Performance of Marine Vehicles at Sea

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Lecture No. # 23

Irregular Sea Waves – I

See, today's lecture is going to cover what we are we call Irregular Sea Waves, and some people call this actually also random.

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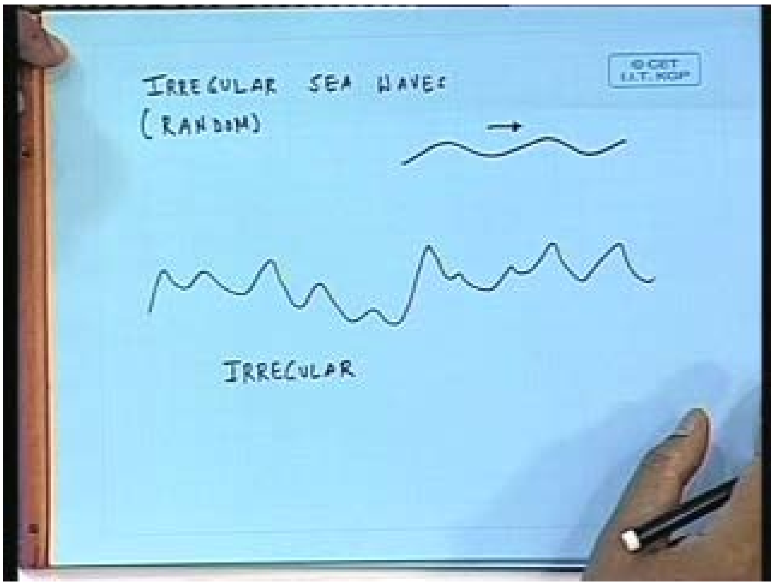

Now, see yesterday we talked about regular sea waves, nice sine curves coming, but as you all know, if you really go to open ocean, you will never find a wave which is very nice and sinusoidal, you will find wave which will probably look something like, something like that. If you try to monitor it at some location, it will look like that or if you try to take a snap shot also will look like that, it will not look like a nice and sine curve. In other words, what will you find is that there is lot of irregularity, it does not at all look like a sine curve, or one can say it is random.

In fact, one can show that, if you keep taking for a long time, it it seems to be a never repeating kind of a signal as called as type of a signal is concerned, it will never be exactly same and this is what we call obviously irregular waves. Irregular, the word irregular is more used by or coined by you know ship people, people never like it as a background, but eventually when this offshore structures are coming up, more of civil engineering people were concerned, they started using the word random waves which is both are same thing. So, this we will use it interchangeably, irregular means random.

Why we are on went to need to study that, because ultimately our aim is to find out, when I have been going to put my ship in a real sea, in a real sea how the ship behaves. Now, a real sea is not composed up idealized single sine waves, it looks something like there. So, there is no point, I mean, my ultimate aim would be not to figure out only how a ship behaves in regular waves, but how it is behaving in this wave.

So, I need a description of this wave that is a very critical thing, because I must describe, what is my environment in which, I am going to put just like, yesterday we talked about one single wave which is environment, but the real environment is irregular waves. So, we are going to talk about this.

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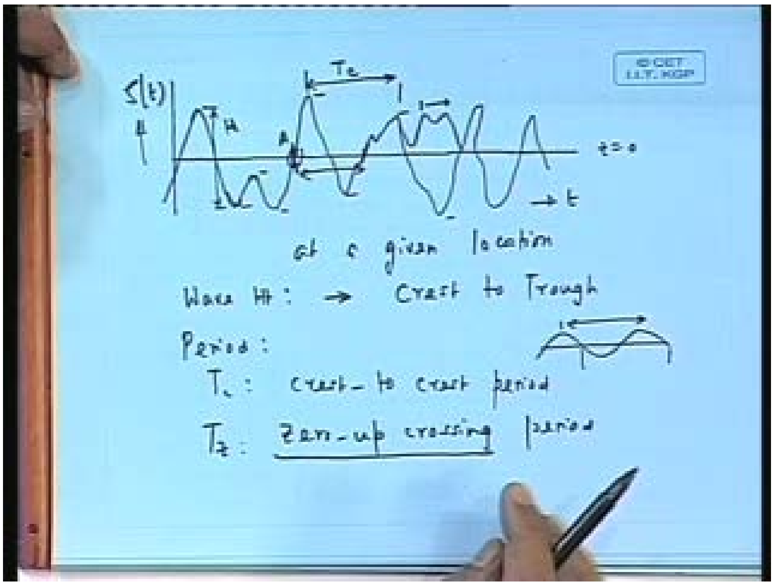

Now, let us see how we can go about it, let me give an example, supposing you monitor wave at some location, you you may find it, it goes like that irregularly. Now, what happens is that, here we first of all we have to define, because it is in random signal, we must define certain parameters by which you may say this curve is characterized, let us say that this is plotted against t at some location, at some given, x at a given location and this is my so called xi as a function of time that is wave height.

Now, what happened here, see we will call this successive crest to a crest to trough this as H, again here this to this as H, like that there is a number of pairs of H, see each each one, if you take crest to etcetera, then you end of getting number of heights, so it is not a constant height, now you call that height which itself is random nature. So, there will be suppose an, I take a record for one hour there will be (()). So, many H is occurring, H equal to you know like say this is 3 meter, 2 meter, 4 meter, 3.8 meter, etcetera etcetera.

This is what I can call height, therefore height is measured not amplitude, but height between crest to trough and trough to crest between the two, where the signal is as changed from minimum to maximum, its successive one. See it is going up and then coming down, coming down, going up, here to here.

Now, let us say a period, now there are two waves I can find period; one of the period would be if I take from crest to crest see this, this I take this as t, this I can, this is one type of period crest to crest period, see first of all wave height which is successive crest to trough, trough to crest etcetera.

Now period, period is basically we saw yesterday, the interval between successive crest location, you may say for a regular wave, that is here to the time taken for that is what we call period. And now this is same in regular waves case from here to here, any you know identical point if you take, the the period is the repetition of that, but in here it is not so.

So, if I take peak to peak, I have got T c, which we can call crest to crest period, but there is one more thing that is introduced which is now called zero up-crossing period, see what happens, some of this crest to crest period there can be a signal, I did not write, but say a signal can be like that. So, here also you get crest to crest period.

The other the period is, what is called zero up-crossing. This is where the signal is crossing the 0 in upward direction, see here the signal is going from negative to positive, excuse me it is crossing the 0 line in a upward direction, see this point A to the next one where it is crossing upward direction, this time interval that is zero up-crossing period.

When the signal is sorry crossing the 0 line in upward direction, from that to the next (( )) interval where the signal is crossing the upward period, this is called zero up-crossing period.

