Strength and Vibration of Marine Structures
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Lecture - 24
Ship Vibration II

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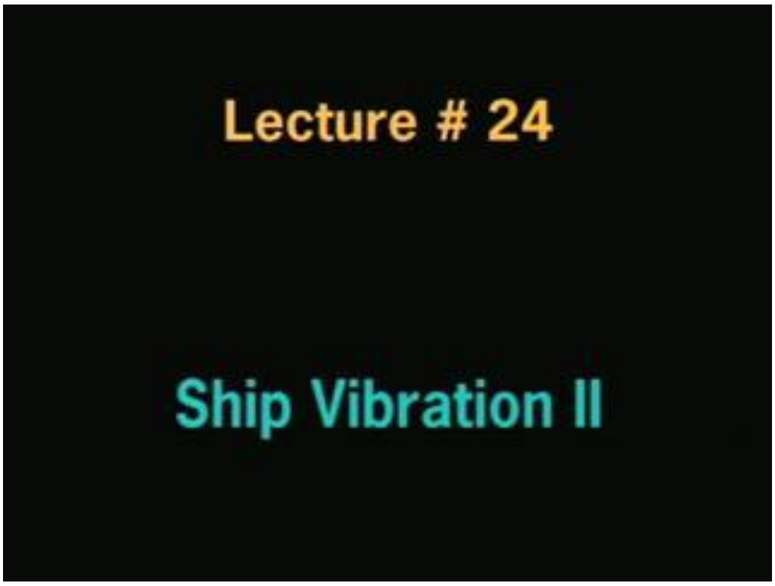

So, after deriving that simple expression for single degree freedom system of vibration, we try to see the response which is except t is equal to x 0 cos pt plus cos 0 dot by p sin pt.

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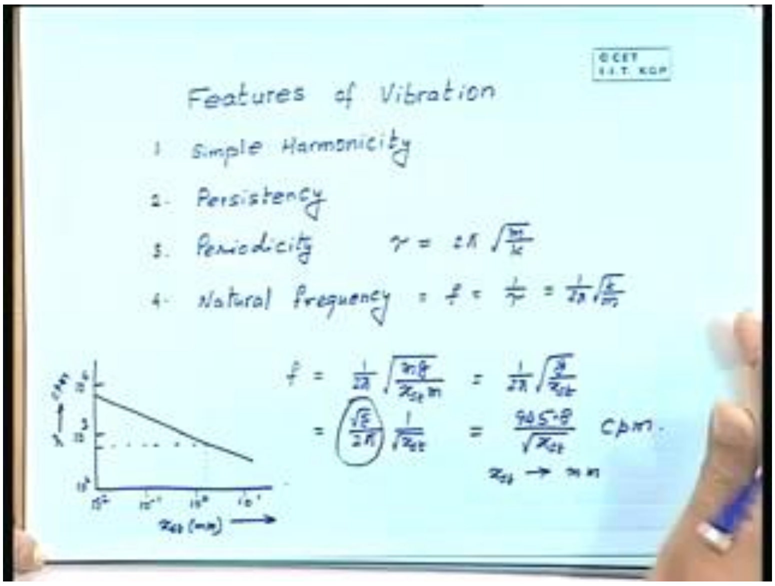

Now, what are the features? We see from here those features we can list it in this fashion. Number one is simple harmonic motion that is to write it simple harmonicity as depending on what are the initial conditions, whether you are having initial displacement present or initial velocity present. One of them is usually 0, either x 0 is 0 or x 0 dot x 0. So, the response will be given by say x 0 cos pt or x 0 dot by p sin pt. Now, this cos term or sin term, they are both simple harmonic motions and therefore, we first of all see, that this particular motion is a simple harmonicity. It possesses a simple harmonicity.

Second is persistency. Now, from this expression, we do not find that if suppose we say that x of t is equal to x 0 cos pt. As t keeps on increasing, the value will fluctuate from plus x 0 to minus x 0 and it will continue for t is equal to infinity. There is nothing which will try to decay this function and once the vibration starts, it will continue forever that what the equations says, but actually speaking we do not see this. There is a simplicity in this because we have assumed there is no damping in the system, but every physical system will have some inherent damping, and due to that what we actually see is that once the system is disturbed, it does oscillate, but the amplitude decays and within a finite time period, it comes back to the stationary condition.

So, that is because of the inherent damping feature of the system, but mathematically we have ignored for the time being. And therefore, what we are saying from the expression that once it is disturbed, it will continue because the energy has been put and there is no means by which the energy can be dissipated or taken out from the system. So, it will keep on going.

