Marine Propulsion
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Lecture - 04
Propeller Theory 1

Welcome to the 4th lecture of the course Marine Propulsion. Today we will start Propeller Theory. So, when talking about propeller theory these theories basically they try to explain propeller action. So, the propeller action of the propeller can be explained using different theoretical concepts. So, under propeller theory we will cover these different theories which try to explain the thrust generation by the propeller by absorbing the input power and also the concept of propeller efficiency.

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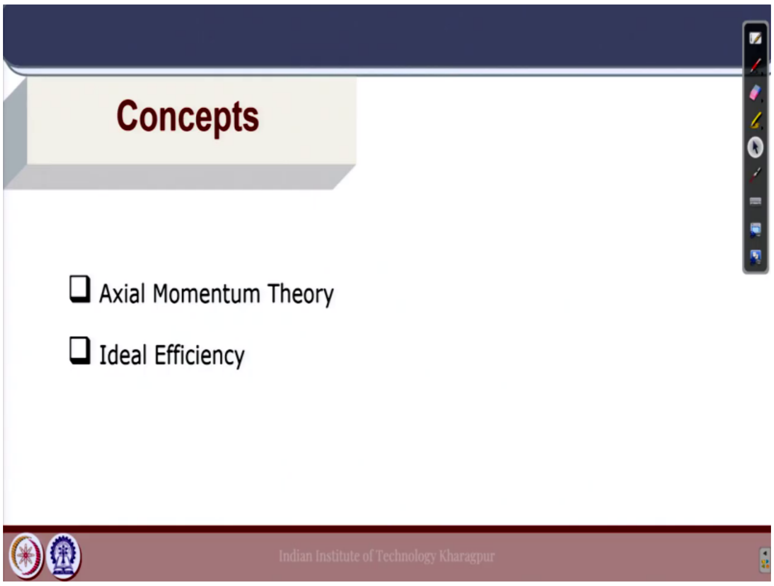

So, in this first class on propeller theory, we will be covering propeller axial momentum theory, which is the basic simple theory for marine propellers which can be used to explain very simply the action of a propeller using some basic fluid mechanics principles. So, we will cover the axial momentum theory and the concept of ideal efficiency.

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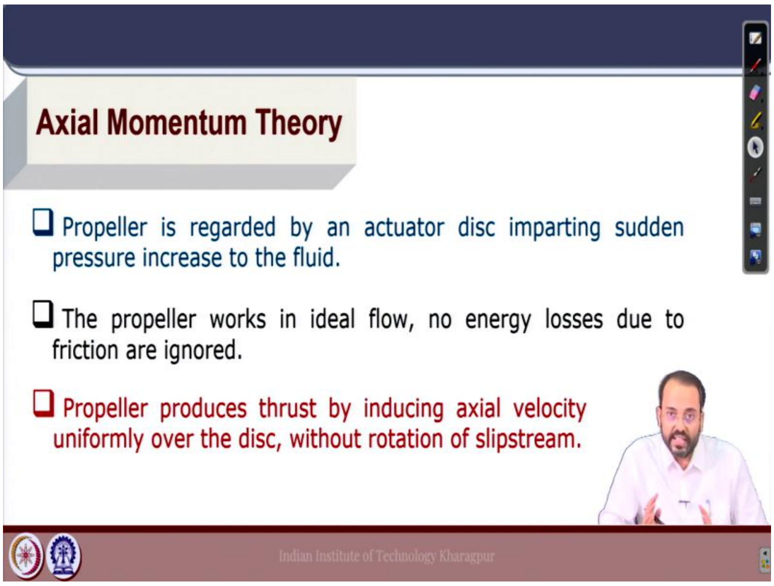

So, for axial momentum theory, there are some basic assumptions that we make and it is very important to understand these assumptions based on which the theory is established.

So, the first assumption is that the propeller is regarded as an actuated disc. So, in this particular theory, we do not have the propeller as such. So, instead of the propeller what we have is basically an actuator disc which imparts a sudden pressure jump and it increases the pressure of the fluid and in that way, it generates thrust. So, here we are not concerned with the entire mechanism of action of the disc.

So, we are assuming that the propeller is a disc and it imparts a pressure increase to the flow. Next assumption is the propeller works in ideal flow. So, ideal flow means, it is the fluid is considered frictionless and there are no energy losses in the system which can come due to friction so they are ignored here. And the third assumption here for the axial momentum theory is that the propeller produces thrust by inducing axial velocity over the entire disc uniformly.

Here by that what we mean is that the propeller is producing an acceleration to the inflow, we will explain all these things with the derivations, but without any rotation to the slip stream. So, the propeller being a disc it is only giving a pressure increase to the flow, by that it is generating a thrust, but it is not providing any rotation to the flow.

