Marine Propulsion
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Lecture - 07
Propeller Theory IV

Welcome to the lecture 7 of the course Marine Propulsion. We will continue with Propeller Theory.

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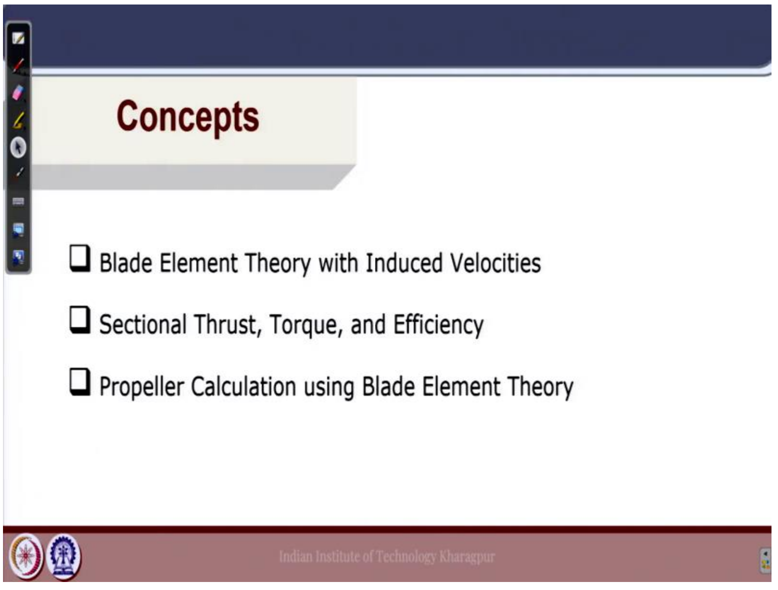

So, the key concepts covered in today's lecture will be; blade element theory with the induced velocities and then how it impacts the sectional thrust torque and efficiency of the propeller blade. And finally, we will do a simple propeller calculation using blade element theory ok.

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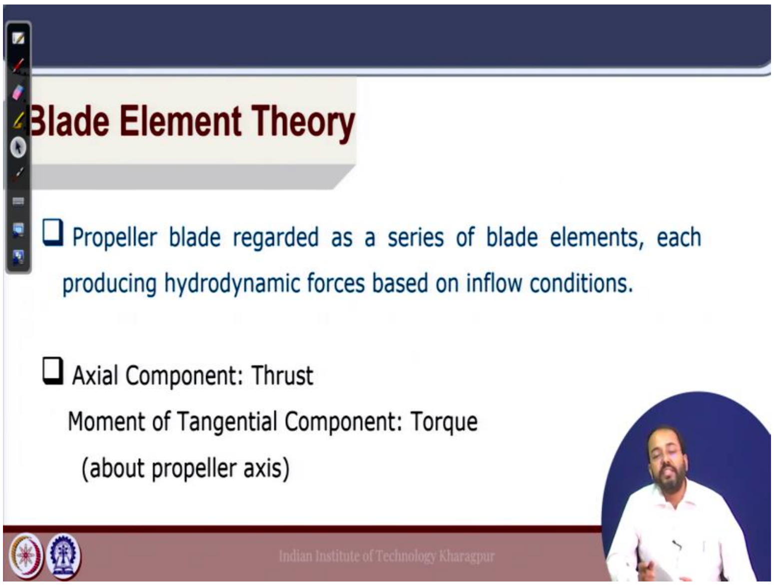

So, just a short recap of the assumptions that we make. In the blade element theory we have the propeller blade which is regarded as a series of propeller blade elements and each of them provides forces hydrodynamic forces based on the inflow conditions. So, the velocity profile of each blade element varies along with the radius and based on that they provide they generate the forces. And the axial component of that force is the thrust and the moment of the tangential component is the torque that moment is taken about the propeller axis.