Then, we also find periodicity. It is a periodic function as we can see and we see that the time period tau can be written as 2 pi root over m by k because we have seen or we have taken p is equal to k by m root over. So, this is the time period you are getting, and the fourth feature is the natural frequency which is given by frequency f is equal to 1 by tau and 1 by 2 pi root over k by m. Taken the proper units, this will give you the frequency either in cycles per second which is hertz or you may convert it to cycles per minute or if you want circular frequency, then p is equal to root over k by m radiance per second.

Now, a little bit manipulation can be done here like we can write f is equal to 1 by 2 pi, and we can say that k is the stiffness. Stiffness is forced by displacement and we assume that the force is created by the weight itself. So, I can write m into g for the force, and I can write some static displacement under this force with the given spring, it will undergo some static displacement. So, I can write that x as t and m is there. Now, this can be written as 1 to 2 pi root over g by m. Sorry, g by x is tm cancels out or we can say root g by 2 pi 1 over root xst.

Now, let us see g is the acceleration due to gravity and at a particular point on the surface of the earth, we find g posses more or less a constant value and if we make that assumption that g is a constant value, 2 pi is a numerical constant and therefore, this term is constant here, this entire term and taken proper units in cycles per minute, we can convert it to 45.8 by root xst cycles per minute. If xst is expressed in millimeter, unit is a convenient unit for the displacement that you measure it in millimeter. Sir, it is cycles per minute.

Now, for every structure under some load, we can find out what is the static displacement and if we try to plot that, I just try to plot it here, you can draw with the (( )). This is your frequency in cpm and this is xst in millimeter. So, if we plot them, then you get it in this fashion. Now, suppose if you are getting a displacement of say how much say 1 millimeter, 2 millimeter. This will be somewhere here and the corresponding frequency can be obtained here which basically comes from this expression here. If we try to plot it, it will go in the long scale like that.

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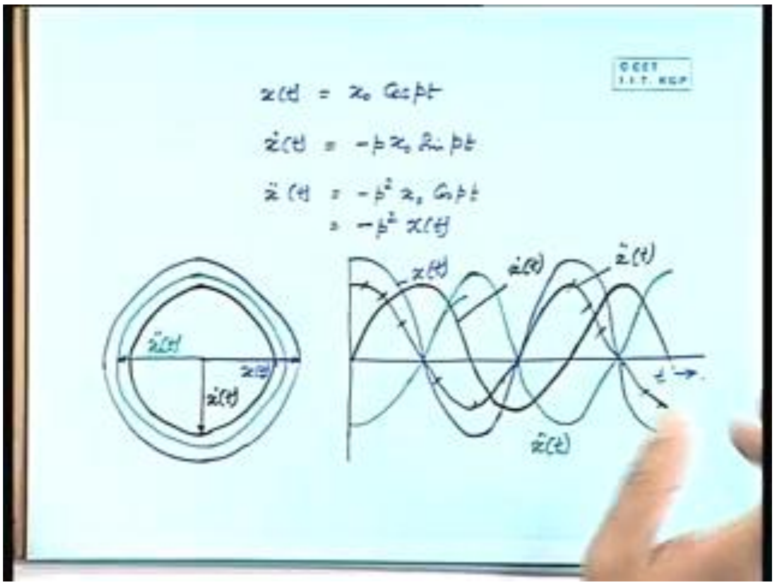

Now, let me write down this expression here xt is equal to say x 0 cos pt. Let me take the first term assuming that x 0 dot is equal to 0. Now, from here we can find out what is the velocity. Velocity is nothing, but d by dt of this and d by dt. This will give you minus p will come out from here and cos pt will give you sin pt. So, this is the expression and if you want to find out what is the acceleration as the second derivative of this, or first derivative of this, this will give you minus p square and if you substitute x 0 cos pt from here. Now, in any equation if you get acceleration is represented by the displacement by some scale factor and it is opposed in the direction, then one can say it is a simple harmonic motion because this is a harmonic term. It gives you that unless this is equal to this, it will not come.

So, that cost pt term repeats here. X 0 comes as the constant, but only thing this minus p square is coming and p is the frequency. So, one can say that p square is nothing, but a scaling factor and negative says that it is directed towards the centre. So, if acceleration works out to be a function of displacement with a scale factor and in the negative direction, then it is a simple harmonic motion. Now, this can be plotted here, and let us see how the plot goes. Our clarity say I take this is one term. So, if you see this diagram, you see that the curves will go in this fashion. I have purposely drawn them separately, so that some scale effect is coming and this is how we can put the equation. Am I right or I am wrong?

