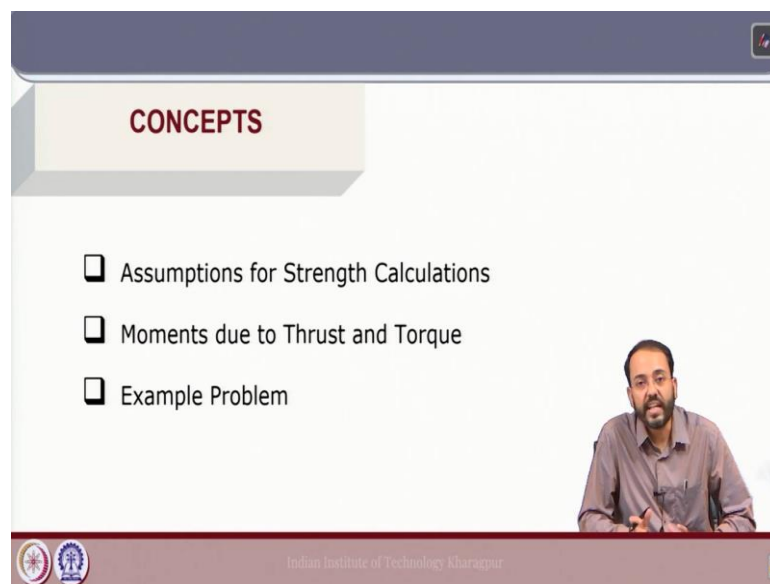


Marine Propulsion
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Lecture - 26
Propeller Strength (Part - I)

Welcome to lecture 26 of the course Marine Propulsion. Today we will start with Propeller Strength.

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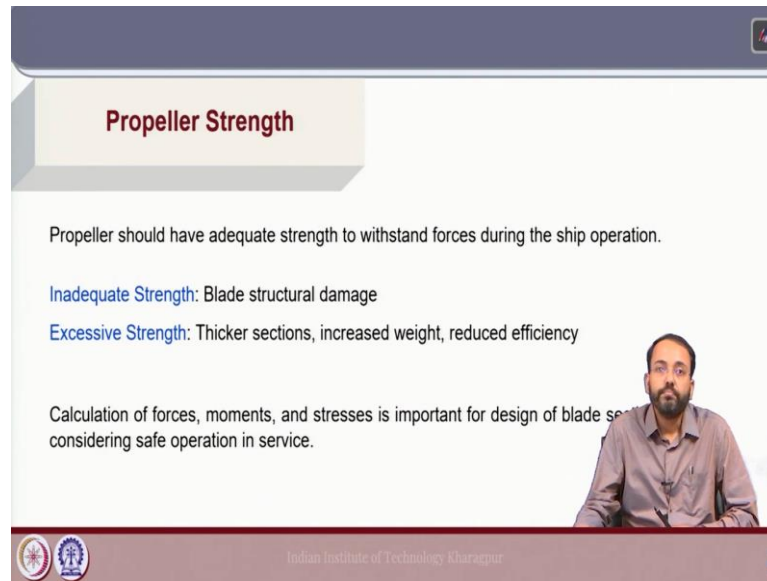
The slide is titled "CONCEPTS" in a red box. Below the title, there is a list of three items, each preceded by a square checkbox:

- Assumptions for Strength Calculations
- Moments due to Thrust and Torque
- Example Problem

In the bottom right corner of the slide, there is a small video inset showing a man with a beard and glasses, wearing a light-colored shirt, sitting at a desk. The bottom of the slide features the Indian Institute of Technology Kharagpur logo and name.

The key concepts covered in today's lecture will be the strength calculations for a propeller and the basic assumptions for the strength estimations. The moments on the propeller blade due to thrust and torque and an example problem based on the moment calculations.

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Propeller Strength

Propeller should have adequate strength to withstand forces during the ship operation.

Inadequate Strength: Blade structural damage

Excessive Strength: Thicker sections, increased weight, reduced efficiency

Calculation of forces, moments, and stresses is important for design of blade section considering safe operation in service.

