Marine Propulsion
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Lecture - 06
Propeller Theory III

Welcome to the 6th lecture of the course, Marine Propulsion. Today, we will be continuing with Propeller Theory.

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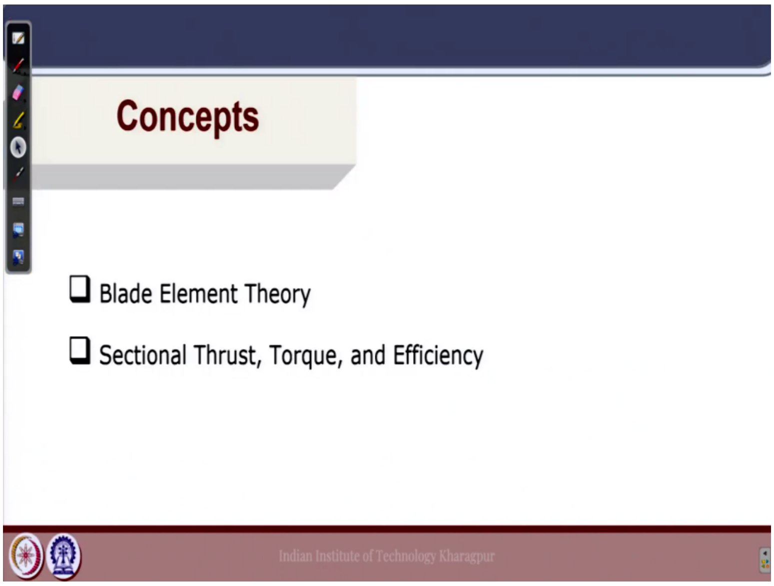

So, the basic concepts which we will be discussed in today’s class will be blade element theory and from that blade element theory how we can calculate the sectional thrust, torque, and efficiency for the propeller blade, ok.

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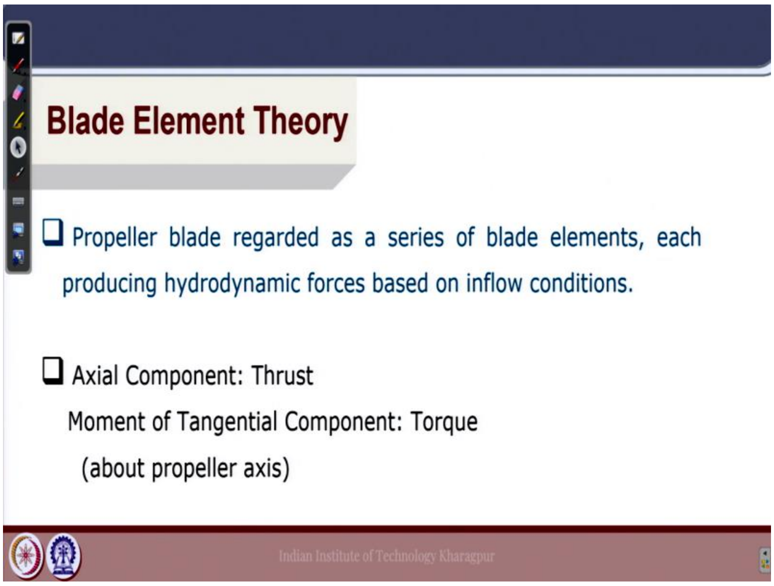

So, before going into the details let us try to understand what are the assumptions of the propeller blade element theory. In this theory, we have the propeller blade which is regarded as a series of blade elements. Initially, we have started with the most simple theory which is the axial momentum theory where the propeller blade was not considered.

So, we had only a disc, we had only a momentum disc which provided the thrust. So, there was no shape or configuration of the propeller blade. But in this particular theory where we will go into slightly more details, we have the propeller blade which is regarded as a series of elements or different sections of the propeller blade and each of them will produce some force depending on the inflow condition.

And from that how will we how we are going to compute the propeller forces? We will have the axial component of that force which is the thrust. And then we have the tangential component the moment of which about the propeller axis will be called the torque, ok. So, for the propeller, we have different sections, at different cylindrical sections which we are calling the blade elements at different radii and each of them will have the sectional thrust and torque depending on the velocity conditions, ok.

