Strength and Vibration of Marine Structures
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Lecture - 2
Introduction to Ship Structures – II

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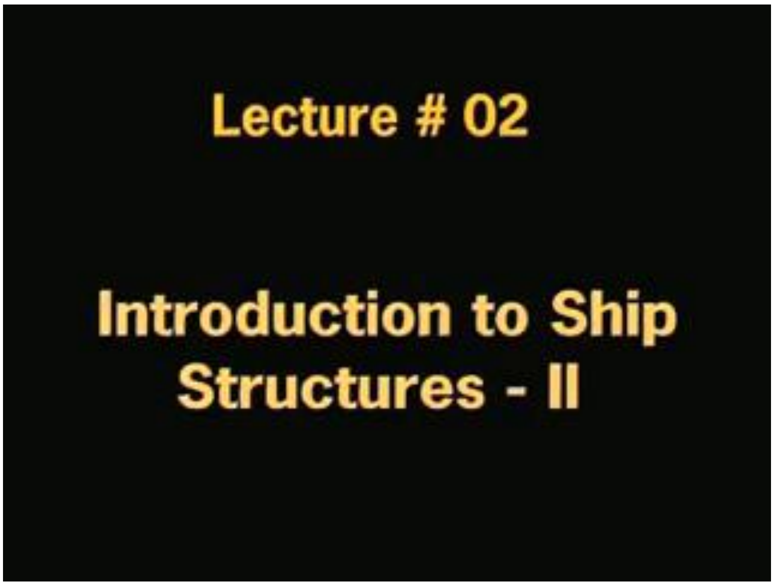

So, we have talked about bending moment in the last class. We can continue with that and switch over to other thing like shear force. So, it is, basically, some component related to bending of beam. For the same problem if we draw here again. So, we took a beam like that; put a load at some intermediate point P; we got some reaction Rb Pa by l; and at a it was Ra Pb divided by l.

Now, one of the component, we have defined as bending moment. And we wrote some expression for bending moment. And we have seen that quantity is not fixed within the member; it is varying; and that was function of x; we have tried to plot that. In that case, it was varying in a linear manner.

Similarly, there is a quantity called shear force. Basically, at this force, if we try to take any section, so it will give some shearing type of stress. So, if you take any section in between and that shear stress will be that force divided by area, more or less it is shear stress, and the total force is shear force.

Now, here also if we define the total beam segment into two regions: one a to this load point, and load point to the b. So, x, one part it is a 0; here another is x a. So, shear force sometimes we define in the form of b. It will be say Ra. So, here there is no other force. Any section if you pick up, the shear force is only Ra. If you cross that load, so b will be your Ra minus P or this Ra will be your Pb by l or here it is Pb by l minus P; Pb by l into P.

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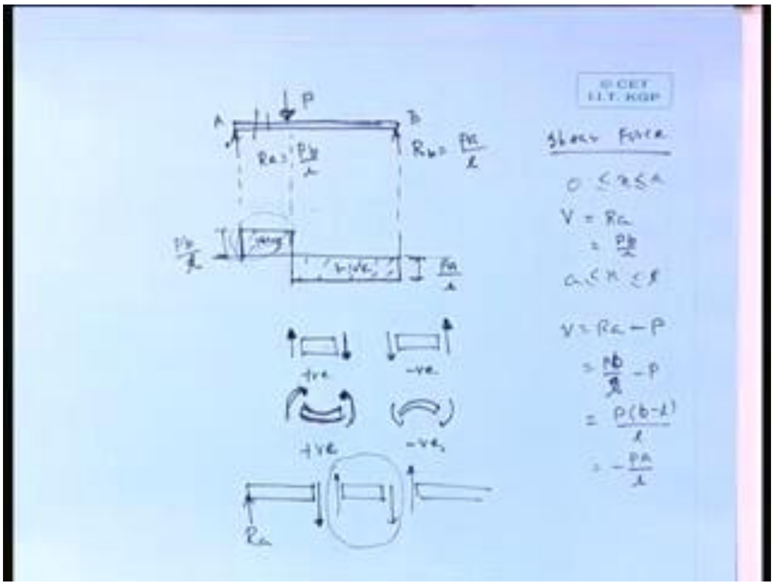

