Performance of Marine Vehicles at Sea

Prof. S. C. Misra

Prof. D. Sen

Department of Ocean Engineering and Naval Architecture

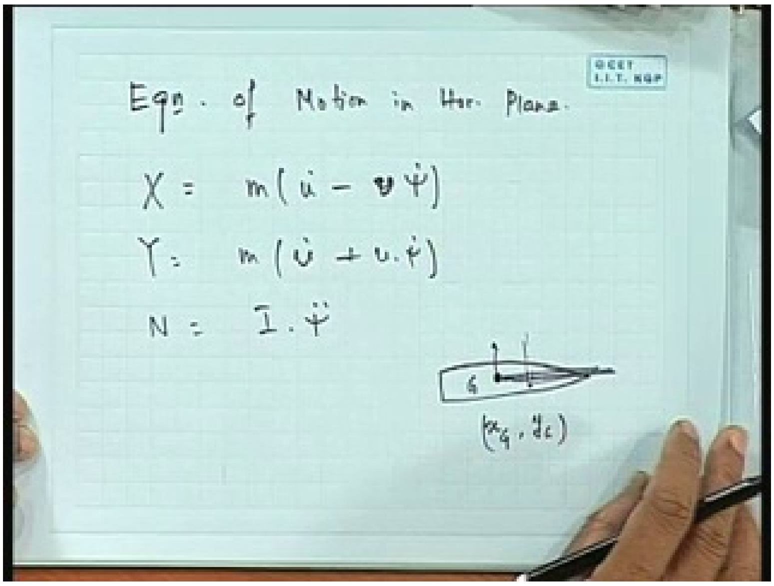
Indian Institute of Technology, Kharagpur

Lecture No. # 34

Equation of Motion in Horizontal Plane

We will continue on this Equation of Motion in Horizontal Plane.

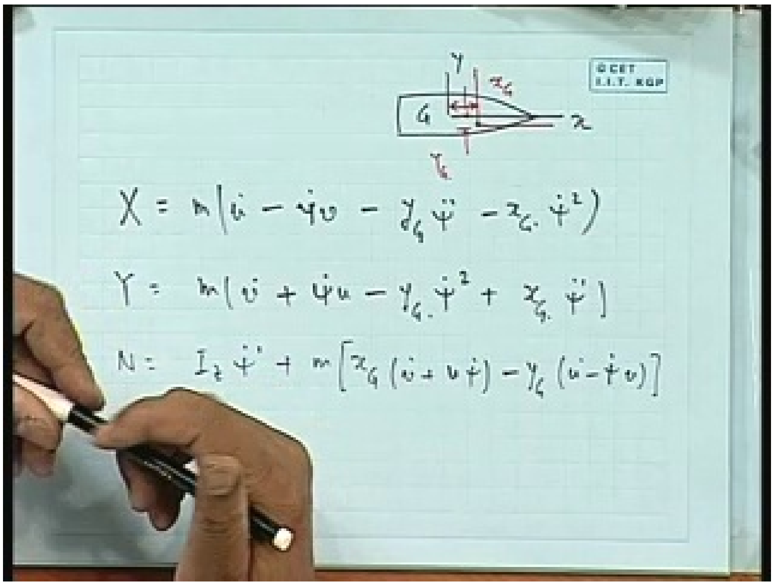
(Refer Slide Time: 00: 58)



See, we had this equation just in the last lecture, I just write that v into psi dot and Y was m of v dot plus I think u into psi dot and N equal to I into psi double dot. Now you see, there is one thing that is, that modification we can do on this is that, some time what happen, this equations are written when the axis was at centre of gravity you know G, sometime what happen, it is easier to define that when the for an arbitrary axis system somewhere else here.

See here, I have taken an, this equation of motion I have taken, where the body system is having its axis at centre of gravity, but sometime supposing it is having an axis which is a some other location a o o here, so that the centre of gravity is the coordinates are x G and y G. In other words, I define it with respect to another coordinate system, then a certain extra terms coming. In fact, it simply gets modified as I will just write down that, some extra terms will come in here or rather, let me do it in a second one.

(Refer Slide Time: 02:28)



Let me let me be draw it here, this is my this is my G, this is my x, this is my Y, but if I took an axis in a general sense somewhere else with this to be the coordinate axis, so that this is my x G. In other words, if I took let me put it, if I took a body coordinate system somewhere else, so that the centre of gravity has a coordinate of x G Y G, in other words, you can say centre of gravity is say 5 meter forward of the mid, say I took a mid ship, so centre of gravity is 5 meter forward of mid ship.

The reason is because, sometimes some of the definitions are easier for a hull with respect to a geometric fixed point, then the centre of gravity, when you are defining lines plan, you are better of taking in mid ship as the origin point, not x G, not the centre of gravity because, centre of gravity can be 2.354 meter forward of mid ship, so it becomes difficult. So, as a result, we if it is if we want to want, we can choose the body coordinate system as some geometric point and then say that, centre of gravity is x G meter forward of that, Y G meter star board of that.

If I do that, this equation of motion actually simply gets modified by some extra terms you end of getting something like I will just write this, we had here psi dot v minus y G into psi double dot minus x G into psi dot square, this two extra term comes in Y equation, m v dot plus psi dot into u minus y G into psi dot square plus x G into psi double dot and N equal to I z psi double dot plus m x G into v dot plus u psi dot minus y G into u dot minus psi dot v.

See normally why I say that is that, most of the ship sometime it will be easier to have some x G, y G will not be there, because normally you would always have the centre of gravity on the central line. So, normally this will not be there, but this will be there because, you might want to use your mid ship as the coordinate system. So, you might have only some kind of value of x G but not Y G, but that this is a trivial thing, never mind this, this is only to tell you that, the same equation of motion can be easily read it in another coordinate system, instead of centre of gravity. In other words, you have shifted it, see I have shifted the axis system slightly forward or slightly on the star board side or whatever, it is only for convenience because,

(( ))

Y G is,

(( ))

x G is obviously the x coordinate of centre of gravity, in other words, it is l c g simple as that, Y G is t c g, but t c g is 0 for ships normally. So, you are in let us say, I am defining this, all this hydrodynamic equation of motion with respect to a coordinate system with originate mid ship.

In that case, my and the l c g is 3 meter forward, therefore, x G will be plus 3 meter, Y G will be 0, remember that the turning is defined with respect to G, but geometry I may defined with respect to another point, this is trivial never mind this, but it is just for completion I am just showing that, this is my one side of the equation. Now comes up, the difficult part of its sea or rather, let me see from the simplified one, I have got now this, force equal to mass into acceleration.

What I want to now know, I want to know that, the ship was moving on a straight line; remember it was initially moving on a straight line. So, what it had; it has u to be a constant, ten knots, what was v; v was 0, because it was not having any v motion, no such motion, what was r, r was also 0.

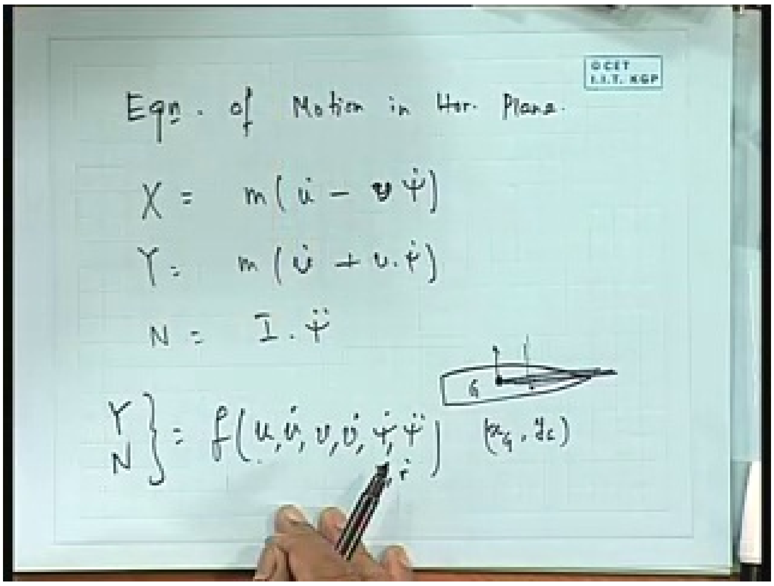
Now what happen, the idea is like this, you have given x also, Y was 0, forget x part, I mean I am just going to talk about these two; let us say these two, because these two are the one, basically looking into Y direction and the moment, x direction is the force going. So, let me look at just these two, let us understand the physics behind it, what we are trying to look at, initially I have Y is 0, v dot is 0, psi dot is 0, so 0 equal to 0, no problem, no force, no motion.