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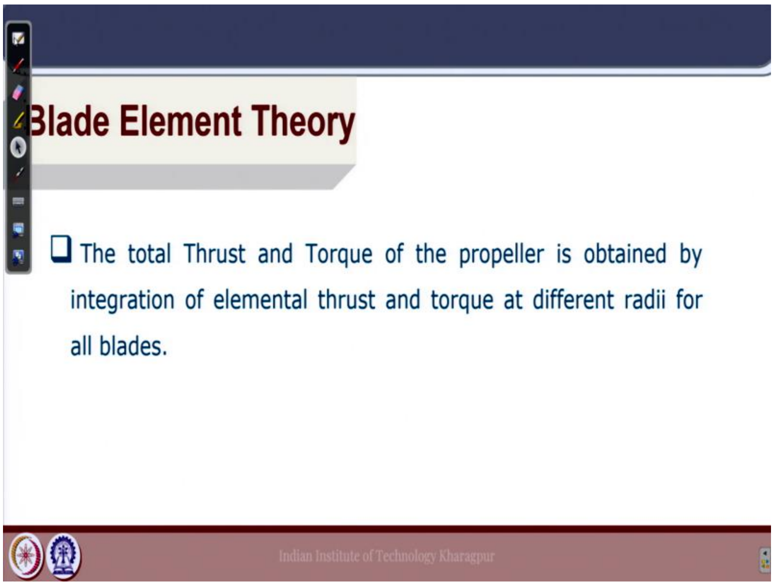

And then, the total thrust and torque of the propeller blade can be obtained by integration of this elemental thrust and torque at this radii. So, what we do is, we for each element we can compute the thrust and torque of the propeller and then we can integrate them over the blade, over the part of the blade which is producing. So, that from the hub to the tip and then that gives us the total thrust and torque of the propeller, ok.

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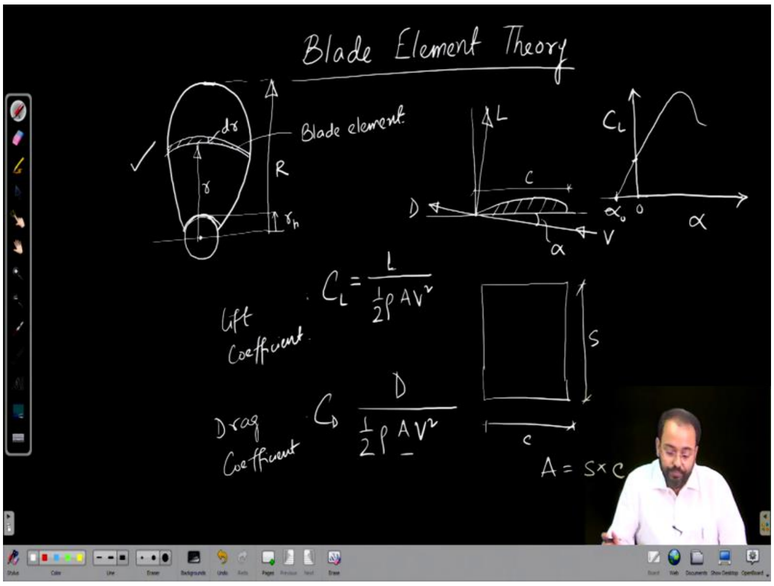

Now, let us try to look into the different aspects of the blade element theory. So, we have the propeller blade. This is the radius of the propeller blade R, and the blade is fitted over the boss or the hub. This is the radius of the hub(r_h),. And let us take any cylindrical section at a radius small r, of thickness dr, ok. So, this is basically the blade element.

So, in this particular theory, we are considering the blade to be consisting of a number of such elements right from the root of the propeller blade, the root section here to the tip of the propeller blade here. And for each blade element depending on the inflow velocities and finally, we will calculate the angles based on that we will calculate the forces from where thrust and torque can be computed, ok.

Now, let us see how a blade element will provide the forces. For that, again as far we have discussed in the propeller geometry. So, this propeller blade consists of different radial sections that we can take at different locations along the radius of the propeller blade. Now, each of these sections typically can be considered as simple air-foil sections and depending on the flow characteristics over those sections the sections generate force.

So, let us say this is a section, we have taken of the propeller blade at any particular radius and we have an inflow to that section at an angle of attack alpha(α), ok. So, what will happen? It will generate a force which is lift perpendicular to the inflow vector and also it will generate a drag force. So, L is the lift and D is the drag of that particular section, ok.