I have given a wrong curve just to check whether you are going to pick up the thing or not. You see this is absolutely wrong here and therefore, I should remove it. How this curve will start? You should try to tell me x and x dot x double dot, they are in opposite direction. How the x dot will start? This should be our x dot curve. If we try to use a phase or diagram, then this works out to be something like this. One circle I draw, here another circle, and then the third circle. Is this the longitudinal view of the same waves? No, this is supposed to be a phase or diagram.

Now, suppose this is my x, then the displacement is here. This is what I have written displacement here and this is my acceleration. The velocity is lagging behind by this by pi by 2 and acceleration is lagging behind by pi. So, this will be the phase or representation and this is the useful representation. Now, if you have any questions or clarification here, we may stop for a time and then proceed. Otherwise, we go to the next topic. That is you have any queries here. No. We will have it during the break time.

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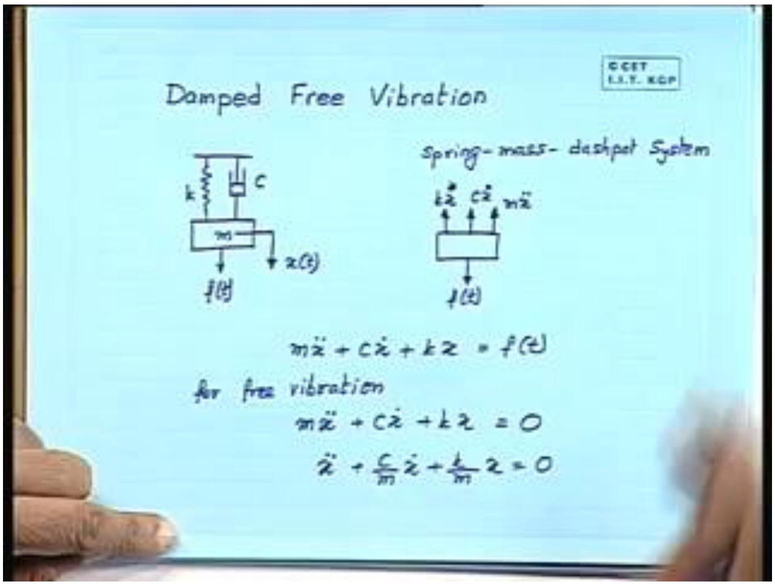

So, we go for damped free vibration. We are talking about single degree freedom systems. So, I am not repeating that over and again and again. So, it is damped free vibration. Now, the model is we saw in practical cases that things die down, and it is always better to consider that damping is inherent property and let us try to mathematically model it. When we talk about damping features, what do we mean by that? We have seen that once the disturbance is given, then it vibrates and disturbance is basically inputting some additional energy. So, the energy of the system increases and due to loses, its stable condition will become unstable, starts vibrating and then through some means or other, the energy will be dissipated and it will come back to the minimum energy level and then the system will again become stationary.

Now, in such mechanical systems, what are the ways by which energy being dissipated? The other day I took a simple raffle example and we saw that quite easily it comes back to the stationary position. Now, the things, which we says the surrounding air. Air is the (( )). So, when the body is moving up and down, it is trying to displace the mass of air which is surrounding it. So, some energy of the system is being consumed by displacing the air or the air column above it or below it which is coming into its way. Then there is an interface between the fluid and the structure. So, some sort of frictional resistance is there between the two. So, to overcome that frictional resistance, some energy is being consumed. So, these are the two things which we cannot see.

Now, as soon as it pushes the air up, immediately down below the object there is a vacuum created and when this liquid is there surrounding some substance, vacuum cannot remain there and therefore, some cross flow takes place, and the air particles or the fluid particle comes and fills it up. So, some sort of a motion is given to the surrounding fluid. So, these are the three major components after energy dissipation. Let us see what are the things happen? All things are made of elastic material and any elastic material for that matter, any material. We know that molecules are there, they are inter-connected and then there are some sort of a bonding is there and so and so forth and then the solid is formed.

So, when we talk about crystalline material then there is some sort of a crystalline arrangement within the element which gives you the material. They are stacked in layer they are stacked in some sort of an arranged the order whatever it is. So, when we are trying to displace the object; that means, the point is getting object and the corresponding crystals are getting moved from its original position.