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The propeller is a very essential component of a ship. It absorbs the power, which is supplied from the engine and it provides the thrust force for the ship to move ahead. Now, because it is a structural entity, it also undergoes certain forces during its operation and that is why proper strength characteristics have to be maintained for the propeller blades. So, that it is able to withstand the forces during its life cycle.

Now if the propeller strength is not adequate, what will happen? The blade will undergo structural damage and the blades will not be able to provide forces and gradually the propeller will be damaged. On the other hand, if the sections are designed in such a way by using thicker sections, if more than required strength is provided then that leads to excessive weight of the propeller and also it reduces the efficiency of the propeller. So, till now under this marine propulsion course we have mainly covered different aspects of propeller hydrodynamics.

This particular part of the course propeller strength will be based on mainly structural calculations with respect to different blade sections. So, the forces, moments, and stresses have to be calculated based on the propeller operation conditions and that is important for designing the blade section considering safe operation in service.

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Forces on Propeller Blade

- Thrust and Torque of the propeller
- Centrifugal Force

Uncertainties:

- Complex shape of propeller blades
- Non-uniform wake field
- Effects of ship manoeuvring
- Stresses during manufacture
- Vibration, corrosion during service

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Now, if we think of the forces on a propeller blade we have the thrust and torque of the propeller and from the blade element diagram we have seen that the thrust and torque of the propeller blade are functions of the radial locations of the particular blade section. On which we are computing a thrust and torque, and the same concept will be used here now for the calculation of the moments due to these forces.

And the next part is the centrifugal force because the propeller blade revolves about the axis and due to that there will be centrifugal forces on the propeller blade and we will observe that the rake and skew of the propeller blade impacts the effect of this centrifugal force in the form of moments.

Now, these are very simple estimations of the bending moments on the propeller blade from which the stresses can be calculated. Now, these forces give a simplistic estimate of the stresses that are encountered on a propeller blade for specific operation conditions.

In addition to that, there are certain complexities and uncertainties that occur during the realistic operation of a propeller. Now, some of them are listed here. The complex shape of the propeller blades which are very much three dimensional that makes the actual stress calculation very difficult.

On top of that the flow field into the propeller is very much non-uniform because the wake of the ship impacts the inflow into the propeller. So, as the propeller works behind the ship

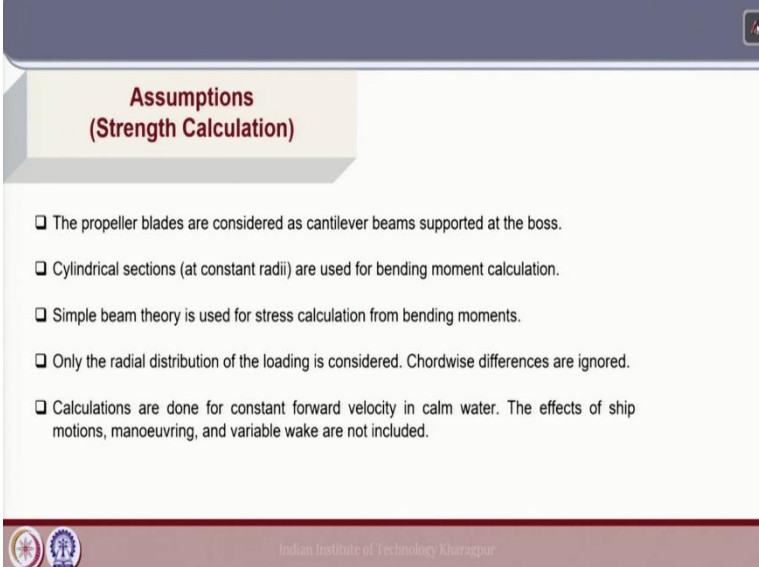
the wake velocities change in both the radial as well as the circumference directions that we have seen in hull propeller interaction.

