Strength and Vibration of Marine Structures

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Lecture - 6
Deflection of Structure Beam - 1V

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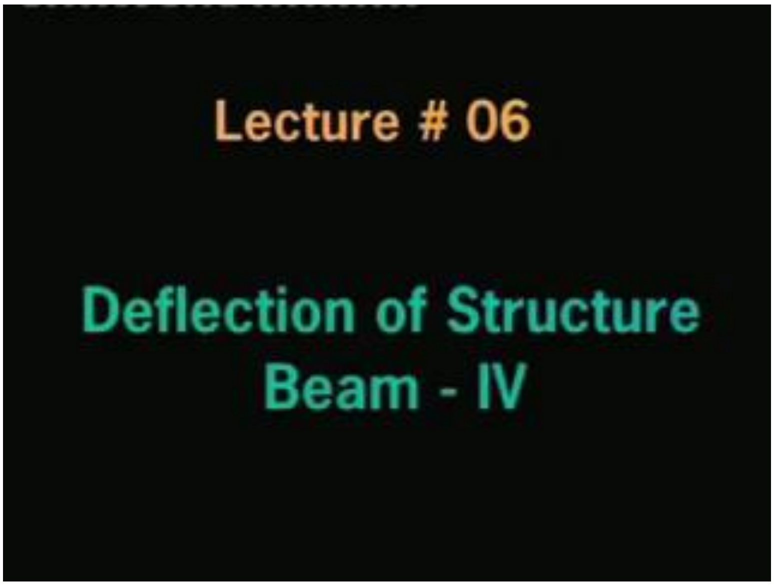

So, we are talking about moment-area theorem, and two cases - simple beam problem, cantilever beam - with some deep load and maximum deflection slope, we have calculated and tried to compare our values with those obtained by your direct integration of the differential equation. And you must have that feeling, that it is much more easier compared to the earlier approach. So, it is not a very general statement, for that problem, it might be easier, I think. So, it depends; problem to problem, you have to choose the method in a appropriate manner. Sometimes the direct method may be much more easier or convenient to get the solution.

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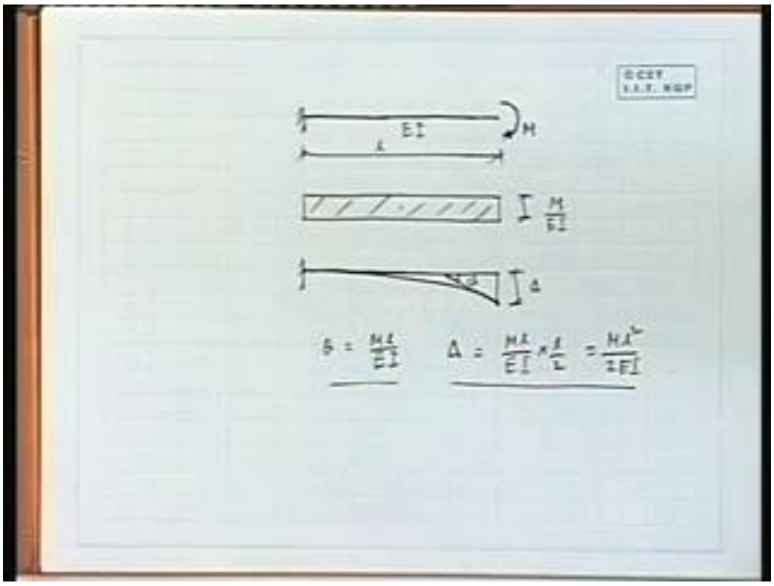

Now, I will take another problem of a cantilever type of beam, because the values are quite important here. So, length is l, and again it is EI. Here, there is a moment acting at the end, say, this moment is a M. Now, for that beam, you can draw the bending moment diagram. So, it is M. Throughout the beam, bending moment will be M or if we divide by EI, we will get the M by EI diagram on the beam. Now, if I draw the deflected shape of the cantilever. So, here, there is one advantage - this point is fixed. So, tangent at that point is the initial line. So, that we have already mentioned. So, at the free end, if we draw a tangent, so both that tangent will have a difference in slope, that will be theta. And incidentally, that value will be the actual slope of the point at the free end.

So, it is what? It is basically area of that curve. So, it is l, it is M by EI. So, it will be simply Ml by EI. And delta is, from this point, if we move in a direction perpendicular to the original line, it is meeting that tangent drawn at that point here. So, that is delta. So, we are moving along this line. So, about this point we have to take the first moment of area of this M by EI diagram. So, it will be ranging from here to here, moment about that.