So, there will be some friction between the two crystals. There will be an intra molecular friction due to which some energy will be dissipated and at the inter phase of those molecules, there will be some sort of a friction and then due to that intra molecular friction, there will be some heat generation, there will be some sound generation. This experiment one can do when we try to break a small piece of wire. We try to bend it up and down like that actually we are giving two end flow motion and what we can see that after sometime, the surrounding areas becomes very hot and you get the burning sensation. If you keep on doing like that, then stage comes when the thing gets broken. That is one of the ways by which we try to separate into two pieces.

Now, why that happens? It happens because of the intra molecular friction. You are trying to take it apart, you are generating the stress in such a fashion that it goes beyond the plastic limits. Strain hardening takes place and all sorts of metallurgical changes. At the same time if you bring it very close to your ear, you can find some sort of creaking voice also. So, some sound is generated, heat is generated which you have felt and therefore, there are number of ways by which the energy input can be dissipated to the surroundings.

Now, the fluids will have one particular law for a mathematical law. For the dissipation of energy sound, we will have another law and the heat will follow some another method and majority of the mechanical system what we do, we try to make use of the dissipation of energy by means of fluid. For example when we design a shock absorber for a vehicle, it is nothing, but we are using a spring to bring it back and there is a piston in the hydraulic plunge. In a hydraulic cylinder, we have a plunger and you fill it up, seal it, fill it up with mineral oils, seal it and then when the compression takes place, the fluid move from one side to the other side and then when the undulation of the road is over, it tries to come back and you have the spring to pull it apart or some such thing, but the motion is retarded.

So, that is how we use the fluid force for damping of the system. So, in majority of the cases, we try to bank upon the fluid and one way fluid is good because it gives a very easy mathematical expression which follows the damping law. So, what we do? The damping of the system is mathematically modeled as a cylinder and piston or in usual way we say it is a dashpot. So, what we say is a spring mass dashpot system and then we use the notation. As it is this is the spring, this is the dashpot connected to the mass and the dashpot coefficient. We consider as seal and we put the axis system once again from the mass enter in equilibrium condition, dynamic displacement pointing downwards like this and acted upon by forcing function.

So, if we draw the free body diagram of this system, this is f of t, the spring force which is pulling it up, sorry kx. There is no eraser here. The damping force cx dot and the inertia force an x double, this is not there. So, the equation of motion we can write now and for free vibration as usual, we will divide all through by m and doing that we get it is. So, once we have done that, then we do little bit of jugglery here.

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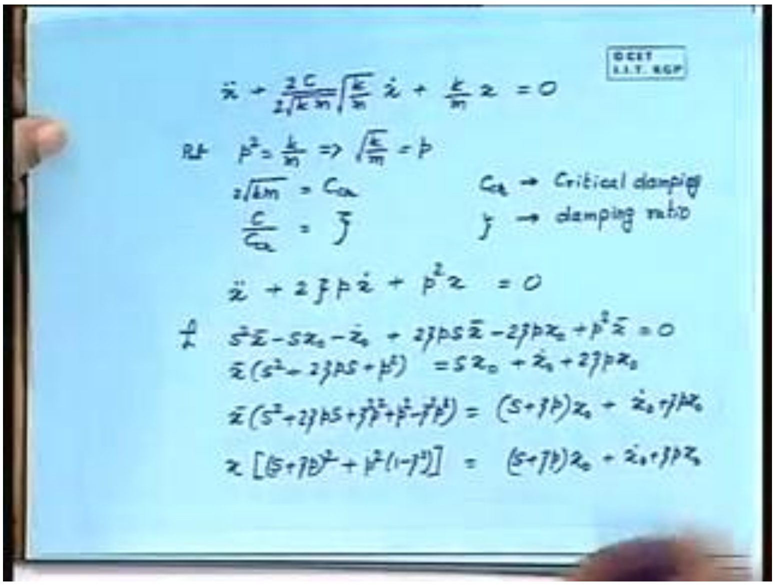

I say jugglery it is a mathematical jugglery because we have to bring the equation in a very decent form, so that we can handle them very nicely. Now, this c by m I will write it like this multiply by 2 divide by 2 multiply by root k divide by root k and split m to root m and root m. So, it is basically c by mh dot. Now, we will make substitution. We put p square is equal to k by m which leads to root over k by m as p naught minus p. We also put 2 root km some quantity ccr and we say that c by ccr some quantity zeta. We will come to the definition of these two quantities. A little later we have already seen that p is the circular frequency. Similarly, the notation, which we are using will also give you a hint, but anyway we will come to it a little later.

So, substitution of this gives me x double dot. Now, this becomes ccr and c by ccr is zeta and this is p. So, this becomes the governing differential equation. Now, let me define it, otherwise ccr is critical damping coefficient and zeta is known as the damping ratio because it is a ratio of two coefficients. One is the structure damping coefficient and another one is suppose to be the critical damping coefficient with the base. So, this is a damping ratio.

