. Please remember this is extremely important for us to understand the particles here does not move. This particle movement I will show later on how it moves. But it is that today what you see like that, this particle will have actually come down here, next second it looks like that, next second it looks like that, it is the shape that looks. You know that if you take this number of this string and some balls, you give a hit here, you see a cluster, you will find a cluster moving like that. Few of them will get together and that togetherness moves. So, each one goes and comes back, goes and comes back, each of this ball. I will tell which fashion they move, that I will it is not they move in a particular fashion, that I will tell afterwards, when we come to particle velocity.

But celerity is the speed at which the form moves, the wave form moves. And this is given by obviously, lambda by t and that can also be written as omega by k. Why? Because omega is 2 pi by lambda, K is 2 pi by T. If you can check that lambda by T is nothing but omega by K. You see omega by K is 2pi by lambda divided by 2 pi by T equal to, sorry no. This is K, it will be your omega. See, lambda by T omega, no omega by K is other way round T and this is lambda this is lambda by T. So, we call it celerity as lambda by T omega by k. This is what we can call.

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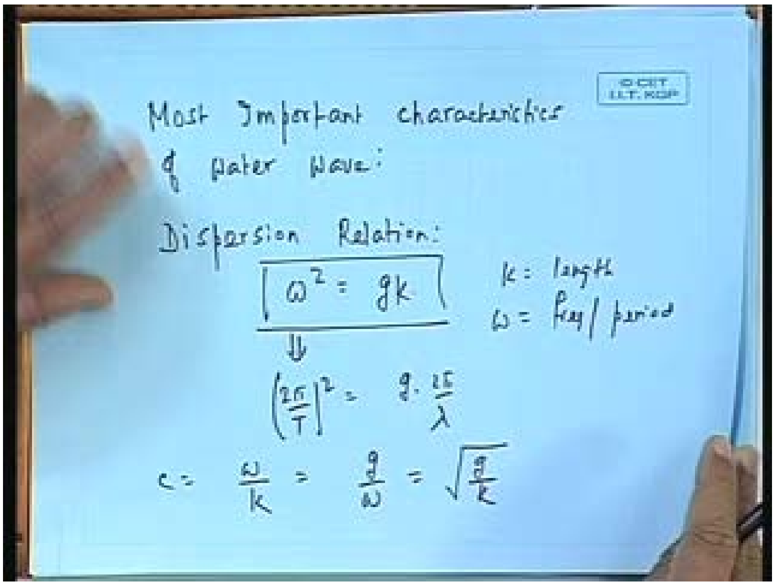

Now, one of the most important thing that I did not mention. One of the most important the most characteristic is that it follows a relation called dispersion relation, that also comes out from the theory. There is a relation, it is so important that everybody should given by this. See, there is a relation that the wave follows, it is called dispersion relation and that can be considered the most important and significant characteristic of water wave. It actually distinguishes water waves with all other kinds of electromagnetic waves and sound waves and all other waves.

This is given by the fact that by an equation omega square equal to g k, what does it mean? Remember omega is actually period, see omega is basically 2 pi by T square and K is 2 pi by lambda. What it means is that k is related to length, omega is related to frequency and period. What this relation tells is very interesting. It says that for a given wave length, there can only be a given period because omega and K are related to each other not independent to each other, not independent to each other. That means if you have omega means T, let us say I have a wave of 10 second period. Then the 10 second period wave will have only so much of length. It cannot have like a 10 second period of wave cannot have length of 100 meter or also 200 meter also 300 meter it cannot have.

Length and period and frequency are all intimately connected to each other, one of them decides the other, that is the most important characteristic. Why I say? You take for example, electromagnetic wave light wave spectrum, you take typical radio waves or you know two x ray. Radio waves, two x ray all travel at the same speed, but radio waves length and period are inverse you see. No sorry. The speed is constant there in radio waves. Only thing is that length and period, frequency and wave length actually h into lambda what they call becomes constant. If the x ray has a wave length longer, it has a smaller period etcetera, but the speed is constant. But in here the interesting part is that, this relation tells me that speed becomes a function of length, How it is so? Let us say speed C, what is this C? C is given by omega by K. By definition, what is omega by K? From this equation is equal to g by omega.