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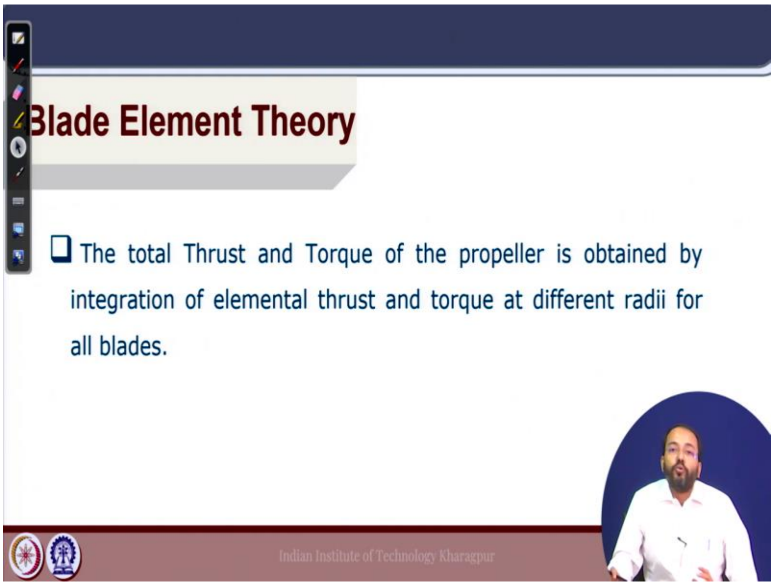

And the total thrust and torque of the propeller blade can be obtained by integrating the elemental thrust and torque at different radius for all the blades.

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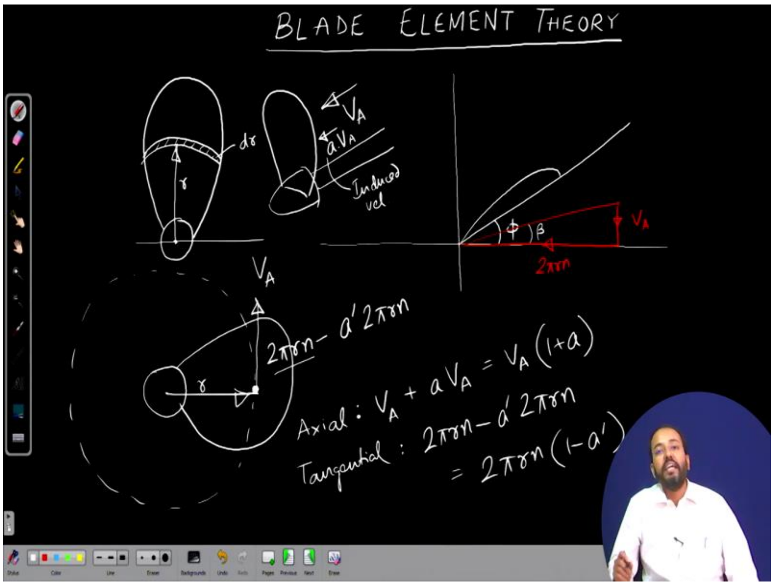

Now, let us move. Now let us start with the blade element theory considering induced velocities. Similar to the basic blade element theory calculation we have a propeller blade for which we are taking a small strip or the element at a radius r of thickness dr ok and for that element we will try to compute the thrust and torque. So, again we will draw the blade element diagram. So, using the same nomenclature we have used for the basic blade element computations.

So, this is the blade element which is having an angle ϕ with the horizontal and that is defining the phase pitch angle of the propeller blade at the radius at which we have taken the element ok. And we have the two velocities V_A and ($2 \pi r n$) the resultant of which initially was computed as V_R which had an angle β with the horizontal. So, here in the initial calculations we did not assume any induced velocities.

The meaning was that if the propeller is rotating it is moving ahead with a velocity V_A and rotating at n rps which is giving the tangential velocity at the radius small r equal to $2 \pi r n$. So, we had only the two components. But let us now go to the realistic scenario what will happen to the propeller blade at a radius r? Let us think of the fluid particle which is around the propeller blade at the radius r.

