Marine Propulsion
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Lecture - 05
Propeller Theory II

Welcome to the 5th lecture of the course Marine Propulsion today we will be continuing with Propeller Theory.

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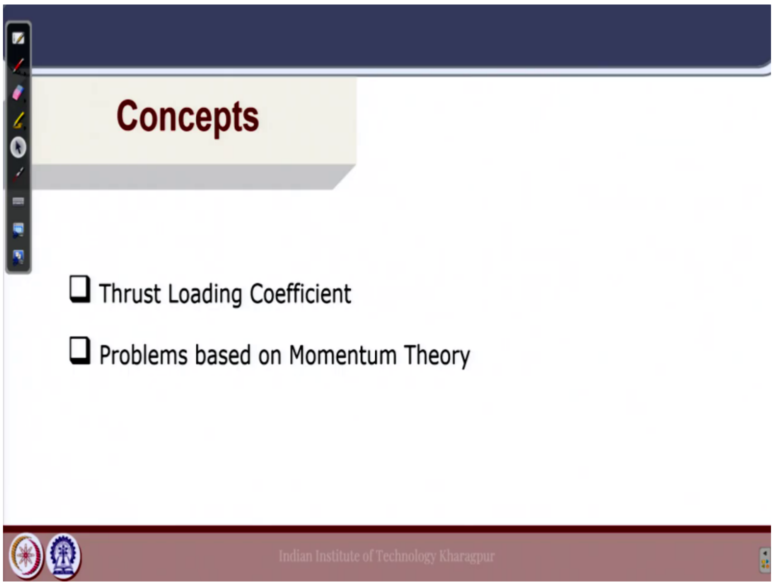

So, the key concepts discussed in today's class will be thrust loading coefficient from the context of propeller momentum theory and we will also do some problems based on momentum theory.

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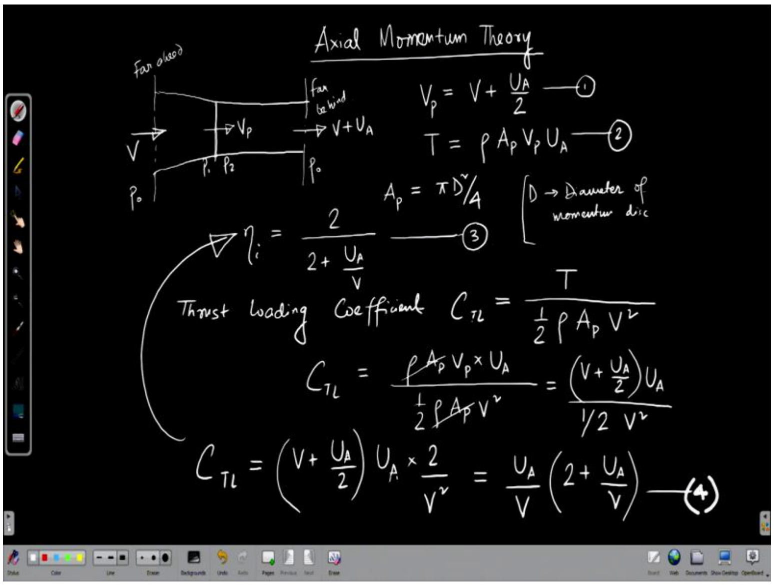

So, let us continue with the equations that we have derived for axial momentum theory of propellers.

So, I am just showing the domain here we had the inlet velocity because instead of the propeller moving ahead with a velocity V we had defined the actuated disc facing a velocity V at the inlet and the disc is fixed at a particular position. So, this was far ahead and far behind the pressure is same in these two conditions and we have the axial induced velocity U_A which adds up to the inlet velocity.

And at the propeller plane so, the propeller is simulated with an actuated disc which imparts axial momentum to the flow here the velocity is V_P at the propeller plane and there is a pressure jump from P_1 to P_2 across the propeller disc. So, the key equations that we have derived in the last class where V_P was equal to $(V + U_A/2)$ where U_A is the total axial induced velocity next thrust is given by $\rho \times$area(A_P) of the disc $\times$ the velocity at the disc(V_P) $\times U_A$ right.