Omega by K, if you write it is g by omega. So, celerity becomes omega by K which becomes g by omega. And that also becomes root over of g by K. How it is? See, here now this relation requires lot of little discussion, I want to tell that. See, omega square is g k this is my dispersion relation.

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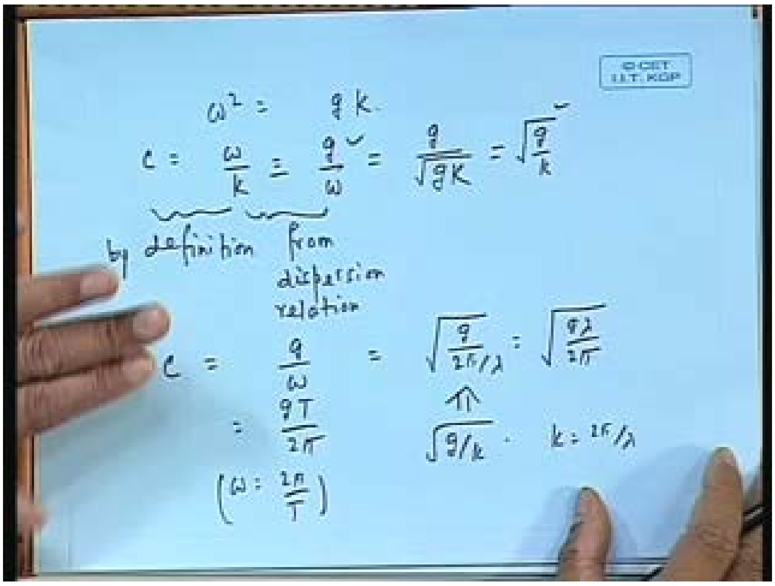

Now, celerity C is given by omega divided by K. How much is omega by K? See, I bring omega and I bring K from down here. So, omega by K becomes g divided by omega because I can write this omega into omega g into K. So, omega divided by K is g divided by omega. This is equivalent to that. This part is from definition, this part is from dispersion. Now, instead of writing omega I can write, see I do not want to write omega. So, I can write this g by omega equal to root over of g K because omega square is g K. So, I can write this to be root over of g k. See, I want to eliminate omega. So, omega square being g K, I can write omega to be root over of g K. So, this is equal to root over of g by K.

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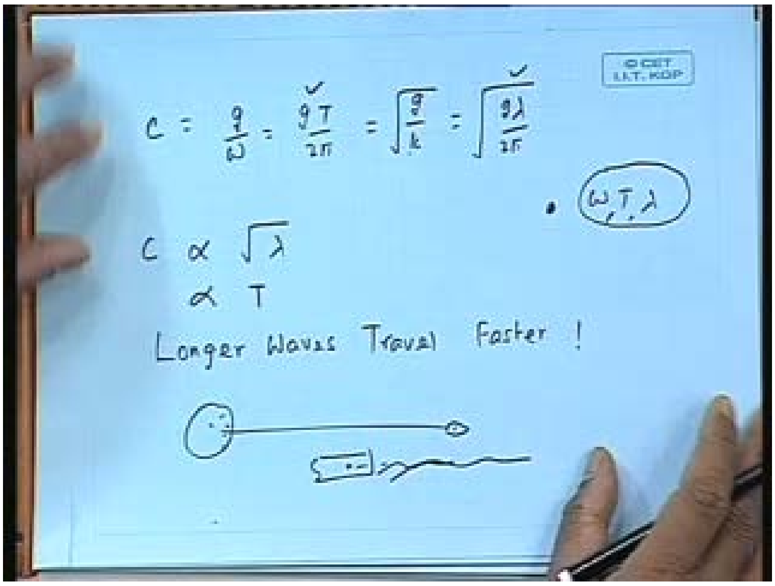

