# Marine Propulsion

**Prof. Anirban Bhattacharyya**
**Department of Ocean Engineering and Naval Architecture**
**Indian Institute of Technology, Kharagpur**

## Lecture - 12
## Dimensional Analysis and Similarity

Welcome to lecture 12 of the course Marine Propulsion. Today, we will discuss Dimensional Analysis and Similarity in the context of Marine Propellers.

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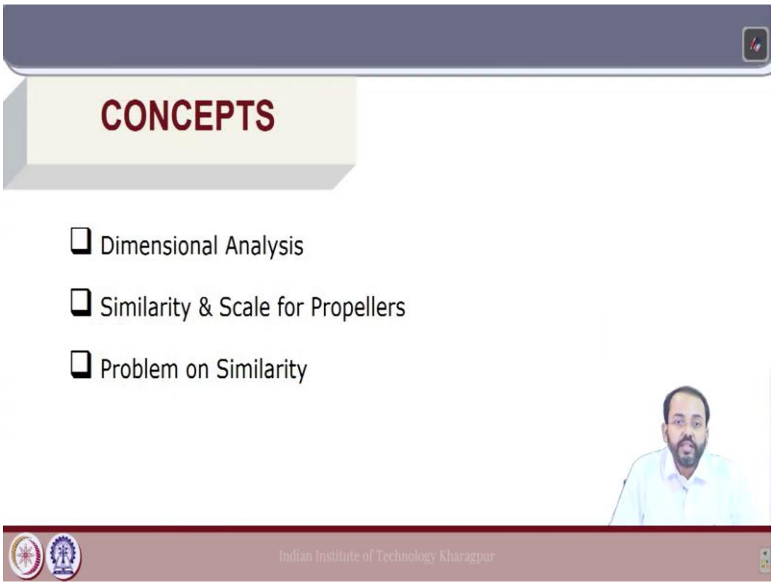


So, the key concepts which will be covered today are dimensional analysis for propellers, similarity and scale and how they are related to the non-dimensional numbers or the coefficients, and a problem on similarity for propellers.

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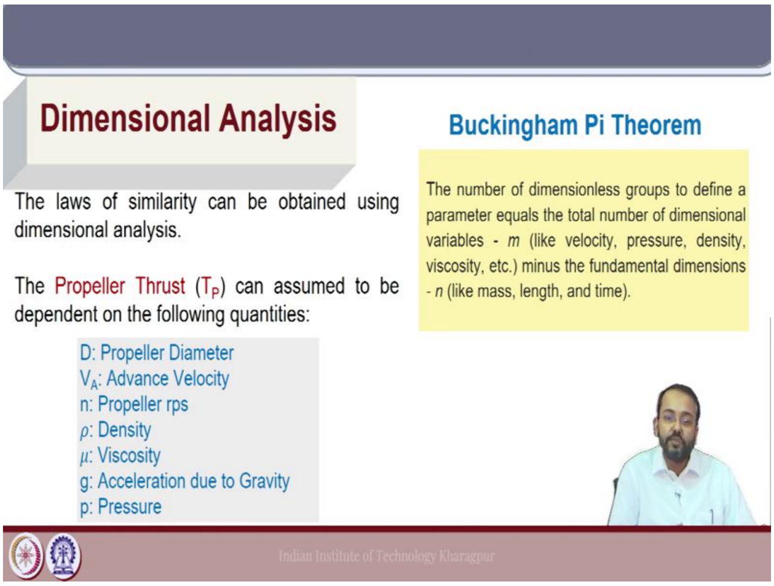


For marine propellers, it is of importance to relate the model scale characteristics to the full scale with the use of certain dimensionless parameters, where we use the laws of similarity to do a dimensional analysis. So, we will use Buckingham Π theorem here, which is a popular theorem used in engineering as well as mechanics problems to relate a parameter with a number of dimensionless groups.

So, if a parameter can be defined with respect to many dimensional variables for example, velocity, pressure, density, etcetera, this Buckingham Π theorem states that the number of dimensionless groups that can be formed with these parameters. Let us say m number of parameters is m minus n, where n is the number of fundamental dimensions which can be used to define a parameter. Fundamental dimensions mean basically mass, length, time which are used to define the parameter.

We will see how we apply this concept in the context of the propeller thrust coefficient with an example. So, if we think about the propeller thrust, what does it depend on, what are the quantities that the propeller thrust depend on? It depends on certain quantities, as we see here, some of which are constants and some of which are variables.

For example, the propeller thrust depends on propeller diameter, the velocity of advance, the propeller rotational speed, density, viscosity, acceleration due to gravity, pressure, all these terms. So, these are discussed in the context of propeller open water diagram. So, we do not use any ship terminology here we are only using the terms which are related to the propeller.

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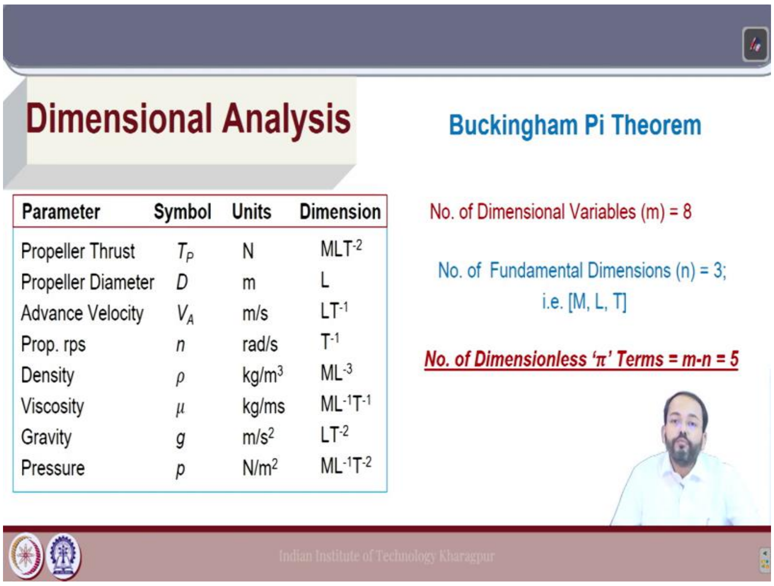


So, we have all these terms and we look into the dimensions of these terms. So, what we will do is to use Buckingham Π theorem to get propeller thrust and torque coefficients as a function of all these parameters.