And also there is a temporal variation of the velocity components, which leads to fluctuation of thrust and torque of the propeller. So, these lead to changes of forces and moments and finally, the stresses on the propeller blades. Then, there are effects of ship maneuvering when the ship is turning or during astern motion the forces on the propeller blades are very much different from the normal case of a head motion, because the flow into the propeller blades will be very much different and the stresses will also change.

Further there are stresses during manufacture of a propeller blade which may be locked in the blades and that may lead to additional forces on the propeller blades. And finally, there are vibration issues and corrosion during the service of a propeller during its lifetime and these change over time. So, the total aspect of the stresses on propeller blades depend on many uncertainties apart from the standard forces that we will calculate as a part of this particular course.

So, one should appreciate that in an actual lifetime of a ship the propeller stresses and the forces change with time due to the operational conditions and also as the propeller blade undergoes corrosion in service the stresses on the propeller blade, the pattern of that will change over time.

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**Assumptions
(Strength Calculation)**

- The propeller blades are considered as cantilever beams supported at the boss.
- Cylindrical sections (at constant radii) are used for bending moment calculation.
- Simple beam theory is used for stress calculation from bending moments.
- Only the radial distribution of the loading is considered. Chordwise differences are ignored.
- Calculations are done for constant forward velocity in calm water. The effects of ship motions, manoeuvring, and variable wake are not included.

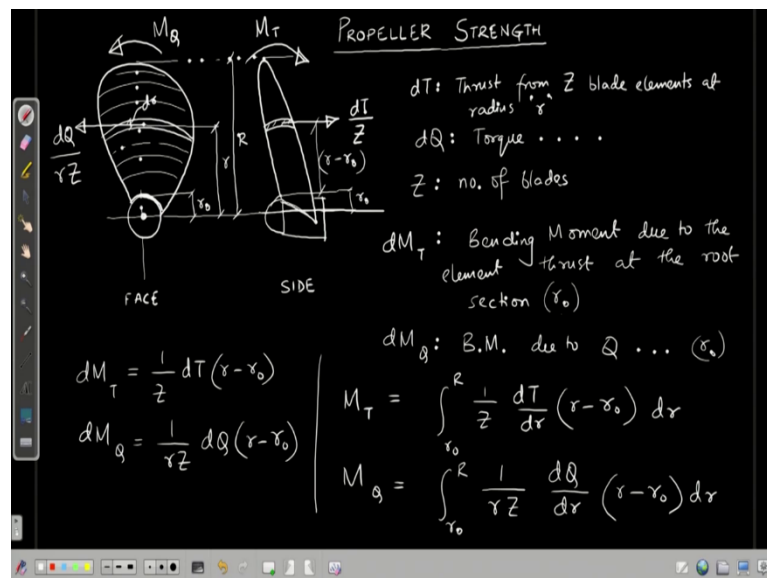
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Now, we will go through some basic assumptions which are made for propeller blade strength calculations. So, we have considered different types of beam based on end conditions in the beam theory. For propellers we will consider that the blades are behaving like cantilever beams supported at the boss or at the hub.

Next, the cylindrical sections are taken at various radial locations on the propeller blade for estimation of the velocity thrust and torque and similarly we will take the cylindrical sections for bending moment calculation for the propeller blade. Simple beam theory will be used for stress calculation from the bending moment and the neutral axis of these sections will be assumed to be along the chord for the expanded blade sections.

Only the radial distribution of loading will be considered chord wise differences in circulation and thrust torque will not be considered for the propeller blade forces and moments. So, the thrust and torque will be considered only as a function of the radial location and finally, the calculations are done for forward velocity in calm water, as we have just discussed the effects of maneuvering, ship motions, and variable wake are not included in the strength calculations.