Now, let me write in terms of these two, this and this relation. So, my C's speed becomes g by omega, but what is g by omega? Omega is 2 pi by t. So, it is 2 pis by T g T by 2 pi because omega is omega is two pi by T. So, I am writing in terms of period. See, if I write in terms of period, it is g T by two pi. And let me see this relation g by K. Now, K is 2 pi by lambda. So, this becomes equal to g lambda by 2 pi. See, this is g by K and K equal to 2 pi by lambda. So, what I am trying to say is that, see C now I will just summarize this. C therefore, is equal to g by omega is equal to g t by 2 pi is equal to also g by K is equal to g lambda by 2 p,i all are related to each other. How?

See, remember g omega T lambda intimately related, one of them can be represent in other forms. So, in here I wrote C in terms of only omega, in here in terms of only T, in here in terms of only k, in here in terms of only lambda. So, same thing. What you find? It is easier to find out these two. See, C take this. If lambda is long, what is C? C is also higher. So C therefore, is proportional to root over lambda. Also C is proportional to T. So, this tells me, longer waves travel faster. This is contradictory completely with any other waves.

Here because length and frequency are interrelated, speed is not constant. Unlike in an electromagnetic radiation where the speed is equal to speed of light, whether it is a long wave length or a short wave length, whether you go to gamma rays of very short wave length or radio waves of very long wave length. Speed is constant only length and, but in here it is not so. Then, what is the important thing? You see from sun I have got all these radiations coming, all of them reach my eye at the same time. So, I am seeing red, white, blue altogether and so I see white.

But here if 10 waves starts from one point, they will immediately space out. A ship is creating lot of waves here. Right now, you met lot of waves you ten stones, but eventually if suppose it met number of waves, suppose you took like various kind of poking and making waves, soon they will separate all. Because the one that you are making at a high frequency wave would have travelled in some time only this much, the other longer one would have travelled more. So, they would have dispersed out. This is why it is called waves are dispersive. Longer wave would have travelled faster. So, if there is a volcanic eruption, it makes all kind of waves, but the longest one will reach your coast fast, shorter one will take long time to come. So, this is very important. See, this part that longer waves travel faster, shorter waves travels slower. In fact, we will find out from this relation what can be the typical speed of a wave. You see you take this part, C equal to root over of g lambda by 2 pi.

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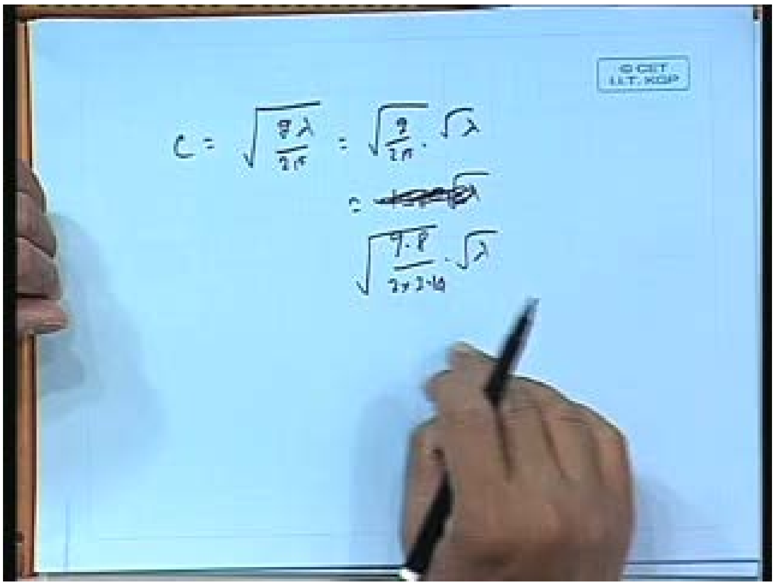

So, C equal to root over of g lambda by 2 pi, can you write g by two pi into root over of lambda? You can write that. This approximately comes out to be 1.56 lambda. 9.8 by

61.5 root over something like that. Let me not write that. Let me write its 9.8 by 2 into 3.14 into lambda.