So, this is as I have explained this is a very simplistic theory, this is a very basic theory which explains the propeller action using some very basic fluid mechanics principles. So, the axial velocity that is induced because of that pressure jump is uniform over the entire disc and there is no rotation. So, next we will move on to the derivation of this axial momentum theory ok.

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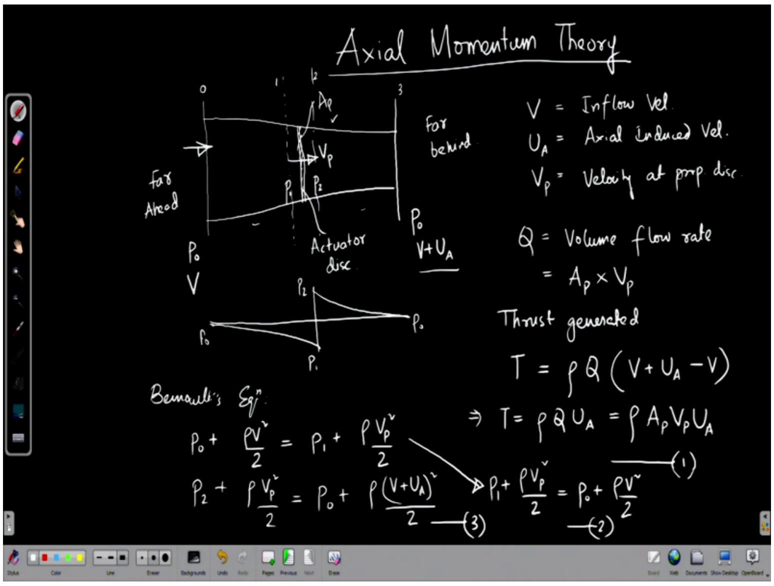

Let us take a flow domain where we have the propeller disc or the actuator disc as we say. So, here we have the actuator disc which is basically giving the thrust here. So, we have taken the domain so that it extends far ahead of the disc and to a zone which is far behind. Now, what are the pressure and velocity components in these regions? Far ahead let us say it is pressure is P_0 and a velocity we put at the inlet which is in this direction which is V.

Now, we have to remember that the propeller ideally should move ahead in the fluid at a velocity V, instead of that we are resolving the hydrodynamics in such a way that we are giving a flow V at the inlet and keeping the propeller fixed ok, that is how typically these problems are analyzed. And because of the propeller giving a pressure here jump we will have an induced velocity at the propeller plane and also far behind, let us say far behind we will have the pressure here again P_0 and $V+U_A$ ok.

Let us say that is the induced velocity far behind the propeller disc. And here at the propeller plane let us say the velocity is V_P, where V_P is the velocity at the propeller and the pressure just in front of the propeller is P_1 propeller or the disc here and just behind the disc is P_2. So, basically what the propeller does is it imparts a pressure jump across the disc here and because of that it generates a thrust.

So, if we try to draw the pressure lines. So, here we have a pressure P_0, it comes down to a value P_1 because of the velocity increase and then at the propeller disc, there is a jump ok and then again it comes to a value P_0. So, if we consider P_0, which is let us say atmospheric pressure at the ends. So, the pressure comes to P_1 here and with a jump P_2 and then we have the pressure coming down in the slip stream.

So, if we see there is a slight contraction of the area here. Here if we see there is a contraction of the area why? Because the propeller accelerates the flow; as the propeller will accelerate the flow through the disc, the it will impart a momentum and because of that acceleration there will be some contraction of the flow because of continuity.

So, that contraction will start slightly ahead of the propeller here ahead and also end at some position behind the propeller. So, we are taking the two ends far ahead and far as behind the propeller in such a way that the velocity is fully developed far behind. Now, V here will be the velocity by which the propeller is moving ahead. So, we can write as the velocity of inflow.

U_A is the axial induced velocity right, V_P is the velocity at propeller disc ok, and P_1 and P_2 are the pressures just ahead and just behind the propeller disc. So, now, if Q is the volume flow rate, through the propeller disc ok how is Q defined then? We can express Q as area of the propeller disc here. Let us say if area is A_P, then multiplied by the velocity of fluid at the propeller disc because in the momentum theory assumption we have that the velocity is uniformly distributed over the propeller disc.

Now, what is the input velocity to the system is? V at the inlet and at the outlet we have $V + U_A$. So, the difference in momentum of the system should be of fluid, it should be equivalent to the thrust generated by the disc. So, thrust generated T should be $T = (\rho Q(V + U_A - V))$, where V was the velocity at the inlet and $V + U_A$ is the velocity at the outlet. So, thrust is given by $(T = \rho Q U_A)$ right.