Now, from that how do we calculate the lift and drag coefficient? If we divide the lift by ($1/2\ \rho\ A\ V^2$), we get the lift coefficient (C_L), ok. Similarly, if we divide the drag force D by ($1/2\ \rho\ A\ V^2$) we get the drag coefficient(C_D), ok. Now, what is A here? If I take the plan from area; so, this is basically we have drawn the velocity with respect to the base line here, the angle shown. So, this is the chord length of the propeller blade, of this particular section.

So, if we take a three-dimensional foil, ok, so the planform area that we have which consists of this called two-dimensions and the other one is the span S. So, this area will be given as span into chord, ok. So, this is the area which is used to calculate the lift and drag coefficient, ok. Now, how does these forces depend on the angle of attack? Specially, the lift is very important for us because that is the part which is most important in calculating the propeller forces that we do from the sectional properties.

So, if I draw this lift as a function of angle of attack, let us say lift coefficient, ok. So, the typical curve will be like this, ok. So, at 0 angle of attack there will be a small value of lift coefficient that will come due to the camber of the foil, ok. We have gone through the basics of foil section, where in the propeller geometry. So, if the foil is a symmetric foil, ok where the foil section is symmetric about the centre line, ok then in that case the foil will not generate any, lift at 0 angle of attack.

But if the foil is cambered it will generate a small lift at 0 angle of attack. And the lift will be 0, the value of C_L will be 0 at a value of negative angle of attack, ok. This is the no lift angle alpha(α) 0, where actually C_L is 0. Now, this is required because when we now go to the blade element theory and calculate the propeller forces, we will require this basic concept of variation of lift coefficient with angle of attack, ok. So, we will use these concepts now to calculate the forces on this particular blade element at a radius small r.

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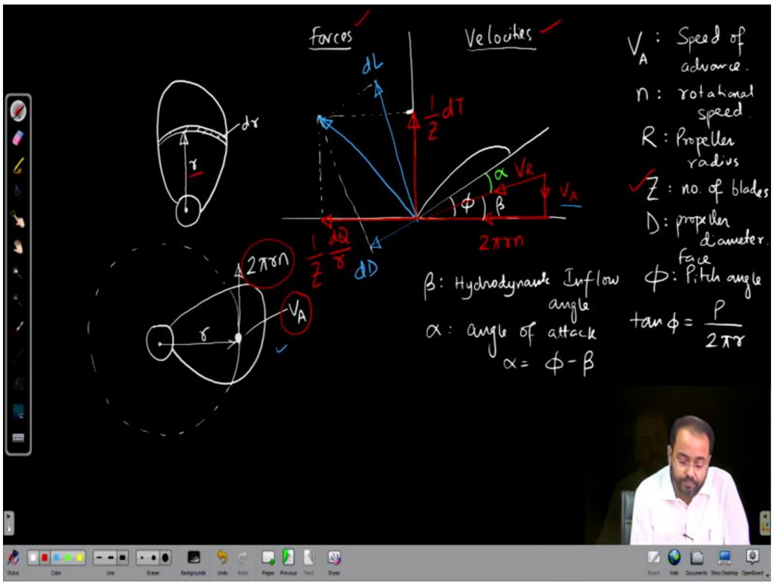

For this case, what will be the forces based on the velocity at that particular section? Now, if I look at a propeller blade, it is rotating as well as moving ahead. Here we have a case where we have a velocity which is coming to the propeller blade which is the inflow velocity or as the propeller blade is advancing, so it is a velocity of advance, we call it V_A.

Now, when we discussed the basic momentum theory, from axial momentum theory point of view there were no other velocities which was considered except the axial velocity. There was no rotational component in the axial momentum theory, the original one. So, in the axial momentum theory we had only the propeller moving at a forward velocity V, it was mentioned as a very generic value V. But here we would prefer to use the term V_A because later we will see that this is not equal to the ship speed, ok.

So, the propeller velocity of advance will be the velocity at which the propeller is advancing. That means, if we are standing in the position of the propeller that is the velocity that we will be experiencing. So, it is you can assume it as the averaged velocity at the location of the propeller in the axial direction, ok. So, if I think of a propeller blade, let us say rotating blade at any radius, ok the velocity in the axial direction is V_A which is perpendicular to the plane of the board here.