So, we can actually calculate this part and we will find out afterwards, now next hour from there what is the kind of C value with respect to lambda. You will find out because you like to know speed, how many meter per second, how many kilometer per second and you will find out which explains why a tsunami. When there is a volcanic eruption or waves of tens of kilometer, why it actually travels the entire pacific coast and reaches the other side in few hours time. Whereas, a small wave it will just not survive. So, we will have a feel about the speed, length etcetera in the next hour class. And then we will be talking about other standard water wave properties today.

Today's lecture we will finish just the basic definition. Why I want to finish that because, if I do not understand how a single wave works, I cannot understand how an irregular wave works. Then therefore, I cannot figure out how a ship will work when you put in the wave. Thank you for your attention.

Preview of Next Lecture

Lecture No. # 22

Regular Sea Waves - II

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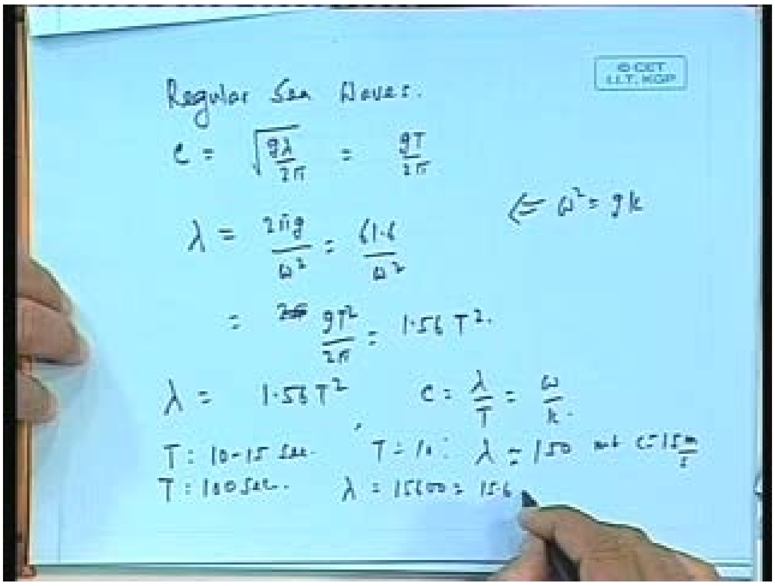

We will continue on our discussion on regular sea waves. See, this ended up at this relation g lambda by 2 pi, that is to prove that or we can also say it is g T by 2 pi. To say that longer waves travel faster and also waves with higher period travel faster. In fact, we can relate g lambda and T also from dispersion, it turns out that if you go by lambda and T relation, see from omega square equal to g k, from this working back we can find that this is equal to 2 pi by g into omega square is 61.6 by omega square or this can be also 2 pi sorry, it is g T square by 2 pi equal to approximately 1.56 T square. So that means, we can say like this lambda becomes 1.56 T square. C equal to lambda by T equal to omega by K etcetera. Why I am writing this is that, let us try to find out some relation between speed, length etcetera.

Now, typical ocean waves, what we call everyday waves. A T typically may vary between say 10 to 15 second. In fact, one can say, lower may be say 8 to 15 second. So now, let us take 10 second to be a typical example. This is what is called in our terminology as every day wave. Some people call an half sheer structure, that is most frequently occurring kind of waves. Now, 10 second will be see T equal to 10 will give you lambda approximately equal to 150 meter and C will become equal to 15 meter per second, which is about 30 knots, quite high.

Now, this is actually everyday wave. Now, let me take for an example a very long wave and find the length. So, let us say that t equal to 100 second. In a very long wave, 100 second wave. Now, if t is 100 second, you find what would be lambda? Lamba is going to be 1.56 into 100 square, that is going to be 15600 or approximately 15.6 kilometer long, isn't it?