Now, what happened I am assuming that, I have introduced a slight Y, small y, delta Y; what happened, moment I give delta y, there will be delta v, delta psi, small psi and small y this thing, moment these two are there, this is going to influence Y again then I have an equation of motion, small change of Y because, I have given a Y, which has causes small v as small psi dot, this in turn would have caused a Y, which in turns would cause v and psi dot. Now, we have to find out, if I made a small change from a straight line motion, what would happen, in which direction my v and psi will throw. Again, I want to tell this, I was going on a straight line, I had no sway velocity, no yaw velocity, and I had no force, sway force, yaw moment, everything is 0. Now, by an external mechanism, I have given a disturbance; what happened by giving a disturbance, I have introduced v dot and v and psi dot because, let us say somewhat it has (( )) caused it to just move, moment I have given this, which is a small number, which is because you have just given say let us say by some disturbance, I have got some kind of a yaw, sway velocity of 0.1 meter per second.

Somewhat, it has pushed it and left it that would give me Y, now this Y in turn would then I have to go back to this equation of motion, I have got a Y, then this Y would in turn this side my v dot and psi dot. Now, suppose the v dot and psi dot that Y I have got, then this is the v dot, which will increase the Y, which will increase v dot, I will become (( )) end up into instability, what I want to find out, therefore when I make a small parted by small change from its straight line motion, the nature of the force is resulting because of a small velocity introduced yaw and sway velocity introduced, should they grow with time or should they diminish with time; if they diminish with time, then it is a stable ship, grow with time it is not a stable ship.

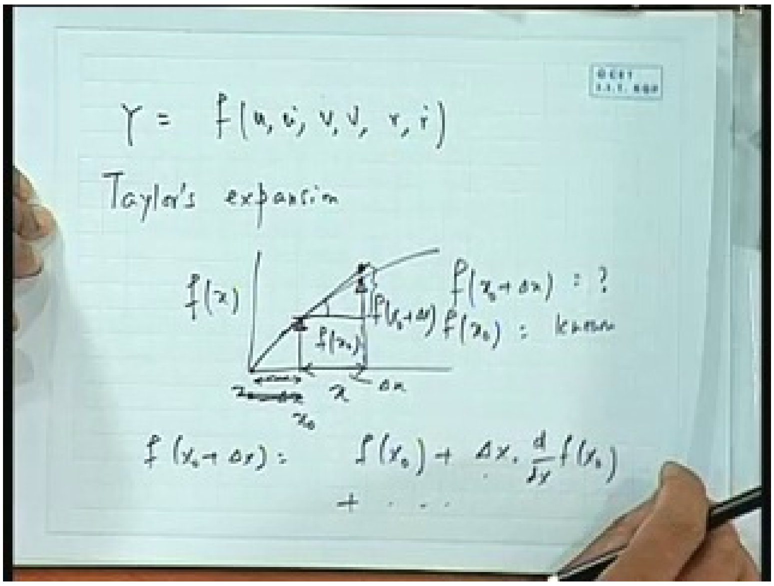
So, I need to actually express this, now what is happening that that is the most important part that will come, this Y force, nobody knows what is Y force but I know that, this Y force depends on v, v dot, psi, psi dot, because it is obviously it will depend in some sense on the forces on the various motion parameter.

(Refer Slide Time: 10:04)



See, I know that Y and N this will be depending on, I know what it will depend on, it may depend on u, u dot, it will depend on v, v dot, it will depend on psi dot and psi double dot or rather r, r dot, it does not depend on psi remember, psi is a location you know wherever, it is there the location. So, it depends on all the three velocities and three accelerations in general, it may not depend on all, but I have to assume that, because you see because I have given a velocity on the hull, I am getting Y and N, velocity and acceleration had there been no velocity and acceleration, had there been no motion in this direction or no motion in this direction, I have no Y and N, so I am getting Y and N, because I have got, somebody from outside gets in and pushed, something has pushed and created a v and v dot, one of those, you know or all of these, and these all of this obviously cause this, so this depends on that, but I do not know, how this depends on that.

(Refer Slide Time: 11:16)



Now, I have to find out, this side of expression in some mathematical way that is what, where this explanation, I am trying to tell now you see, this we understand, let me take one by one, one part let me say Y, Y is actually let me say, it is function of u, u dot, v, v dot, r, r dot.

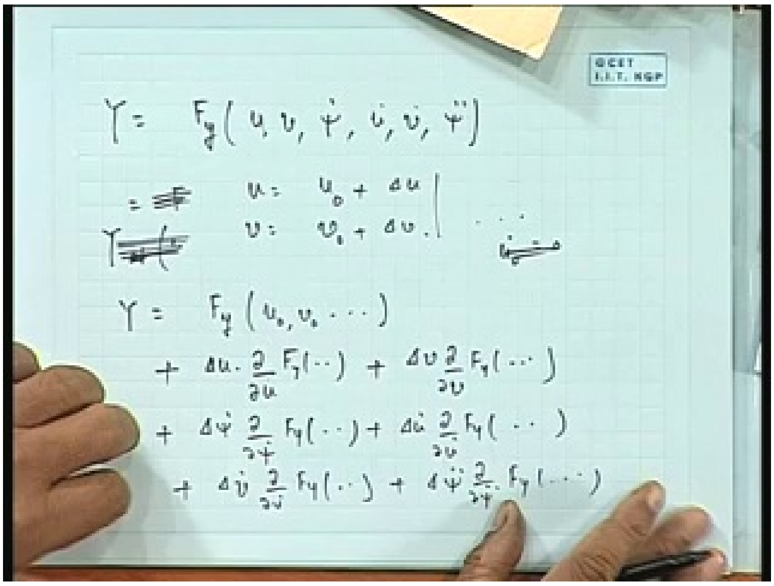
Now, we use a concept called Taylor expansion; what does it tell, suppose as a graph here, graph f here against x, say there is an f x, it looks like that, this is my say x 0 and this is my delta x, I want to find out, what is my f at x 0 plus delta x how much? I know, that means I or rather let me put with the other way round, may be it is better to put this as x 0, so this is my f x 0 and this as delta x, so that this is my f x 0 plus delta x, it is very small, is it in writing or you can make out still you can more or less make out.

See the question is that, if I know mathematically function value at some point and I want to find out, what would be the value of the function at another points closed by, near by, small distance away, in the first approximation I can tell that, it is equal to this value f x 0 plus delta x is equal to f x 0 plus delta x into d by d x of f x 0.

In other words, this slope see I can tell that, this value is equal to I mean I simply assume that, this part is a straight line, then what happen to f x basically, I am saying that, estimated value is this point, what is this point, what is this much, this much is delta x into a slope, it is delta x into the slope at that point plus my original value (Refer Slide Time: 13:20).