So, you can have as far as period is concerned crest to crest was zero up-crossing why I mean is that, if you take a long signal there will be again number of T c, number of T z, number of H. So, you will have a long series of what is called a data file, we I will just come to that in in a kind of statistical thing in a minute. Now, this is with respect to your period and height, now supposing instead of period I have the see, I change that with respect to this side instead of this thing length, then this side will become instead of T c lambda c and this will become instead of T z lambda z.

In other words, then you can call, you will have crest to crest distance to be one kind of length or zero up-crossing to a next zero up-crossing to be another kind of length. In other words, the the concept is that as far as period is concerned, as far as length is concerned, you have got excuse me I do not know what is you have got two kind of a definition; it can be measured from crest to crest whether for period or for length or it can be measured from the point where zero up-crossing to next 0 up, this is called up crossing not down, not this point, this is down crossing typically taken as up crossing to up crossing.

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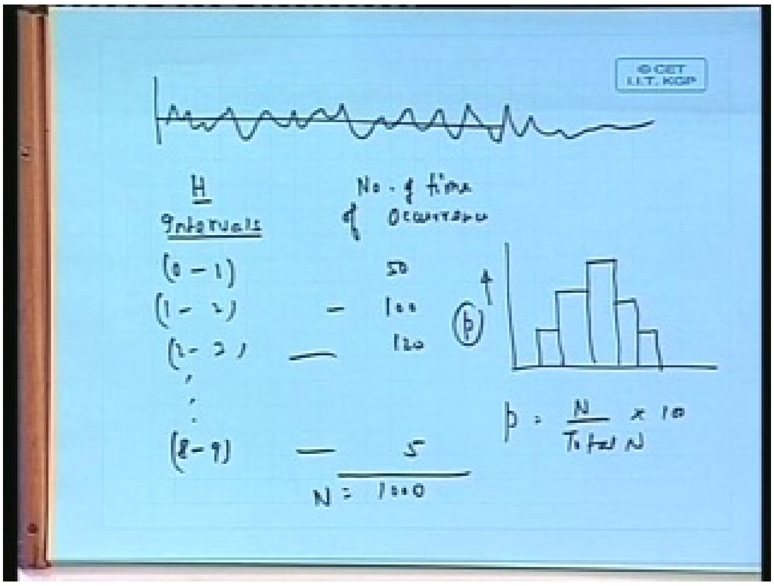

Now, let us say you took a signal, long signal now, so you have got now let us say long signal, you take it for a number of hours signal is here, so what happened? You are going to make a statistical analysis; you will try to find out wave height H number of number of time it occurred.

Now, what it means, you have got a signal. So, let us say two hour, you have got now, what is this happening I do not know 1000 H values, you have got 1000 H values, H equal to 0.5, something 0.25, something 0.5, etcetera, now what happened? You have taken a statistics of H, but normally no H is repeating you are we are having 2.357 and 2.912, 8.25 it is not convenient to do that, instead of that, you can have an interval of base, say you can have interval of base 0 to 1 meter, 1 to 2 meter, 2 to 3 meter.

So, I can have H intervals and then find out, how many times the waves are occurred in that intervals otherwise, if you take each one there will be 1000 difference, you cannot have a plot. So, let us say it this is got some number say 50 times, this is got some number say 100 times, this is got some number 120 time, like that you go, you have 8 to say 9 say this is 5 times. So, now you have got total number N observation height. Now, you can plot what is called a histogram, I can plot here, what is called N here against the interval, so this interval occurred so much, this occurs so much like that.

Now, this actually this interval can also be written in terms of percentage, see 50 signals out of N equal to let us say 1000. So, it will be, if an, if I write in a percentage 50 by 1000 percent you know that is 5 by 100, 5 percent, then this can 10 percent, this is 12 percent, and like that I can draw here. Then, probability which is a percentage or the number itself p is a better thing because p will tell me, the number of the signal occurrence by the total number into 100 (( )) into all actually for some reason.

So, you see you can actually plot here, what is called a probability distribution of the occurrence of wave heights in interval, see you could not do it, now this is typically known as a histogram.

Sir (( )) finding by N.

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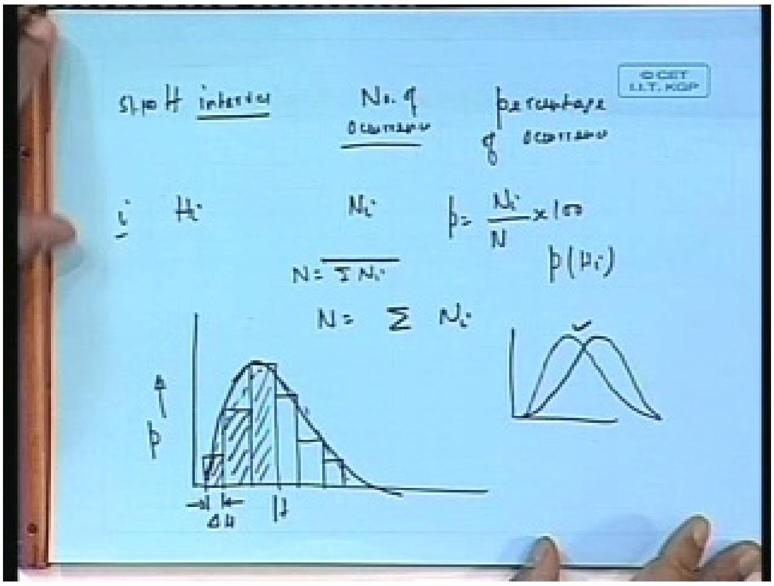

I will now come to that here, I will now come to that, now here what happened next one? So, I have got here H interval and here I have got number of occurrence, then here I have got percentage of occurrence.

This is this is some interval say H i, this is say number of occurrence N i, this percentage this become N i by N into 100 when N equal to sigma N i. See, what it happens is that, I have a certain interval say i, the interval say serial number let me put. So, i is the interval is for H i; that means, it is between say 5 meter to 6 meter some interval that number of times the waves within their interval occur is N i say 600 the total number that occurs is N which is equal to sigma N i.

Naturally this has occurred how many times of total N i by N times as a factor and if you multiply with 100 it becomes a percentage. So, I can call this to be p in a percentage or I can call this to be p of H i. Percentage occurrence of the wave height H i, what is H i? H i can be the middle value of the interval, see when I have an interval I can take middle value for plot, you know say 5 to 6 meter. So, I can write that has 5.5 meter. So, percentage of occurrence of waves 5.5 meters plus minus 0.5 meter would be p H i nothing but, it is trying to tell us the percentage occurrence of certain waves. Now, you cannot hit an exact number, so we have to go in intervals that is all I am trying to say, you can actually have small intervals. So, I can then plot that in an interval I can say this H here, so you see it occur this much, this is H.

This now, now this would be what we could do is at this area we can make it to be equal to the number N i divided by N, why we are doing that? I have given many ways by which we can do that. So, supposing you do this way, now basically total area under that should be the total number of occurrence divided by N which means 1, I can, if I call this be p, this area should represent, what is the occurrence of this wave height. The area therefore, basically what you are plotting is p, this values are p divided by delta H you may say if if I call this to be delta H.