Or if you write P it will be b minus L by l or you can write P into a by l, because l minus b is basically a. Now here ,there is one quantity Pb by l; another quantity Pa by l; and they are practically matching with Ra and Rb. Pa by l is basically your Rb and Pb by l equal to Ra, but here it is a plus sign, it is a minus sign. And the bending moment expressions, what we have written earlier, it was positive in both the cases, and we want a positive moment bending diagram. So here, I will first draw it then we will try to define what is the meaning of this plus, minus. So, we have to define some sign convention; based on that, we have to draw the bending moment diagram and shear force diagram.

So, here, if we try to draw the shear force diagram, so we draw a line. So, this region is basically your Ra or Pb by l, and here it is negative; this part will be Pa by l. So, this is a positive part; this is a negative part. If we take that is a 0 line, this is above is positive, below is negative. So, it is plus and minus.

Now what is happening? Shear force is changing; up to certain point, it is uniform, then it is changing; going to the negative side; then again, it is remaining constant; negative. So, at this level, I should introduce the concept of sign convention.

Now, if I take shear force. Now, if we take a shear force like this, we can define this is positive; if we get shear force in that manner, it is negative. Here shear force and bending moment, it is not a single force; a reaction is a single force. So, when there is a single force, we follow some vector direction, say, it is in the positive direction of x, positive direction of y, or positive direction of z, with that we define the plus or minus.

But when it is shear force… shear force is basically, there are… it is a system of two forces; it is a balance system of force; bending moment there are two forces. So, it is it is trying to bend like this. So, this side there is a moment; this side there is a moment or shear force. So, there is a counter force. So, if it is one of the force is along the positive direction of x, other thing will be negative direction of x. So, normally it is not defined according to the direction of vector; it is defined in terms of the type of the deformation of the structure.

Now, here this is upward, this is downward; but as a whole, this sense is defined as positive or this sense it is defined as negative. Similarly, if we talk about moment - this is clockwise; this is anticlockwise. So, we will not define as clockwise, anticlockwise. If it bends like that, if the bending phenomenon is like that, we say it is positive; and if it bends like this, we say it is negative. Not necessarily, this negative, this is this positive, this is negative; it is a very universal thing. In some cases, in some books, you may find it is positive, it is negative; in some cases, it is positive. Some have written it is positive; it is negative. But whatever sign you will follow, you have to follow consistently throughout the problem. Any problem if you handle with this type of sign convention, you will follow up to the end of the problem. Next problem you can change your sign convention, but in between if you change, it will be total mix up.

Now, bending form, we say it is… sometimes that is defined as sagging type of moment; if the member is horizontal, if the loading is vertical, gravity is downward, if the tendency is like that, so we say it is a sagging moment. But if it is horizontal plane, there is no gravity. We cannot define it is sagging, it is a very general case, but anyway in a particular case we can say it is sagging moment; that is a hogging moment. So, plus and minus, and shear, this is plus, this is minus.

Now, this type of, say here, up to from the left end to the application of the load, if you cut any section, say, if you cut here and remove the left part, so this reaction Ra, it will be acting on the upward side; or this left part if you take separately, you will find to balance that load, that load will be acting downward. So, anywhere if you cut, so there will be upward and downward forces. Now when you will split, the right side face it is downward, and left side face it is upward; or I can draw, say, this part little bit I can put it in a bigger manner.

So, this is your Ra; if you cut here, so this part it will try to give one balancing force. So, as a reaction, it will get some opposite force. Here, again, if we cut then there will be a force, and if you continue, it will be like this. So, if you combine, so it is just like electric plus minus charger. So, there is here one negative charge, one positive charge; if you combine they will be balanced. So, there is no external force there, but internally it is there. It is just like fluid pressure; some disturbance is there inside, it will be transmitted through interaction between the particle. So, this is the force. So, if you cut here, only one force; if you think that face, it will be upward. So, in that process, if you take a small element, so it is more or less like this. So, that type of phenomena, we say it is a positive type of shear force. If it is opposite, if you come from the right side, it is your opposite phenomena. So, though both the reactions are upward - Ra and Rb - but this side shear force is negative; if you take shear force on the left side, is positive.