Now, as the propeller blade rotates and also moves ahead the fluid particle will have a relative velocity with respect to the propeller blade which is V_A which is the velocity of advance of the propeller blade as the blade is moving ahead. And as the blade is rotating the relative velocity in the other direction will be 2 π r n, but the propeller not only moves ahead, but also induces a velocity to the fluid because of its action because the propeller blade generates a pressure difference.

So, it also induces a velocity as we have seen in the momentum theory. Here we have not initially considered the induced velocities in the original derivation in the last class. So, if we think of the induced velocities, what will be the final velocity that will come in this blade element diagram? Now V_A is the velocity or the speed of advance of the propeller blade. Now as the propeller blade is advancing it is also inducing a velocity to the flow due to the suction effect.

So, the fluid velocity with respect to the propeller blade is more than V_A because V_A is the original velocity of advance plus the induction component plus the induced velocity which comes as the propeller is advancing in the fluid right. Now, what happens to the rotational component? As the propeller blade rotates at any radius r the relative velocity with respect to the fluid particle was 2 π r n because of the rotational speed n.

Now if we think of the induced velocities as the propeller rotates it will also induce a velocity to the fluid particle in that direction ok. Now initially think of let us say as we have drawn the propeller blade it is rotating and we are taking a point here ok and V_A was perpendicular to the plane of the board here. So, initially it was V_A, now basically it is the velocity at which the if I think of the draw it in a slightly different way. Let us say in an isometric view if this is a propeller blade ok which is facing a velocity V_A.

Now, as it induces a small velocity also to the flow that will also be accelerated towards the propeller. So, that induced component, let us say is a × V_A we express it as a non dimensional fraction of the actual advance velocity V_A. So, we write it we write the induced component a V_A the induced velocity as a × V_A. Similarly, the original velocity at a radius r due to the rotational component was 2 π r n right.

As the propeller rotates it also induces a tangential velocity to the fluid around it. So, not only the axial velocity, but also we have a tangential velocity now that fluid is also having a tangential velocity in the same direction as the propeller rotates ok. So, in that case what will be the new relative velocity due after considering the induced velocity? So, that will be reduced by the amount of the tangential induced velocity ok.

So, that tangential component if we express it again as a non dimensional fraction of the actual tangential velocity it will be $2 \pi r n - a' \times (2 \pi r n)$ ok. So, the effect of induced velocities is different in the total calculation of the axial and the tangential component. In the axial case because the propeller accelerates the flow the induced velocities will result in a higher velocity component which is adding to the axial velocity or the velocity of advance.

So, the total will be so, the axial part will be $V_A + a V_A$ ok which is $V_A \times (1+ a)$, but for the rotational part. When we consider the tangential velocity the fluid particle now has a relative velocity of $2 \pi r n - a' \times 2 \pi r n$ where a' is due to the induced tangential velocity at that point. So, the resultant now velocity in the tangential direction will be $2 \pi r n - a' \times (2 \pi r n)$ which is basically $2 \pi r n \times (1- a')$.

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Let us try to understand the propeller velocity triangle with the induced velocities with the help of a model propeller. So, I again have a model propeller where we will try to understand how the velocity pattern on a propeller blade section look like in the velocity diagram. As the propeller rotates it will create a pressure jump across the propeller disc between the suction and pressure side.

So, it will accelerate the flow. So, because it rotates and moves forward. It moves forward with a velocity V_A and on top of that it creates a pressure difference by accelerating the flow. So, that acceleration component $a \times V_A$ or the axial induced velocity will be added because it is accelerating the flow as well as moving ahead. So, the fluid particle will be having a relative velocity V_A plus the acceleration part which is the induced velocity as the propeller is creating an acceleration in the flow.

So, that induced velocity part a × V_A or small V_A will be added to the velocity of advanced V_A for the total axial velocity now let us think of the tangential part as the propeller blade rotates. Now, the blade as it rotates will also induce a rotational velocity to the fluid around it. Later we will see that the trailing vertices shed from the propeller blade actually lead to the induced velocities of the fluid in the downstream of the propeller.