Where area is the disc area ($\pi D^2/4$) where D is the diameter of the disc diameter of momentum disc. Now next what we will do is we have also derived the ideal efficiency of the momentum disc system and it was $2/[2 + U_A/V]$. Next we will define the term thrust loading coefficient in the context of the axial momentum theory.

So, this coefficient is basically non-dimensionalizing the thrust generated by the disc with respect to the parameter 1/2 $\rho \times A \times D^2 \times V^2$. So, thrust loading coefficient which we call as C_{TL} is basically the thrust generated by 1/2 $\rho \times A_P \times V^2$. So, here V is the reference velocity which we will use to define different parameters of the axial momentum theory.

So, for the thrust loading coefficient we use it to non-dimensionalize the thrust and we will relate it to the ideal efficiency in this particular calculation. So, what is T? T will be $\rho \times A_P \times V_P \times U_A$ right and in the denominator we have 1/2 $\rho \times A_P \times V^2$ right. So, $\rho \times A_P$ cancels out. So, what is V_P? V_P is $(V + U_A/2) \times U_A$ and we have $1/2 \times V^2$ right.

So, now C_{TL} becomes $(2 \times (V + U_A/2) \times U_A)/V^2$ ok. We can rearrange these terms and write it as U_A/V and take the 2 inside the bracket $(2 + U_A/V)$ ok. So, this becomes our 4 in this way what we have done is we have expressed C TL as a non dimensional velocity equation.

So, we have U_A and V where U_A is the axial induced velocity and V is the velocity at which the momentum disc is advancing in the fluid. So, given that ratio we will be able to calculate through the thrust loading coefficient. Now how do we relate this to the ideal efficiency? This is very important in the context of understanding the efficiency of a propulsion system.

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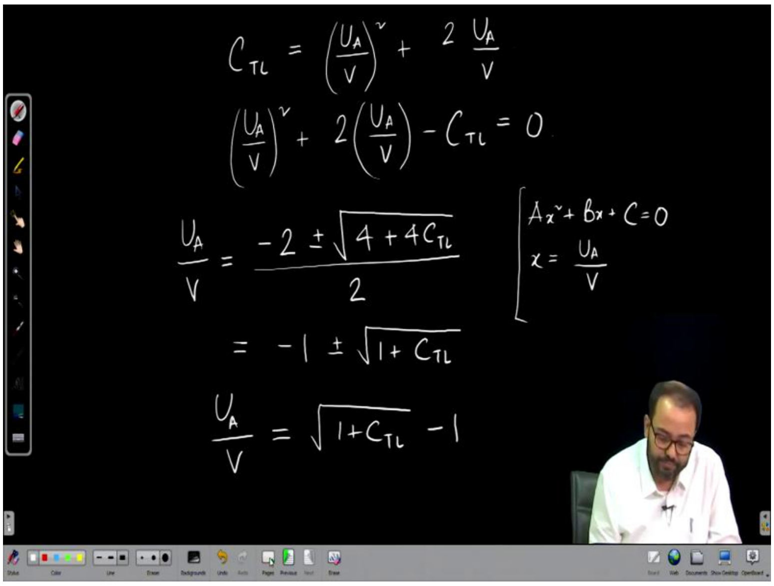

So, if we expand this equation C_{TL} becomes $[(U_A/V)^2 + 2\ U_A/V]$ right. Now, we can write it as a quadratic equation $[(U_A/V)^2 + 2\ U_A/V] - C_{TL}=0$, which is basically of the form $A\ x^2 + B\ x + C = 0$ where the variable we can calculate x here, which is U_A/V.

So, what we do here is we apply the Sridharacharya method and calculate $U_A/\ V$ as - b which is 2 ± root over of (b^2- 4 ac). So, C_{TL} is basically -c. So, plus 4 C_{TL}, and a which is one here by 2 right. So, this becomes -1 ± root over($1+C_{TL}$). So, 4 comes outside the root and divided by 2 it is 1 right ($1+C_{TL}$).