So, area already we have written Ml by EI. And its centroid will be at the midpoint. So, that will be l by 2. So, it will be Ml square by 2 EI. Now these values are quite significant values. Like the case of a cantilever subjected to a load and the free end, we have written Pl square divided by 2 l or Plq divided by 3 EI. Similarly, if there is a moment, it will be Ml by EI or Ml square by 2 EI for deflection and slope.

Now, cantilever type of problem it is very easier to handle, because one of the end is fixed, and there we can draw the tangent, and that will be identical to the initial line. So, at any point, if we draw a tangent, difference between the slope of this tangent and this second tangent is basically the actual slope of that point; deflection also directly you are getting in a similar manner. But if you take a simply supported case, problem is little different.

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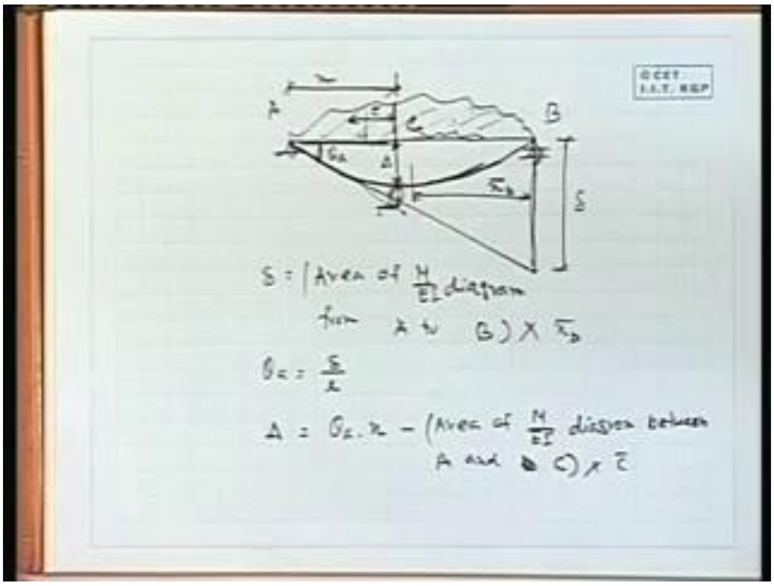

Now, how will we handle a case of a simply supported type of problem? Now, it will undergo some deformation like this. It may be some loading. Now, if you are interested to find the slope at the supported end or deflection at the middle of the span or some intermediate point, just like cantilever, we cannot get it; but here, more or less we will follow that type of procedure.

About this point, suppose if you draw a tangent, it will go like this. Now, here to here, we can calculate. What we are doing? This is the deflected line and from this deflected line this is a typical point on the elastic curve. From this point, we are moving along a path normal to the initial line and it is meeting the tangent drawn at that point. So, here to here, it is basically, it may have some M by EI diagram over it.

So, we have to take the M by EI diagram from here to here, about this point we have to take the amount, that will give this value. So, if the area is this one, and it has a centroid, a distance is this, so this area multiplied this by the centroidal distance from the right end, that will be the value of this. And once that is there, that divided by l will be the slope here. So, we can say this is your some value delta. So, delta is your area of M by EI diagram, from A to B. Say it is A; it is B. So, that multiplied by, say, x bar. So, x bar will be this; this will be x bar; say, we say it is x bar B. So, x bar B is centroid of the M by EI diagram from your point B and that will be the delta according to our second part of the moment-area theorem.

Now, if delta is there say theta a. So, theta a will be straightaway delta by l; l is the length of the total beam. Now, at some intermediate point, if we want to find out the deflection, it can be calculated like this: theta a we can calculate, similarly, that side we can draw a line, and this side we can find out something, that divided by l will be theta b. So, end rotation we can find out.

Say at some point intermediate point here we want to get the deflection; this deflection we are interested in. Now, this is a point. So, from this point we can move vertically downward, perpendicular to the initial line. So, it will meet the tangent drawn on a. So, this part is basically about this point we have to calculate the first moment of area of this diagram; so, M by EI diagram; here to here, about this point if you calculate the first moment of area, you will get this part. And this slope into that length you will get the full part; this theta into this one will be the total length. So, total length minus this length will be the actual deflection.

So, if we are interested to find out deflection at a distance of x, so this will be the actual deflection. So, actual deflection that will be your theta a into x; theta a into x will be the here to here; theta a into x, minus this part we have to put there, so that we will get the actual deflection. So, here to here it will be the actual deflection. So, it will be minus your area of M by EI diagram, between A and say it is C, this point is C, distance is x, into, say, here it will have some centroid, and that centroid, that distance, there might be something say C bar; so, it will be C bar. So, that part is only the small part and this part is the total component. So, if you make it minus, the effective value what you are interested at any point you can find out.