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So, now let us look into the forces on the propeller blade, which are the thrust and torque and how they lead to the bending moments on the propeller blade. Here, we are drawing a propeller blade from two different views. We have the view from the face, and the view from the side. Now, let us take a radial location on the propeller blade at a radius s r and

let R be the propeller radius, and the thickness of this section as taken before is dr . Similarly, the section is also visualized in the side view. Now, how will the thrust and torque force for that particular blade section act. The thrust force is dT/Z where dT is the thrust produced by Z blade elements at radius r and each of these elements have thickness dr .

So, for one blade element if we have Z as the number of blades the thrust component is dT/Z . Now, due to that we will have a moment calculated at the root of the propeller blade due to the thrust force we call it M_T r_0 , which is the root section here, the same one taken here ok. Similarly dQ will be the torque from Z elements each of thickness dr at the radius R and where will we have the force that causes the torque is $dQ/r Z$. Now, this force causing the torque is perpendicular to the propeller axis as shown here.

On the other side, this thrust force from the element is parallel to the propeller axis which is this and in the first diagram the propeller axis here is perpendicular to the plane of the board. So, if we consider this as a right handed propeller, the moment due to the torque reaction on the propeller blade will be in this direction M_Q .

Now, let us look into the bending moments caused by the elemental thrust and torque at the root section. So, let dM_T be the bending moment due to the element thrust at the root section at r_0 , similarly dM_Q is a bending moment due to elemental torque Q at section r_0 ok.

So, what is dM_T ? It will be $(1/Z) dT \times (r - r_0)$ simply the force into the momentum because this is $r - r_0$. The r and the same thing for $dM_Q = (1/r Z) dQ (r - r_0)$. Now, if we try to get the bending moments due to thrust on the propeller blade at the root section; that means, all these elements we will have different elements right from the root to the propeller blade tip.

So, if we combine the moments caused by all these elemental thrust and torque and calculate the total bending moment on the root section due to thrust and torque we will have $M_T = \int_{r_0}^R \frac{1}{Z} \frac{dT}{dr} (r - r_0) dr$ and $M_Q = \int_{r_0}^R \frac{1}{rZ} \frac{dQ}{dr} (r - r_0) dr$. So, we have calculated the expressions for the bending moments due to thrust and torque for the entire propeller blade about the root section.

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$$M_T = \int_{r_0}^R \frac{1}{Z} \frac{dT}{dr} (r - r_0) dr$$

$$M_Q = \int_{r_0}^R \frac{1}{Z} \frac{dQ}{dr} (r - r_0) dr$$

$$M_T = \int_{x_0}^1 \frac{\rho n^2 D^4}{Z} \frac{dK_T}{dx} \times \frac{D}{2} (x - x_0) dx$$

$$\Rightarrow M_T = \frac{\rho n^2 D^5}{2Z} \int_{x_0}^1 \frac{dK_T}{dx} (x - x_0) dx$$

$$M_Q = \frac{\rho n^2 D^5}{Z} \int_{x_0}^1 \frac{dK_Q}{dx} \left(\frac{x - x_0}{x} \right) dx$$

$T = K_T \times \rho n^2 D^4$
 $Q = K_Q \times \rho n^2 D^5$

$x = \frac{r}{R}$
 $r = R \cdot x$
 $r_0 = R \cdot x_0$

Now, we will express thrust and torque in the form of the standard non dimensional thrust and torque coefficients. $T = K_T \times \rho n^2 D^4$ and $Q = K_Q \times \rho n^2 D^5$. In a similar way, we will also make the propeller radial location non dimensional as we have done previously.

So, we will take a non-dimensional factor x which is given by r / R . So, that $r = R \times x$ and this particular location at $r_0 = R \times x_0$, which is the fractional radius at the root of the propeller blade. Now, if we use all these non-dimensional entities into the equations for the bending moments we will have $M_T = \int_{r_0}^R \frac{\rho n^2 D^4}{Z} \frac{dK_T}{dx} \frac{D}{2} (x - x_0) dx$ i.e (from the root to the tip of the propeller blade). So, $M_T = \frac{\rho n^2 D^5}{2Z} \int_{x_0}^1 \frac{dK_T}{dx} (x - x_0) dx$ becomes when we take the constant term out of the integration.