Now, when we have a velocity V at which the momentum disc is advancing forward the induced velocity U_A induced by that disc has to be in the same direction as V. So, U_A by V is definitely positive. So, we will take only the positive root here U_A/V will be root over of ($1+C_{TL}$) -1 right. So, for all positive values of thrust loading coefficient you will have U_A/V will be positive here ok. Now how do we write η_i?

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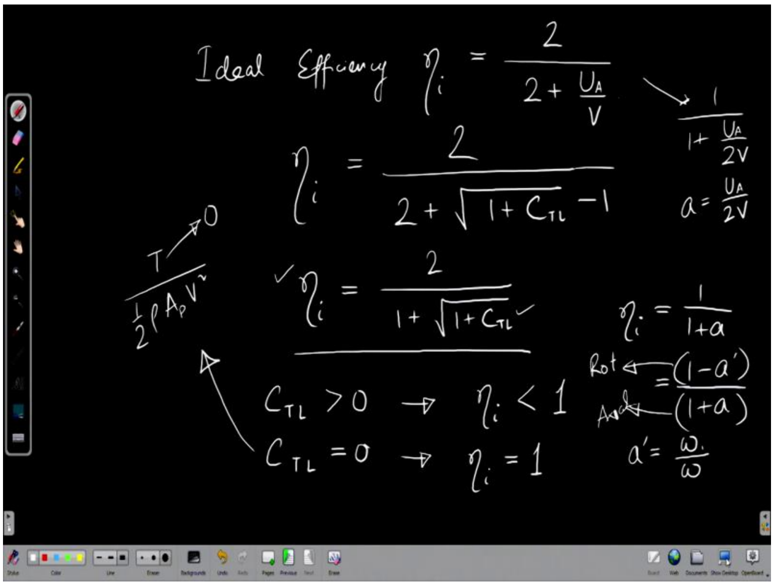

The ideal efficiency η_i was 2 /(2 +U_A/V) right and from the previous page we see that U_A/V is root over ($1+C_{TL}$) -1. So, we can write it as 2 / (2 +root over ($1+C_{TL}$) -1). So, your ideal efficiency becomes 2/ (1 + root over ($1+C_{TL}$)) this is a very important result from the context of axial momentum theory of propellers which relates the ideal efficiency to the thrust loading coefficient.

Now what does it imply? If you want to get an ideal efficiency of 1 let us say. So, what will be the value of the thrust loading? For any value when C_{TL} is greater than 0 let us say for any positive value of thrust loading coefficient what will happen to this denominator? 1 + root over C_{TL} will be greater than 1. So, η_i will be 2 by something which is greater than 2. So, η_i will be less than 1 ok.

So, the only condition where we can get an ideal efficiency of 1 is the case where C_{TL} is 0 then only from this equation we will have the ideal efficiency equal to 1. Now what does it imply? If C_{TL} is 0; C_{TL} is defined by thrust(T) /[1/2 ρ A_P V^2] right. So, if C_{TL} is 0 then thrust has to be 0; that means, the momentum disc is not generating any thrust.

So, if it does not generate any thrust then there is no point in the calculation of efficiency here ok. So, for any practical value where the momentum disc is generating a finite value of thrust we will have the ideal efficiency of this system less than 1 ok. So, another important thing should be mentioned here is this is a very simplified assumption that in this axial momentum theory the propeller is or the disc is not imparting any rotation to the flow.

Now, what happens if we consider a rotation in the flow? If we do that then the ideal efficiency will be slightly reduced by another fraction which will come from the rotational value. So, here we have the ideal efficiency given by 1 /1 + a if you remember where a is the ratio of the axial induced velocity at the propeller disc.

So, this was basically 1 by 1 plus $U_A/2V$ which is the ratio $U_A/2V$. So, similarly we can define another ratio if we take care of the rotation ok then we will see that it will come to another value (1 – a')/(1 + a). similarly where a' will be the rotational component induced at the propeller plane divided by the rotational speed of the momentum disc (i.e., $a'=(\omega'/\omega)$) and a here we already know $a = U_A/(2V)$ ok.