So, this is up to first approximation, if you want, you can actually keep on expanding that further, but half row everybody knows that perhaps. So, I need not go through, this is my first order Taylor expansion. What we are going to do is basically that, because I want to find out, what is my Y and what is my N, about the initial value which was 0, initial value was 0, so my Y 0, N 0. Initially at u 0, u dot 0, v 0, v dot 0, r 0, r dot 0. Now, I have got u plus delta u, v dot v dot plus delta u dot, etcetera; I have got some changes.

(Refer Slide Time: 14:19)



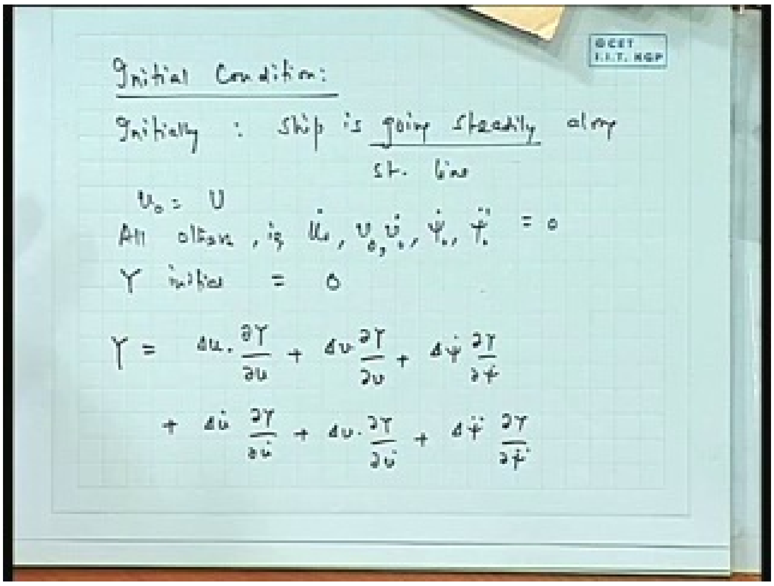
So you see, if I use a Taylor expansion for this, my own notes only, so I can go then, I can say like this, I will just take the Y value, Y is equal to function of, we can write this way, u, v, psi dot, u dot, v dot, psi double dot. So, I can say, this to be equal to, see now what think of this I am trying to find out Y, initially or let me put it this way, therefore let me say Y at, see Y is a function of all that Y at difficult to say, no let me not write that way, see I have this Y, now I want to expand that, now that is what I am trying to get, I want to expand that or I want to find what is Y, which is this side about its mean value, initially I have got u 0, v 0, psi dot 0, u dot 0, v dot 0, I am calling this 0 to be the initial values, the suffix 0 here, refers to initial values. I had initially initially this values, so what happened when these are the values. So, in other words, my u equal to u initial plus delta u, v equal to my v initial plus delta v, etcetera etcetera. So, I had got initial value plus the small disturbance, what is my Y; obviously, Y is going to be basically delta y, this is this value at all that you know u 0, v 0, etcetera, plus u delta u into d by d of u of F, you know F y all that plus delta v of d by d v of F y, all that.

I have to take one by one, because there are six terms, delta of psi dot d by d psi dot F y, all that plus now delta u dot d by d u dot plus let me write it on fully, delta v dot v by d v dot delta psi dot dot d by d psi dot dot F y of like that.

See, what I have done? I want to find out, what is my value of F y in these expanded about the initially value of u 0; that means, I have initially u 0, v 0, psi 0, u dot 0, v dot 0, psi dot 0 that is what my initial condition. Now, I want to I have made them, slightly different by adding a delta u, what is my value of this full thing, when I know this, initial part, see when I know this initial part, what is my full thing, that is what I want to know, this is actually my total Y at that point. this is my What is this, this is my Y under initial condition, this is my Y under final condition, so I have got Y at final condition, Y at initial condition plus all the small changes, this is the physics behind it.

Now, come to an interesting part of it, now you see, what we are doing is here, is that the initial condition we are doing, what is my initial condition; the initial equilibrium condition was, the ship going in a straight line initially, so initially I have got or rather, let me write it in another another piece of paper, that will be easier.

(Refer Slide Time: 18:15)



So, we have got this condition, now let us see, initial condition; initially the ship is going; that means, I have got u 0 to be U, because forward value is U, but all others are, all are 0 because, obviously I am going on a on a sort of steady line, so therefore, what happen also remember that, Y initial is also 0, there was no Y initial. So, if you look back at that see here, if you look back at the equation, this part was initially 0 (Refer Slide Time: 19:30).

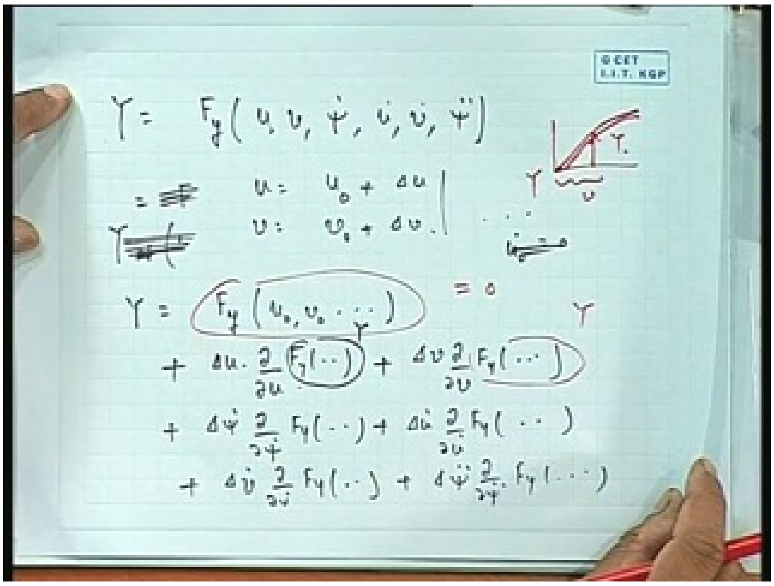
And the Y that I am getting is, the small Y that has got disturbed; now here, delta u is the change, delta v is a change of velocity, delta v is etcetera, but you know all this thing there is some kind of that that I mean this initial condition you are putting it. Actually, we could have put u minus u 0, etcetera, but if I put this initial condition here, in this equation what I end of getting is something like, we will we will end of getting some kind of relation. Now, there is a way of looking at the part, see this is F y, this part is nothing but Y.

(( ))

So now what I initial Y, etcetera, now what happen that, I can actually rewrite this equation in terms of d Y by d u, d Y by d v etcetera, because it is easier to write that way, I will come to that, in a in a minute.

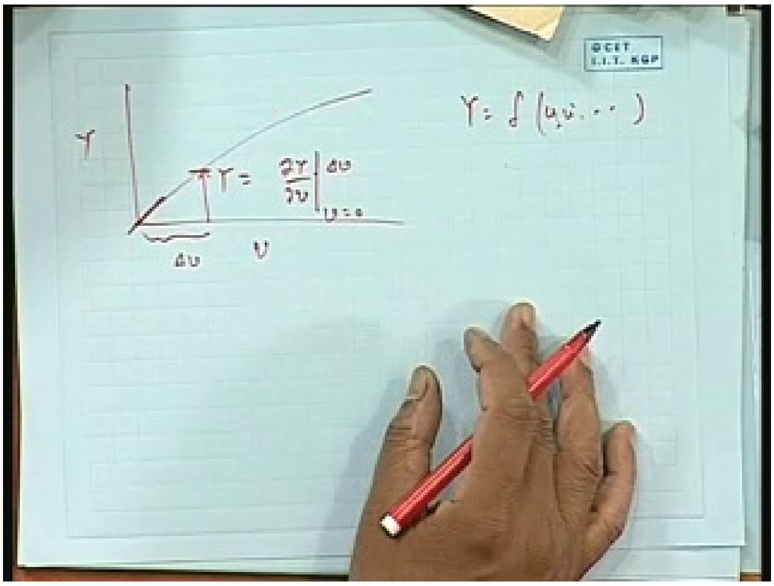
So, I wanted another color of pen, the color is important. I have got this Y here, etcetera. Now, I end of getting this disturbance Y, again looking my at this is 0, so I have got all this extra terms and I am going to write this extra terms at delta u into d by d u of instead of F Y, I will write them Y because, that this part is nothing but, basically Y whatever the Y is, so what we will end up writing is, that we have got here see, what I am doing is that all I am doing is writing this same thing again, where I have made this 0, because now looking back at that, this part is equal to 0 this parts, this parts are nothing but Y, so instead of writing this F Y full thing, after all F Y of whatever is nothing but Y, you know Y equal to F y, so I can write Y instead of writing F Y full thing, it is d v of in fact, what we have done, see its more easily seen Y value at a point, later is Y value at 0 plus that small change into d Y by d v plus d you know etcetera, Y value on various thing.