Now, with that idea, we can take a beam problem, with some loading, having some support condition. That support we can remove with some appropriate support reaction. We can apply the equation of statics; get the support reaction; once that will be known, it will be member with some loading. So, from one end, we can go on adding the load. So, summation of the vertical forces will give the shear force.

Say this is a member, some loading are there, so at this station, from this end, if you go on adding the forces here. So, summation of all the forces will be the shear force here. And incidentally, if you calculate from this side and that side, it should be equal, because whole object is in equilibrium. So, if you cut here, the entire force, and entire force this side, this would balance each other. So, you can proceed from this side or you can proceed from that side; both the cases you are supposed to get the same shear force. Normally, we try to adapt one procedure.

If we want to calculate shear force here, we will see which end is narrow. So, our calculation will be little easier, if the less number of forces we have to count. So, we can calculate shear force.

Similarly, bending moment is we have to take moment of all the forces acting on one side above that section; that is the bending moment. So, if you count from left end, if you count from the right end, you will get the same bending moment. So, this way, it may be clockwise, this end it will be anticlockwise; but clockwise anti clockwise; they will give a sagging type of moment, so it will be positive; or it will give a hogging type of moment, it will be negative.

So, that is the basic idea for finding out bending moment, shear force at any station within that beam, but you have to start with finding out the reaction. Then we have to proceed inside for getting the expression for the bending moment, shear force; then drawing out the bending moment diagram. Once you will draw the bending moment and shear force diagram, from the diagram itself, here say your Ra normally it will be more, because a is less than b. So, this reaction should be more. So, it is nearer to this support. So, at this side shear force will be more compared to shear force of this side. Naturally, the shear stress produced by the shear force here, that will be more here compared to that.

Say bending moment. We have got some diagram. From there you can get the maximum bending moment at some point. If we take a very complex problem, your bending moment, some where you may get positive, some where you may get negative. Among the positive part, there will be a maximum value; within the negative part, you will get some maximum value or minimum value, whatever you can say. So, those stations will be the most severe stations. So, we have to investigate the stresses there; what will be the bending stress generated due to that bending moment or whatever the shear stress generated due to the shear force. And that will be our guideline for the design or you will get the idea, your stress will be within certain limit, that is your allowable stress. If that is there, your safety will be assured; otherwise, you have to change your design; you have to strengthen your structure, or basically, you have to make some alternative measure, so that your structure will be within your safe region; that is the basic purpose. So, what should be our next job? Next job will be once bending moment, shear force we can find out, we can go for finding out the bending stress and shear stress calculation.

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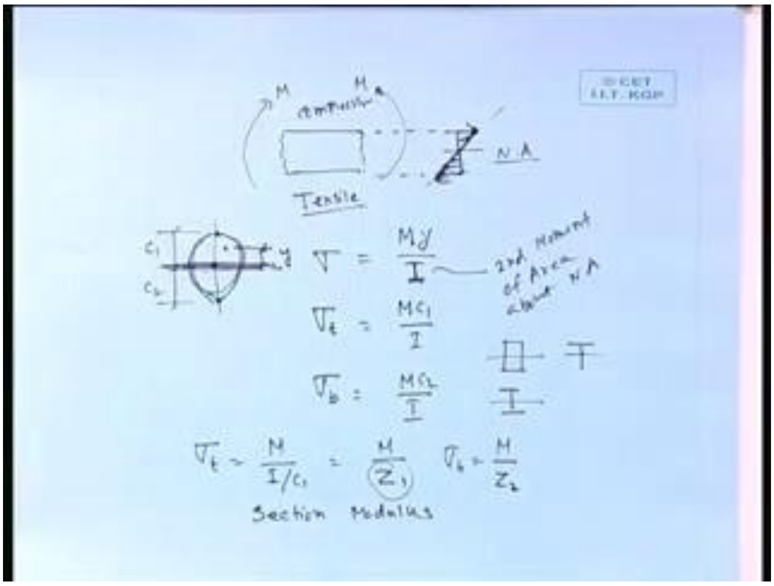