So, if we consider both the axial and rotational induced velocities, η_i will be reduced further; that means, both the numerator here will be lower and we have a already higher denominator due to the effect of the axial induced velocity. So, this here the numerator is the rotational component (1-a') and the denominator is due to the axial component (1+a) of the induced velocity ok.

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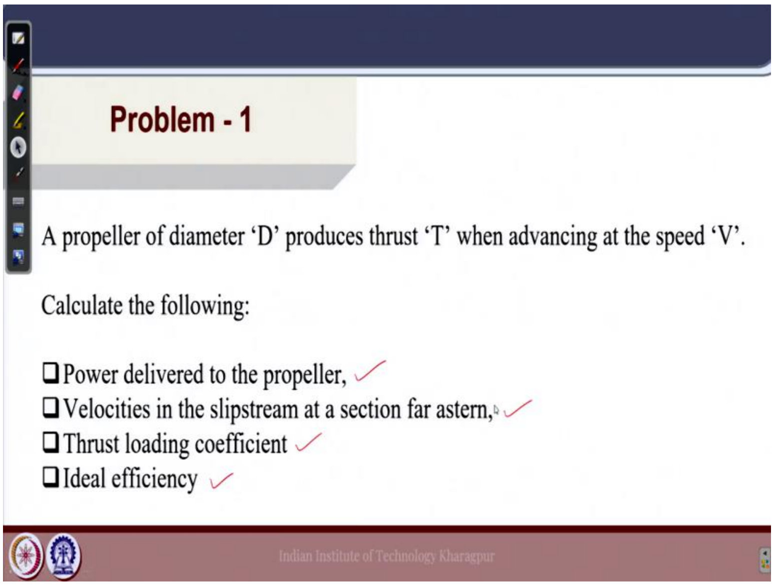

So, let us try to look into very simple problems, which we can do applying axial momentum theory ok. Suppose in this particular problem you are given a propeller of diameter d which produces a thrust T when advancing at a velocity or speed V ok. Calculate the different components like power velocities in slip stream at a section far astern; that means, far behind thrust loading coefficient and ideal efficiency.

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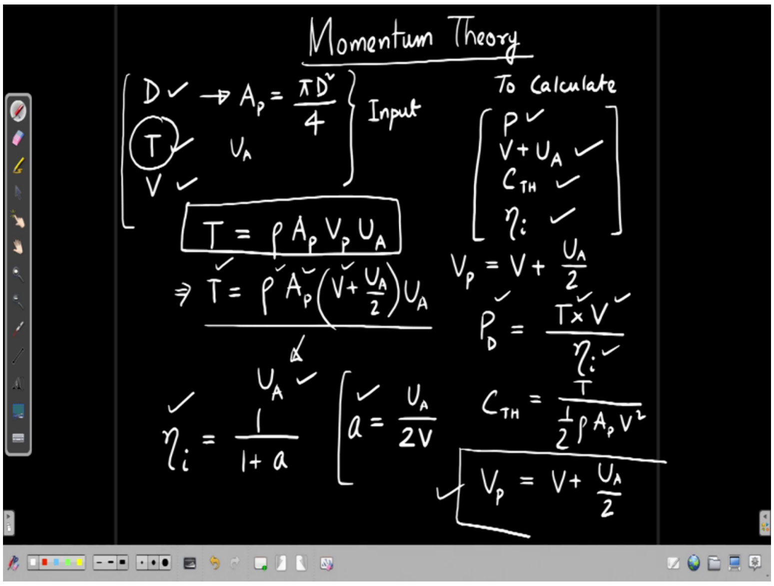

So, in this given problem the diameter of the momentum disc is given which is representing our propeller.