So, according to this theorem, we have a number of dimensional variables which is 8. We have all these variables which we want to link together to give the coefficients. And number of fundamental dimensions are n equal to 3, mass length time. So, we will have dimensionless terms, number of π terms is m minus n is 5.

So, we will try to calculate for each of these terms and we will use that to get the propeller coefficients.

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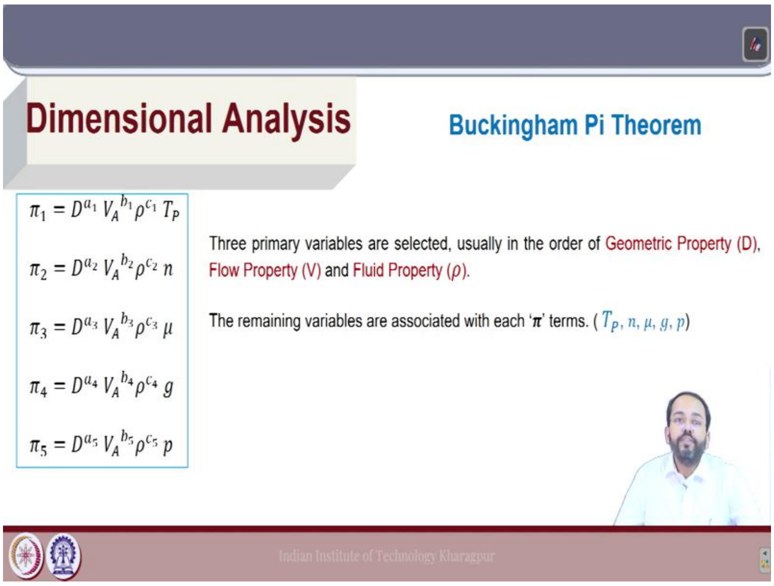


Let us look into the first dimensional coefficient $\pi_1$, where we will have 5 terms and each of them raised to a certain power which we will try to solve using these similarities.

So, similarly we will have $\pi_2$, $\pi_3$, $\pi_4$, and $\pi_5$, the different dimensionless terms. In each of them, 3 primary variables are selected which consist of a geometric property which is the diameter, a flow property which is the velocity of advance here, $V_A$, and a fluid property which is the density ($\rho$). And we combine them with the remaining variables in the $\pi$ terms which are the propeller thrust, n, μ, g and p. And in this way, we have constituted the 5 $\pi$ terms.

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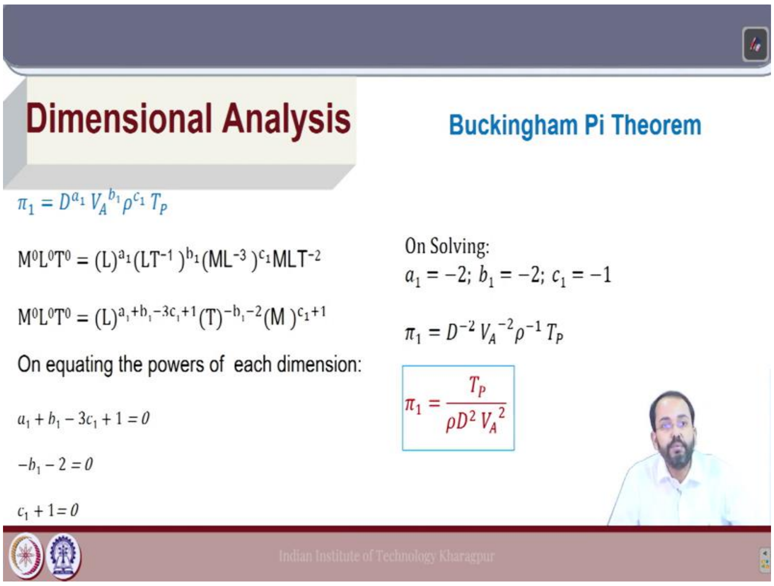


Now, what we will do is each of the π terms will be solved for and then we will try to combine them together to get the thrust and torque coefficients. Now, let us look into the first π term. Each of these π terms are non-dimensional by definition. So, we will do a simple dimensional analysis of the main terms, so that we have the mass, length, and time for the π term which are 0 because it is non-dimensional, and take all these terms D, $V_A$, ρ, and $T_p$ raise to their respective powers, we will solve for them and get the values of the coefficients $a_1$, $b_1$, and $c_1$.

So, we have these equations and unknowns, and on solving we get the values of $a_1$, $b_1$, $c_1$ and write $\pi_1$ as a function of the thrust, ρ, D, and $V_A$, ok. In a similar way, we can do dimensional analysis for the other π terms. So, the basis here is that each π term consists of a number of dimensional terms like D, $V_A$, ρ, and T. Each of them raise to certain powers, but the combination of them is dimensionless because π by definition is dimensionless. And in this way, we try to get these different π values based on the relations between these parameters.