Why I am doing that? The reason is of doing that is that, the area under that represents the total percentage of occurrence which is equal to 1 in a probabilistic sense, it does not matter how you do that, because we are not going to do this. It is only to show, see if you look at this diagram, it is to tell you that look, this wave occurs much more times than this wave, this wave occurs less times than this wave and you can figure out the relative number of times of occurrence in a in a particular wave signal. So, this is nothing but, the probability distribution of the wave heights in a given signal.

Now, it turns out that if you keep doing that this graph looks like, you know if you now draw a graph here it looks more like this, I mean in other words it does not look equal, but it looks more like this it looks more like this. If you do a histogram and you know if you make a small what happen is that, tomorrow I will draw it another one, this histogram part say I can make a smaller and smaller, I had initially taken interval as this.

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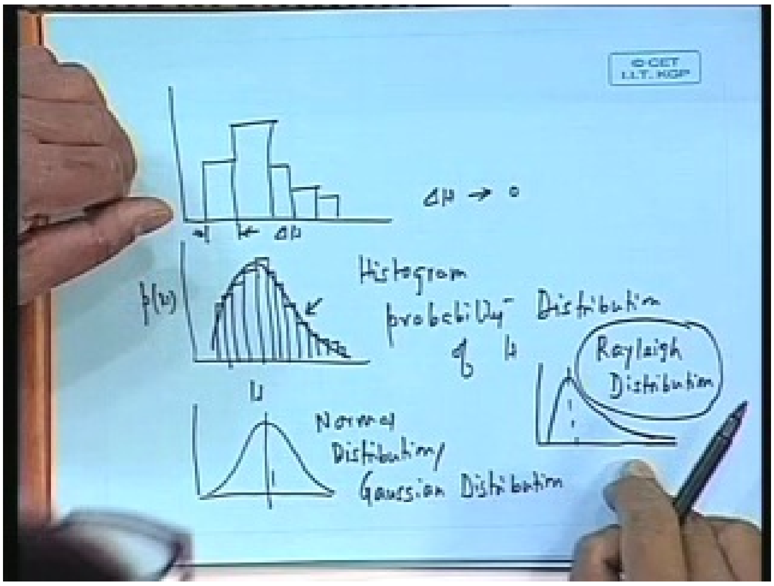

So, one percent has taken this has interval, but I can make these even smaller intervals, I can take it at more refining curve.

More if I if I actually break the signal into much more number of small part, I get depending on how I have taken H i, see if I instead of taking number of occurrence of 1.2 meter I say 1 to 1.5 meter, 1.5 to 2 meter like that, I can continue. So, I can actually keep refining this delta H this, I can make delta H tending to 0 a small in fact, when delta is actually become 0 you will end of getting a continuous graph.

In reality it is not possible, it it is like telling what the number of occurrence of height is between says, you know 25.25, you know centimeter 25.26 centimeter does not make sense. Normally, you will go in the interval of half a meter to one meter, not anything beyond that half a meter is ok, even quarter meter is very rare.

Now, my point is not that, my point is that having drawn this p h by H 3 is a graph it is called histogram and this and or you can say probability distribution wave height. Similarly, you can do a same curve for period, same curve for wavelength, same curve for all the parameters that we measured. Normally, that is not what is done; height is what is most important. Now, here there is a very interesting point see, if I took a the age of a number of student in the class, you will find out that the distribution of that, this is normally what is called normal, that is it has a mean and it is there is a equal chance both sides.

You know, this is what is called a normal curve, some time one can you can call also Gaussian distribution, it is a kind of distribution, which is most normally occurring and you will find in most places as I said like age of a my student etcetera, etcetera, where the chances there will be a mean value say in a class age of a student is 20, but there will be people, less people more the chances of people, less is more or less same as chances of people more, so it is equally spread. This is a bell shaped curve about a mean having both side same, but the wave had curve turns out to be always something like this, it always tends to follow what is known as a Rayleigh distribution; that means, chances of its occurring I mean about the mean value, the the it is not symmetric it actually tappers down like that.

What it means? It basically means that, the chances of occurrence of height is very high or very very low compared to chances of waves which are lower and this makes perfect sense; you know why it should be so. This is why I wanted to discuss a physically. See, if there is a smallest wind possible, you know it is the wind which causes waves, wind blows and that will excite a wave. Now, even if there is a small wind say 5 knots it is going to create waves, but what it will create? It will create the lower waves; you know waves are 1 meter, 2 meter, etcetera.

Now, if there is a 20 knot wind it will also create this 1 meter, 2 meter, but it will now create also some number of 10 meter waves. But this 80 meter also, it will create 1, 2, 3, 4 meter going up. So, the lower ones are always occurring, no matter what the situation is. Therefore, there is a tendency of lower, you know always having more (()). This is exactly why, the chances that a wave of 2 meter will occur is always more than the chances that wave of 10 meter will occur.

Because, number of high ones occurrence that phenomena that causes 10 meter will also cause 2 meter, but the one that cause two meter exactly therefore, that is (( )). This is almost like I am telling my classes that if you take a book, try to find out the number of digits 1, 2, 3 how many times they occurs.

You will find the number of 1 occurrence is more than number of 2, a number of 2 is more than number of 3. You know, if you just take why because many chapters will start 1, 2, 3, 4 at n such may be 9 or 7.

So, 1, 2 are always occurring the 12, 13, but on the other side chances become kind of less and less. See, any number you start in a class or anywhere you know say it goes to 57. So, there you see, if you see number of 6 occurs much less than 5 or 7 occur like that.

This is also like this; because 1 and 2 are occurring always no matter how you start similarly, here lower heights are occurring always. So, it becomes a Rayleigh distribution. So, this is it turns out there, this is following a Rayleigh distribution. This is very important property that we will find out later on.

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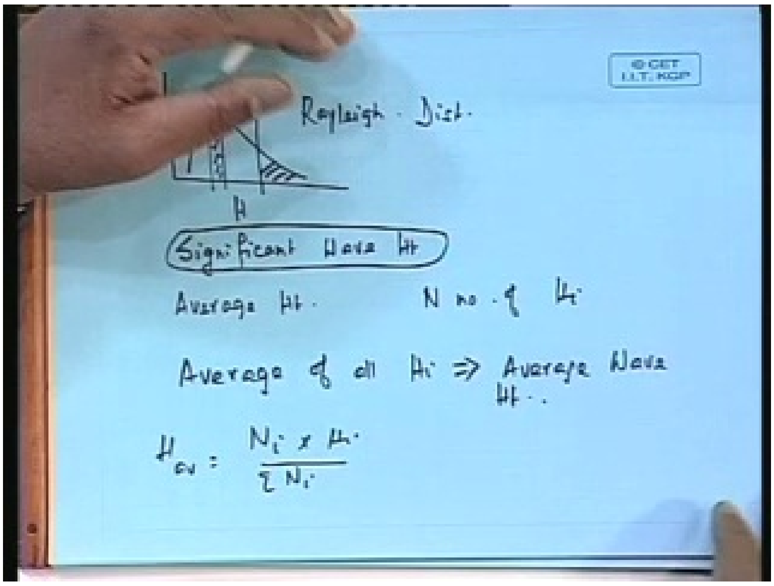

Now, actually what happened? Theoretically, it turns out that if you have a distribution known you know statistical distribution, Rayleigh and if you can find out this, if you if you can fit this curve you can actually find out all statistical property. For example, what is the chance that my wave is going to be more than certain height that is an area under this curve? What is a chance that the wave would be no more than this, but less than this, that is area under this?