So, these induced velocity in the tangential direction will be in the same direction as the rotation of the propeller blade. So, if we take a fluid particle here again and consider only the tangential velocity. Initially because of the rotation of propeller blade it had a tangential velocity ωr which is 2 π n r. Now as this blade also induces a tangential velocity to the fluid at that location which is in the same direction.

Now, what will be the relative velocity? It will be ωr minus the induced component. In this case if we express as a non dimensional fraction a dash it will be ωr – a’×ωr. So, these two induced velocity components in the axial and tangential direction will be added and subtracted from the original V_A and 2 π n r for the propeller blade to give the blade element diagram with the induced velocities.

This understanding is very important to calculate the blade element thrust and torque considering the induced velocities ok. So, based on these two axial and tangential components we will now move to the blade element diagram again and compute the thrust and torque from the velocities. Now let us try to draw the induced velocity components on the blade element diagram. So, V_A will be increased and 2 π r n will be decreased due to the induced component. So, 2 π r n will be decreased and V_A will be increased.

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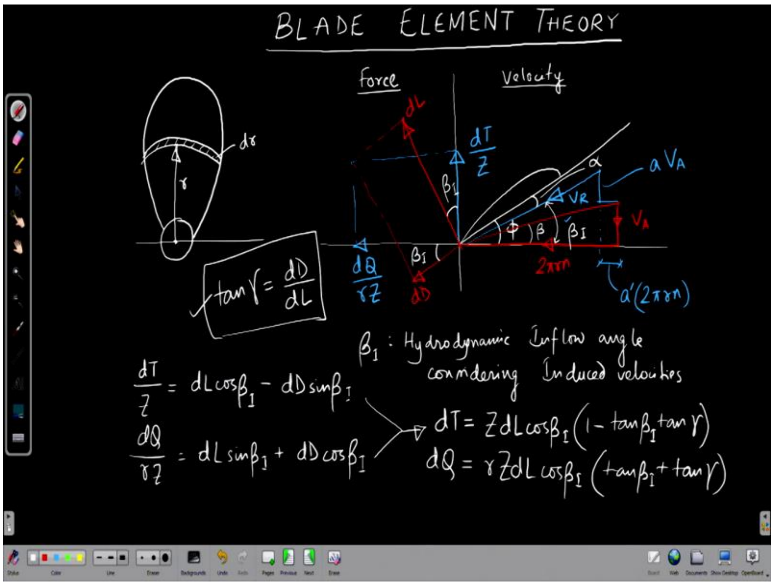

So, the resultant this length by which 2 π r n will be decreased is a’×2 π r n and by the value by which V_A will be increased is a×V_A ok. So, the new resultant velocity will be aligned with respect to the final velocity components in the axial and tangential directions. So, this is V_R the resultant velocity after considering the induced velocity components. What was our original V_R? Original V_R was directed along this red line ok where we did not use the induced velocities ok. So, the new angle that comes up after considering the induced velocities this angle is called β_I ok.

This is also the hydrodynamic inflow angle after considering induced velocity. So, β_I will be the hydrodynamic inflow angle considering induced velocities ok. So, this is our velocity part of the blade element diagram. And let us now concentrate on the force part. Now what has changed here? Instead of the old resultant velocity along the red line we have the resultant velocity along the new V_R with an angle β_I.

So, the new thrust and torque forces will be based on the new lift and drag forces which is based on the angle of attack after considering the inflow angle β_I. So, the angle of attack now is this α ok. So,$\alpha = \phi - \beta_I$ ok based on this we will draw the sectional lift and drag forces dD, and dL. So, in the same way as before we can compute the sectional thrust and torque dT/Z again why? Z because the propeller has Z number of blades.

So, if dT is the thrust produced at by a blade element over the entire propeller blade across the entire propeller covering all the Z blades then for each blade element for one single blade it will be dT/Z. Similarly this will be d Q/rZ as before right. So, what are the angles now? Here, this angle and this one are both β_I instead of β ok. So, we can write in the same way $dT/Z = dL \cos(\beta_I) - dD \sin(\beta_I)$ ok.