So, this shown as a dot will be V_A, and as the propeller blade rotates, so if the propeller blade is rotating because of the rotation there will be a tangential component of the velocity which is $2 \pi r n$ because we have taken the section at a radius small r, where n is the rotational speed of the propeller blade, ok. So, let us define a few terms before we start to draw the diagram related to the velocity components. So, V_A is the speed of advance, n is the rotational speed in rps, and R is the propeller radius, ok and let us say Z is the number of blades of the propeller and D is the propeller diameter.

Now, for any section at taken at a radius small r of the propeller blade because of the propeller blade pitch the section will have a pitch angle, right. So, ϕ is the pitch angle or here in this case it is the phase pitch angle because we have drawn the line simply on the phase of the section, ok. So, which is given by $\tan(\phi) = (P / 2 \pi r)$, where P is the pitch of the propeller blade at that location, ok.

So, it can be a fixed pitch propeller or the pitch can be varying with radius, that is a different thing. But here we will have the propeller, phase of the propeller blade at the section at radius r at an angle ϕ based on the pitch, right. Now, what are the velocity components? We have the velocity V_A here, perpendicular which is shown here V_A. And we have another velocity which is $(2 \pi r n)$. Now, V_A and $(2 \pi r n)$ are mutually perpendicular to each other. So, this is V_A and we have $(2 \pi r n)$ from the tangential component, ok as the propeller blade rotates at the speed rotational speed of n rps.

So, the resultant velocity to this section can be drawn using this vector diagram which is basically V_R, the resultant of V_A and $2 \pi r$, right. We call this particular angle β, where β is called the hydrodynamic inflow angle because β depends on the two velocity components V_A and ($2 \pi r n$). So, β is basically a hydrodynamic inflow angle depending on the hydrodynamic characteristics, depending on the velocity values.

Now, on the other hand ϕ this phase pitch angle is a geometric characteristic of the propeller blade. So, ϕ does not depend on the velocity here, ok. Now, what is the angle of attack? If V_R is the resultant velocity and the phase pitch angle is at an angle phi here, so the angle of attack will be α, α is the angle of attack at this propeller blade section the angle which V_R makes with the phase pitch line of the propeller blade. So, α is given as ϕ-β, ok.

Now, in this part, we have drawn the velocities, ok. On the other half which I have left empty, we will draw the forces, ok. So, we have seen for the airfoil section, what are the forces that are generated due to a resultant velocity. We will have the lift and drag forces, the drag forces, the drag force along the line of the resultant velocity V_R and the lift force perpendicular to this line. Because these are sectional lift and drag forces we will name it as dL and dD, ok.

Now, finally, what do we need for the propeller blade section? We will need the thrust and torque. Now, the thrust which is the resultant force in the axial direction and the torque is the moment of the tangential component about the propeller axis. Now, which is the axial direction? V_A is the velocity which is coming here also. So, the thrust should be directed opposite to the direction of V_A, because V_A is the velocity which is coming on the propeller plane and the propeller is moving ahead. So, thrust should be directed vertically in the upwards direction in this.

So, we will have thrust along the vertical axis. Now, what is the resultant of these two forces? If I try to draw this the resultant of dL and dD will be directed in this particular direction. And now if I try to draw the components on the two mutually perpendicular axis, we will have the thrust (dT) and the other force which leads to the torque. So, because the moment of that force, where r is the arm is the torque, so we call it dQ/r, ok, because torque is a moment.

Now, here one more thing will come, we will assume that dT is the elemental thrust provided by the propeller blade over all the blades. So, the propeller depending on the design may have 3, 4 or 5 number of blades, ok. So, in this case the blade number is Z, ok. So, for each blade if dT is the thrust produced by all the blades, taken to all the blade elements at a radius r taken together, then the thrust produced by 1 blade at the radius r is given by 1 by Z dT, ok. Similarly, the torque will be (1/Z) (dQ/r), ok.

So, now, we have both the velocity diagram as well as the force diagram which forms the basis of the blade element theory. From here we will now compute the thrust and torque of this particular blade element. So, here the aim is to calculate the elemental thrust and torque and finally, calculate the efficiency of the propeller blade element.