So, if you actually have a statistical curve you can figure out many statistical properties, if you can actually fit this curve by means of some air, some formula. Then you can tell, what is the chance look, what is my chance of wave height occurring at least once 10 meter you can make it out etcetera, etcetera.

But this is a different thing, I will come to that later on, this is about the height. See, now a question comes, a very important question comes, a definition which is known as significant wave height, people would have or before significant let me first write average wave height. Now, you see you have got now N number of H i values; that means, you have said that this height occurs so many times, this height occurs so many times, this height occurs so many times, etcetera.

If you simply average them then what you get is an average wave height, just average of it is as simple as that, you have got say 0 to 1 meter occurring 100 times, 1 to 2 meter occurring 200 times, etcetera. So, you take the mean value as 1, simply multiply the number of I mean just a take a geometric average. So, you would tell that, what is the average height this is what is called average height? You will can actually tell average height to be simply a number of occurrence into H i divided by total number that is an H average.

The question is that, most people use the significant wave height more than average wave height and so, we have to know because whenever you describe a sea, you will see here the term significant wave height or some people will call one-third significant wave height, that is the most common thing what is the meaning of that one-third significant wave height.

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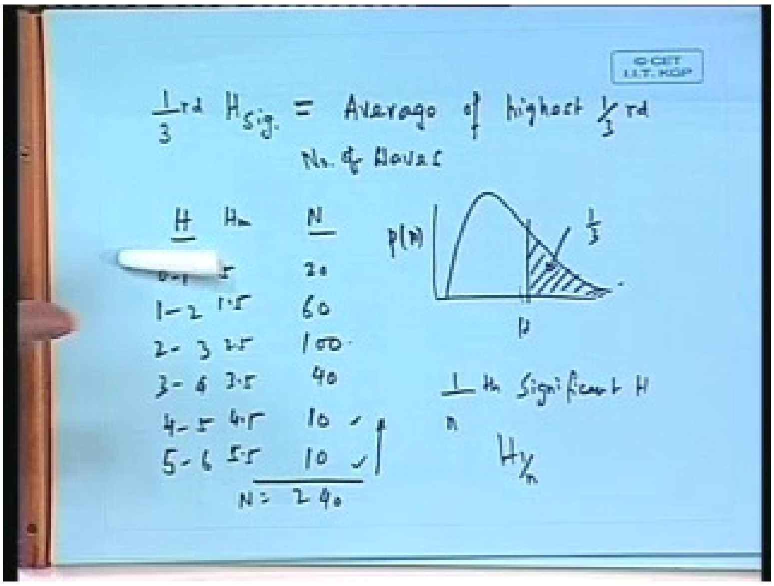

Now, it I will tell you that one-third H significant. In fact, people call it one-third wave height or significant wave height, both are same them by definition, this you see the average of now this is important here, average of highest one-third waves.

Now, this requires some description. Now, you see again I will go that, this is an H here this is N here. So, this H is 0 to 1 occurring say 20 times, this is 1 to 2 occurring say 60 times, this 2 to 3 occurring say 100 times, 3 to 4 occurring say 40 times or 4 to 5 occurring say 10 times.

So, 100, this is 81, 81, 80 plus 50; 230 and say for 5 to 6, I want to make it 240 that is why this is 10 times. So, the total occurrence has become 240. So, 240 records I have collected of its 0 to 1 as happened 20 times etcetera.

Now, what is one-third? Now, if you have to take an average wave height I will simpler to multiply this, I have to take an when the H mean value that is here may be 0.5 here 1.5, 2.5, 3.5, 4.5, 5.5, I will multiply this with this and they can average.

That is an average height, see 0 to 1 meter is occurring because it can be 0.1. So, we will take a mean value the to be the representative height say 0.5 meter occurred 20 times one and half meters occurs 60 times, like that I will tell, the N i will multiply and taken average this is my average wave height.

But one-third significant wave height would be that I have to take top highest one-third wave means what I have got total 240 waves I have to take 80 number of waves now 80 means this ten this 10, 20 plus 40 will become 60 and 20 from here. So, 10, 40 and 20 from here we will make it total 80.

So, from this onwards, I must take one-third of N number of occurrence of waves; that means, I as if the signal consist of only from here only 80 number of waves,,, but from here wherever eighty is it, this is an important concept. See here I have to take them that 10 number of this wave, 10 number of this wave, 40 number of this wave, and I have now left 40 because you know total has to be 80. So, 20 numbers of these waves.

So, now if I take that part and then take a simple average of that wave, then what I get is called one-third significant wave height. In other words, it is average of the highest one third number of waves you can say number of waves.

This requires some sort of thinking that it is basically an average, but average of the top on that. So, again in a spectrogram formula see I have got this kind of formula. Here this is probability of H, this is H occurring.

Now, the area under that is 1; that means total number of occurrence now, I take this much of area which is 1 by 3; that means, these are the waves occurring the top one-third wave. See this area under that is one because you know it is normalize the total number by number that is how we have done it.

So, if you take 1, 3 you know 0.3, you take from this side a line where the area becomes 0.3, this area is 1 by 3. See, the probability of H by H if you take the area under that full it is equal to one because it what it tells me is that some wave height must occurred it is 100 percent sudden.

So, it is like if you take probability of a tail and head is half plus half,. So, it is 1, because if you do except in the movie it is going to either be head or tail it is not going to fall like that, it is going to be the head or tail. Similarly, here it means that there will be some height whether 0.1 meter or 100 meter whatever.

So, area under that is actually 1, but from this other side, this is the higher waves. You take a curve where the area is 1 by 3. Now, this would represents the top one-third wave, now you take this curve and take an average of that; this would be average of the highest one-third waves.

Now, you see instead of one-third, you can have 1 by nth significant H or you can say H 1 n; the definition is same this would be average of the highest one, N the number of waves. Suppose you want to find out what is one-tenth significant wave height or H 1 by 10 it would imply in this diagram the top 24 waves average if you say 1 by 100 then it will be imply top two and half waves average that will be 5.5.

So, you can actually end up getting from this side the top. So, much percentage of waves, see if I want to do this, then this is area is 1 by n. So, if I take an average of that, I will get average of 1 nth wave height.