In a similar way we can write the equation for M_Q there is an additional r in the M_Q equation here.

$$M_Q = \frac{\rho n^2 D^5}{2Z} \int_{x_0}^1 \frac{dK_Q}{dx} \left(\frac{x - x_0}{x} \right) dx$$

So, that r will cancel out the two term here because r will be x times diameter / 2. These two equations give the bending moments due to thrust and torque as a function of the thrust and torque coefficients of the propeller blade.

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$$M_T = \frac{\rho n^2 D^5}{2Z} \int_{x_0}^1 \frac{dK_T}{dx} (x-x_0) dx$$

$$M_Q = \frac{\rho n^2 D^5}{Z} \int_{x_0}^1 \frac{dK_Q}{dx} \left(\frac{x-x_0}{x}\right) dx$$

$$M_T = \frac{\rho n^2 D^5 K_T}{2Z} \cdot \frac{\int_{x_0}^1 \frac{dK_T}{dx} (x-x_0) dx}{\int_{x_0}^1 \frac{dK_T}{dx} dx}$$

$$M_Q = \frac{\rho n^2 D^5 K_Q}{Z} \cdot \frac{\int_{x_0}^1 \frac{dK_Q}{dx} \left(\frac{x-x_0}{x}\right) dx}{\int_{x_0}^1 \frac{dK_Q}{dx} dx}$$

$$K_T = \int_{x_0}^1 \frac{dK_T}{dx} dx$$

$$K_Q = \int_{x_0}^1 \frac{dK_Q}{dx} dx$$

Now, we can write K_T as the variation of the K_T over the radial location integrated in this manner and K_Q similarly using the variation of the torque coefficient over the radial location. Now, we will use these expressions for K_T and K_Q into the equations for the bending moments.

$$K_T = \int_{x_0}^1 \frac{dK_T}{dx} dx , \quad K_Q = \int_{x_0}^1 \frac{dK_Q}{dx} dx$$

So, if we multiply the bending moment due to thrust with K_T and divide the term under the integration with K_T again, but replacing with this particular form we get we have the additional K_T outside and the same form we have divided by K_T which is this. Similarly for M_Q we multiply by K_Q outside and divide the term with integration by the integrated value of dK_Q , so this is also 1.

$$M_T = \frac{\rho n^2 D^5 K_T}{2Z} \frac{\int_{x_0}^1 \frac{dK_T}{dx} (x-x_0) dx}{\int_{x_0}^1 \frac{dK_T}{dx} dx}$$

$$M_Q = \frac{\rho n^2 D^5 K_Q}{Z} \frac{\int_{x_0}^1 \frac{dK_Q}{dx} \left(\frac{x-x_0}{x}\right) dx}{\int_{x_0}^1 \frac{dK_Q}{dx} dx}$$

Now, there is a reason of expressing the moments with these values related to the change of the thrust and torque coefficients over different radial locations because in circulation

theory calculations, these values will be having certain assumed variations on the propeller blade. And we can substitute these particular factors with the variations to do the calculations for the moments for thrust and torque on the propeller blade.

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Using approximations,

$$\left. \begin{aligned} \checkmark \frac{dK_T}{dx} &= K_1 x^2 (1-x)^{0.5} \\ \checkmark \frac{dK_Q}{dx} &= K_2 x^2 (1-x)^{0.5} \end{aligned} \right\} \begin{aligned} x &= \frac{r}{R} \\ [K_1, K_2 \text{ are constants}] \\ x_0 &= 0.2R \\ x_0 &= 0.2 \end{aligned}$$

$$M_T = 0.2376 \frac{K_T \rho n^2 D^5}{Z}$$

$$M_T = 0.2376 \frac{TD}{Z}$$

$$M_Q = 0.6691 \frac{Q}{Z}$$

So, using approximations which are standard for different types of circulation theory calculations this term the variation of the thrust coefficient with the radial location in a non-dimensional fashion can be written in this form.