So, moment-area theorem in cantilever form we have tried to explain, in a simply supported condition, how we will get deflection at some intermediate point; it may be mid span or some other point; you can calculate in this manner. But initially you have to find out the slope at the two ends. Say, you have to calculate some small delta in this form, that divided by l will give the slope at the end. So, we will get the initial slope; from there if we follow that procedure, deflection at any point you can find out. We have already, I mentioned you, it is not the only method, there are different methods; you have to apply your method according to the type of problem. Now, we can take one example and try to utilize some of the standard values to get the deformation of a structure.

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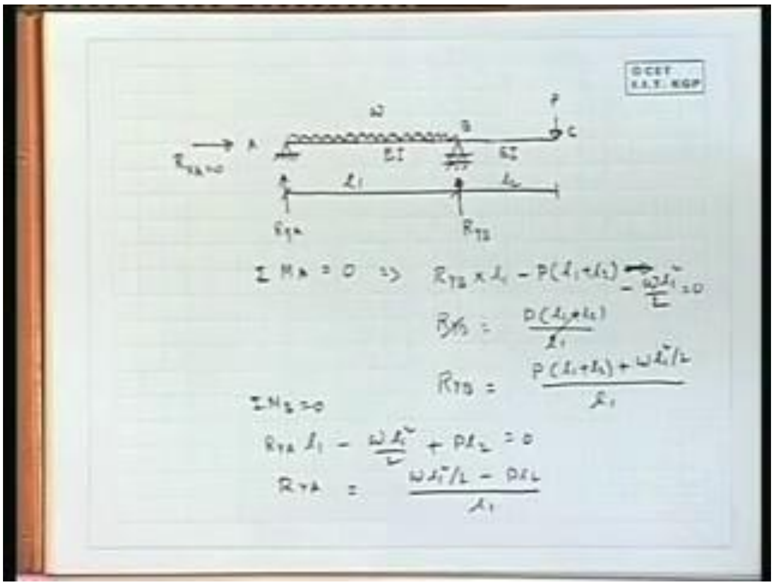

So, the beam is like this ABC; at A there is one support - simple support; B another simple support; C is free. So, free end, there is a load P. A to B it is fully loaded with some uniformly distributed load; intensity is omega. AB length say it is defined as l 1; BC it is defined as l 2. For simplicity, entire beam having a identical cross section EI. Now, for that beam we are interested to find out the form, say, for the deflected shape of the beam.

Now, here beam is a statically determined beam, because you can find out the support reaction. Here there are two components of reaction, because it is hinge, one is vertical, and another horizontal; here one vertical; and horizontal part will be 0, because the external loads are all 0. So, there are two quantities; two quantities you can find out summation of moment, summation of vertical forces. Now that part is basically our first job work. So, we can say there will be a reaction; it is RXA - it will be 0; and here RYA - that we have to find out; and here RYB - that also we have to find out. Now this is a problem, straightaway you cannot say RYA will be this, RYB will be something like this. Now, you can take moment about A of all the process, so it is moment about A, due to all the process summation it will be 0. So, A means both the process will cancel. So, from here we can say RYB into your l 1 minus P into l 1 plus l 2 that will be equal to 0. So, RYB will be Pl 1 l 2 divided by l 1.

The uniform load - that part - we have not taken I think. So, it will be additional problem. So, here, that part also we have to consider. So, we cannot write here 0. So, that additional part will be minus L 1 square divided by 2; that will be equal to 0. So, RYB it will be Pl 1 plus l 2 plus omega l 1 square by 2 divided by l 1. So, if that is the value, summation of RYA and RYB will be equal to the total force. So, from there also we can bring or you can take another moment of about BS, whatever you like. So, I think if you take moment about B, we will get the another force. So, here this will be 0; automatically it is 0. So, here it will be RYA into your l 1 minus omega l 1 square by 2 plus P into l 2 that will be equal to 0. Say RYA if it is clockwise; P is also clockwise. So, both are positive; this is negative. So, from here, RYA, we can write omega l 1 square by 2 minus Pl 2 divided by l 1.

Now, the reactions we have determined or evaluated in the form of your l 1, l 2, omega, and P. Now, it depends which method we will apply. In some cases, it is necessary you have to find out these reactions. In some cases, it may not be necessary. But how it can be determined? That we have shown it. Now, here there are two loads: one is P; another is omega. Omega is acting from here to here, fully distributed, for this entire span and these two loads are acting on the entire structure, and we want to find out the deformation of that.