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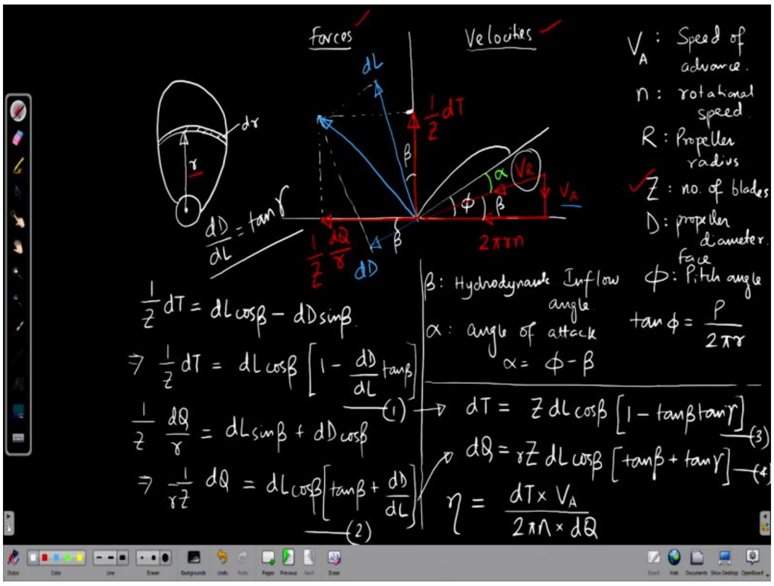

So, we can write 1/Z dT equal to what? Here this angle is equal to β, right that is how lift and drag are defined with respect to the resultant velocity V_R, ok which is inclined at β with respect to the horizontal line, and similarly this is also β, ok. So, 1/Z dT from the components we can calculate will be dLcos(β) – dDsin(β), ok because dT the component, the component from dL will increase dT and the component from the drag dD will decrease dT, ok.

So, we can write this as 1/Z dT equal to dLcos(β) (1 – dD/dL) tan(β), ok. Similarly, we can write 1/Z × dQ /r = dL sin(β) + dD cos(β). Now, here interestingly for the torque, we have both the components of lift L sin(β) and dD cos(β) which will add up to give the total one from this diagram, ok. So, this is plus here. So, 1/Z × dQ /r =dL cos(β). We will take dL cos (β) as common and write the first as tan (β) + dD/dL, ok. So, let this be 1 and this be 2, right.

Now, we will use a simplification. We will write dD/ dL the ratio of the sectional drag by the sectional lift force as tan γ, ok for ease of computations. So, we will write dD/dL equal to tan(γ). Now, from 1, we have $dT = Z\,dL \cos(\beta)\,(1 - \tan(\beta)\tan(\gamma))$. So, this is let us say 3. And from 2, we have $dQ = r\,Z\,dL \cos\beta\,(\tan\beta + \tan\gamma)$. This is 4, ok. So, this gives the sectional thrust and torque based on the lift and drag of the blade section, ok.

Now, how is efficiency defined? Efficiency is defined by the output power divided by the power absorbed by the propeller blade section here. Now, how is it defined? We have efficiency (η) is defined by dT is the thrust of the propeller blade element multiplied by V_A by ($2\pi n$) × dQ, ok. So, we will try to use section to calculate the efficiency of the propeller blade element. So, this efficiency is the efficiency of the blade element at the radius r, ok. So, if we write the efficiency here then will be $(V_A/(2\pi n)) \times (dT / dQ)$, right.

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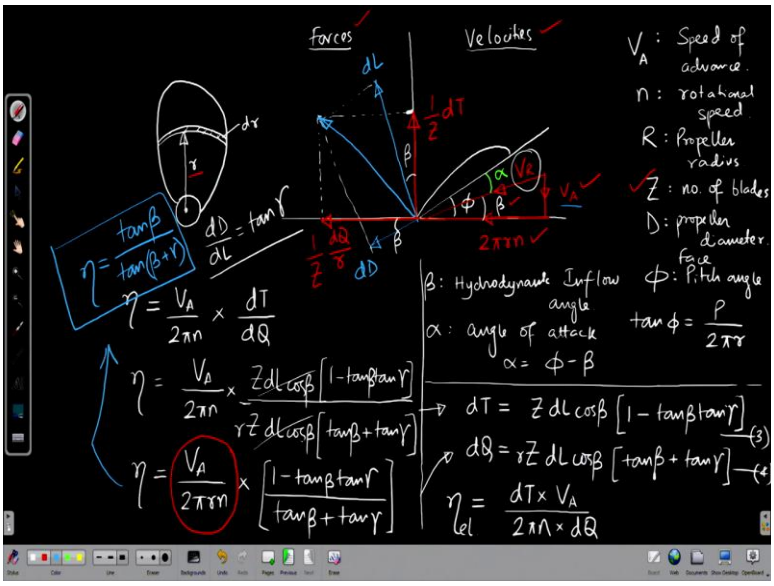