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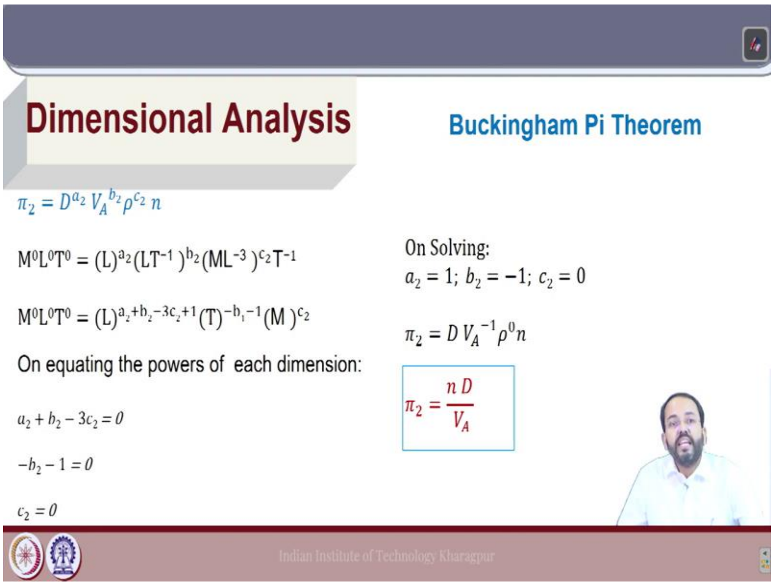


Similarly, if we solve for $\pi_2$, here we have the additional term other than D, $V_A$, and $\rho$ we have n as the 4th term. So, in a similar way we use the indices $a_2$, $b_2$, and $c_2$ for D, $V_A$, and $\rho$, and do a dimensional analysis based on the basic dimensions and equating these powers we get in a similar way $\pi_2$ as $nD/V_A$, ok.

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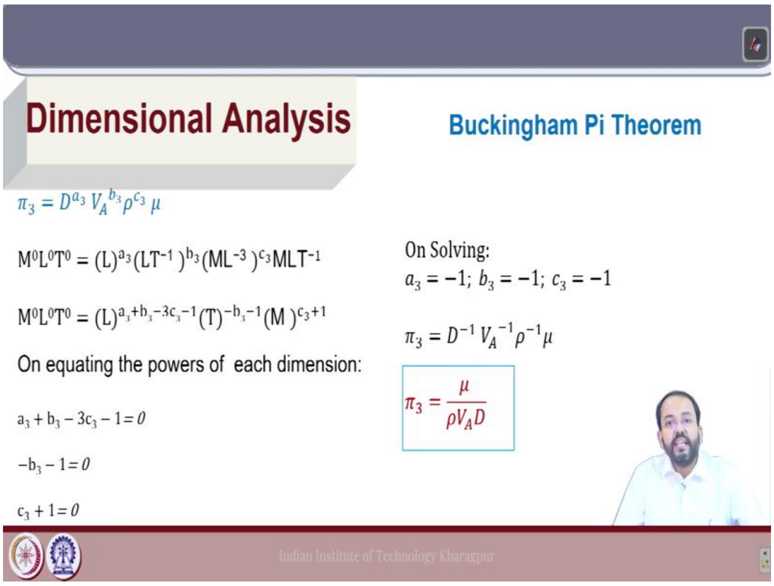


Again, we do the same thing for $\pi_3$. We have to keep in mind that D, $V_A$, and $\rho$ are kept same the first 3 terms, but the powers of them will be different because the $\pi$ terms are different. So, we solve for $\pi_3$ in the same manner.

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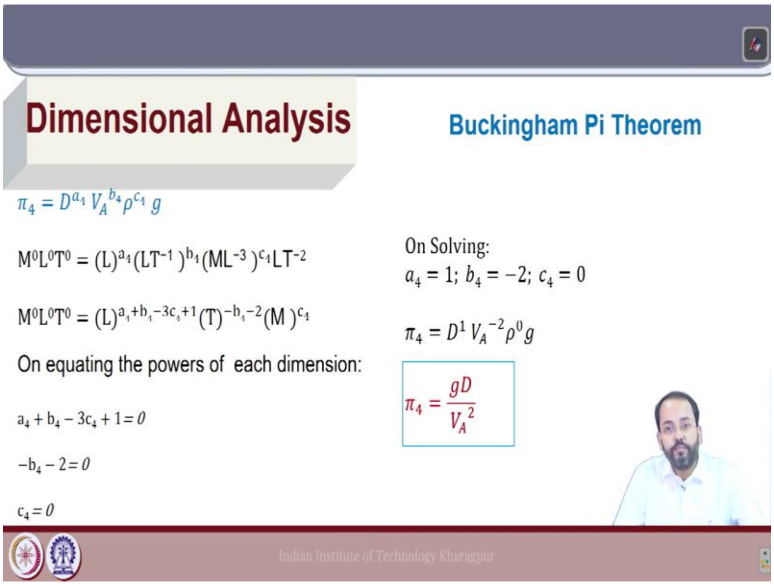


In $\pi_4$, we have g as the 4th term. In the same way we solve for $\pi_4$.

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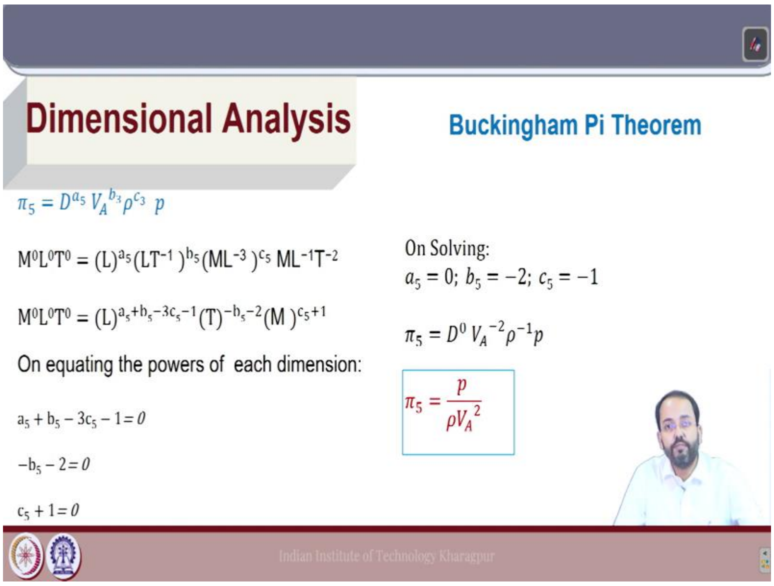


And followed by $\pi_5$ which involves pressure p as the 4th term.