So, this is the concept of basically one-tenth, one-hundredth, and one-third wave height. By default significant wave height is referred to one-third wave height. See, when I normally you know strict mathematician will say that, when he was tell it is H one-tenth or H one-hundred or whatever.

So, the word is to use one-tenth significant wave height equal to H one-tenth or whatever, but normally we do not use the word that one Nth, we simply use the word significant wave height.

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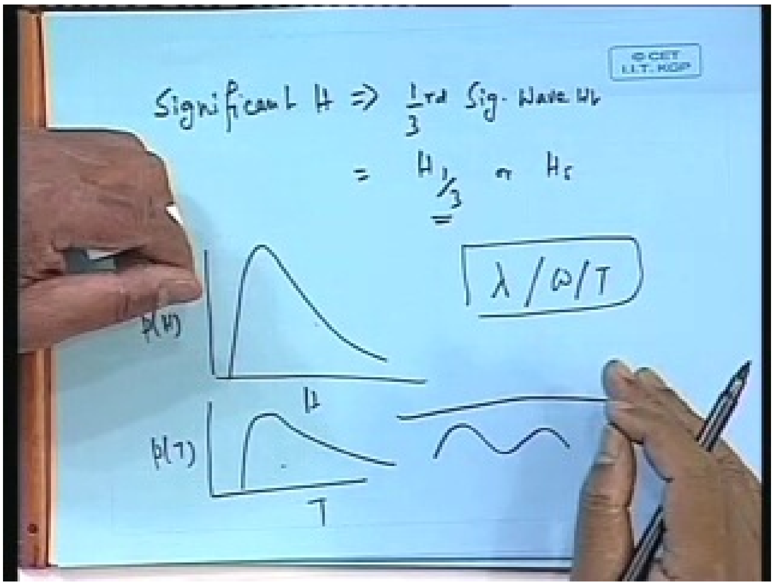

So, you see significant wave height by default, by default means; one-third, you call that. So, somebody did not tell you the you know the number one-third, I just told you that look significant wave height is 6 meter, you can safely assume that what is referring is one-third significant wave height.

Now, a question comes, you you will ask why it is at one-third are very important why not average why why I will be not taking average wave height? You see, this is the question why the sea? A sea people likes to use or whether people like to describe or use one-third average average of one-third higher waves as a mechanism or a means of representing severity of the waves rather than average value.

The answer is again simple, answer is that, if you take average it tends to get pull down because any small wave, any small wind will cause repels. So, there are number of small waves that are coming 1 meter, 2 meter, half meter, and etcetera. So, if you take an average you tend to pull down and it does not appear to give you basically a sense of really severity of the waves even an average sense.

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Not only more, even in a without see even there is no storm, simple breeze there will be a small waves. So, always are. So, much there you know like if you take a record there is always small small waves they are trying to pull down the average.

So, if I say average of 2 meter, it really do not indicate the severity of the wave; this is why average is felt not to be a number a kind of indicate to a person the severity of the waves.

But one-third seems to be slightly higher than that and therefore, it seems to be better. This is why, at convention as exact people use one-third highest waves rather than average wave height. The that main main philosophy is that, that you do not want to take a average because there are too many small numbers you know like like a kind of making it a bias in the lower side.

But after all, you would like to have a higher estimate because your ship has to survive the weather. So, if you design it for surviving a kind of lower value, it may you know like break up whatever. So, you want to that, but you do not want to have a too much high,. So, you want to have a kind of an average slightly more than average this is why one- third is used. Afterwards, we will find out that for our design wave you have to design for things like, H 100th or for the wave height return period of 100 years means, what is the wave height that may come once in 100 years?

That we will see later on, that extreme events for which here design must are survived. Just like when you have done, say our new calculator bridge then you will have to find out what is the wind speed it must survive. So, there will be this similar term it must survive a wind which may have a return period of 100 years; means, once in hundred years there can be a storm with 200 knots. That is a different issue; we will come to that afterwards.

Now, this is the concept of significant wave height. Now, the problem is that so far what we did that we have found out from a wave? This histogram of this thing wave height occurring, but you see we talked yesterday if you look at the regular wave, you will find out that waves are not only decided by height, but also by length or frequency or period; this three are interrelated to each other.

Now, this distribution is not going to tell me this information. See, I can have another one for say period versus p T, but you know I can have two different sort of a in a histogram, but this does not tell us the relation between the H and the T? You see, this is independent, here I am telling how many times this height is occurred, here I am telling how many times this length is occurred but I have not anyway telling what is that time a certain height of a certain length is occurred? Because, you see, after all occurrence of say 10 meter wave, a 1000 meter long 10 meter high 1000 meter long wave is very small slope, is much less severe than occurrence of 10 meter wave of 100 meter long wave, because see one would be a occurring like this wave another is occurring like this wave. See, if I just take height as a parameter, I do not know what the length is associated then I do not really get a feel of the ocean, because I can have say 10 meter height I can say the 10 meter occurred many times fine, but the wavelength is a 101 kilometer. So, it looks still like that.

Histogram do not give the (( )) No, histogram cannot give that other side it can either give only height it can give only period, but the relation between the two is not there. So, it can you see suppose, I give an example here supposing you take the height. So, there is a height like that another signal, which has same height but spread.

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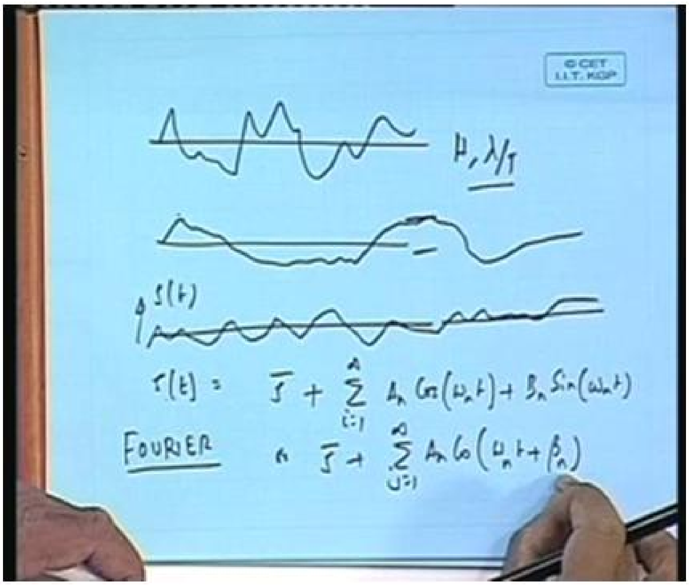

Both of them, the heights are same. So, you get a same h average but obviously, they are not similar waves. You see now, if you take a length now you get the here longer length, but you do not know what is associated height. So, if this H and lambda or T are not compiled any histogram and you do not get a feel of that. So, we have to have another means of actually representing ocean waves where, information on both height and length are they are together, this is a necessity now, because otherwise you do not understand what kind of wave.