(Refer Slide Time: 22:46)



So, basically the Y value, this is my Y, this initial Y was actually 0 and I want to find out, what is this Y, this Y equal to the initial Y plus d Y by d v into the small changes. See, if suppose it is actually v, so it is this Y equal to this Y plus d Y by d v into v plus d Y by d v into etcetera etcetera. When the d Y by d v are the slope of the curve, Y at origin, may be it is which is a one more diagram I want to explain that because I think, this might have been slightly difficult for you.

(Refer Slide Time: 23:24)

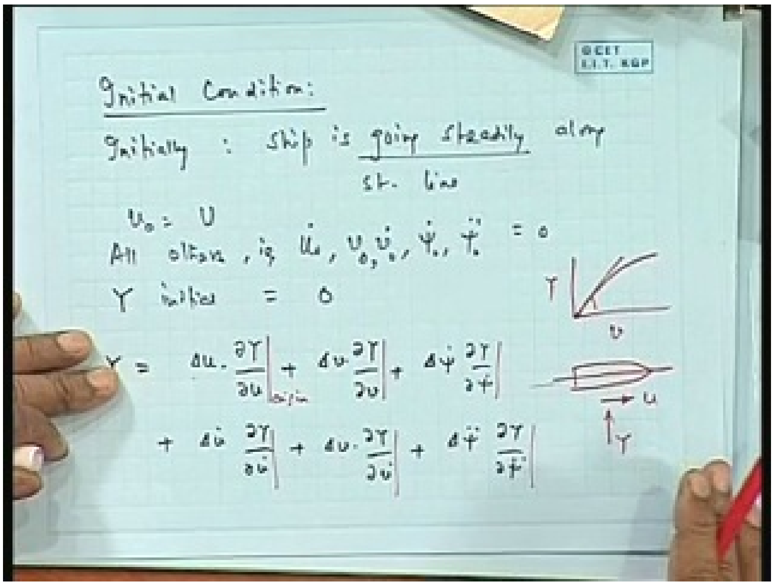


See, what we have done here is, something like that you see, I have got this here, Y force this is my Y, Y of course is function of many things. So, let me take that, Y is a function of u, u dot etcetera. So, let me take Y one by one, let me take this as a function of v, then I have take Y as a function of you know v dot, six of them I have to take.

Now, what happen I want to find out, this part is my delta v, what is my this Y in terms of this, this Y is nothing but, d Y by d v into delta v, this is to be taken of course at v equal to 0; that means, d Y by d v that is slope at this line into delta v, this is exactly what we have did, that is what exactly is my Taylor expansion. In fact, all that we have done here was that, we wrote F y but we replaced them by Y (Refer Slide Time: 23:57).

So this expression that I have written here, is straight forward, you have got delta u d Y by d u all the slopes are to be evaluated, remember that at the origin, this slopes to be at origin, all this slopes, all this Y slopes are to be evaluated at the point of origin, slope can be evaluated it is something like your g m concept, what is g m, it is d g z by d theta at theta equal to 0, but if you have theta equal to 0, there is no g z, but you can always evaluate the slope.

(Refer Slide Time: 24:57)

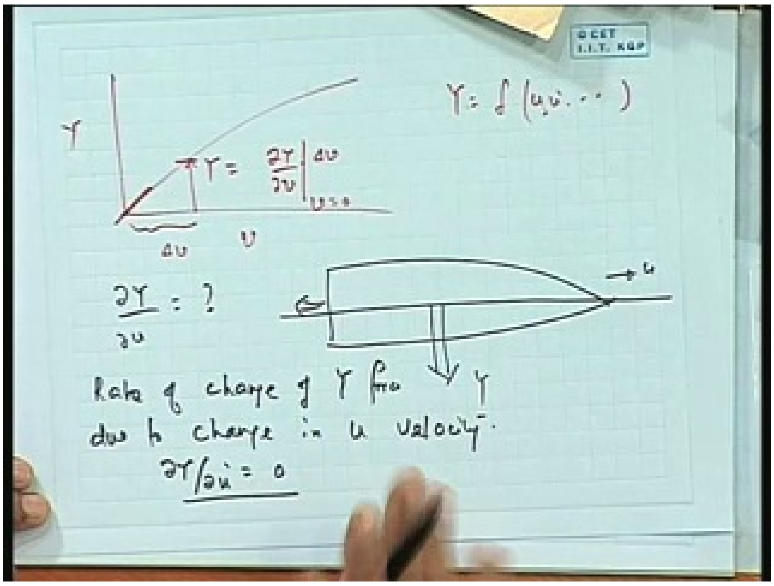


You see, if you look at this g z curve also, I have got here g z curves going like that, g z is 0 here, but g m is slope of the curve at the origin right, see exactly here, I have got the slope of the curve Y equal to 0 at v 0 remember, there is no Y force, but there is a slope existing, moment you can slight free there is a some value of Y.

I just want to know, what is that, if I give a small v how much Y has come; obviously, it will be equal to the slope of the Y curve into that change, that is exactly what. What would be my Y force if I have given a small change about its mean, it will be delta u into d Y by d v plus delta v into this, this is the concept that is most important that we have to understand, this is the most important concept in our this calculation.

Now, come to an some more interesting point of this reduction, you see what is d Y by d u, there is a ship here, which is symmetric about the center line, now I have given a velocity here, this is my u and this is my Y, now the question is that, if I give here a change of velocity will it give me a change of force here, that is the question, this d Y by d u means, what is my rate of change of Y force because of rate of change of velocity in the u direction, but the point is that, in a u direction if you change it Y force does not change, because the ship is symmetric about its x axis.

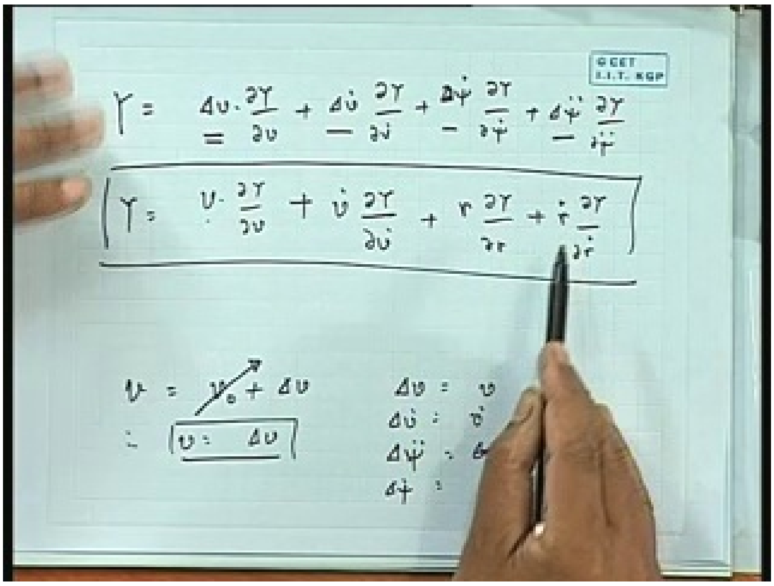
(Refer Slide Time: 26:34)



Now, I will make this bigger one, so here d Y by d u is what, I have a ship here, this is my u, and this is my Y the question here d u by d Y is rate of change of Y force due to u velocity, this is the definition Now, the question is the ship is symmetric about this axis, so if I change the u velocity if I see let us say I was going at 10 meter per second, I just go to 11 meter per second what would happen, it is going to give me a large difference and force this side, will it give me force this side? no, because it is symmetric, therefore, d Y by d u becomes equal to 0; similarly, d Y by d u dot will become equal to 0 because of the simple fact that, u direction the ship is symmetric both sides; similarly, the moment will be also 0, because what would happen if you change that, whatever comes this side, will get balance in the other side.