So, D is given which gives us the area of the disc or propeller we can say is $\pi D^2/4$. Next the thrust generated is given and the velocity of advance V is given these are the input parameters the parameters to be computed. So, I write to calculate are power delivered to the propeller velocity far astern which is $V + U_A$ the axial induced velocity the thrust loading coefficient(C_{TL}) and the ideal efficiency η_i

We start with the given value of thrust and the equation for thrust with respect to the velocity components is given by $T = \rho A_P V_P \times U_A$ where U_A is the axial induced velocity and V_P is the velocity at the propeller which is the velocity of advance or the inlet velocity plus half of the induced velocity far astern right. So, we can write this as T is $\rho A_P(V + U_A/2)\times U_A$. Now in this particular equation thrust is known and on the right hand side all the terms except U_A are known.

So, when we put the values we will have a quadratic equation with U_A as the unknown and we can solve for U_A. So, this gives the value of U_A from this we can now get $V + U_A$ as the velocity far astern because V was already known. So, this solves one part of the problem next η_i is given by $1/(1 + a)$ where a is the fraction given by $U_A/2V$ which is $U_A/2$ the axial induced velocity at the propeller plane divided by the advanced velocity V.

Now, once we have computed U_A we can calculate this fraction a and this gives us the value of η_i so, this part is done. Next we can compute power delivered or power P_D; however, we may write now here as thrust (T) × velocity(V) divided by η_i we can use the output power which is thrust into velocity divided by the efficiency to get the power which is delivered to the propeller. So, this the same as P_D which is mentioned here.

We know all these terms $T \times V$ and η_i. So, P_D can be calculated and finally, the thrust loading coefficient C_{TH} can be computed as $T/[1/2\ \rho\ A_P V^2]$ in this way all the terms can be calculated for this problem. Now the same problem can be given in a slightly different way where instead of this thrust value the any of the velocity component can be given for example, either U_A or the total velocity far astern may be given.

So, we can use the same set of parameters to calculate the value of V_P right. So, for example, if $V + U_A$ is given we can calculate $(V_P/V+U_A/2)$ from the value of total $V + U_A$ we can calculate only U_A and then we can calculate the value of V_P from that and we can use this value to calculate the thrust, which is unknown if the problem is given in that way. So, in this way we can use simple momentum theory calculations to do basic powering estimate for a momentum disc where only the axial induced velocity is considered.

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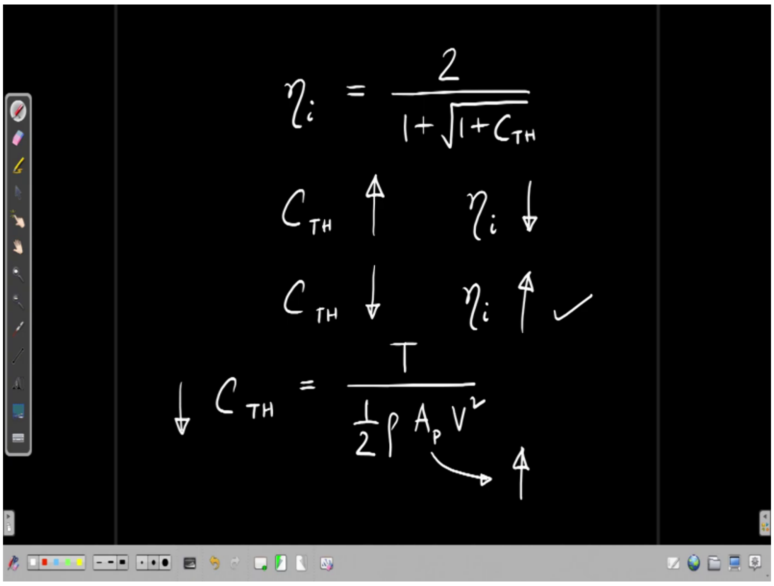

Another very important aspect of this momentum theory is the relation between the ideal efficiency and the thrust loading coefficient. So, as the thrust loading coefficient C_{TH} increases the ideal efficiency of the propeller or the momentum disc here will decrease. And only for the condition where this C_{TH} is equal to 0 then the momentum disc will have an efficiency of 1; that means, when the disk is not producing thrust at all that condition only will lead to an efficiency of 1.