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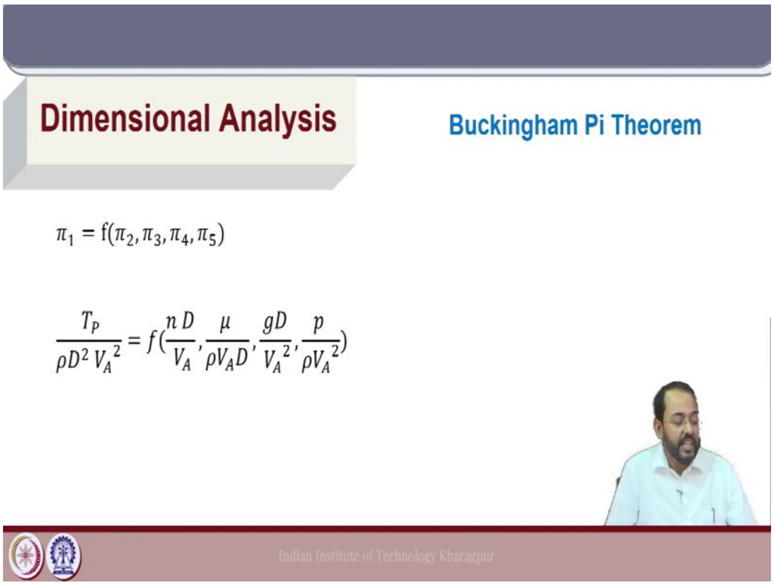


As a consequence of our calculations, we can express $\pi_1$ as a function of the other non-dimensional quantities $\pi_2$, $\pi_3$, $\pi_4$, and $\pi_5$ that we have just computed. So, if we write it in terms of the values that we have obtained, we will get this particular equation, $\pi_1$ as a function of the other non-dimensional quantities. So, each of these quantities are non-dimensional, right.

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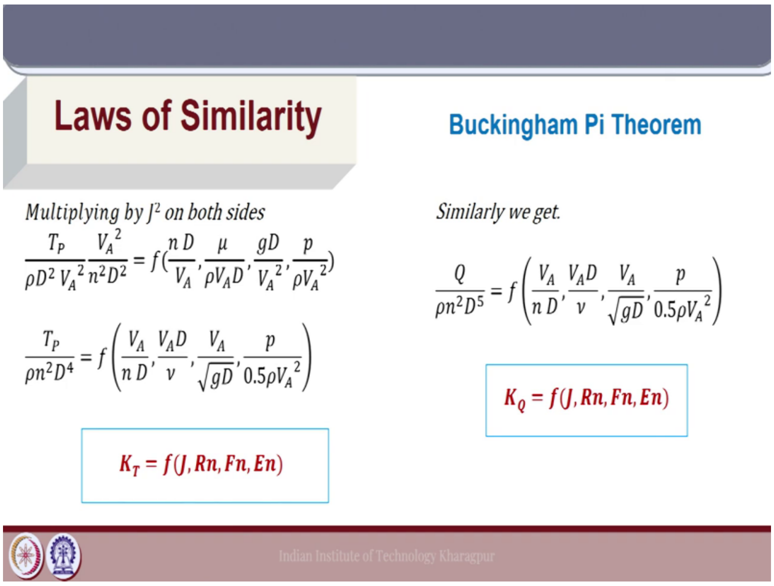


Now, as the terms on both the left hand side and right hand side of this equation are non-dimensional. We multiply both sides by $J^2$, where J is the advance coefficient $V_A/nD$. Why are we doing this? Because we want to express these non-dimensional quantities with relation to the concepts of Reynolds number, Froude number etcetera. So, if we do this we will get $T/(\rho n^2 D^4)$ as a function of these 4 non-dimensional quantities.

Now, one by one let us see what these are. $V_A/ n D$ is nothing, but the advance coefficient J itself. $V_A D/\nu$ is the Reynolds number, where D is the characteristic length if we take it as the propeller diameter. $V_A$ /root (gD), D is again characteristic length if we consider for the Froude number, then $V_A$/ root(gD) will be the Froude number. And $p/(\frac{1}{2} \rho V^2)$ is the Euler number related to the pressure.

So, finally, we can write the propeller thrust coefficient as a function of these non-dimensional numbers. The advance coefficient Reynolds number, Froude number and Euler number. This is how we use Buckingham Π theorem to do a dimensional analysis and relate the propeller thrust coefficient to a set of non-dimensional numbers which we have previously defined.

We can do the same exercise for the propeller torque coefficient. So, instead of D to the power 4, we will have D to the power 5 in the denominator to get the propeller torque coefficient, and we will get the same set of numbers to define the propeller torque coefficient. So, we can relate the propeller thrust and torque coefficients to the dimensionless numbers, the advance coefficient Reynolds number, Froude number and Euler number.

Now, we will see gradually which one we will take as important in the context of analysis for propeller performance.

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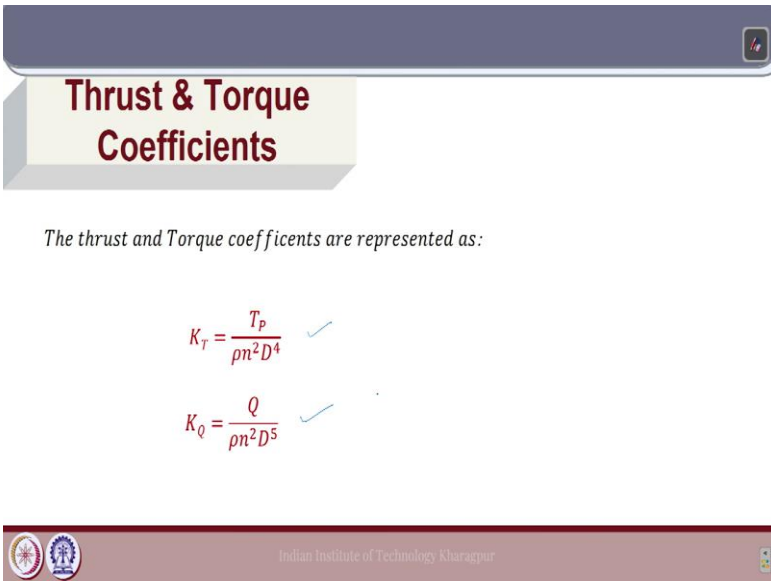


So, the thrust and torque coefficients of the propeller after doing this dimensional analysis can be written in this particular form. In the propeller open water characteristics, this is the primary form of expressing the propeller thrust and torque in non-dimensional form, using $K_T$ and $K_Q$ which are the thrust and torque coefficients.