And $dQ/rZ = dL \sin(\beta_I) + dD \cos(\beta_I)$. So, from these 2 equations we can write $dT = Z\, dL \cos\beta_I (1 - \tan\beta_I \tan\gamma)$ where $\tan(\gamma)$ is the ratio of the sectional drag and lift force and $dQ = r\, Z\, dL \cos(\beta_I)(\tan(\beta_I) + \tan(\gamma))$. Now from this sectional thrust and torque we will compute the efficiency of the propeller blade section.

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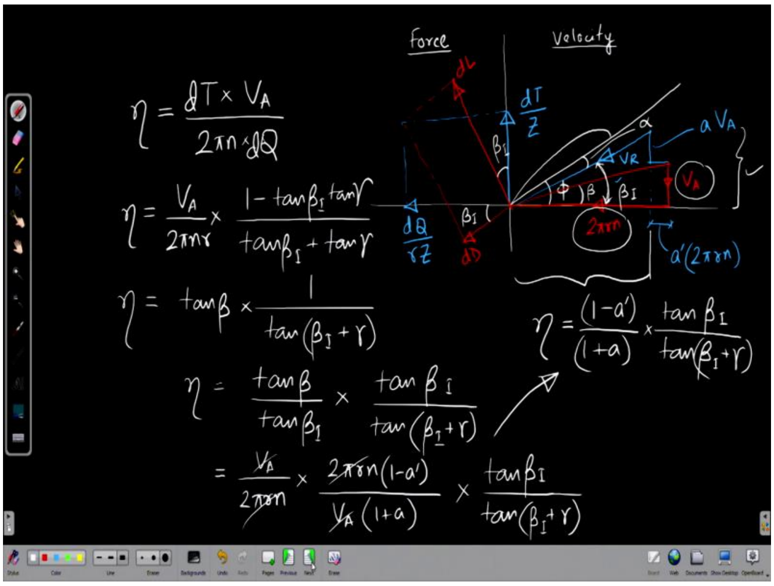

Efficiency is given by thrust into velocity of advance by $2\pi n \times$ torque (Q) ok. So, everything with a d here because we are computing the efficiency of the propeller blade section ok. So, η will be $V_A/2\pi n \times dT/dQ$. So, $Z\, dL \cos\beta_I$ cancels out we have only the tan part and r $(1 - \tan(\beta_I)\tan(\gamma) / (\tan(\beta_I) + \tan(\gamma))$ with an r ok.

Now, what is this? $V_A/2\pi r n$ if we look in this particular blade element diagram V A by $2\pi r n$ is nothing, but $\tan(\beta)$ which was the original angle before we consider the induced velocities. So, this is $\tan(\beta) \times 1/\tan(\beta_I + \gamma)$ this is the efficiency after considering the induced velocities. Now, we can simplify this in a way we can write this as $\tan(\beta)/\tan(\beta_I) \times \tan(\beta_I)/\tan(\beta_I + \gamma)$, why are we doing this? Because we need to relate this efficiency to the induction factor.

That means, a and a' the induced velocities that we have computed non dimensionalized with respect to the V_A and $2\pi r n$ we need to use this in the efficiency term. So, what is tan β? tan (β) = V_A / 2 π r n. What is tan (β_I)? tan β_I will be V_A + a V_A the vertical value here by what is the base for β_I this full angle is this which is 2 π r n × (1 – a'). So, this is (V_A/2 π r n) ×(2 π r n×(1 – a'))/ (V_A × (1 + a)) × tan β_I /tan (β_I +γ) ok.

What do we have finally, from this? Efficiency will be equal to V_A and 2 π r n cancels out (1 – a')/(1 + a) × tan β_I /tan (β_I +γ). So, what is the difference with the case where we did not consider the induced velocities? Instead of β we have β_I the hydrodynamic inflow angle after considering induced velocity and we have these terms a and a' which are coming due to the induced velocities.