So, for any practical condition where a finite thrust is being generated there is a value of efficiency which will be less than 1. So, if this thrust loading coefficient increases the ideal efficiency will decrease on the other hand if C_{TH} decreases the ideal efficiency can be increased. Now this has a simple practical implication for example, if we have a propeller of higher diameter as compared to another propeller which has a lower diameter, then for the same thrust output the propeller having the higher diameter can operate at a higher efficiency.

So, C_{TH} is basically T / (1/2 $\rho A_P \times V^2$). So, if the propeller diameter is higher then A_P will be greater right if A_P is greater for a propeller of higher diameter it will lead to a lower value of C_{TH} that will lower the thrust loading coefficient. And if the thrust loading coefficient is lowered the efficiency will be higher that is why it is preferable to have a propeller of higher diameter to obtain a higher efficiency. If all the other geometrical aspects are similar.

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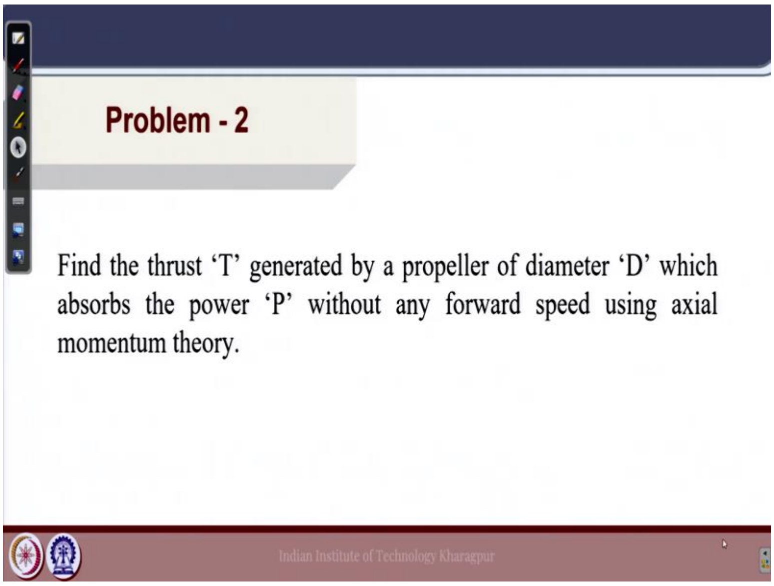

Another problem which is slightly different is like this you have to calculate the thrust generated by a propeller ok where the diameter is given and it absorbs a power (P) without any forward speed using the axial momentum theory ok. So, basically the input power is given or thrust may also be given whatever any one of them can be input and another one you may be asked to calculate and we are having a propeller of diameter (D) and we would want to calculate at the condition where the forward speed is basically 0.

This condition is very special for ships because later when we discuss propeller in open water condition we will see that this is called the bollard pool condition where the forward speed of the ship is 0 and we have the propeller working in a static condition and we would want to know the characteristics of the propeller ok. So, using axial momentum theory how can we do that this is the next calculation example.

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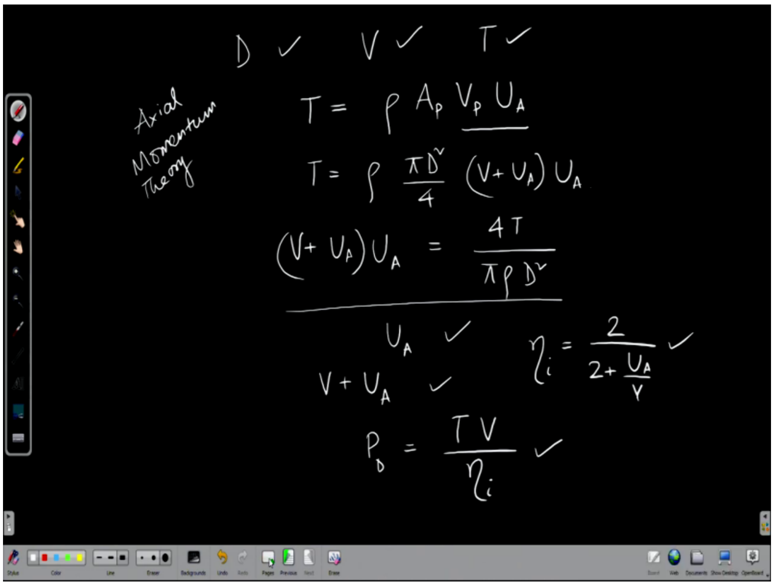