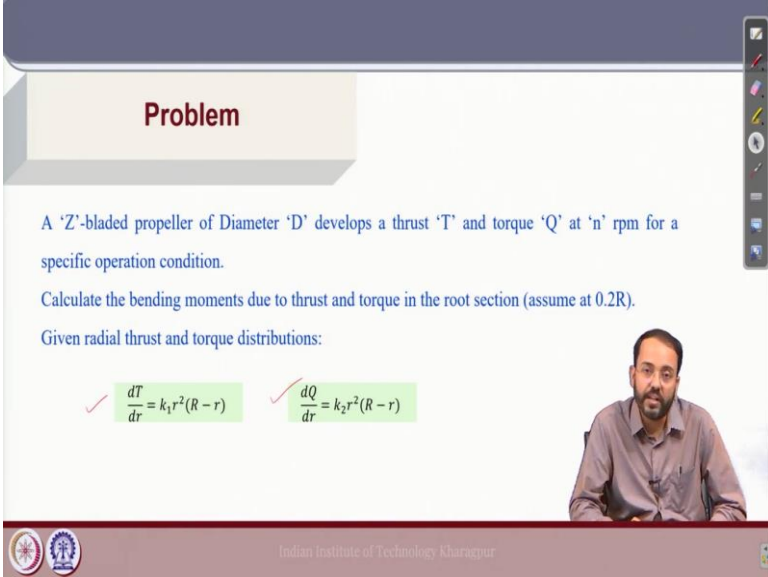
$$\frac{dK_T}{dx} = K_1 x^2 (1-x)^{0.5}, \frac{dK_Q}{dx} = K_2 x^2 (1-x)^{0.5}$$

Where x is the variable radius non dimensionalize by the propeller radius and K_1 is a constant. Similarly, for torque it is written in a very similar way using K_2 as the constant. So, K_1, K_2 are constants ok.

So, if we substitute these relations in the equations for bending moment due to thrust and torque and we simplify assuming that the root location is at $0.2R$ ok; that means, x_0 equal to 0.2 then we have $M_T = 0.2376 \frac{K_T \rho n^2 D^5}{Z}$ in this form, which gives us a simple formula based on the thrust the diameter and the number of blades $M_T = 0.2376 \frac{TD}{Z}$. Similarly for M_Q , if we assume that the root location is at $0.2R$ after simplification using these distributions of the thrust and torque over the propeller blade we get $M_Q = 0.6691 \frac{Q}{Z}$.

So, these approximations can be used for simple calculations of the bending moment due to thrust and torque at the root section of the propeller blade. If we have the thrust and torque coefficient distributions expressed in a function of radius using these equations. Now, if the equations change or the location of the root section changes then we will have different expressions for the bending moments due to thrust and torque.

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Problem

A 'Z'-bladed propeller of Diameter 'D' develops a thrust 'T' and torque 'Q' at 'n' rpm for a specific operation condition.

Calculate the bending moments due to thrust and torque in the root section (assume at 0.2R).

Given radial thrust and torque distributions:

$$\frac{dT}{dr} = k_1 r^2 (R - r)$$
$$\frac{dQ}{dr} = k_2 r^2 (R - r)$$

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Now, we will look into a simple problem, which represents these calculations for the bending moments due to thrust and torque on the propeller blade. So, we have a propeller with Z number of blades having diameter D, where the thrust T and the torque Q are given for the entire propeller blade and it is having an n rpm for a specific operation condition. So, for this particular problem we need to calculate the bending moments due to thrust and torque of the entire propeller blade about the root section.

And the given radial thrust and torque distributions are as per these two equations. Now, to do the calculation the first step should be to calculate K_1 and K_2 based on the given parameters. So, the thrust and torque are given for this particular condition and we have to calculate K_1 and K_2 . So, that we can use the expressions for bending moment due to thrust and torque that we have just derived and get the total bending moment for the propeller blade.