Now, we take Laplace transform of this that what I said that will use Laplace transform. Now, the equation is slightly more complicated and we find that if you use this method, then other equations also can be solved very nicely. So, this is the transform of this part. Transform of this part collect the coefficients of x bar and all minus quantity I take on the other side. Now, on this side, I will do this here. You see what I will try to do here. These two terms gives me look of a whole square term. You know this is an algebraic equation and I would like to do some algebraic manipulation. So, s square 2 s. So, zeta p square zeta square p square term should be there, but I have only p square term here.

So, I do not see any problem. I put zeta square p square and then minus zeta square p square. So, I add and I subtract this side. What I will try to do is this x 0 term is there. So, what I will do? Here I will take one of them and the other one I put it along with this. Now, let me write these three terms together and from here I take p square common. So, I get 1 minus zeta square. Now, this side if it would have been 1 minus zeta whole square, it had been much better, but anyway let us see what we can do.

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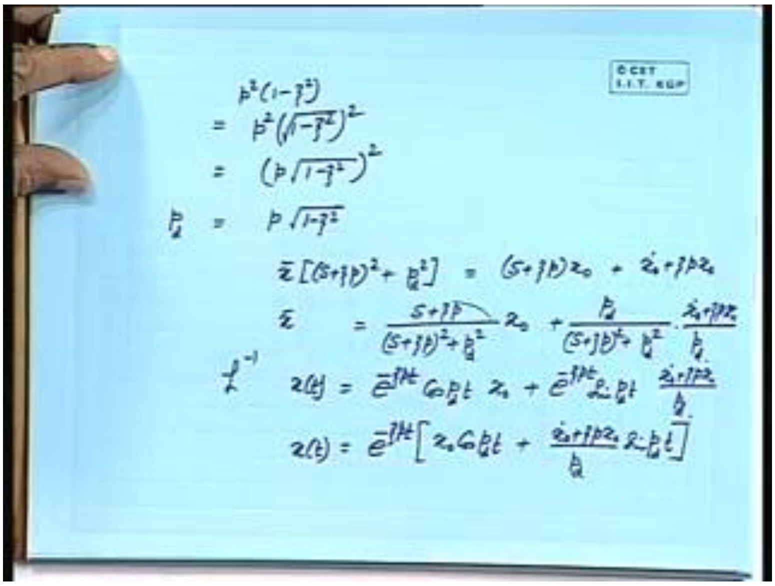

So, I will try to take out that part here, and I will redefine it that this is this and then I will write it like this. So, I want to put it as some whole square term. I have brought it and then I will define that this quantity, this is the natural frequency of the system. This is some sort of damping ratio factor, which is coming. So, the natural frequency is getting reduced by some factor. So, after all this is a frequency only, and it is getting reduced because of some sort of a damping present in system and therefore, I redefine it and I say it is the damped frequency. Pd is damped frequency.

So, with this definition I rewrite my algebraic equation and then this takes the shape like this. So, I want to take the response and therefore, I divide by this part here. Here what I do? I multiply by this and then I divide it by that quantity. Now, I try to take the inverse of this. Inverse of this will be the response x of t and this response will be e to the power minus zeta pt and this will give me cos p dt. Similarly, this will give me e to the power minus eta pt sin p dt. I will just write it in a neat manner. This term is there. So, this becomes the expression for the response.

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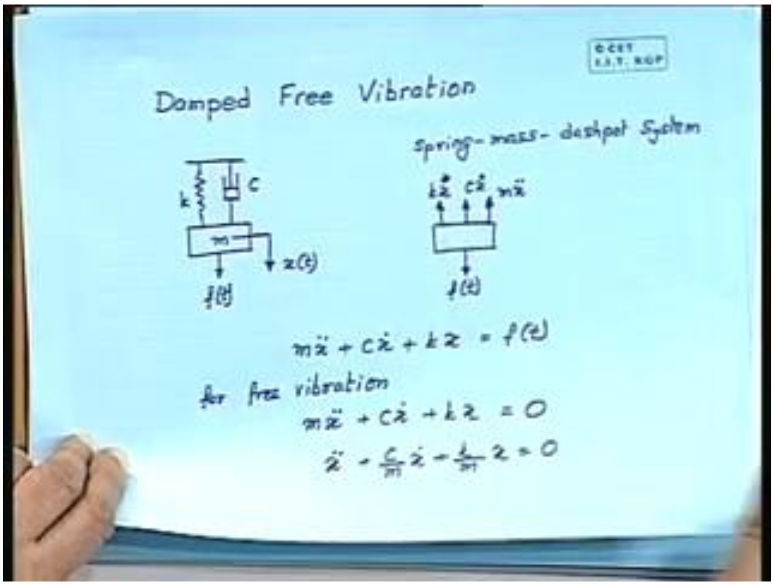