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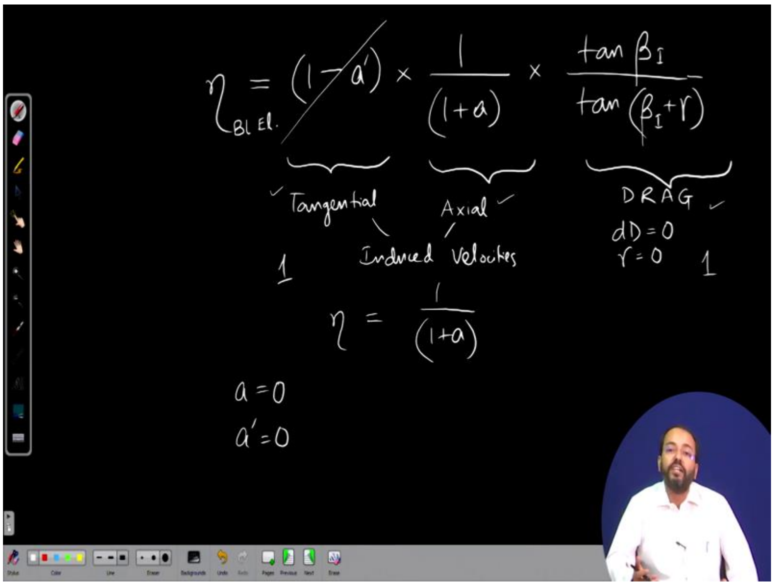

Now, if we look into the efficiency term very simply, the efficiency is a multiplication of three components. (1 – a') ×1/(1 +a) × tan β_I /tan (β_I +γ) right. So, we have this first component second component third component. Now, if again we consider drag equal to 0 then this component will become one; that means, gamma will be 0 and this part will be 1 if we neglect drag, but still we have the component due to the induced velocity.

So, this is the part coming due to drag this is the axial and this is the tangential induced velocities both of them. So, these three parts in total give the efficiency of the blade element is if we do not consider any tangential induced velocity and think of only the axial part then this a' will be 0 then this part will become 1. And also if we neglect drag then this part will also become 1 then η will be 1 /(1 + a) which is the same value we had obtained from the simple axial momentum theory ok.

So, if we consider the axial momentum theory with the rotation we have the combination of the axial and tangential and if we consider drag on top of that we will get the three components of the efficiency of the blade element. So, this is the efficiency of the blade element ok. So, let us say we can write like this and if we compare with the other efficiency that we had obtained neglecting the induced velocities in that case both in the previous condition both a was 0 as well as a' was 0.

So, both these parts were 1 we only had the effect of the drag. So, we will see that all these components will bring down the efficiency from the ideal value of 1 to a realistic value. Still there are some assumptions which we have used here for example, at a particular radius the we have assumed that there is no variation of the force or the velocity in the circumferential direction also there are certain other assumptions this is a very simplistic theory. So, this as of now this forms the basis of the circulation theory, but this cannot be directly used for realistic propeller calculations yet.

But this gives an estimation of the blade element efficiencies as well as the thrust and torque. And we can do simple calculations based on the velocities to get the thrust and torque of the propeller blade element and compute the total value for the propeller blade theorem.

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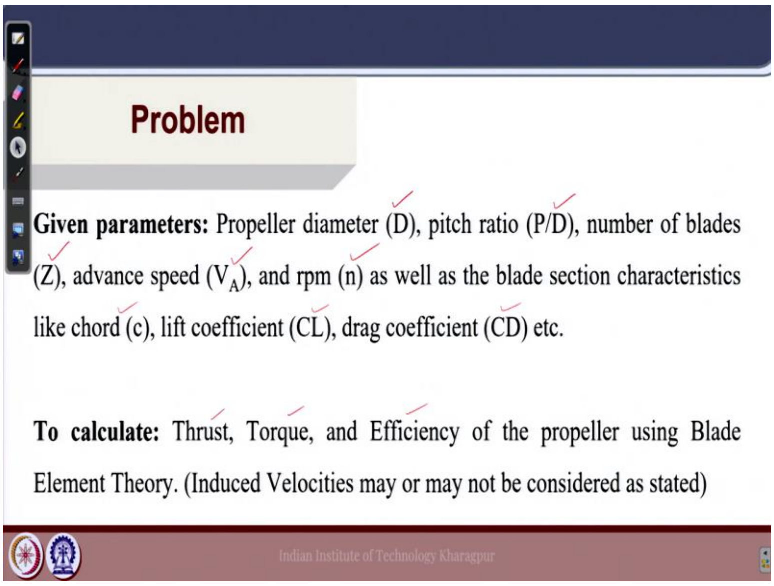