Now, say, I am drawing only a part of the beam. Say some moments are here; some moments are here; say, this is the M; M it is a typical station; and that is one of the maximum bending moment. Now, when moment will be acting on that, it will generate some stress, and those stresses we say bending stresses, because bending moment - these are moments - and that moment is generating bending, we say it is bending moment. And that bending moment will generate some stress, we will say it is bending stress.

And which way it will generate bending stress? So, bending stress, its distribution is something like this. So, if we go from the centre line, a little away, our bending stress value will increase and it will be maximum at the extreme (( )); at the top it will be maximum; at the bottom it will be maximum. Maximum means one side it will be tensile; another side it will be compressive. And in between there will be a whichever, that part, we say it is the neutral line or neutral axis. So, neutral axis there will be no stress, there will be no strain; it will be under neutral condition; so we say it is a neutral axis.

So, if you go beyond that, more you will be away from the neutral axis, more will be the stress; and the stress it carries a linear path. So, it follows absolutely the linear path. So, here linearly it will increase. So, extreme point, extreme stress, this side extreme point, extreme stress; and you can visualize which side will be the tensile stress, because any member, if we apply moment or if we take our finger apply moments, so this side will get stretched; stretched means tensile stress will be developed. So, this will be tensile side and this will be your compressive side. So, this side will try to compress. So, compressive stress will be generated, and this side tensile stress will be generated.

So, one will be tensile; another will be compressive. So, in between there will be a switch over smoothly from in a linear manner, crossing the 0 point through neutral axis. Now, so here I have shown like this; I have shown like that. So, stress at any point, it can be obtained with that relation sigma is your My by I.

Now, this diagram is taken from side of the beam. If we take a diagram along this, means if we take the cross section of that beam, so it may be any arbitrary shape. Say, that is the neutral axis; now here, say, any point, that distance we are defining as y. So, if y equal to 0, My by I it will be 0. So, at neutral level it will be 0. If y is maximum at the extreme level, it will be maximum stress; here also it will be plus or minus. If we define tensile as positive; compressive will be negative. Here also there is a sign convention; we can follow anything; either tensile positive or compressive positive; other thing will be negative.

Now, My by I is the sigma or this is the bending stress here. Now what is I? I is the moment of inertia of the section above the neutral axis. So, I is moment of inertia or sometimes we say it is second moment of area. So, above neutral axis if we calculate moment of inertia of the cross section or sometimes we say it is second moment of area; so, that will be I. So, that will give M is fixed for that station; I is fixed. So, it will just vary on, it will dependent on y, which is a variable quantity.

So, y it is varying from 0 to some value; 0 to some value. So, accordingly our stress will change. So, we will get maximum stress. So, sigma top, that will be M; say this is C 1; this is C 2. So, it will be MC 1 by I and the bottom part will be MC 2 by I. I am not putting plus and minus here; only numerical value I am putting here. So, sigma top and sigma bottom will be MC 1 by I, MC 2 by I. Now it will depend which one is bigger - C 1 is bigger or C 2 is bigger - based on that sigma t or sigma b it will be maximum.

Again, there is another aspect, another requirement that one part it will be tensile, another part it will be compressive. Steel normally tensile compressive part not much varying, unless the buckling part we consider it; but there are some material where compressive strength is different compared to tensile strength.

So, might be one of the component is small sigma t or sigma b, but in that mode, the capacity of the material may be less. So, which ever will be guiding that depends on that nature of stress, and its value; both will be important. But this extreme fiber stresses are quite important. Now at this moment let us introduce one term, say this one - this I MC 1 by I; we can sometimes write as Mt equal to MI by C 1 or we can write M by Z 1. Similarly, sigma b it will be M by say Z 2.