Now, what is Q? A_P times V_P. So, we can write equal to Thrust (T)

$T = \rho A_P V_P U_A$,   (1).

Now, let us take two sections of the fluid, one just ahead of the propeller disc and one just behind the propeller disc. Now, if we apply Bernoulli's equation, between let us say section 0 at the inlet and section 1 here, what do we get?

$P_0 + \rho V^2/2 = P_1 + \rho V_P^2/2$. Here the assumption is we are taking the plane section here just ahead of the propeller disc.

So, it is the velocity here is equal to the velocity that is on the propellant disc ok. Now, if we write again the equation between the section 2 and the section which is far as done let us say 3, what do we get? The pressure here at 2 is P_2. So, $P_2 + \rho V_P^2/2 = P_0 + \rho(V+U_A)^2/2$, here the pressure is P 0 at 3. So, this second equation we are writing behind the propeller disc $P_0 + \rho(V+U_A)^2/2$ right.

So, we can write this equation by exchanging the terms on the left hand side and right hand side in this way,

$$P_1 + \rho V_P^2/2 = P_0 + \rho V^2/2 \qquad (2)$$

$$P_2 + \rho V_P^2/2 = P_0 + \rho(V+U_A)^2/2 \qquad (3)$$

if we see here on the left hand side we have $P_1 + \rho V_P^2/2$, we have P_2 here . And on the right hand side we have P_0 and the velocity terms square with rho. So, if we go to the next page.

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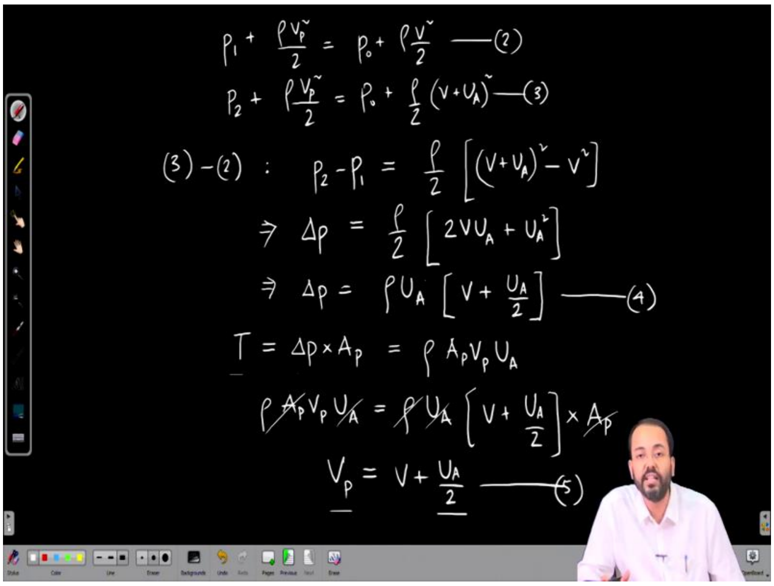

Now, if we subtract 2 from 3. So, if we write 3 minus 2 we get

$P_2 - P_1 = \rho[(V+U_A)^2-V^2]/2$

$(\rho V_P^2)/2$, this term will be cancelled equal to again P_0 will be cancelled. Now, what is P_2 minus P_1? P_2 was the pressure just behind the propeller disc and P_1 was the pressure just ahead and because the disc is providing a pressure jump, we can write it as ΔP which is the pressure jump created by the disc.

And we have to remember that in axial momentum theory we are assuming that this pressure jump as well as the velocity is uniformly distributed over the disc. So,

$\Delta P = \rho[2VU_A+U_A^2]/2$. Now, again if we write ΔP equal to. So, this pressure jump can be written as $\Delta P = \rho U_A[V+U_A/2]$ because this half goes inside the bracket ok.

Now, let us write it as 4 right. Now, this pressure jump is related to the thrust produced by the momentum disk. How? The thrust produced will be this pressure jump into the area of the propeller disc right. So, how is the thrust related to the velocity U_A we have already derived it in the last page here.

Thrust is rho area of the propeller disc terms $A_P V_P U_A$, right equation 1. So, if we write $T=\rho A_P V_P U_A$, right. So, if we compare equation 1 this is nothing but equation 1, if we compare that with 4 what do we get? $\rho A_P V_P U_A$, is equal to $\rho U_A[V+U_A/2] \times A_P$, because thrust is equal to $\Delta P \times A_P$ area of the disc. So, ρA_P will cancel out on both sides and U_A will also cancel out.