So, if we if I that that is interesting because, when I look back at that I can further deduce the equations and I can say that, this goes to 0 and this goes to 0, so I end of getting only this part, this is a very very important and interesting part (Refer Slide Time: 27:52).

(Refer Slide Time: 28:08)



So, what I end of getting is, therefore, is that I end of getting that Y force becomes equal to delta v into d Y by d v plus delta v dot d Y by d v dot plus delta delta psi dot d Y by d psi dot plus delta psi double dot d Y by d psi double dot, now this is what, we end of getting.

Now here, I want to I have to say something more at this point of time. Now, you see what is my delta v, delta v dot, delta this thing, these are actually the small changes, there is no point of writing them as delta v, because after all initially you see, I have a v velocity of 0, now I have got v velocity of small value of say 0.1 meter per second, but that 0.1 meter per second is just change in v and I call that delta v, but I can also call that as a v itself, because after all the ship was going at a velocity here without without any sway velocity. Now, I have got this sway velocity small number regardless, so I can call that as instead of delta v, I can call that as all also v, because delta v makes it more confusing, so I can call directly this as v. So, I can call that as v of d Y by d v because, when the v is understood to be a small number, a ship cannot be see, a ship goes at 10 meter per second here, it cannot go 10 meter per second this side, in this side is only small number, it can only be a change with respect to the 0.

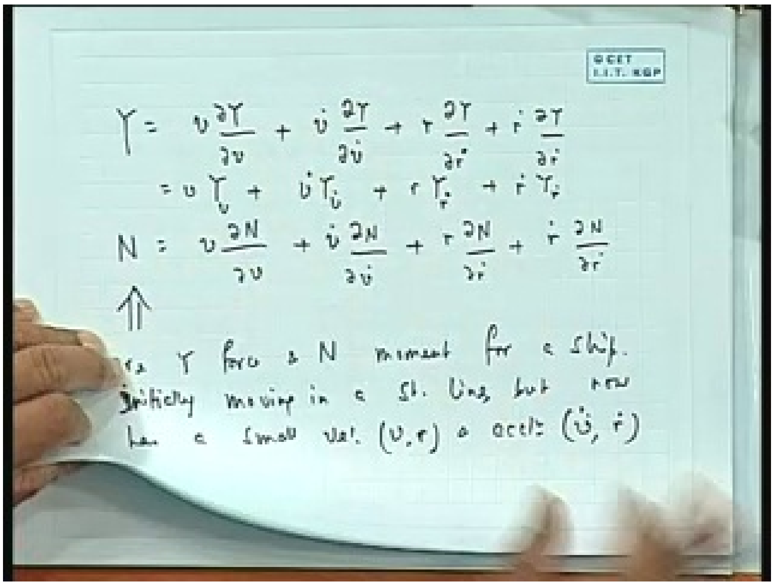
And that I change initially to illustrate that, we call delta of v, see in other words, you can see this way v equal to v 0 plus delta v, but v 0 is 0 therefore, delta v is v. So, whatever small number you get, it is the sway velocity. So, I use this nomenclature because, delta v is no point of. So, whatever sway velocity I get small number, if the change of sway velocity because my initial sway velocity was 0.

Initially, I had a 0 velocity sway velocity as well as yaw velocity similarly, you will find out that, I can call delta v as v, delta v dot as v dot, delta psi dot dot at delta as psi dot dot and delta psi dot at delta sorry as psi dot or this is actually r dot, this is actually r, why I can call like this, it follows from here even otherwise, you can know that the change of sway velocity is itself the sway velocity because, the change is with respect to 0 mean.

Initially was 0, now I am having 0.1 meter per second, so 0.11 meter per second is my delta v as well as v, this is all question now as nomenclature. So, I can write this as v v dot delta d Y by delta v dot plus, let me call this to be now r delta Y by delta r plus r dot delta Y by delta r dot. If you look at this I come back to little more this thing, you find that Y force is essentially is very simple expression, it is because there is a sway velocity small number into the rate of change of sway, what is the see I have got now a sway velocity of some 0.1 meter, but I know what is d Y by d v I know that per unit change of sway velocity I have I get this much.

So, therefore, you have got a change of sway velocity of v then, my total force is this, see let me say, d Y by d v is equal to 1, which means I get 1 unit of Y per unit of v, but now I have got v of 0.1 meter, therefore 1 into 0.1 is my force that is all, like that. I have got Y because of some v, because of some v dot, because of some r, because of some r dot, I do not have Y for u and u dot because, u direction do not give me v force, this is the simple explanation that we end of getting for Y force for a ship, which as initially going steadily which is always the case.

(Refer Slide Time: 32:31)



So, now I will explain that too by writing the physical meaning at more detail. So, I am end of getting this as, because I am writing these two together, so that is why I am writing again. Similarly, if I do N, I will get exactly the same thing, it will just get changed by N here I will end of getting like this, if I if I do that, basically I am going to end of getting like this, what is this we must understand, what are this Y and N forces, these are Y force and N moment for a ship initially moving in a straight line, but now has a small velocity, v, r and acceleration, v dot, r dot, initially it was going v 0, r 0, v dot 0, r dot 0, Y 0 everything is 0.

Now, I have got a small value of v, small value of r, small value of v dot, small value of r dot, what is my Y force and N force; obviously, it is that small value into the rate of change against that value, that is all.

So in fact, you can easily find out, this is nothing but an expression of Y and N in terms of rate of change, Y changes with respect to v in this rate, therefore, if v has become so much, my Y is going to be v into d that rate. But, Y changes also, because of change in v dot, Y changes also, because of r, Y changes also, because of r dot. So, it basically changes for these four parameters; two velocities, two acceleration. So, I have got this expression as simple as such.

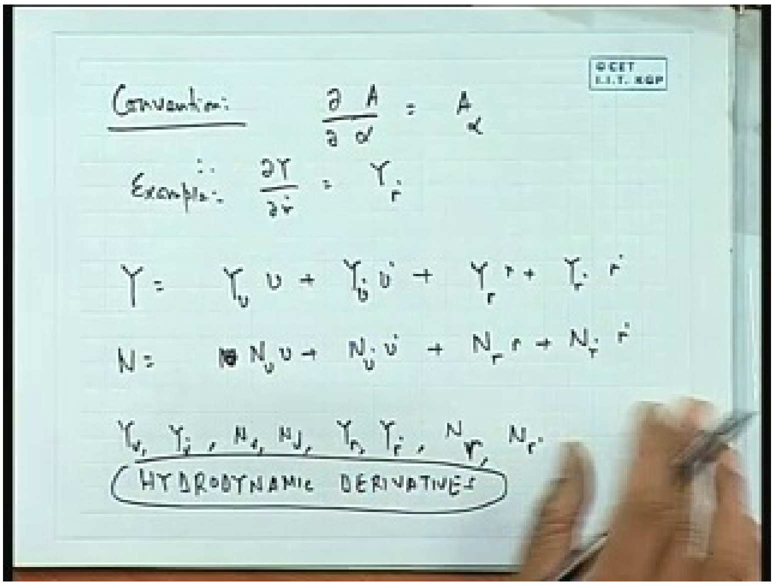
(( ))

No, no motion in this direction in fact, it can be, but we are not complicating that, they are the force may come because of propulsion characteristic, but we are not right now considering that for the simple case, we are thinking of a ship having a propeller fixed not changing, you are going in a straight line this is my expression of only change.