See you want to know there are N number of waves of height 10 meters, length 100 meter. Another 200 number of wave of height of 200 meter half length 200 meter I mean you want to have both together so that you can understand about the slope that is what you want to know you do not have it. Now, before I go to that, you see it turns out that if you take a random signal, any random signal can be actually what is called analyzed by a mathematically by a Fourier series you know you can you can say I will just write it first, then I will this thing.

See, I mean I just wrote some equation, what I wrote here I will tell you, now random signal tells us now xi t is a random signal now this is xi t now what I have done is that, I have taken any random signal you know, the height has been taken and it is obviously occurring.

Now, the the there is a theory of signal processing that tells us that a random signal can be written as a sum of N number of harmonic or N number of sine curves. See, this is what Fourier has said many years back that any random signal can be broken down into N number of sine curves or in in other words, if you add N number of sine curves you will end getting a random of waves provided of course, this curves have some kind of a random phase.

This is why here what happened this xi t can be expressed this xi t is a random signal can be expressed as a mean of set xi bar plus a sum of A n cos omega N t plus B n sine omega t in this A n and B n, you do not know some unknown sort of coefficients.

But it is a sine curve and cos curves you can add why I write sin and cos here is that, basically this can be also written as cos omega n plus beta. You know cos theta plus beta is you know sin theta plus cos theta A sine theta plus B cos theta.

Basically a signal a signal with a phase can be written as sum of two signals that is the idea, but this is only in going by mathematics we need not worry about that. So, what this Fourier theory says is that, it is like this.

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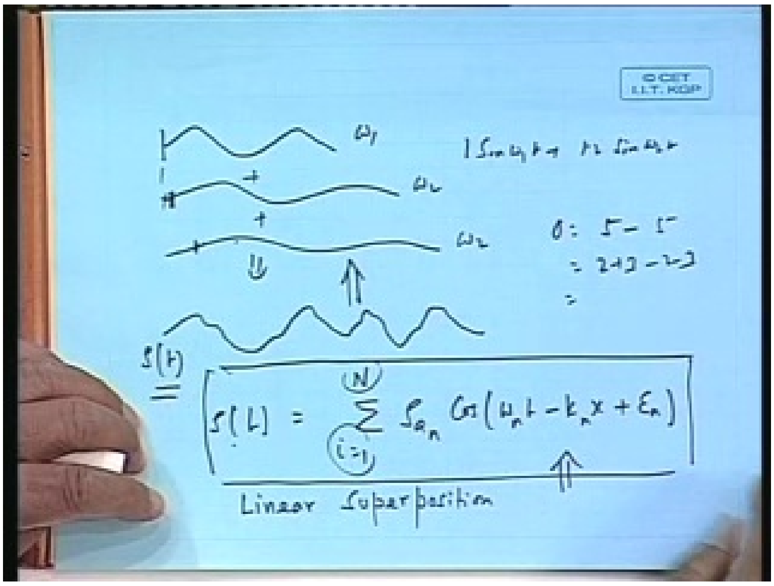

If I have, I take I give an example, if I take a sine curve of omega 1, another sine curve omega 2 and another one of omega 3, you add these three together what you will find? You will find something, some kind of random signal.

So, this is what theory behind the Fourier analysis is, that any random signal can be broken down into a theoretically infinite number of sinusoidal signals in practice some discrete number of sine signals.

This is actually in terms of random process. See in other words, conversely if you can just check, if you take say sine you you take 1 sine omega 1 t plus 1.2 sine say omega 2 t, some value some t you try to find out the behavior you will find that it is basically become random and spatially if you have a random phase between the two.

You start this from here, next one you start from somewhere else here with a gap another one from somewhere here. In fact, if you just add two sins with a random phase you will find that it will look random.

Conversely therefore, any random signal can be broken down into N number of sine signals this is what Fourier analysis says. This becomes a powerful tool therefore, for our purpose here because, we have an unknown signal wave height that we have taken this xi t is a random.

So, I can therefore, break it down and tell that look, this xi t can be thought of as if it is some of N number of sine waves with some random phase, with some there are waves having some phase which we do not know of, this is basically extremely an important event.

So, the wave height that we got therefore, can be expressed as if it is nothing, but N number of waves, I can write it in this fashion just by following Fourier analysis. This is actually, you can say linear superposition. In other words, I can say a random signal is linearly adding number of sine curves.

See here, I am adding number of sine curves to get this, this is a, we are not going to the detail mathematics, but the concept is extremely important. So, I am repeating many times; that a random signal is electrical, mechanical whatever kind it has actually come from signal processing in electrical engineering. When you take some record, now the signal comes; it can be random Fourier actually try to say that you can you can represent that as if it is sum of a number of sinusoidal waves with some phase gap between them which you do not know.

We do not know the amplitude of the sine curves, you do not know the gap between them, but if you add number of sine curves you will get that is that random signal there is now a technique available that if you measure that from there you can find out what are the sine curves that will cause this signal. You know this is what is called Fourier transform and trough like that by which any random signal can be broken down to which constituent sine curves here again you may say what are the sine curves what how do you choose N, it is actually up to us supposing I have a random signal, I want to break it down to 100 100 sine signals. Then I will get one hundred omega values and corresponding amplitude if I somebody says no I want to break it into thousand you will get same thousand it is a question of reverse breaking a signal. In other words, either those 1000 or those 100 if you add them you will get them sine signal.

So, in other words you can is something like this this 0 can be set 5 minus 5, you can also say it 2 plus 3 minus 2 minus 3, you know something like 0 can be broken down to five plus minus five or 2 plus 3 plus minus 2 plus minus 3, it is up to you to break it down you see the breaking add gives you like this. So, same thing here I have a random signal and I can break it down to number of components in such a way that when you add them up together you get the signal back.

This is what is called what what do you call a method of linear superposition, because you are linearly adding remember, linearly adding means; you are adding 1 plus 2 plus 3 etcetera. Now, you see here if you look at that why it is so tantalizing, why it is so interesting? It is because we have found out that regular waves are sine curves. Now, I take the signal and I break it into number of sine curves.

Therefore, I can think that an irregular wave is nothing, but sum of n number of regular waves. So, if I can add number of regular waves I end of getting irregular wave. This is the most interesting part of the full phenomena because, we will find out even now, next, that the entire concept of how do we represent irregular waves changes on the fact that irregular wave is nothing, but can be taken as sum of sine of regular waves; this is exactly why we talked of regular waves and we will find out that the the complication will actually rest on finding out regular wave response not irregular wave, irregular wave will simply the sum of regular waves.

You see, in other words irregular appears very complex, but it is actually really very simple it is nothing, but just sum you break it down to some regular wave, that is all. If you can break it down which will be a simple job we will find out that is it, you know is a plus b plus c plus d that is all.