So, what is Z 1? Z 1 is nothing but your I by C 1. So, second moment of area divided by the distance of the extreme fibre from the neutral axis. So, that quantity we define as section modulus. So, it is called section modulus. So, if we think in terms of our design problem, we will be mostly interested about the maximum stress.

So, any section if it is not symmetrical, it will have two section - two values of section modulus; one will be based on C 1, and another will be based on C 2 - the top one and bottom one. So, we will get Z 1 and Z 2, but if it is a symmetrical section, say, if we take a rectangular section, up on I section. So, here it is absolutely symmetrical about the neutral axis. So, C 1, C 2 identical; it will have only one value of section modulus; only nature is different one will be tensile and compressive, but the stress evaluation part will be only one step.

But if the section is different if we take a t section, so it is not symmetrical about the neutral axis. So, top flange will be very nearer to the neutral axis; this part will be little away from that. So, as well this numerical value of the stress will be more here compared to the stress at the flange, but again it may be dependent on the nature plus or minus. Now, if we can get the bending moment, we can find out the bending stresses. Now, the next step is shear force. So, shear force will give some stress and that stress we say it is a shear stress. So, similarly, My by I, there will be something for finding out the shear stresses.

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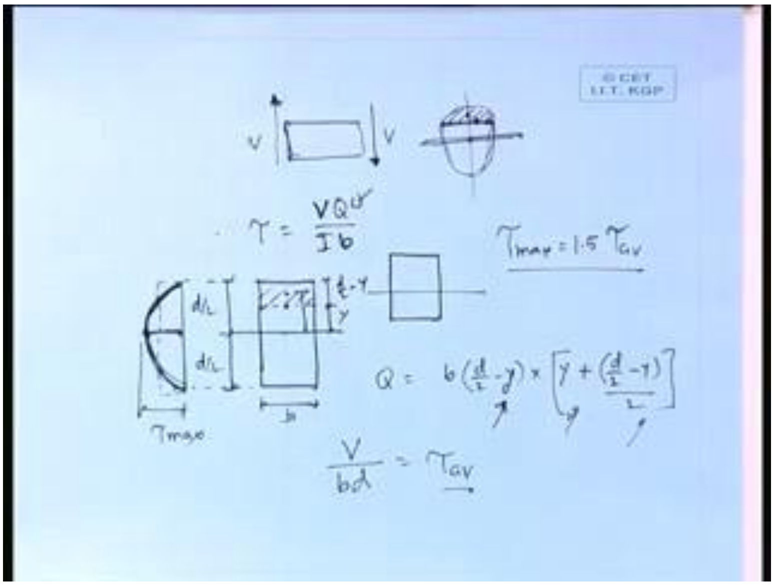

Now, if we take say this side is V; this side is V; and some cross section here. So, this cross section may be looking like this. Now, this V will be responsible for generating the shear stresses. So, shear stress it can be calculated as VQ by Ib.

Now, V is the sheer force; I already have defined. So, b and Q these are the two terms if we know, we can get the shear stress at any point. Now, b is, say at any level, and say here we want to find out the stresses. So, this is the b; b is the width of the section at that level. See here this is the b. And Q; Q is basically the beyond that line whatever area you will get, you have to find out the first moment of area of that about neutral axis. Now b is clear. Q is, say, here we are interested, we can draw a line, we can get b. If it is a rectangular section all the time it will be b, fixed. Now, Q is where we want. So, if we draw a line, above that some portion we will get; and that portion if we consider, we have to took the first moment of area of that region about that neutral axis.

So, what is that? That is basically area of that portion multiplied by the centroid of this shaded part, say, it may be here. So, from the neutral level whatever is the length that we have to multiply. So, Q part, say, I am trying to define with a rectangular section; it will be easier to calculate. So, it will give a better understanding.

So, that is the… let us draw in a little bigger manner. So, that will be the neutral axis. Now here if we are interested. So, we have to take a line. Now this part can define as say it is d by 2; it is d by 2; d is the depth of the beam; and b is the width that is uniform throughout. Now at this level we are interested to find out the stress.