Now, another thing that happens is that this d Y by d v etcetera, this expressions are written as Y v, I mean d Y instead of writing d Y by d v always, you write at Y suffix v; Y suffix v means, d Y by d v something by something.

D x see d Y by Say, d a by d alpha is, a suffix alpha that is how we are writing, that is a nomenclature. So, you write that like this, you write at v dot of Y v dot you call this r Y r dot, you call this r dot sorry this is not r dot r Y r like that, see you can write like that.

(Refer Slide Time: 35:47)



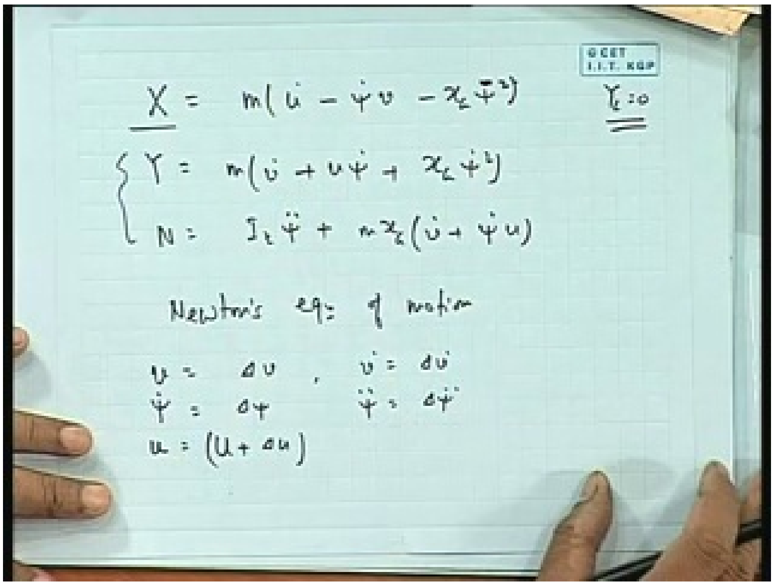
In other words, the convention I will write the convention d of d say A by alpha is written as d alpha A you write this convention, derivative against something is that value by the suffix, because it is makes easier. So, therefore, that is d Y by say d r dot will be Y r dot, as an example.

(( ))

You can say, you can say you can say an operator or a coefficient actually basically, you are rewriting that you know it is just writing in a short form, no its not, you are not simplifying you are writing using a different convention that is also I have end of getting

Y as Y v into v plus Y v dot v dot plus Y r r plus Y r dot r dot, N as N let me write it with respect to Y here, N v v plus N v dot v dot plus N r r plus N r dot r dot, you have just got this, what we have done here, I am just rewriting that this is my one part of the equation. See, these terms Y v, Y v dot, N v, N v dot, Y r, Y r dot, N v sorry N r, N r dot; these eight numbers, these are will be called as hydro dynamic derivatives. I will tell you later on about this, later on.

(Refer Slide Time: 37:52)



We will come back to that little more later, little later; let me now, complete the equation of motion, so I have got this. Now you see, this was the last, we started with this expression of a Newton's equation of motion, which we have got u dot minus psi dot v minus say x G psi dot square no sorry psi dot square what I am using Y G equal to 0, because as I said that, most ships will have no l c g, t c g is 0.

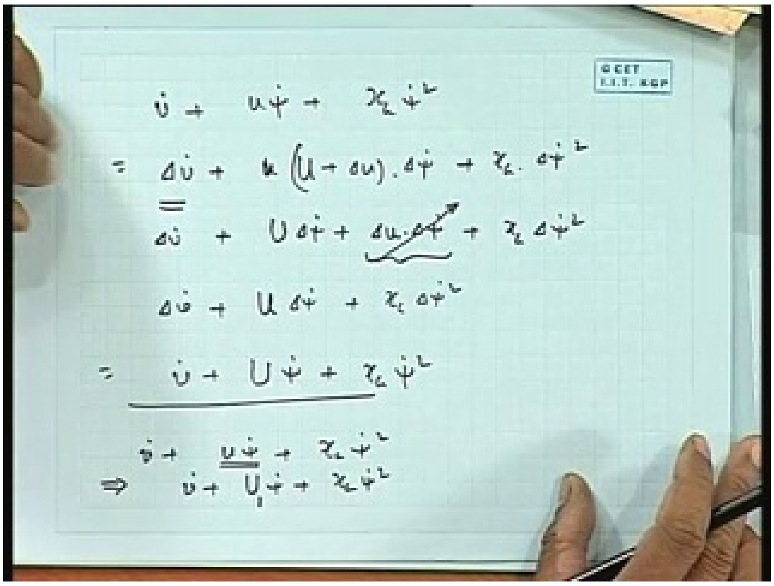
We have Y equal to m of v dot plus u psi dot plus x G psi dot square, and N was equal to I z psi double dot plus m x G v dot plus psi dot u this was my Newton's equation of motion, we had that know.

Remember, I have got two systems; one is that, this side X Y N force is or rather, forget this one, this two I express them if the ship was going on a straight line and if it made a small change, what happen to my Y and N force, what is the expression and I found out, that the expression is something like this, I found this expression.

On the other hand, I have got this equation, which is, what is my Y and N force, if I have got a value of v dot, Y dot, etcetera etcetera all that. Now, I must combine that two, but in combination that two is in this right hand side I should use the fact, that my v dot is a small value, psi dot is a small value etcetera, because remember that, this values are nothing but, the small values. You see because, that the expression with the force that I have used remember that, expression for the force this u etcetera, here it is the exact value of u and v, but there it is not so. So, I must use that, fact that these expression is meant for those values of u, v, w which are small values. Now, why I say that because of this term I will explain to you, there is a there is a reason why one one does that.

See here, I have got v equal to delta v, I have got psi v dot equal to delta v dot, psi equal to delta psi sorry psi dot, psi double dot is delta psi double dot, but u equal to u plus delta u; the u was much larger you see, why we are doing that now I want to express that in this fashion and see because I want to use this to this part (Refer Slide Time: 40:04). What happens, we want to see, what happens to that because there is, you remember there is u psi dot; u is a not a small number, u is a large number remember u is a large number, is it; this u is a large number, because it is not delta u, because the ship is travelling in the direction u at 10 knots and now it is making small changes on that 10 knot, 10.2 knot, 9.9 knot etcetera. So, u is a large number, we have to remember that.

(Refer Slide Time: 41:13)



Now, let me see what happen to this term this v dot plus u psi dot plus x G this is because delta v dot, because v dot is delta v dot this becomes u is u plus delta u into delta psi dot see, u is this thing and this becomes of course x G into delta psi dot square. Now, you see this of course remains like this delta v dot, but here I have got two terms; one is u into delta psi dot plus delta u into delta psi dot plus x G into delta psi dot square, but this term happens to be very small, because it is small into small. So, we neglect that, so, therefore, this has this becomes delta v dot plus u here this is my steady velocity into delta psi dot plus x G into delta psi dot square, which I can rewrite back as v dot because delta v dot is again v dot.