Now, you see from here we wrote down the equation in the usual manner without doing any hassle. I did a little bit of algebraic manipulation here and by substituting all these quantities, we had written another simplified equation here. Then we have taken the transpose. Sorry, the Laplace transform and rearrange the old terms, put the means, some sort of an ordered manner. So, this is what we have tried and finally, we have got this expression here.

Now, we can say seeing this expression here, zeta is damping ratio c by c critical, where we have defined c critical to be 2 root km, k is a physical quantity, m is a physical quantity. You can have any numerical value for any mass. Even the material chosen will have some sort of a damping coefficient. So, these are all realistic values, but we do not know how they will behave. So, with this expression, what we can say that you can divide into some cases.

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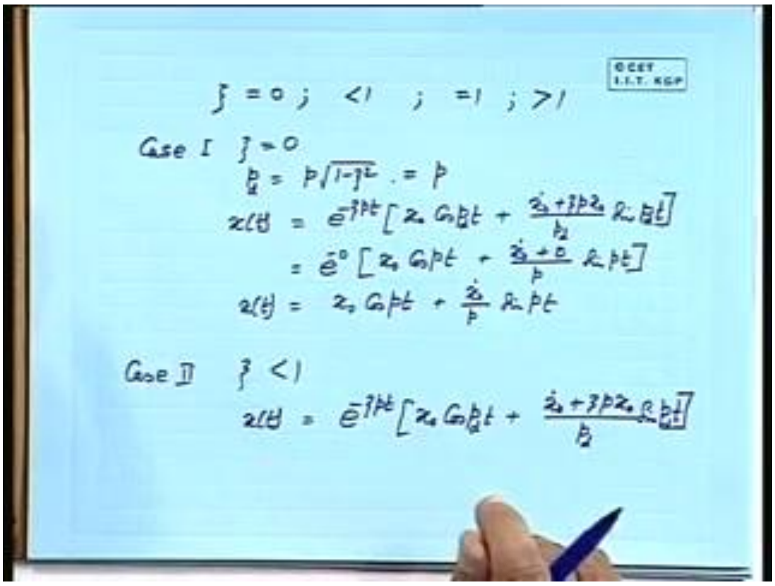

You can say zeta can attain a value of 0. It can attain a value which is less than 1, it can attain a value which is equal to 1 and it can also attain a value which is greater than 1. So, we can define these four cases and try to see that under these four cases, how the system behaves and what response we get. So, let us take the case 1. Zeta is equal to 0. As soon as we put zeta is equal to 0, our definition of pd which we are saying is p 1 minus zeta square, we will find that zeta being equal to 0, pt is equal to p and substitute in the expression xt, e to the power minus zeta pt x 0 cos p dt x 0 dot zeta px 0 by pd. Now, you put pd is equal to p, what we get? Zeta is equal to 0. This is p, this is t same as free vibration response.

So, zeta is equal to 0 means undamped condition and we should have got that response and we are getting it. Then let us say case 2 less than 1. So, what happens is this form remains as it is and therefore, x of t. So, this is the form which remains now under this form what happens.

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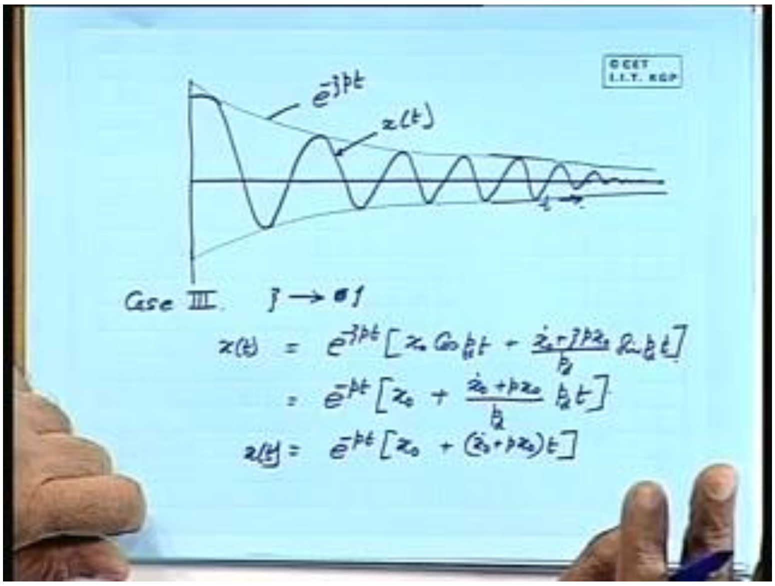