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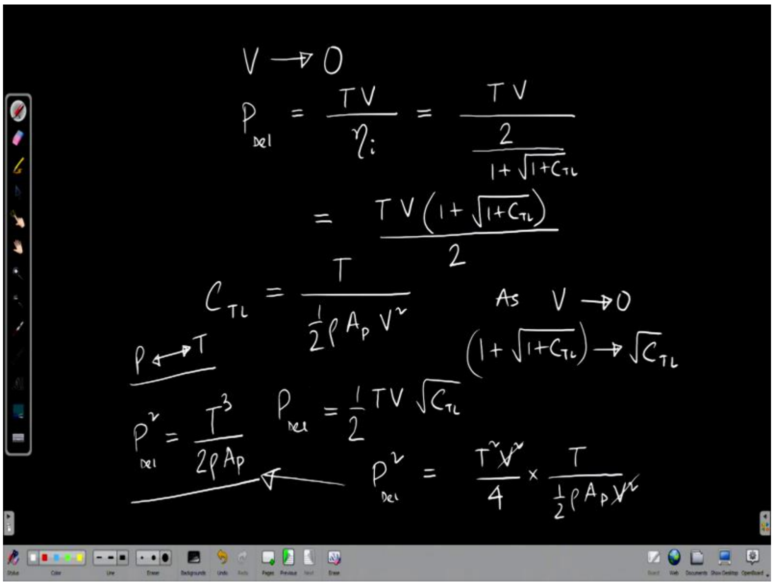

Now let us say we have the forward velocity V is approaching 0 we take a limiting condition where V is approaching 0. Now what is the power delivered = T×V /η_i. Now, if we write this η_i as a function of C_{TL} the thrust loading coefficient what will it be? It is 2 /(1 + root over(1 + C_{TL}) ok.

So, the delivered power can be calculated as [T×V ×(1 + root over(1 + C_{TL})]/2. Now when the velocity the forward speed of the propeller in this case the momentum disc is 0 what happens to the thrust loading coefficient C_{TL} is given by T /(1/2 ρ×A_P×V^2) right.

So, when the velocity V is approaching 0, C_{TL} will be extremely high right at velocities close to 0. So, in those conditions we can write that as V is approaching 0, (1 + root over(1 + C_{TL})) will approach root over C_{TL}. This is a small trick we need to do to cancel out the velocity components so, that we will be able to relate the thrust to the delivered power ok. So, we write the same equation we delivered will be [T ×V ×root over C_{TL}]/2 ok. Now if we take square power delivered square $(P_D)^2$ will be $(T^2 V^2/4) \times C_{TL}$.

Now what is C_{TL}? C_{TL} is $T/(1/2\ \rho \times A_P \times V^2)$. The V^2 term can be cancelled out so, that we can get this equation power delivered square is $T^2 \times T = T^3/(4/2)$ is 2; ρ A_P; A_P is the Area of the momentum disc here $(T^3/2\rho A_P)$. So, we can use this equation simply to relate the power to the thrust. The delivered power to the thrust can be related using this equation where we have the forward velocity approaching 0 using the axial momentum theory ok. So, in this condition if the thrust is given we can calculate the power and vice versa ok.

So, these are some simple problems we can do using axial momentum theory and this gives a basic idea of thrust loading and other efficiency calculations that can be easily performed using axial momentum theory, which is a very simplified theory, but it gives some understanding of the efficiency that a propulsion system can have at a given conditions which involve the velocity and basically the thrust that the propeller is producing with respect to the diameter which is given finally, by the thrust loading coefficient ok. So, this will be all for today’s class.

Thank you.