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Given: T ✓
 Q ✓
 D ✓

$$\frac{dT}{dr} = K_1 r^2 (R - r)$$

$$T = \int_{r_0}^R \frac{dT}{dr} dr = \int_{r_0}^R K_1 r^2 (R - r) dr$$

$$T = K_1 \int_{r_0}^R (Rr^2 - r^3) dr$$

$$T = K_1 \left[R \left(\frac{R^3 - r_0^3}{3} \right) - \left(\frac{R^4 - r_0^4}{4} \right) \right]$$

$R = \frac{D}{2}$
 $r_0 = 0.2R$

Calculate K_1 ✓
 K_2 ✓

Let us look into the solution approach: The given parameters are thrust, torque, and the diameter which will be used for the calculations and the variation of thrust with radius is given based on this expression. One should note that this expression is slightly different from the one that we have used for our calculation for derivation of the moments. Now, any problem can be given based on any given variation for the thrust and torque and that can be simply used to solve for the K_1 and K_2 values for thrust and torque.

So, I am writing only the thrust part here. The thrust will be integration of $T = \int_{r_0}^R \frac{dT}{dr} dr = \int_{r_0}^R K_1 r^2 (R - r) dr$ over the entire propeller blade which is given. So, $T = K_1 \int_{r_0}^R (Rr^2 - r^3) dr$. So, if we do the integration where K_1 is the constant we have here, R^2 integration is $R^3/3$. So, $T = K_1 \left(R \frac{(R^3 - r_0^3)}{3} - \frac{(R^4 - r_0^4)}{4} \right)$. Now, in this particular equation the diameter of the propeller is given. So, $R = D/2$ and the location for the root of the propeller blade is given as $0.2R$.

Now, in this particular equation T is known R , r_0 is known. So, we should be able to calculate K_1 in a similar way, we can use the equation for torque depending on the same variation of dQ/dr on the propeller blade to calculate K_2 . Now, once we have calculated the two constants we can go ahead for the next step, which is the calculation of the moments.

Again, we will simply look at the moment for the thrust and it can be written as this integration $M_T = \int_{r_0}^R K_1(1/Z) r^2 (R - r) (r - r_0) dr$ which is the given value multiplied by this expression and in a similar fashion we can take K_1/Z constants outside and integrate this particular equation. So, it will $M_T = \frac{K_1}{Z} \int_{r_0}^R [r^2(Rr - Rr_0 + r^2 + rr_0)] dr$

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$$M_T = \int_{r_0}^R \frac{1}{Z} \frac{dT}{dr} (r - r_0) dr$$

$$= \int_{r_0}^R \frac{1}{Z} K_1 r^2 (R - r) (r - r_0) dr$$

$$= \frac{K_1}{Z} \int_{r_0}^R [r^2 (Rr - Rr_0 - r^2 + rr_0)] dr$$

M_Q K_1, Z, r_0, R

So, in a similar way as before we can express M_T as a function of K_1, Z, r_0 and R after the integration is done, all these terms are now known because we have already evaluated the value of constants. So, in this fashion we should be able to calculate the value of the bending moment due to thrust, and similarly the value of the bending moment due to torque M_Q can be calculated by writing the torque equation.

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Problem

A 'Z'-bladed propeller of Diameter 'D' develops a thrust 'T' and torque 'Q' at 'n' rpm for a specific operation condition.

Calculate the bending moments due to thrust and torque in the root section (assume at 0.2R).

Given radial thrust and torque distributions:

$$\frac{dT}{dr} = k_1 r^2 (R - r)$$
$$\frac{dQ}{dr} = k_2 r^2 (R - r)$$

Solution steps: Calculate the constants k_1 and k_2
Calculate Bending Moments M_T and M_Q

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So, this particular problem can be solved in two steps. The first step is to calculate the constants K_1 and K_2 and we will use these constants in the equation for the bending moments to calculate the bending moment due to thrust M_T and due to torque M_Q at the root section, due to the thrust and torque of the propeller blade. This will be all for today's class.

Thank you.