Now, using equations 3 and 4, ok efficiency can be written as $V_A / 2\pi n \times (Z\,dL \cos(\beta)\,(1-\tan(\beta)\tan(\gamma)) / (r\,Z\,dL \cos(\beta) \times (\tan\beta + \tan\gamma))$, ok. So, this efficiency becomes Z dL cos(β) is cancels out, so it is $V_A/2\pi r n \times (1 - \tan(\beta)\tan(\gamma)) / (\tan\beta + \tan\gamma)$, ok. Now, one final line; what is this fraction $V_A / 2\pi r n$?

V_A is the velocity of advance and $2\pi r n$ is the velocity due to the rotational component, ok. So, $V_A / 2\pi r n$ from this figure is nothing but tan of this angle β, ok. And what is $\times(1 - \tan(\beta)\tan(\gamma)) / (\tan\beta + \tan\gamma)$? That is $1/\tan(\beta+\gamma)$. So, this efficiency becomes a multiplication of tan (β) and 1/tan(β +γ). So, we can write the final efficiency is tan(β)/tan(β +γ), ok from here, right.

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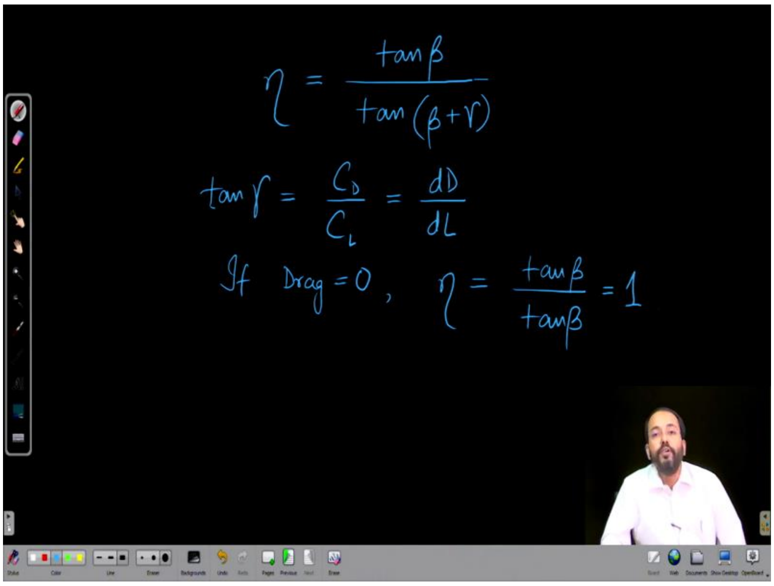

Now, let us look into the efficiency equation directly, equal to tan(β)/ tan(β +γ). Now, how is γ defined? Tan(γ) is defined as C_D/C_L, ok or the sectional drag by sectional L, because C_D and C_L are obtained by dividing dD/dL by the same quantity which is $1/2\rho AV^2$, ok. So, when is tan(γ)=0? When if I assume that drag is 0; if there is no drag then tan(γ) will be 0. So, in that case, the efficiency of the blade element will be tan(β)/tan(β) which is 1, ok.

This is a very curious observation from the blade element theory, where if we neglect drag we can obtain an efficiency of 1, ok. So, in the axial momentum theory, we have considered the induced velocity, the axial induced velocity and that is how we have computed the efficiency. But in this blade element theory the basic calculation that we have done right now, we have not considered any induced velocity. That is why the efficiency computed here has come to 1, if we neglect drag, ok.

So, we will continue with Blade Element Theory with the induced velocities, in the next class.