Now, when I use my pen for, so the response x of t is given by this whereas, this envelop is nothing, but governed by this exponential decay function. So, this is the exponential decay function which is being governed by e to the power minus zeta pt, and this is your response function and this explains that why majority of the system when it is disturbed comes and attains the stationary position after few cycles of vibration.

Let us also see what case 3 is. Case 3 is zeta equal to 0. Zeta equal to 0, but mathematically we do not write zeta equal to 0. We will say zeta tending to 0. Now, let us see what happens. Sorry, if zeta tends to 1, this tends to e to the power minus pt. What happens to cos p dt? Pd is equal to 1 minus zeta square into p. So, if zeta tends to 1, then under the square root term will tend to 0 and therefore, cos pdt will tend to cos 0 and cos 0 will tend to unity. So, x 0 into unity x 0 dot as it is zeta is tending to 1 is there. X 0 pd why I have taken tending to 1? It is because of this term here. This is not tending to 0 here. So, let it be in this form. Now, when pd is tending to 0, what happens to sin p dt? This will disintegrate to only p dt’s sin theta is equal to theta is equal to tan theta.

Now, these two pd will cancel off and the limiting case and we write. So, in the third case when zeta is tending to 1, zeta is equal to 1, we find that in the final expression for the response there is no sin or no cos term here, nothing of that sort. This is not an oscillatory term, but e to the power minus pt is there. So, there is an exponential decay term present in the system, but the motion becomes non-oscillatory. So, as soon as zeta is tending to 1, the system is going to obtain from oscillatory motion to a non-oscillatory motion, but exponential decay is there.

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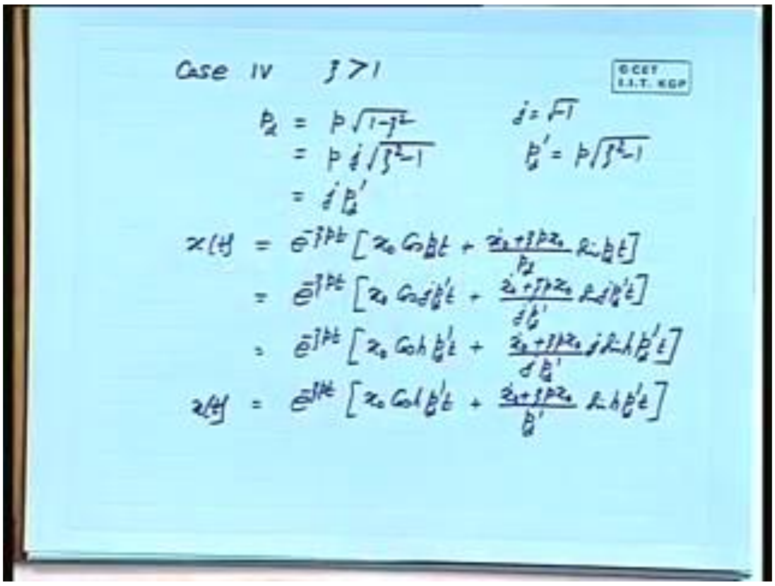

Case 4, we say that zeta is greater than 1. Let us write the expression for pd zeta is greater than 1. That means, inside the bracket there will be a negative term square root of that imaginary quantity. So, I write and I redefine and therefore, I put here. It is j is nothing, but let me put j is equal to root minus 1. Now, let me take that response expression pd is to be substituted by j pds. Pd is equal to jp dash. What is cos jp? Jp dash t is nothing, but cos hyperbolic p dash t. This term comes out as it is. I do not say anything here. What is sin jp d dash t? It is j sin hyperbolic pd dash t 1. J is coming out from here, another j is there. So, jj will cancel out. See that imaginary quantity j is out from the expression.

Now, it is a physical condition. You can have a system in which you can expect that the damping ratio is greater than 1 and therefore, you cannot have a damped frequency or the response in an imaginary way, but what does this expression say. This expression again says that this is a non-oscillatory motion, hyperbolic terms or non-oscillatory terms. It is an exponentially decaying term and non-oscillatory and therefore, even if you have frequency as an imaginary frequency, it does not vibrate at all. You give a displacement and displacement decay down to 0. So, this explains you mathematically of course one will find that these expressions are bit complicating, but if you really go through that thing, it is not that complicating, but it explains what is what.