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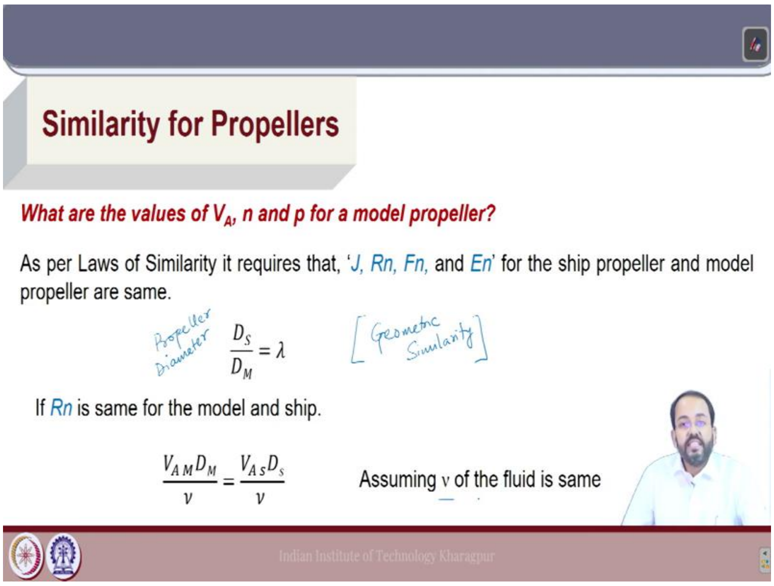


Now, if we look for similarity of propellers what will be the value of $V_A$, n and p for a model propeller. Because we have discussed that the idea of ship and propeller model testing is to estimate the hydrodynamic performance in the model scale and to extrapolate it to get the full scale properties. So, what will be the values? If we consider all these similarities that we have discussed till now, how will we get the model values of VA, n and p for a model scale propeller?

So, as per the laws of similarity, we have to keep advance coefficient Reynolds number, Froude number, Euler number same in the ship and the model scale for the propeller. Now, the first part J which is $V_A$/n D comes from the kinematic similarity, and these numbers Reynolds number, Froude number and Euler number come from the dynamic similarity based on the ratio of forces.

So, first we start with the basic geometric similarity, which is the ratio of the diameter in the full scale to the model scale; so propeller diameter. So, this is basically geometric similarity which says that all the dimensions are uniformly scaled between the full scale and the model scale. So, let us consider λ as the scale ratio.

Now, if we consider Reynolds number same for the model and ship, what do we get? For the propeller we can take Reynolds number as V D / ν. Again, we have seen that for a representative blade section, a more accurate expression of Reynolds number will be based on the resultant velocity and the chord length of the representative section. But as of now we are simply writing this Reynolds number in the form of V D/ν using the velocity of advance and the propeller diameter, just to see the relation between the model and the full scale.

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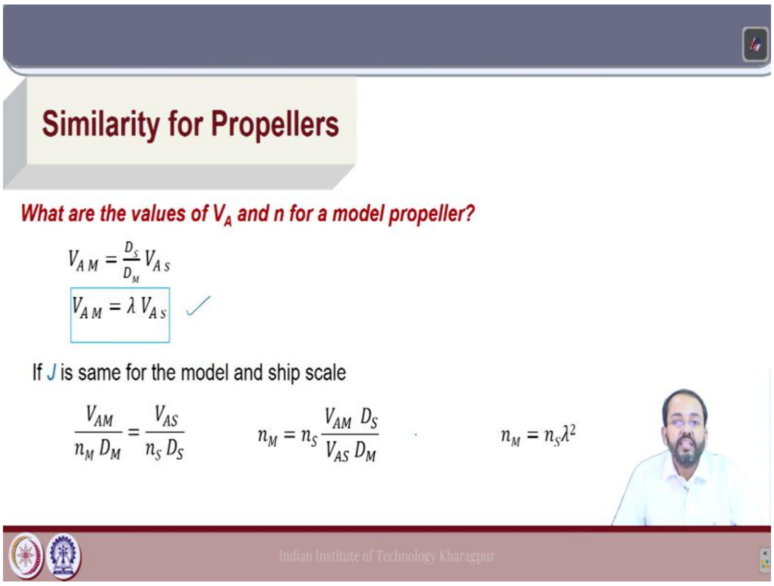

If we do that and assume that the viscosity is same in the model and full scale, we will get the ratio $V_A$ in the model and full scale as a ratio of the diameters in an inverse way. The model scale propellers advance speed will be λ times the advance speed in the full scale, where λ is the ratio of the full scale diameter by the model scale propellant diameter.

Now, if we keep J which is the advance coefficient same based on kinematic similarity which must be maintained, so that the hydrodynamic inflow angles are same in the model and full scale. So, based on that similarity J is $V_A$/ n D.

Now, we already know from Reynolds number similarity that the ratio between $V_{AS}$ and $V_{AM}$ is given by this equation. If we put that value and the same ratio λ between $D_S$ and $D_M$, we will get that the value of $n_M$ is $n_S$ or the full scale rotational speed multiplied by $\lambda^2$. This is based on Reynolds number similarity.