Now, once the reactions are available, due to this P and omega, we can find out the expression for this span. Expression for the bending moment, for this span, because RYA into x omega x square by 2 will be the bending moment here. Here also, you can find out the expression for the bending moment. And you can apply your differential equation technique. So, there will be one segment, another segment; load is started means we have to apply throughout, then we have to add something, in this range one equation, another equation step function, then two constant, we have to put boundary condition.

So, if you start from there, you are basically try to solve the problem from your basic equations. You need not remember anything, start from your initial equations; or other option is some of the standard parameters already you have derived and those values, if it is in our mind, so we can utilize that, and try to get the solution of the problem. Now, that is utilizing some of the known parameters; some known information.

Now, here there is a technique called method of superposition. Method of superposition. The method of superposition is the effect of this and effect of that, it can be combined here, because our analysis is a linear type of analysis. So, we are assuming stress is proportional to strength. So, stress strength-relationship is falling in a linear manner. And our deformations are very small. So, whole problem is a linear problem. So, if it linear problem, means, load and deflection relationship will follow a linear curve. So, if the load is 100, say, only this load is acting, whatever slope we will get, if we make the load 200 instead of 100, slope will be just double; means 100 plus 100 effect will be just added; if it is 300 it will be 3 times. So, all the effect will be simply added with this. So, effect of w and effect of P, if we can study separately, put the deformation, we can simply superpose or simply algebraically add to get the total effect. The reason is load deflection curve is linear. So, this is called your method of superposition.

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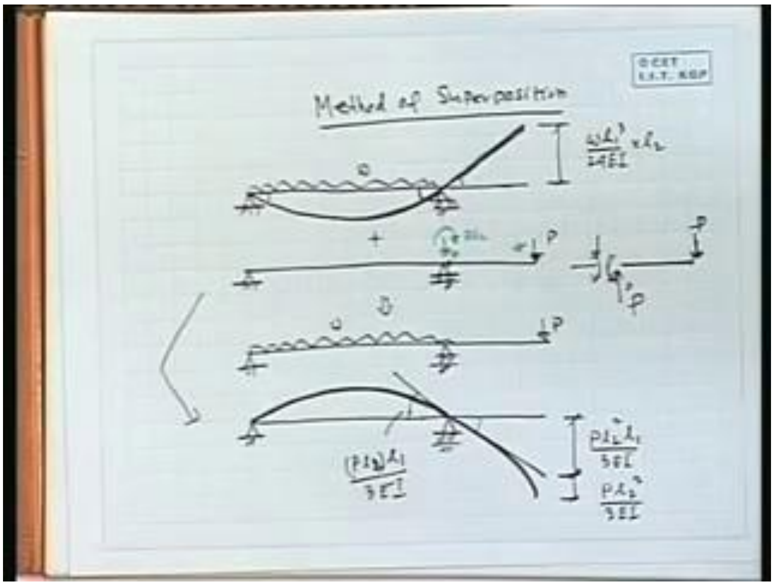

So, method of superposition is valid for a linear system and the type of problem we are handling it is a linear problem. So, if we have a number of loads or system of loads, we can get the response of a load or a system of load, and get the response of second load or a second system of load. If we combine or algebraically add both the responses, it is the response of the structure when it is subjected to both system of load or both are loaded. So, that is one of the principle; we said is principle of superposition, sometimes method of superposition, that we normally apply here.

Now, we have started with this problem. Now will take the effect of w in one step; second step we will take the effect of P; and we will combine, to get the total effect of P as well as w, when they will act in a simultaneous way.

Now, let us take the first case; say it is only w. So, this is the actual problem. That can be split into two forms: one with a load; another with a distributed load. So, response of that plus response of that will be the response of this structure. So, that we are trying to define as a principle of superposition or method of superposition. So, we will utilize that to find out the separate effect and combine.

Now, if we take this w, what will be the type of deformation? So, here to here it is a simply supported beam, under distributed load, and decide there is no load. So, it will be a deformation like this. And we have that information; already we have solved that problem; and this part will be, it will just follow that line; there will be no bending of this member.

So, here to here it is a simply supported case with full load. We have the idea what will be the maximum deflection and what will be the slope here. And here also it will be the same slope. And there is no bending. So, this slope will be continued; there will be no deformation of this member. Straight. So, this information we have already in the form of maximum deflection and maximum slope. So, slope into that l 2 will be the deflection here and slope here.