Look into a simple problem of blade element theory which we can do using the equations that we have just framed. So, the given parameters here are the propeller diameter pitch ratio number of blades Z advance speed rpm. So, the both the geometry as well as the operational values are given and on top of that some blade section characteristics are given like chord lift coefficient drag coefficient etcetera ok.

And finally, we are asked to calculate the thrust, torque and efficiency of the propeller ok. So, it the problem can be framed in 2 ways 1 is to calculate thrust and torque and efficiency of any particular section given section or it can be for the entire propeller ok. So, using these equations that we have developed ok or and induced velocities may or may not be considered ok in this particular problem as given.

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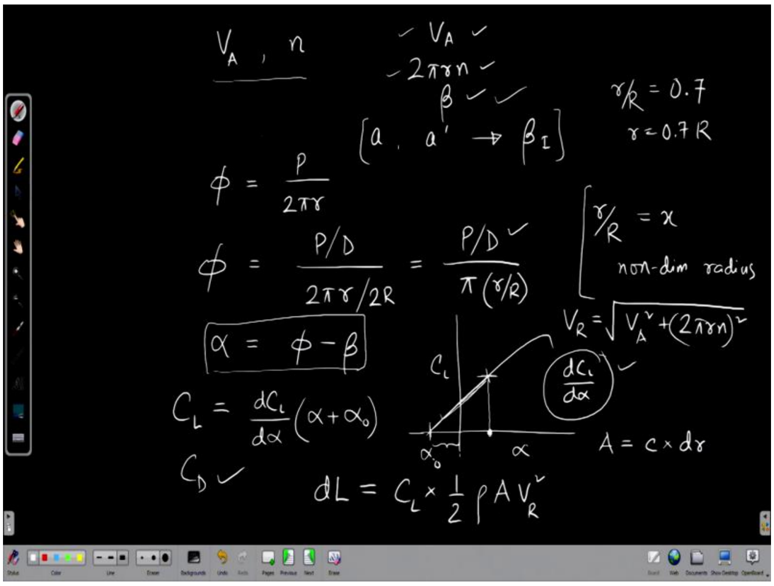

So, if we go back from the input values we have V_A given n given for the propeller blade from which we can obtain the velocities at any section. So, at any radial section we can calculate V_A is already given and 2 π r n in the velocity diagram we can calculate. So, from that we will get the value of β right. If the induction factors a and a' are given if a and a' are additionally given from that we will be able to calculate the β_I if they are not given we will not take that into account in this problem ok.

What else? We have to calculate the blade pitch angle because to get the thrust and torque we have to calculate the angle of attack and angle of attack is nothing but (ϕ –β). So, ϕ is given by P/(2 π r) right that we have seen from propeller geometry now in this problem P/D is given let us divide both by D; D is 2 × radius = 2 R. So, this becomes (P/D) /(π×(r/R)) ok. So, this r/R is sometimes expressed as x which is basically the non dimensional radius non dimensional radius; that means, let us say at a value of $r/R = 0.7$ ok or $r = 0.7$ R.

If we want to calculate the thrust and torque we can use this r/R value and the given P/D value which is given in the problem pitch ratio to calculate the ϕ. And now we can calculate the α the angle of attack as $(\phi-\beta)$ which was already calculated here ok now the sectional characteristics of the propeller blade are given in terms of the lift and drag. So, it can be given in many ways for example, the lift curve slope.

If it is given typically the as we have seen the C_L versus α looks like this ok. So, this part is often linear and in that case $dC_L/d\alpha$ can be given in the problem ok and we know that the lift coefficient C_L is 0 at a negative angle of attack which we call the α_0 ok 0 lift angle. So, we can calculate for any angle of attack finally, I have to calculate the lift dL which will come from the C_L.