So, it is you know this is a very interesting thing, but the question here comes what is the logic behind that? The logic is of course, you are actually adding sine waves. Now, how we got sine wave yesterday? Based on linearity assumption, sine wave was a result when the wave amplitude was small. So, this full theory are breaking down is obviously, based on the fact that individual waves are of small amplitude, individual waves are linear.

If the individual waves are not linear, you could not have find out, but this turns out to be you know in the the way that we will next lecture I will cover the spectrum part of it.

We will find out that the the beauty of this is that, even though we say the linear waves are added up, we will find out that the resulting wave can be non-linear, can be stiff. You see now, this this I want to examine this linear superposition slightly more for this next 5 minutes or so.

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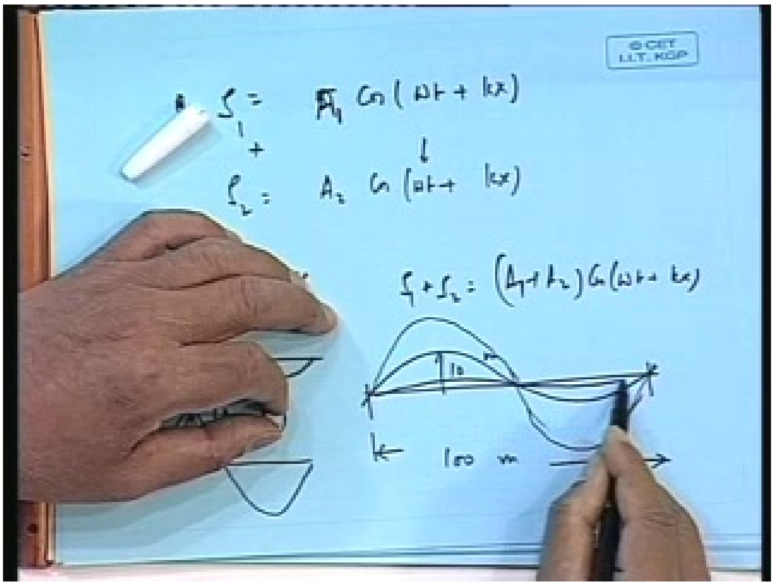

Now, you take 1 1 wave see you take one wave which is eta or xi we are calling now xi equal to say 5 cos omega t plus k x some with an some certain amplitude or let me call it with A 1 a 1 now another one this is xi 1 take as 2 cos omega t plus k x same same amplitude that means I have a wave here like this, another wave here also like this, same frequency and different, this is my A 1 this is my A 2.

What happen if I add them up together? What will be xi 1 plus xi 2? It is going to be simply A 1 plus A 2 into cos omega t. So, it is going to be if you add them up it is simply going to be, where this is A 1 plus A 2.

So, you see if you take therefore, exactly same wavelength wave 1 meter wave, another 1 meter wave, you can add them up and say it is two meter wave. Now, conversely, supposing I find out that there was a 10 meter long wave, you know let us say I have a 10 meter long wave there sorry 10 meter high wave.

I can think this 10 meter high wave is nothing, but 10 number of 1 meter high waves added together; This is what is called breaking it up in pieces, this is only possible for linear waves now you see what happens? This length is same.

Now, I have another 10. So, it becomes 20, then it has changed the stiffness. So, you see they although the wave was very stiff, I can think that the stiffness come because I have added n number of linear waves. See, supposing let me give an example; This is 100 meters, now in 100 meter a height of 10 meter is called quite stiff because by the time we will play. Now, suppose, I have a 100 meter long wave 10 meter high wave hundred meter long wave I can say that look it is nothing, but sum of ten number of 1 meter high waves.

Now, 1 meter high wave a 1 meter high wave is obviously, small amplitude it follows linear theory. So, now, I have an adding 1 linear wave, small amplitude wave plus another one plus another one plus another one, the resultant wave I have got is actually not so small amplitude.

So, conversely in this, when I break it down it just turns out that even though sometime the waves are of very, you know high stiffness. The the principle of linearly superposition or breaking them to linear waves also walk somewhat well.

So, it it kind of it is not too bad you know. You just cannot get this this kind of wave all right, but you can give high wave of this type by breaking it down you knows. So, anyhow, this is the principle of linear superposition.

I will close this lecture now and the next one we will talk about the more interesting part of how we can combine and how we actually represent the wave irregular wave.

Thank you.

Preview of Next Lecture

Lecture No. # 24

Irregular Sea Waves – II

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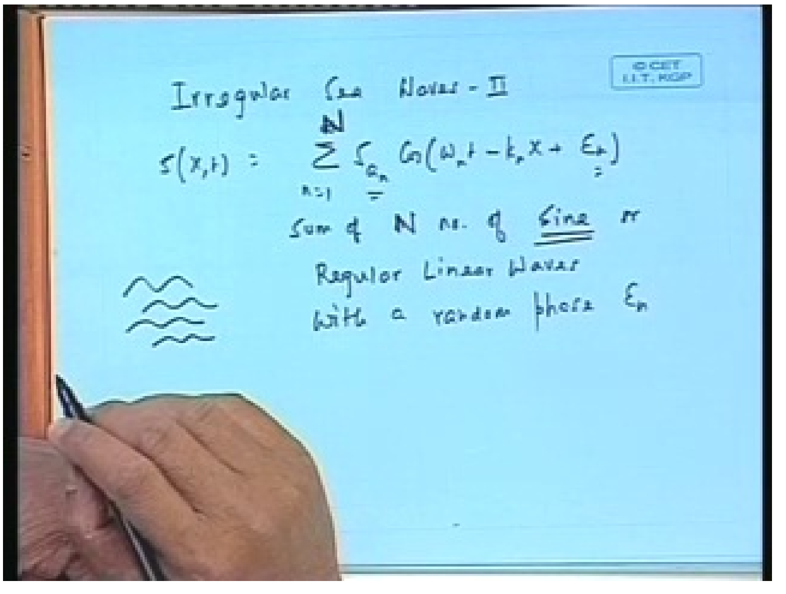

See, we are going to continue our talk on irregular sea waves. So, what we found out in the last lecture is that, you can have by Fourier analysis, a random wave signal represented as sum of N number of sinusoidal waves.

See, this represents this side, sum of N basically here I wrote infinity, but we can make it n, N number of sine or regular linear waves. See, in other words the random signal that we had could be thought or could be broken down into N number of sine curves, each of them representing a small amplitude regular wave with a random phase, this is my random phase.

In other words, phase we have no information on, you are adding n number of waves one is sine omega 1 t, sine omega 2 t, sine omega 3 t, but the phase mean the starting point is different.

You know one is starting from here, another may be starting from here, and another may be starting from here etcetera, etcetera. That is what decides the phase. So, we can get that. We can get a random signal by synthesizing that into this.