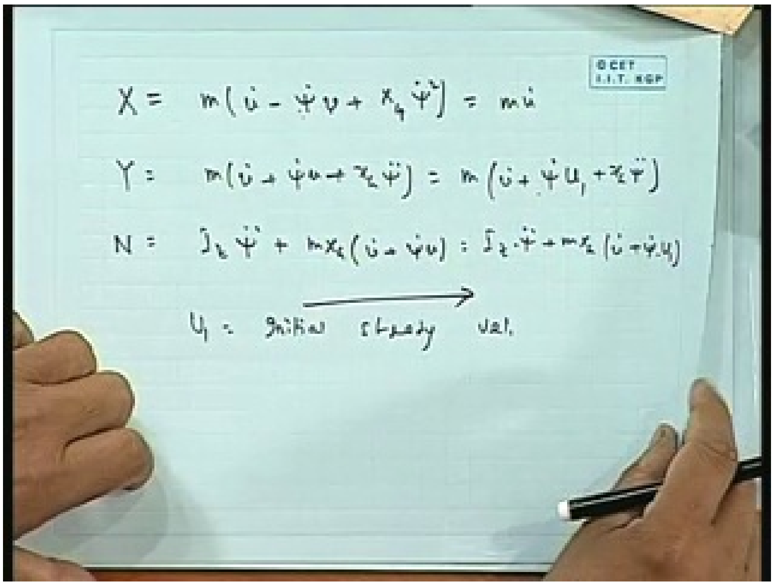
But only thing that is small u has changed to now big U into psi dot plus x G into psi dot square. So, you see what happen that is why, that is what I want you to tell, I had the expression of v dot plus u psi dot plus x G psi dot square this has now changed to v dot plus U psi dot plus x G psi dot square. This term has become capital U psi dot, when I am taking small values of u dot, v dot, psi dot; that means, if it is making a small perturbation, then this term, this forward term becomes the steady forward velocity into the psi dot that is very important to realize that what we are trying to tell.

In fact, we can call this u as u one if you want, but this is my steady velocity or initial velocity may be I can call it u one that is, it becomes v dot plus initial velocity, steady velocity into psi dot plus x G psi dot square whereas, this term was instant instantaneous u velocity.

But this is called as very large magnitude.

That is yes, it is called a large magnitude, yes that is, that it be there, but u dot was, see this is say 0.1, this was 10 plus 0.1 into 0.1 this is again 0.1. So, 10 plus 1 into 0.1 becomes you know 10 into 0.1 plus 0.1 into 0.1 that 0.1 into 0.1 part we are neglecting. So, 10.1 into 0.1 it is equal to 10 into 0.1 to that approximation, this is what we are doing or this is what is logical, because we have actually gone up to that point only.

(Refer Slide Time: 44:07)



So, if you do that then I think what I will end up, up to this. So, I end of getting this to be for this, let me write this, u dot minus psi dot v plus this is actually was not necessary to write any how, still for some completion we are writing. In fact, this will turn out to be actually m u dot only that I will explain that later on little it is not necessary right now, Y becomes equal to m into v dot plus psi dot u plus x G psi dot dot, this becomes equal to m of v dot plus psi dot u 1, I am getting u 1 here.

x G into psi dot square. I think Y sir.

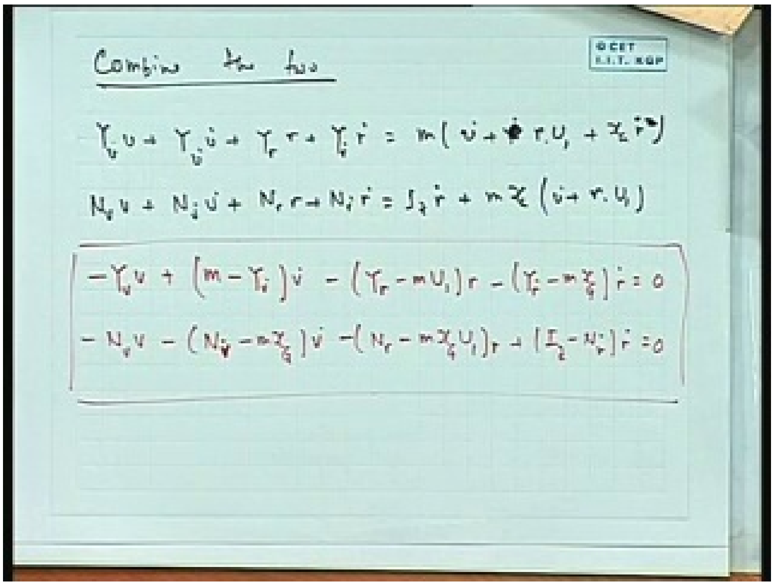
Now, there is no, no let me, let me see no, then Y no, I think that Y I think I made a mistake Y should be x G no, it should be x G psi double dot, this is not square. I think this is this is the mistake, you please correct it, it will be actually double dot, not square. In fact, the square term will not exist actually, in fact I made a mistake psi dot square actually will go to 0 because; psi dot is a small number, so square goes to 0. So, actually these will not be here. In fact, this expression, this is small, this is small, small into small goes to 0, small into small goes to (( )), so that is why it becomes only m u dot, here this is small, this is small but this is small into large plus small, so therefore, it remains small into large (Refer Slide Time: 45:31). That is what happens. I am sorry, this was actually double dot yes; this is not the Y velocity double dot, x minus square. In fact, it is other way round, it may be make this correction N becomes I z of psi double dot plus x G, and again here this will change here into psi double dot plus m x G v dot plus psi dot into u 1.

So, I have got this site, this is reduced to this, this has reduced to this, it is a you do not actually have to go through that the all the mathematics, this is very simple, this is small, this is small, small into small is too small, small into small is too small neglect it, small, this is small, so we have to keep it, this is small into large plus small, so it is small into large plus small into small. So, you maintain small plus large small, small into large you know small into large.

Like here also, small into large plus small, that is small into large. So, this U 1 is actually is the initial steady velocity. Now, we are in a position to combine that two, now we are able to combine that two side that, the I have got this I mean forget the x part, we will not probably do or we can do I have got this Y part N part and I have got here Y part N part, x part also we can do but we will not do the x part now, because we are not interested.

So, what happen I have got this Y this side here, I have got Y from other side here. So, I am going to combine that two, I am going to write here, Y equal to this from that side equal to that from this side.

(Refer Slide Time: 47:34)



So, by doing that by combining that two, I will end of getting this relation, now combine I will end of getting relation is Y v v plus Y v dot v dot plus Y r r plus Y r dot r dot equal to m into v dot plus psi dot actually instead of psi dot, we should write r may be, let me write here r U 1 plus x G r dot square, that is right sorry not square, r dot. I think I will rewrite again, may be as a anyhow, you can understand this is this is dot here, then this is become N v v plus N v dot v dot plus N r r plus N r dot r dot equal to I z r dot plus m x G v dot plus r into U 1, this we get.

Basically, you know this is from one side that is from other side, so we can combine that two if we combine that two, I mean I will just bring them and make them to be 0. So, I will end of getting this relation, which will look something like this, minus Y v v because, we are bringing in on that side so does not matter plus m minus Y v dot v dot. I think Y v v will go that side, then m v dot then this we are just going that side, minus Y r minus m into this is minus Y, it is minus here, then minus plus minus makes it plus fine U 1 r minus Y r dot minus m x G r dot equal to 0 this is one and we get minus N v v minus N v dot m x G U 1 into r plus I said minus N r dot. These are my classical equations of motion in the horizontal plane for a ship initially moving an along a straight line, you see these expression that we have got here, this is N v dot, this expressions are my equations of motion in the horizontal plane.