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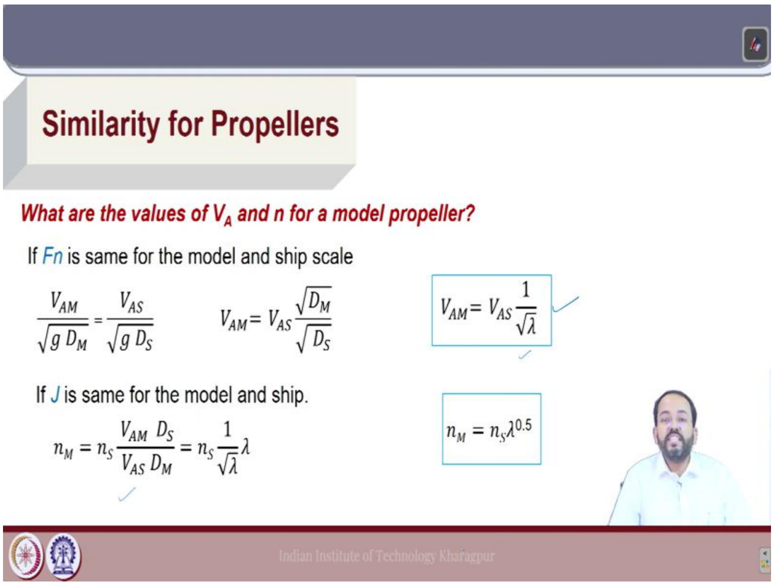


Next let us check the condition for Froude number similarity between the model and the ship scale. If we do that we will have V / root(g l), which is the Froude number same between model and full scale. Again, we take $V_A$ the advance speed as the characteristic velocity, and for length, we take the propeller diameter D as the characteristic length in the Froude number equation.

So, we get this relation between velocity in the model and the full scale as a square root of the ratio of the diameters between the model and the full scale. This gives the ratio between the model and full scale velocity as 1 / root (λ). So, the model scale velocity is the full scale divided by square root of the scale ratio.

Now, again like before, we will calculate the equivalence of J between the model and full scale based on the Froude number similarity, where we use this 1 / λ ratio for calculating the J similarity. And we have the rotational speed in the model case. So, here we will have that the rotational speed in the model scale is the full scale rotational speed multiplied by square root of λ, where again λ is the scale ratio.

So, we see that in the Reynolds and the Froude number similarity, the ratios between the model and full scales in terms of velocity of advance and rpm are very much different.

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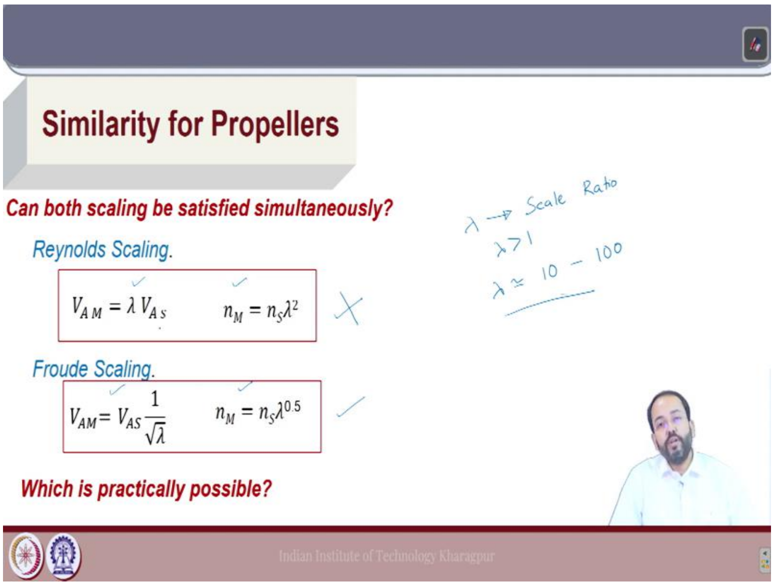


So, for Reynolds number scaling we have the velocity in the model scale will be higher than the full scale.

Now, we have to remember that λ is the scale ratio, right. So, depending on the size of the ship and the scale at which we are conducting the model test which depends on the towing tank capability, we will have a scale ratio which is definitely greater than 1. So, depending on the model size with respect to the full scale ship, this scale ratio can be anything in the range of 10 to 100, ok.

So, if we apply Reynolds scaling, we are having a velocity of the model which is that scale ratio times the full scale speed of the ship or the in this case the advance speed for the propeller. And the rpm of the model propeller as $\lambda^2$ times the full scale rpm.

On the other hand, if we use Froude scaling we have the velocity of advance in the model scale as the velocity of advance in the full scale divided by square root of the scale ratio and similarly the rotational speed is the full scale rotational speed multiplied by square root of the scale ratio.

Now, the big question here is which one is practically possible. In an ideal condition, we would want to keep all the numbers same in the model and full scale, but because they give totally different values of the model and full scale ratios in terms of velocities and rpm, we cannot keep both of them same. And it is obvious from these calculations that it is very difficult or rather impossible to keep Reynolds number same in the model and full scale.

For a very high value of $\lambda$, we will see that the model scale speed will be extremely high and the model rpm will also be very high. So, we will not be able to keep the Reynolds number same in the model and full scale. This can be visualized better with the help of a problem that we will do in this class.

On the other hand, if we do Froude scaling the values that we have for velocity and rpm can be well maintained in terms of the capability of the towing tank where we do ship model testing. So, we use Froude number similarity when we do ship propulsion tests and we compute the values in the model scale and they are extrapolated to get the full-scale powering.

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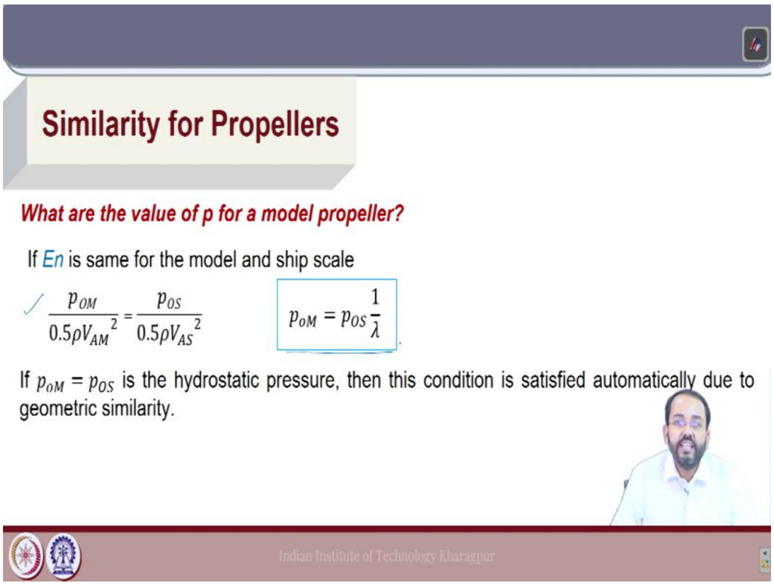


On the other hand, if we look towards the pressure component, if we keep Euler number E p or the pressure coefficient here, same between the model and full scale, we get that the pressure in the model scale is the full scale pressure divided by λ.