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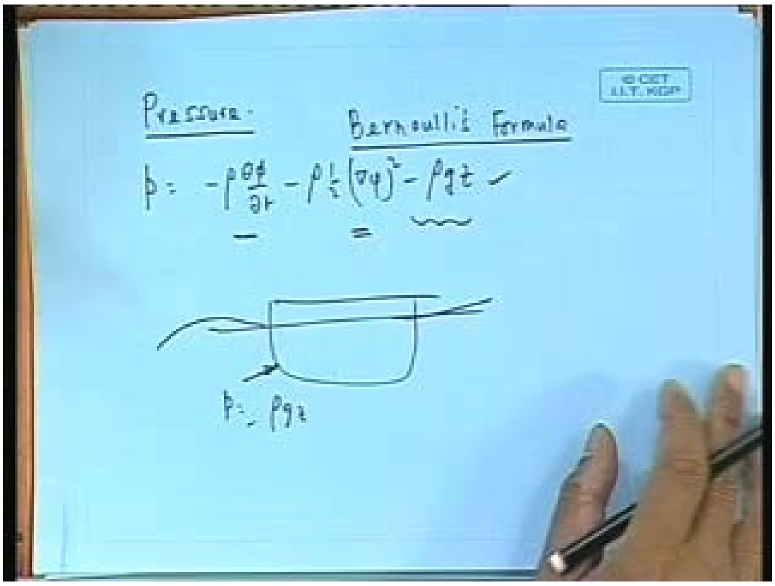

Now, pressure is given by the formula of minus row into D phi by T, actually it is minus row half del phi, I am just writing a formula. See, this is a formula I am writing, we would not go through this much detail, but this is what Bernoulli has said many years back. A famous fluid dynamic is Bernoulli called Bernoulli's theorem, you would have heard this in some context.

The pressure is given by like this, of which this part is called hydro static pressure because this is the pressure that I talked in our earlier class, P equal to rho g Z and integration of that gives me everything. See, now earlier what happened? I have this body here, I want to find out what is my pressure, this is my water surface, I say that this pressure is given by rho g into z. If you recall that is the pressure, not a pressure, pressure is also the force is like pressure acts on the normal direction of this magnitude.

Unfortunately, here I have got waves now. So, I have got this pressure of course, always there, but now additional pressure is there. Why? Because the water is not static, water is moving. Now, there are Bernoulli's equation tells me two parts, one is this part, one is this part. Now, this part is called the linear dynamic pressure because this is the highest the main pressure, this is a pressure coming because of square of the velocity, because this part is velocity.

We will neglect that for time being because this turns out to be much smaller compared to this part. So, pressure can be written as minus row d phi by t, that means it is a time derivative of phi. See, everything comes again to phi.

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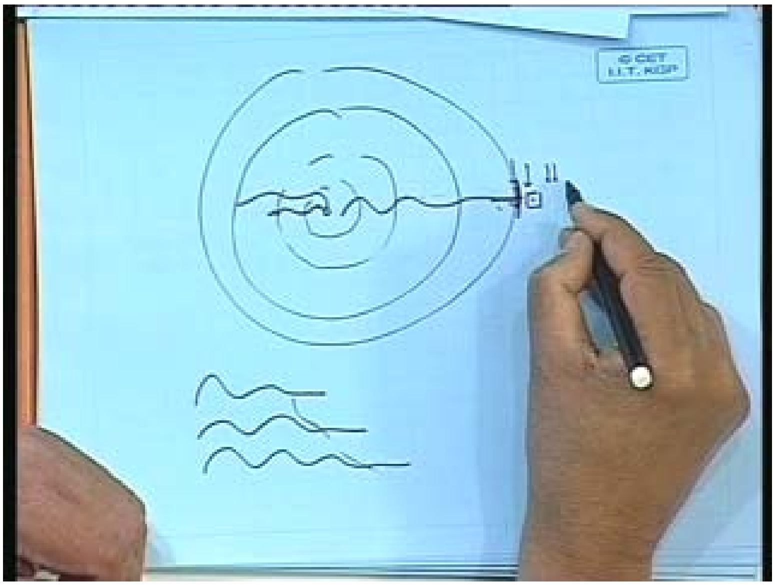

So, look at this the phi formula and then pressure we will write. See, now again we have to look at this phi formula and then we will go to the pressure. Let me write the another formula. Phi omega was given as xi a g by omega sine. Now, d phi by d t. Basically, one row you can multiply, afterwards there is a minus row of that is how much?