Now, if you look at that, you will see here that this has got the amplitude part, this is also got the frequency part frequency see k is after all connected to the omega because k you know omega square is g k therefore this and this are connected. So, here you have, got this. So, I have got omega, I have got this both information are embedded supposing therefore, if I took a random signal and if I could find out this and this together then I will know for which omega what was my corresponding xi a.

Earlier, I got what is my height or amplitude just number of amplitude that is all. Again I had separately found out what is the number of omegas or you can say period or length, but I did not know which waves and how many omegas.

So, this one I will find the say look; I have got in wave number one height so and so, omega one so and so, wave number two height so and so, omega 2 and 2 so and so.

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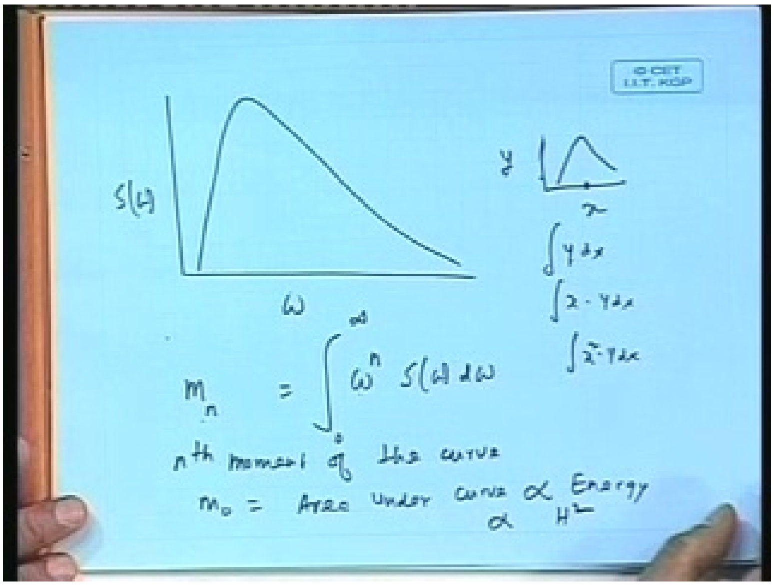

S omega again omega now area under that curve, how do I represent area? See, there are number of wave of representing area. Let me take this quantity define moment what is m n? Now, you see tell me, what is m n? m n represents the nth moment of this area.

Because, see this see if if if if there is a graph here, this is x here, this is y here. Integration of y d x is the area under the graph integration of x into y d x will be moment of the graph about this line, integration of x square into y d x would be second amount of area about the graph.

Therefore integration of this ordinate power into y d x that is why this ordinate power y to the power N into this d x, this d omega represents nth moment, nth moment of the curve. Why I say it is nth moment of the curve? Because, we will find out that many of the statistical parameters of this graph are related to that nth moment. Now, you see when you take m n to be 0; that means, if you take m 0.

What is m 0? It is nothing but the area under the graph m 0 is nothing but the area under the graph because m 0 is s omega d omega. So, m 0 which is proportional to energy, which is proportional to height square. So, the m 0 becomes proportional to height square, now we will come to that this in a in a minute this is very important and see this now we must realize that; obviously, this is true, because what happens now, I took a spectrum, I took a I synthesis and I found out this graph and I know now that the area under this graph is suppose to be proportional to that total energy of that random wave.

That means if I took a one square meter area and I kept on measuring the energy of content of that surface, the energy would be equal to m 0 that is the area under the graph. And that energy is obviously, proportional to the average height square or some statistical height square because, more energy means; there will be higher waves, less energy means; lower waves, that they they may not be 1, 2, but there will be random waves. So, average average wave height or significant wave height would increase if there is more energy. So, we will find out later on that this m 0 is directly connected to various kinds of wave heights.

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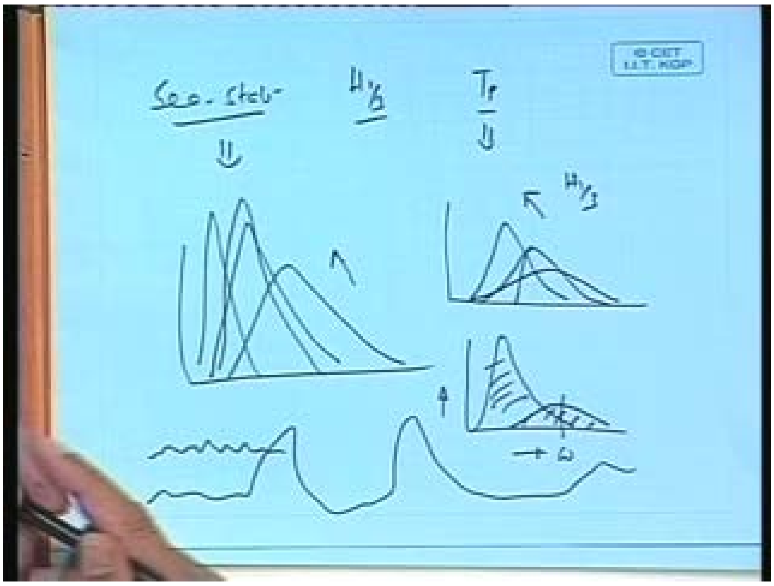

Let me just concentrate right now concentrate right now on this part now, it turns out people you know, what people have done? People have actually been collecting data versus H one-third versus T, so what happened? I want to find out what is my wave spectrum in sea state 5, I will go back to the table and find out what is my H one-third T, then I go to the formula and I I will draw this graph, this become my spectrum for sea state 3.

See, what happen, I go to this picture, I find out for sea state 1, my H one-third are T p, so and so. I go and use that formula it turns out that actually what happen is that, the spectrum you know as as wind blows this diagram is not both I should draw it just last and then quit.

This as the wind starts growing up; the spectrum actually begins to shift on this side. In other words, what happen? As H one-third goes up, as wind starts blowing more, your significant wave height will be more, you will end up seeing a spectrum which will be having more number of long waves, I I will pick up on that on the next class.

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It will be more so area. So, you take two things; small height here, and very high wave here. So, the area under that is much smaller, because obviously, area under about that is proportional to significant wave height which is say 2 meter, but there will be more. See, when there is a 2 meter wave it is all connected, I have got a wind blowing at 10 knots, and 10 knots will give me number of small length wave fifty hundred meter long wave with a smaller height.

So, average height is less. So, the spectrum is shifted on higher frequency side small wave length side. As wind begins to blow more, I will have higher waves, also longer waves. So, it will shift on this side and area will also become more, this will happen. This has a very significant effect on the ship motion, we will find out, see small boats are operating in very moderate weather, but they are operating this small sea state. What happen? When the typhoon starts the blow, you will have more number of 200 meter long wave or 300 meter long wave very long.

You you will in one case you will have more number of small waves, another case you will find much more number of longer waves. So, these are the two extremes and we have to typically that happens.

We will discuss more of that on next class. So, we will we will stop today here, we will we will do a little bit of this elementary spectrum part also in the next class. Thank you.