Now, this part we can take and we say it is y. So, this part will be d by 2 minus y. So, what will be the area of that? So Q. So b is the width. G minus 2 minus y, that will be the height. And that multiplied by the centroid of the shaded part from the neutral axis. We get this. So it will be y plus d by 2 minus y divided by 2. It may be simplified. It is just for our understanding we are writing all these terms. So this part will give Q and b here it is uniform. Now, you can follow that expression that tau equal to VQ by Ib. We can get the shear stress at any point.

Now, theoretically we can say we can calculate. But we should have some idea: which way it will vary; where it will be maximum. Bending stress, it was clearer; it was varying in a linear manner; and it was maximum at the top and bottom. And it was 0 at the mid level. Now b for this section it is some constant value Ib, I will be a constant, but b it may change. For a rectangular type of case, it will be constant, but it will take a circular one. b will be changing; at the centre it will be maximum, gradually it will be less. Or if you take a triangular type of section, it will vary in the V part.

The Q part is quite cumbersome. This b d minus 2 minus y, there is one y; here there is a y; here there is a y; it is depending on y. Now here, this part is y and this part is y. So it will be multiplied. So we will get some term. It will be Y squared. Say we are taking this case only; very simple case - rectangular case. Here in b d minus 2 minus y, there is one y; and this part we are getting y. If we multiply, we get some constant term; some term related to y; some term related to y square. So here, if we put the expression of Q, in b Q by I b, we will get one expression. There maximum power of y you will get y squared; then you can get y; then we can get some constant value.

So, that will indicate our shear force will be not varying in a linear manner; it will not be constant throughout. So, it will be constant, then linear, then it will be quadratic; means it will follow a parabolic manner. So, it will follow a second order curve; second order curve means it is a parabola. Now, here the maximum value will be interestingly at the centroid, and it will be minimum, and it will be 0 at the end. The reason is very simple, because what is Q ? Q is the first moment of area of this region. So, if you put y is equal to d by 2. So, area will be 0. The shaded part will gradually reduce; at the top, it will be

0. So, Q part will be 0. So, at the extreme level shear force will be 0, and if you come nearer to that, area will be more, and your first moment area also will be more, and it will maximum at the neutral level.

Now, if we want to plot the sheer force here, say this is the section, if we plot this, this side, so here it will be maximum, and it will be like this. So, it will follow a parabolic pattern having a maximum value at the neutral level and it will be 0 at the top and bottom.

Now, if I really put the value of Q here, and we will get the expression of tau, and that will give the equation of this curve, but you can more or less put it and simplify. So, you will get the expression of this curve and you can plot it. So, at this level I am not going to find out the final formula, expression, any time you can determine, but this value at the beginning of your last class we were talking about shear stresses. So, we have taken two plates connected by a rivet trying to split by two forces. So that force it was passed through shearing of that rivet. So, we told P divided by area was the shear force, but actual case it will not vary in a uniform manner. So, it will be starting from 0 here, and it will diminish at 0, in between at the center it will be maximum, and it will go in a parabolic manner.

So, actual stress distribution will be little different; it will be little complex compared to the first side, we just… the entire load, entire area we have divided. So, that area, that stress, we got rather some average stress, and here we will get the actual stress distribution. So, in that case, if we calculate, say V is the shear force, and b into d for the rectangular case, it is the area, we will get tau average. And this tau average, this value, and if we actually plot the shear stress distribution, definitely this tau average is throughout uniform; that will be something like this.

So, some part it will be less; some part it will be more. So, the maximum possible stress we will get, they are neutral axis and this stress if we calculate, it is tau max; it can be verified, the tau max, we can write here tau max equal to 15.5 times of the tau average value, at least for the rectangular cross section.

So, if you really put the value of Q and simplify we will get the expression of tau, and at y equal to 0, you will get the maximum stress - tau max - and if you calculate the average shear stress - simply V by bd V by area - so we will find that maximum shear stress that will be 50 percent more than the average shear stress. So, up to this we have the idea about bending moment, shear force, bending stress, shear stresses. So initial part more or less we have covered today. So, in the next class, we will take some further problem, which are little advance compared to this.