So, we have $V_P = (V + U_A/2)$, this is a very important result coming from axial momentum theory. Now, the implication of this result is that the velocity at the propeller disc is the velocity V plus half of the velocity that is induced far behind the disc. That means, at the propeller disc we have the induced velocity component is half of the induced velocity which is far behind the propeller plane.

So, this can be proved easily using momentum theory given the assumptions that we have made. Now, next we will try to find the efficiency of the momentum disc.

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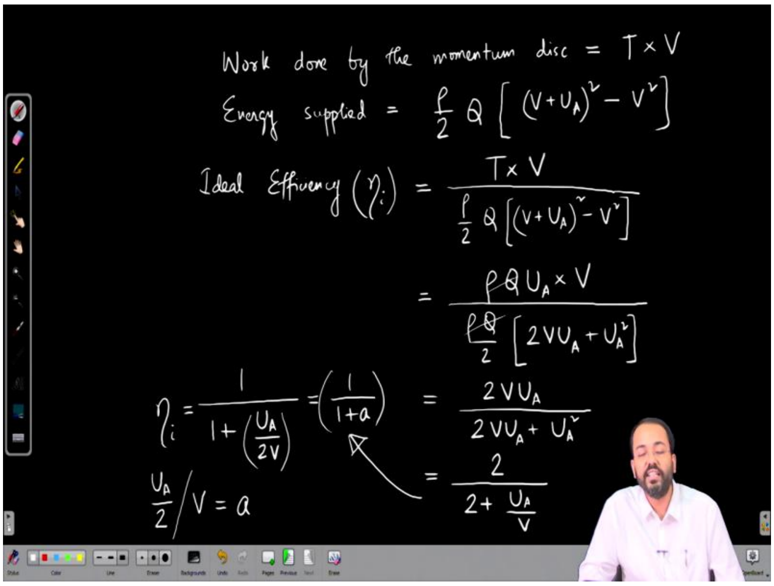

For that we have to calculate the work done by the momentum disc is given by the thrust that it generates, multiplied by the velocity against which it generates the thrust. So, here we will not take V_P or any other velocity, it has to be the velocity at which the momentum disc is moving ahead or the fluid is coming here in this case the from the inlet velocity. So, thrust times the velocity. Now, what is the energy supplied? Energy supplied is basically the difference of the kinetic energy at the outlet minus the inlet ok.

So; that means, $(\rho/2) \times Q$, where we have the mass flow rate $Q \times (V + U_A)^2$, the kinetic energy is basically $MV^2/2$. So, instead of the mass we have the density times the flow rate here per unit time because we are calculating the energy supplied per unit time. $(V+U_A)^2$ at the output minus the input energy V^2 here.

Energy supplied = $(\rho/2) \times Q \ [(V+U_A)^2 - V^2]$

So, the ideal efficiency η_i is given by the work done by the energy supplied.

$\eta_i = (T \times V)/[(\rho/2) \times Q \ [(V+U_A)^2 - V^2]]$

So, basically it is the output that we get which is again the thrust times the velocity right divided by the input ok. Now, how is thrust defined with respect to the mass flow rate? It will be $\rho Q U_A \times V$ here and in the denominator we have $\rho Q/2[2VU_A+U_A^2]$, the expression within the bracket will be $[2VU_A+U_A^2]$, right.

Now, ρQ terms will cancel out each other right so, we have $2VU_A/[2VU_A + U_A^2]$. Again we would want to make this expression non dimensional ok. So, it is already non dimensional, but we will try to make both the numerator and denominator non dimensional so that we can express the velocities as non-dimensional ratios, each of them.

So, we divide the numerator and denominator by $V \times U_A$. So, what we have here will be $2/[2 + U_A/V]$, this can also be written as divided by 2, it will be $\eta_i = 1/[1 + U_A/2V]$. Now, why do we call it ideal efficiency? Because in momentum theory, we are neglecting friction losses and other losses due to rotation and all so, because of neglecting of these losses we call it ideal efficiency because this is the maximum efficiency which the system can achieve, if we disregard all these losses.

Now, if we see this term here, we only have a ratio of the velocities here, in this efficiency term and basically $U_A/2$ is what? It is the induced velocity at the propeller plane because the velocity at the propeller plane V_P was $V + U_A/2$, where $U_A/2$ was the component coming from the induced velocity part, due to the action of the momentum disc. So, that divided by the input velocity if we write it as a ($U_A/2V = a$). So, let us say it is a non-dimensional fraction.

So, then we can write $\eta_i = 1/[1 + a]$, where a is basically the induced velocity at the propeller disc non-dimensionalized using the velocity of moving forward or the inflow velocity. So, this will be the ideal efficiency from the axial momentum theory. So, this will be all for today. So, we will continue with the thrust loading and other calculations regarding the momentum theory.

Thank you.