Now, you think of this flaunt. This flaunt has just a flaunt of that, water here by definition is calm, it is just the calm water surface. Just here it is the having the wave. Therefore, this particular point has no energy, this point has wave energy. Therefore, the rate at which it is moving is the rate at which energy is propagating. Exactly. Because if it was not so, see just this line, this point has no energy, this point has energy. So, next second this point has no energy, this point obviously, this h by which it is moving forward is the speed by which energy is travelling.

And this is an important part because it turns out that this speed is not same as phase speed. Another interesting example I will tell you. Before that let me tell you one thing, that the fact that the waves actually, when you throw a stone, the waves actually amplitude decays as you keep going further is also explained by the fact that the wave energy is equal to amplitude square. See, energy is proportional to square of that.

Now, what happen? Energy should be constant. Now, if you look at this circle the energy is over this area, if you look at this circle energy is over this much area. Both xi square into the perimeter should be constant. Therefore, as the area it grows bigger, the amplitude becomes smaller. This is why when you throw a stone the wave actually decays. It goes smaller and smaller as we go further. About the wave energy, the last thing I will tell you that is very important for wave energy is that, you think of this case there is a wave maker in our tank, there is a point here that moves up and down, makes wave. So, you tuned it today.

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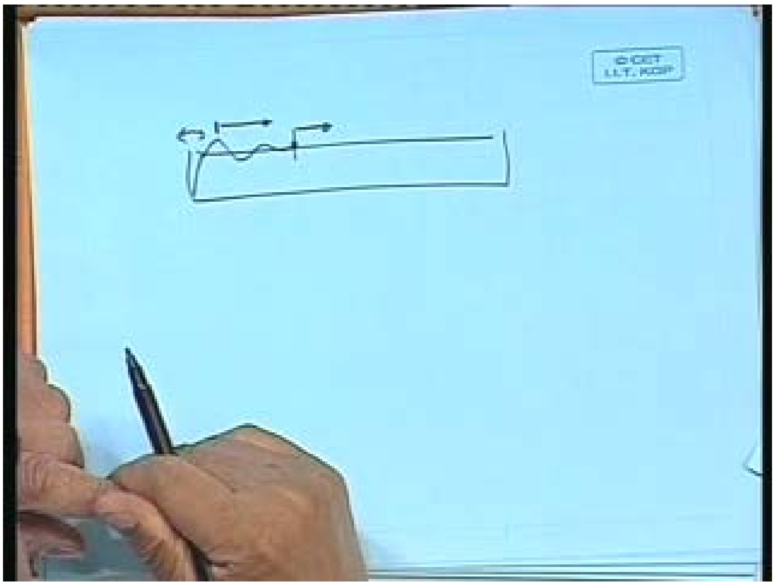

So, what happens? This wave would start like that. Now, I tell that any one of you that you walk along with the edge and another person along with this, you will very soon see that you are walking at a speed much lower than he. He is walking twice as fast. But as soon as he comes to you, collides, you disappear you become one. This is a very interesting phenomena that you will never realize unless you next time carefully observe a stone throwing on the edge. You see the edge moves much that circle is growing much slower rate than those waves are moving ,but they come to the edge and disappear. It is a fascinating phenomena if you ever observe next time, you will see.

And why it becomes important for our ship case? That is because we will find out later on that the ship created wave speed and the wave speed have lot of relation. If so called the two speeds, energy of travelling of the ship wave and the waves are same, then the energy gets trapped and it is exactly same as what happens in a sonic boom.

Just like if the ship is, vehicle is moving at a solar speed, when the sound it creates wants to go at the same speed as the plane. So, the energy gets trapped. Similar kind of phenomena occur. This is why group speed is important as a concept. Anyhow, so I will today stop about linear wave part, if there is little more then next class we can pick up.