So, if this pressure is the hydrostatic pressure, then this condition is automatically satisfied because the hydrostatic pressure is proportional to the depth and because we have scaled all the linear dimensions with respect to the scale ratio λ. So, the hydrostatic pressure is also automatically scaled. So, this is already satisfied if we take the Euler number same between the model and full scale using the geometric similarity λ here.

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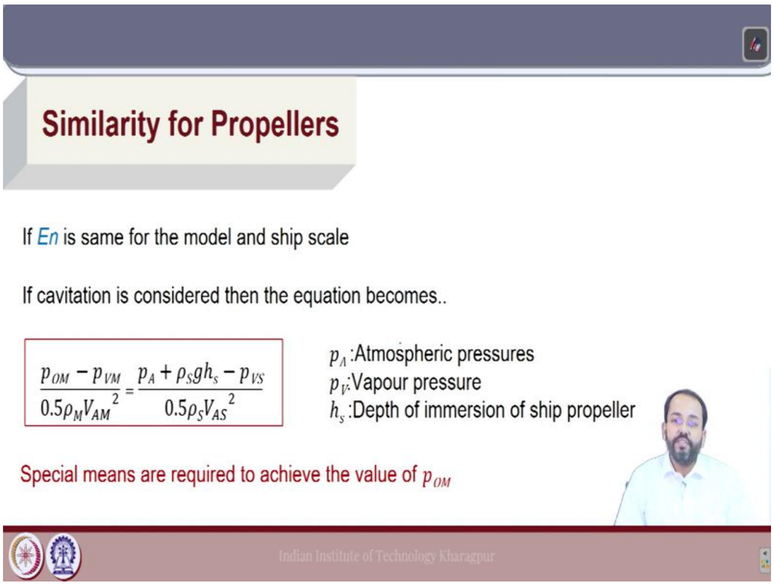


On the other hand, if we consider cavitation which again we will study later, then we will see that this pressure is not only the hydrostatic pressure, but there is also a vapor pressure consideration. In those particular cases, we will require special means to achieve the value of the pressure.

So, these cavitation tests are tyπcally done in cavitation tunnel where the pressure can be regulated in order to study cavitation phenomena.

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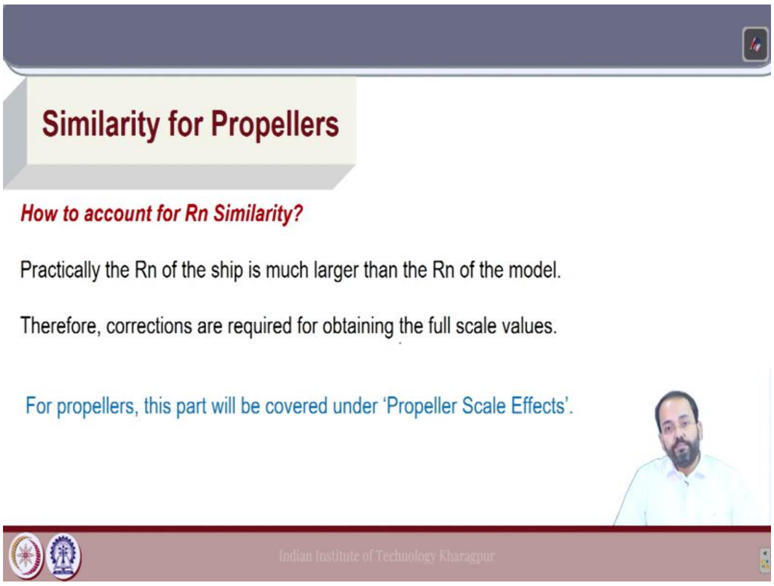

Now, we can maintain Froude number similarity between model and full scale, but we see that the Reynolds number similarity cannot be maintained. It is not feasible because of very high values of velocity and rpm in the model scale if we try to go for Reynolds number similarity.

Now, what is the meaning of that? What is the implication? Reynolds number involves viscosity term. So, we will not be able to maintain similar ratios of viscous and inertial forces in the model and full scale. This is the reason why corrections have to be used to get the full scale values in order to correct for the viscosity terms.

In ship model testing this is done using ITDC friction lines where the viscous resistance coefficient is separately scaled. For propellers this part will be covered under propeller scale effects later in this course where we will see how we correct the propeller coefficients or apply certain correction methods to get the full-scale propeller coefficients from the model scale using some corrections due to viscosity, the standard corrections which are used for propeller thrust and torque coefficients.

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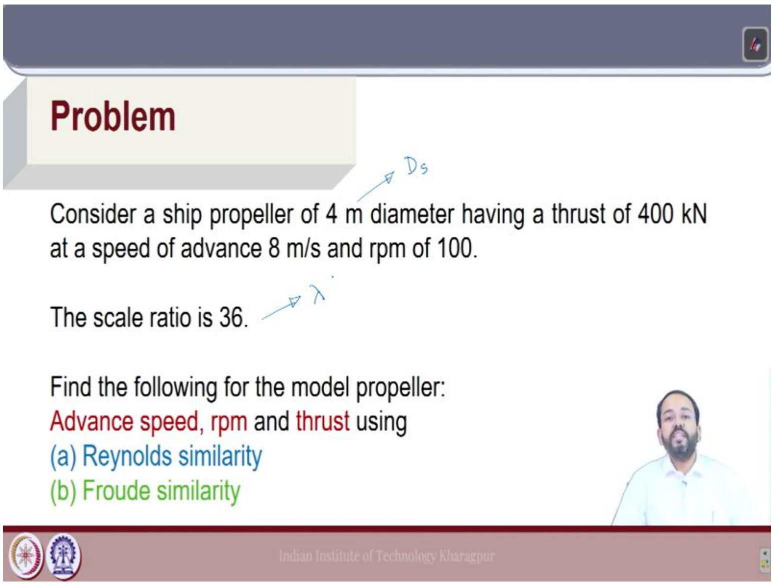


Now, we will do a simple problem based on the model and full scale values of propeller velocities and forces, and how we can use these Froude number and Reynolds number similarity to calculate the values.