So, how do we calculate C_L? From this given $dC_L/d\alpha$ if this value is given, so for any case if this part is assumed linear. So, the value of C_L at any particular value of angle of attack can be $dC_L/d\alpha \times (\alpha + \alpha_0)$ this is the value of α_0 ok right and this is the positive value of angle of attack that we have obtained using this equation $(\phi - \beta)$. So, if α_0 and $dC_L/d\alpha$ is given we can compute the C_L.

Now, using C_L and C_D is or C_D/C_L will be given in the problem and then we will be able to compute the force D, L which is $C_L \times ½ \rho A V^2$ ok. Now, what is V? In this particular problem V is the final resultant velocity. So, V will be the resultant of V_A and $2 \pi r n$ and what is A will be the chord length × dr because we have assumed that strip of thickness dr for the propeller blade element and C the chord length will be is given in the problem. So, that will be the plan form area considering that strip.

And this V will be nothing, but V_R. So, where V_R is equal to root over of $(V_A^2 + (2\pi rn)^2)$ oh sorry where V and $2 \pi r n$ are both known for a particular radius. So, we can calculate the forces dL and dD based on these values and from that already we have the values given of the other parameters we have the beta.

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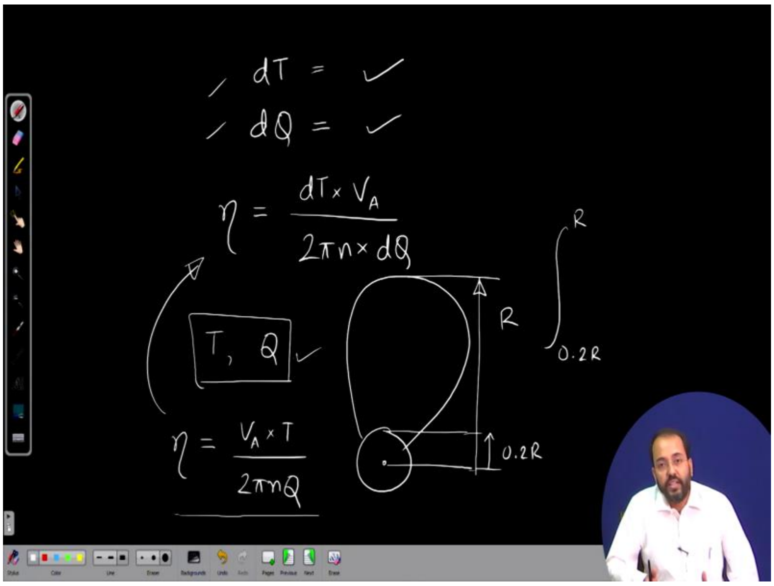

So, we can calculate the sectional forces dT should be possible to calculate and dQ ok. As per the given equations from that we can calculate the efficiency of the blade section as $(dT \times V_A)/ (2 \pi n \times dQ)$. So, the sectional thrust torque efficiency should be calculated we can also calculate the thrust and torque of the entire propeller blade by integrating these sectional thrust and torque over the blade section. So, let us say for a propeller blade we have the root section at 0.2 R ok and the entire radius at R.

So, the blade is only between 0.2 R to R. So, if we integrate the values of thrust and torque between two values between the root of the propeller blade and the tip we should be able to calculate the thrust and torque of the propeller blade and any numerical integration technique can be used to. Once we get the thrust and torque of the blade elements we should be able to calculate the thrust and torque of the entire propeller blade ok.

And finally, the efficiency of the entire propeller. In the same way just like this equation instead of dT we can use the values of T and Q. So, $(V_A \times T)/ ((2 \pi n)\times Q)$ for the entire propeller T and Q as we have calculated here already after integration we can use that to compute the efficiency of the propeller blade ok. This is how we can use simple blade element theory equations to calculate the propeller thrust and torque using the elemental thrust and torque values and efficiency ok. So, this will be all for the blade element theory. We will continue with the propeller circulation theory in the next class.

Thank you.