Therefore, what happens is that, if I know this hydrodynamic coefficients, actually if I know the solution of the this equation, I have to find out, how does v and r grow with time, what is the nature of my solution for v and r, see v and r is my yaw velocity and yaw acceleration and v dot and r dot. We want to know, does v and r with time increases or decreases; because here, I have given a small value I have remove the force, it has a set up v and r, v dot and r dot and those v and r, v dot and r dot set up my forces, now this force equal to you know a motion I now I want to see this equation of motion will tell me the solution of that, that forces that sorry that velocity is that was generated because of an disturbance, which in turn generates a force, does this generates the force at an increasing order or decreasing order because, if that force you think of a loop type for some reason I have given v that this v gives me a Y force. If this Y force is such that, this Y force increases my v, then it would have made them my v larger, then the this v would have made my v even larger, that Y would have made my v even more larger it would have been unstable, but if it is the other way round I have got an v, which produce an Y force, this Y force is in such a direction that it tries to reduce my v, then that v would have become little small, then the Y would have become still smaller, then that would give me my Y even more smaller, it will go to 0. So, this is what is, what is called the study of stability, that we will we will talk next class is given by the characteristic characteristic solution of this equation.

Here I have got v, v dot, r, r dot and all these coefficient, I want to know v, v dot, r, r dot, do they grow with time or do they reduce with time, I have to see and my ship would be stable if they reduce with time, this is exactly why you need to find out the forces. We could have done that ship is stable, unstable we cannot do that in this case for the very simple reason that, the forces are necessarily created because of the velocity in acceleration which are fluid forces.

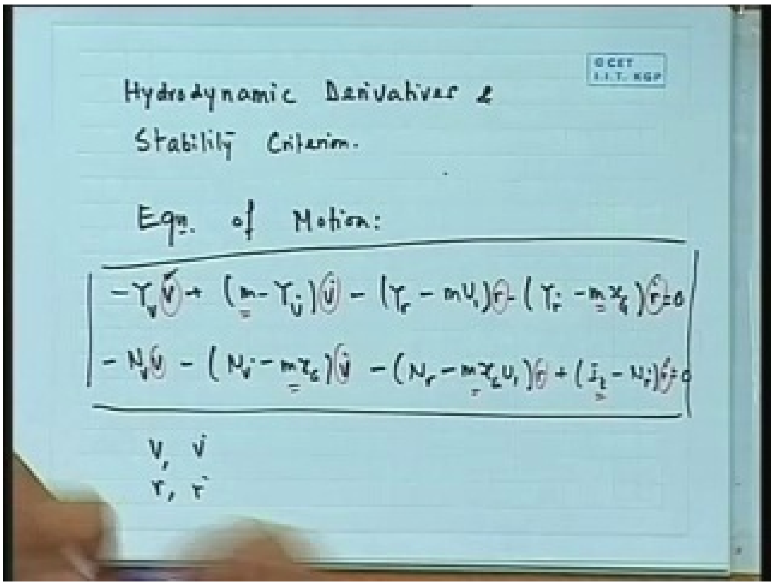
And it is the force and the velocity is interpolated that tells me whether the ship is stable or unstable. So, we will pick up from this point and next class, study the properties of or the characteristic of or the requirement for a ship to be stable. From that solution for that we would not go to the detail solution, just some preliminary ideas, I will I will stop it here for today, thank you.

Preview of Next Lecture

Lecture No. # 35

Hydrodynamic Derivatives and Stability Criterion-I

(Refer Slide Time: 53:27)

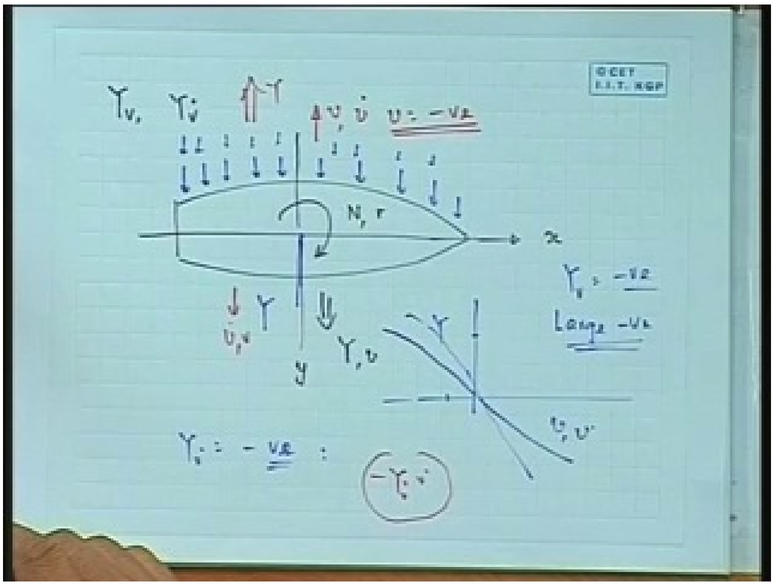


Today, we will talk about; see yesterday, we actually ended up in what is called Equation of Motion. Now, the equation of motion we actually had, I am just write it write it again it was something like Y v v plus m minus I think Y v dot v dot minus Y r minus m U 1 r minus this was one, the sway and we had see, we had this two set of equations yesterday, that we have found out, represent a body or a ship moving initially along a straight line and was disturbed slightly, they are governed by this equation, when v and v dot, r, r dot, sway velocity, sway acceleration, yaw velocity, yaw acceleration.

Now, what is meant by stability criteria and of course I will talk about this hydrodynamic derivative also is that first of all, if you see that this equation, you see this is the velocity, in fact let me put a different color this one, this one, this, this, this, this, this, and this, are the two velocities and two acceleration (Refer Slide Time: 55:36).

This is my rigid body mass, this is my rigid body mass, this is my rigid body moment of inertia, and these are rigid body mass, rigid body mass. So, I have got the blue ones, let me put them as blue, this rigid body mass, this is rigid body mass term, this is rigid mass and you know the geometric term, rigid body mass and centre of gravity.

(Refer Slide Time: 56:09)



I think I mean, the entire fluid is trying to push it on this direction, this push acts over the entire part of this longitudinal length, the full length gets push down.

Now, let us say we can call it separately, say there this plus, this plus, this plus, this, all are added up; obviously, when you add them all up, then you get a large number supposing this was pushing and this was pulling and it was this plus that, that could have been a you know, some number minus some number could have been either positive or negative.

But in this case, it is not so, the entire hull each point on the side facing gets a push. So, I have got, I get a large force, when you when you add them I think I will explain that more with a v dot rather than v because that will be more easier. See, if I give a v dot here see if I give v dot here same thing I am giving an acceleration, so I am again going to get, you know all this forces on this side, all of them. See, I am trying to accelerate that accelerate that side, so I get all the forces this side, what would happen if I draw v dot, also I gets a large Y, so I get Y v dot to be large Y negative, so I get Y v dot to be a negative, now is it large or small I want to answer you this question; see Y v dot is nothing but, like an added mass, Y v dot is actually because I have given acceleration this side, there is a reaction force, so you can say that, the full fluid gets trying to get accelerate on that side, which will give you an inertia for opposite side.

Because you see, if I took this body and I try to accelerate on this negative acceleration then, I get a mass into acceleration force on the negative direction. So, that is why I get minus Y v dot v dot, I get a force of minus Y v dot, v dot, this is like added mass into acceleration and added mass in sway and yaw are of the order of the mass for this, because I…

(( ))

No, no it can be positive I because I give an example where if I give a v dot negative, then I get Y positive, you give a positive v dot, you will get a force on the other side, this is a more debate I will I have to end now. See, if you give v here or v dot here, then you gets a force on this side the question here is that,

It will be in an opposite direction.

Whichever direction you give v, Y is on the other direction, anyhow we will pick up on that again in a next you know like class because, this lecture time is up, and we will pick up on that again. So, I will end this one, at this point.