So, what happens here is that if we are considering a damped vibration case, then damping must be a light damping. You design a system in which you would like the system should vibrate by damp. So, the damping must be a light damping. It has to be less than 0. It cannot be 0 or greater than 1. It has to be less than 1 and therefore, we try to design the damper wherever is required which we say is a light damping coefficient. You say 5 more minutes. So, should I explain something else or I will take up after 5 minutes break?

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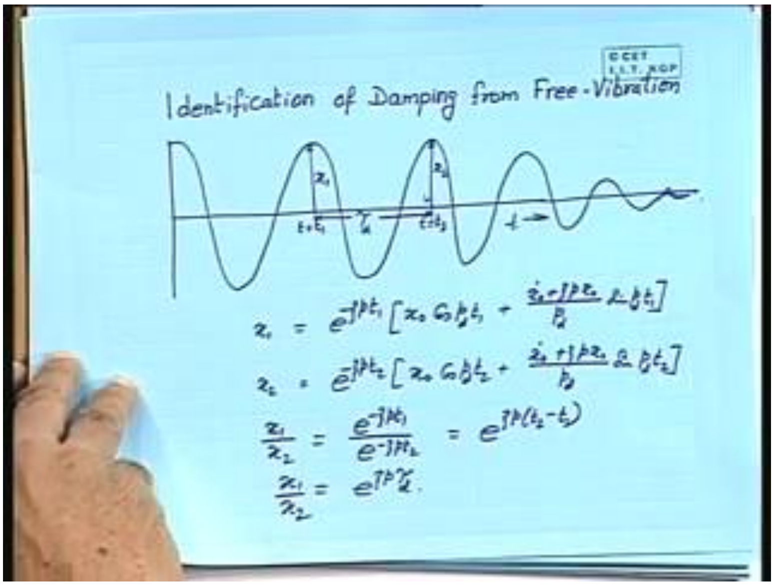

Next we will try to see identification of damping from free vibration record. Now, one thing we have realized that structures do have some sort of damping in it and therefore, the response will decay down. Now, how much of damping is there, how do you measure it because once we have made one gross assumption that we are replacing it by a dashpot, all the system do not follow the same, but we have to assume that. So, now let us try to take the response. So, some response is there and it is decaying down. So, how do we try to find out what is the system damping coefficient here.

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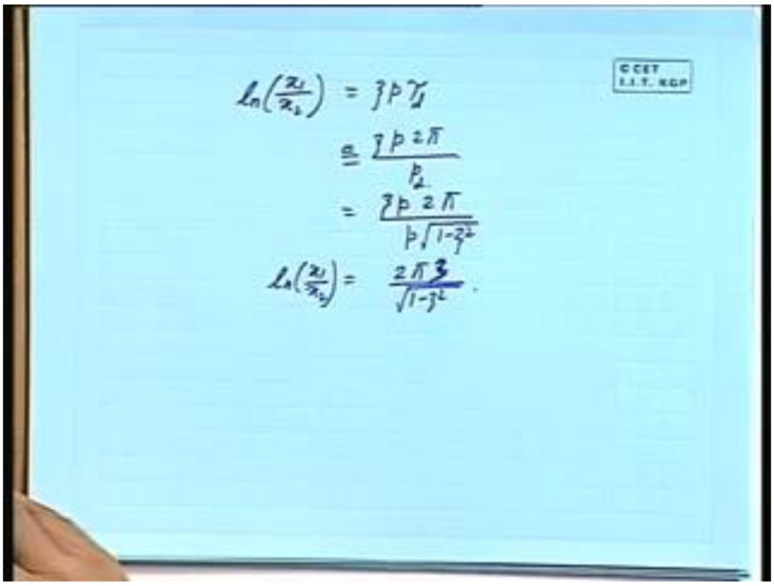

So, now, let me assume that this is the maximum amplitude at t is equal to t 1, and we say that this amplitude is x 1 and at t is equal to t 2, the amplitude is x 2 pi. Then frequency is equal to p by 2 pi and time period is equal to 1 by f. So, if I write the reverse order. So, p will cancel out and I can write 2pi zeta by 1 minus zeta square. How do I get zeta from here? This is a non-linear expression and therefore, we have already said that zeta has to be between 0 and 1. So, we will make one assumption for lightly damped system. We will say zeta square tends to 0.