Now, this part. This part there is a simply supported force and without any load. And there is a part extending beyond the support and there is a load here. Now, this load, we can shift from here to here. So, it will come as a force and a moment. So, if I shift the load , so its equivalent part will be P into P into l 2 is a moment. So, instead of putting the load, we can shift the load here as P plus a moment. Now, this P will be consumed by the support, because it is on the support. And this moment is a moment acting at the hinge of a simply supported beam. So, it will be a simply supported beam problem with some moment. Now, this problem if we just redefine or if we draw the deformed shape, so due to this P and the moment Pl 2. So, P, it will not contribute anything, because it will be just possible by the supporter, only this moment part. So, there is a moment means, it will try to rotate like this.

So, this P if I shift here, P into moment, P will not do anything, only the moment part will give a slope here. And this part will be simply extended. Now, I have done only one violation, the P part we have shifted from here to here. Now, regarding this part, you can put the load here or here with a moment is a same thing, because it is a theory hierarchy. If we just cut here, so it will be a portion, there is a P and reaction will be P, and there is a moment. And that basically we are taking it here. And due to this part there will be a additional deformation as a cantilever. So, there is a load about this point, there will be a bending like a cantilever.

If I cut here will get a reaction and a moment, so this extended part is just like a cantilever. So, it will have a cantilever type of deformation, but this is not fixed. So, whole thing is rotating, say, a cantilever's support is there; support is yielding. So, there will be a normal deformation of the beam plus the support will yield. So, whole thing will be yielded by certain extent. So, that part is basically the yielding of this part.

Is it clear this part? This part. No second part. Left part.

Because it is a simply supported; it is not fixed. So, this is a continuous member, it is just supported.

So, it is resting on the top.

Yes, it is just resting at the two points. So, if you try to put the load automatically there will be a bending of that. So, right part if we consider, separately, so here I have drawn it. So, there is a load P. So, if we cut here to balance that load there will be a P and there will be a moment M. So, if we cut that part as a reaction there will be a force here; this is upward and this will be downward; and that will be that. So, this part I have basically drawn here; or you can think - this part is strong part, it is a rigid - what will happen? There is a load; this part will be rigid one. So, this load will be simply shifted P into this one; this part will undergo deformation or this part is rigid. So, this part will not deform; under this load, this part will be only bend. So, when it is rigid, this part will deform; this is this part. When this is rigid, this part will be this one. So, both the part are flexible; it is not rigid; this part or that part. So, if you combine, you will get a combined deformed shape of the structure.

Now all these information are known; most of the information; say, this one already we have calculated. If you go through your earlier note, you will get a beam with a fully distributed load, what will be the slope? It is already there in a note or I can supply the value from here. It is beam fully loaded; it will be omega l cube divided by 24 EI. So, this is a same slope. So, what will be this deflection? Omega. So, your slope is omega l cube means l here is l 1 cube divided by 24 EI. That will be the slope into l 2; that will be the deflection.

If we are interested for finding out the deflection only, we can find out slope or other parameter in some other place. Now, here this slope, this slope or if I draw a tangent, this slope will be how much? So, it is a simply supported beam; if we apply moment at one end - this is one of the standard value - so, here, the value is given this is ml by 3 EI. So, it is, what is M? M is your Pl 2 into your l 1 3 EI. So, it is P Ml by 3 EI. So, M here is P into l 2 and here l is l 1 divided by 3 EI. So, here to here, this value will be Pl 2 l 1 divided by 3 EI multiplied by l 2, means another l 2 will come here.

And this part it is just a cantilever type of case; there is a load P. So, it will be Pl 2 divided by 3 EI. Pl cube by 3 EI. So, l is, l is equal to this 3 EI, and this part is this theta into l 2; and this part is theta into l 2. The idea is we have some information and we try to explore all those standard values in different places.

Now, in a similar manner, if we can take some other problem, and tick the different loads separately, and try to utilize the standard values, and get the effect for, say, displacement we have tried to find out. Say, if we are interested for slope at that end. So slope at that end is same slope, and here slope is this slope plus here there will be some slope, that we can calculate; it will be Pl 2 square divided by 2 EI.

So, in that manner, we can get the information about the slope; go on adding; we will get the final deflection, final slope. It may be at the free end; it may be at this point; it may be here; it may be central deflection; or some other parameter.

So, we have tried to show you with some simple, simple problem and tried to find out some standard values. Those values can be utilized in this manner to carry out a practical type of problem with some complex support and load system. Now, we can take up some numerical example in our tutorial class and go ahead with different type of situations to have a better understanding of how we can tackle different type of situations case to case. I think with that we can conclude now.