So, let us consider a ship propeller of 4 m diameter. This is in the full scale. So, this is basically $D_S$, and having a thrust 400 kN at a speed of advance 8 m per second and rpm 100. The scale ratio is 36, this is the value of $\lambda$, the scale ratio.

So, you are asked to calculate the advanced speed rpm and thrust of the model propeller using Reynolds similarity and Froude similarity separately, ok. So, we have the diameter, advance velocity, rpm and thrust for the ship propeller given, and we have to compute these values for the model propeller using two different similarities.

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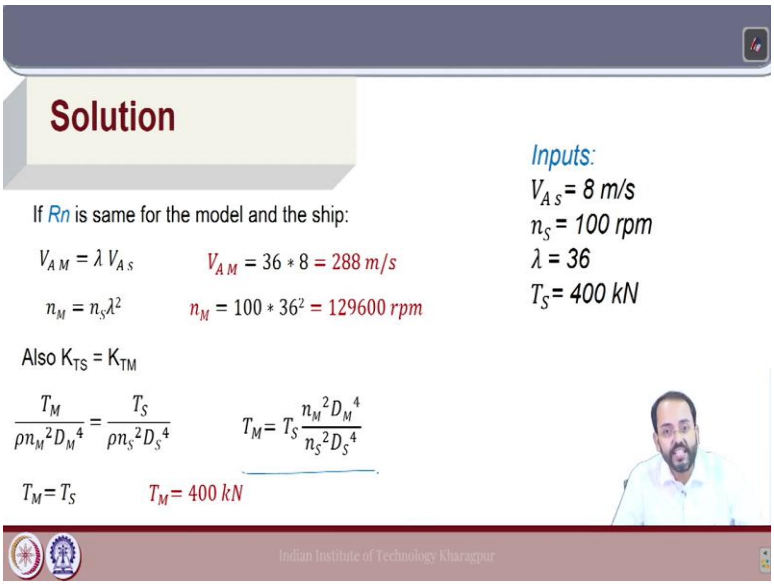


So, using these inputs let us start with the Reynolds similarity. We have seen that for Reynolds similarity the velocity in the model scale, the advance velocity will be $\lambda$ times the advance velocity in the full scale. So, that gives a model scale velocity of advance as $36 \times 8$, where 36 is the scale ratio. So, we have a very high model scale advanced velocity of 288 m/s. Similarly, using the equation for rotational speed we see that the model scale rpm is also considerably high.

For the propeller thrust we will take the thrust coefficient $K_T$ that we have computed to be same in the model and the full scale. So, if we use that similarity, we have this thrust $T_M$ in the model scale and $T_S$ in the full scale related by this particular equation which gives $T_M = T_S$, because $n_M^2/n_S^2$ is basically $\lambda^4$ and $D_M/D_S$ is $\lambda$, and that to the power 4 cancels out. So, we have the thrust in the model and the full scale will be same.

So, if we use Reynolds similarity we have a model scale thrust which is equal to the full scale thrust of 400 kN during model testing.

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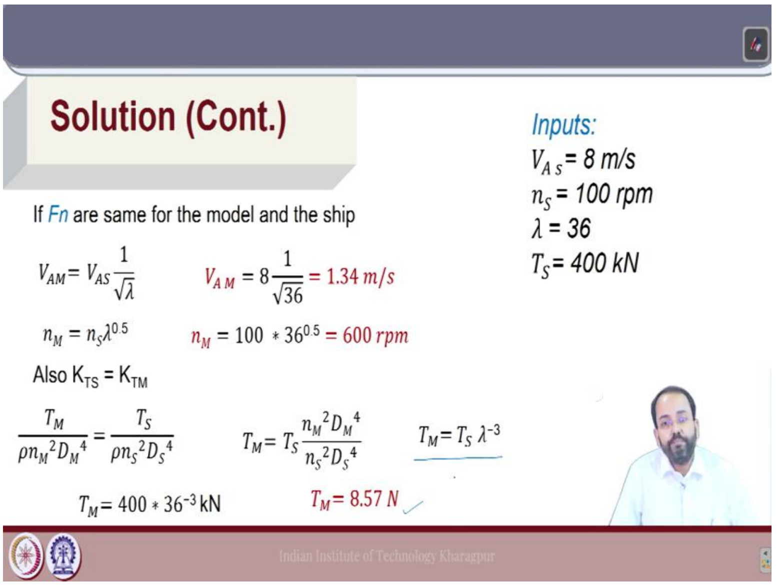


On the other hand, let us look into the condition where we keep the Froude number same between the model and the full scale ship. And we have the same inputs. In this case, model scale velocity is full scale divided by square root of the scale ratio which gives a model scale velocity of 1.34 m/s and for the rpm it is 600 rpm which is higher than the full scale rpm, but much lower than the condition where we used Reynolds similarity.

So, progressing towards the thrust coefficient similarity, here we will have the thrust in the model scale equal to thrust in the full scale divided by cube of the scale ratio. So, the final thrust in the model scale will be very much lower than the full scale thrust which is equal to 8.57 N in the model scale.

So, here we see that the values are very much achievable in the within the limits of the model testing facility. So, we use Froude number similarity for ship model testing. This particular problem gives a very good example why Reynolds number similarity cannot be maintained for practical purpose of ship model testing.

Using Froude number similarity, we will calculate the powering of the ship propeller system in the model scale and based on the extrapolation principles which we will cover in the lecture on ship propeller experiments we will get the powering in the full scale.

So, this will be all for today’s class.

Thank you.