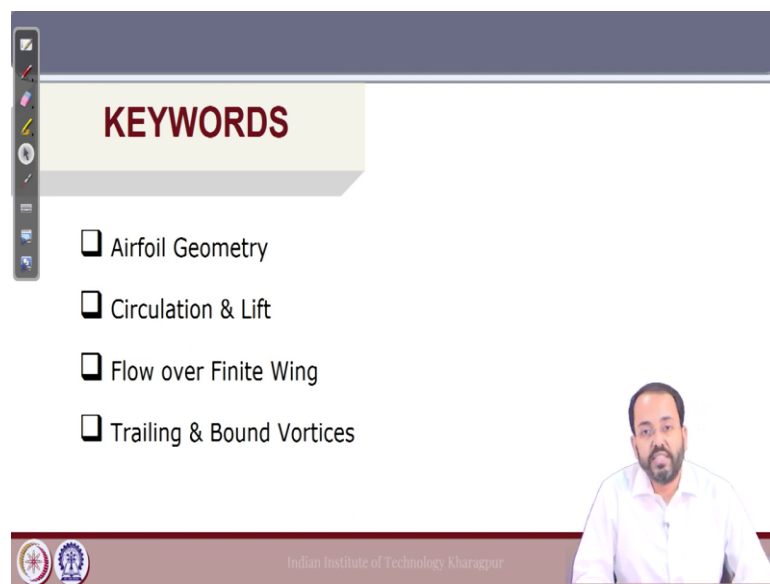


Marine Propulsion
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Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 08
Propeller Theory V

Welcome to the 8th lecture of the Marine Propulsion course, today we will continue with Propeller Theory.

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The image shows a presentation slide with a dark blue header and a light blue footer. The main content area is white. At the top, the word "KEYWORDS" is written in bold, dark red capital letters. Below it, there is a list of four topics, each preceded by a small square icon with a white checkmark inside a dark square. The topics are: "Airfoil Geometry", "Circulation & Lift", "Flow over Finite Wing", and "Trailing & Bound Vortices". In the bottom right corner of the slide, there is a small video inset showing a man with a beard and glasses, wearing a white shirt, looking towards the camera. The footer of the slide contains the Indian Institute of Technology Kharagpur logo on the left and the text "Indian Institute of Technology Kharagpur" in the center.

So, in this particular class we will discuss about airfoil geometry, because the propeller blade sections are basically foil sections. So, we will discuss also on the relation between circulation and lift and the flow over finite wing and the concept of trailing and bound vortices.

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Airfoil Geometry

Numbering system developed by The National Advisory Committee for Aeronautics (NACA)

four-digit series (NACA 2412, NACA0012)

Digit 1: Max. camber in 100th of chord
Digit 2: Location of max. camber in 10th of chord from LE
Digits 3&4: Max. thickness in 100th of chord

** An airfoil with no camber: Symmetrical airfoil

Five-digit series
Six series

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The geometry of a 2D airfoil section is shown here we have the leading edge and the trailing edge. So, the flow direction will be from the left to right and this is a cambered airfoil; that means, it has a finite value of camber which changes according to the location along the x axis; if we look along the longitudinal direction along the chord and take it as the x axis. So, the camber varies with the location and the foil also has a finite thickness and the chord line is the line connecting the leading and the trailing edge of the foil section ok.

So, the national advisory committee for aeronautics NACA, they have defined certain airfoil sections and given nomenclature based on certain geometrical characteristics of these 2D sections of airfoils. So, because we will be extensively using airfoils in general in naval architecture, for example in propeller blade sections, rudders et cetera. It is good to know some basic characteristics of airfoil sections.

So, one of the most popular airfoil series is the 4 digit series where the nomenclature is in this way after NACA we have 4 particular digits representing certain aspects of the airfoil sectional geometry. So, the first digit means the maximum camber in 100th of chord. So, if we look at these 2 we have taken 2 airfoil sections NACA2412 and 0012 to illustrate the difference in geometry between them with respect to these digits.

So, the Digit 1 here which is 2 is the maximum camber in 100th of chord; that means, this maximum camber here is 2 by 100; that means, 2 percent of the chord length of the section

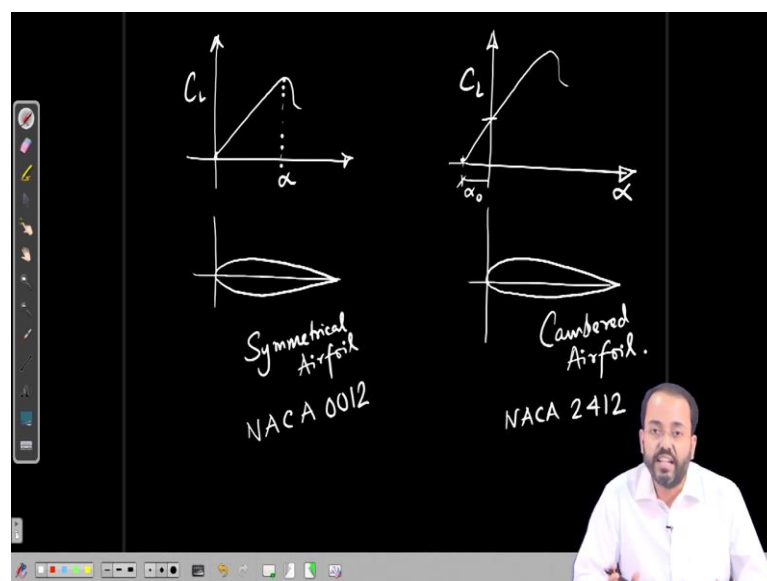
and for the next case NACA0012 the maximum camber is 0; that means, the foil has no camber. The second one Digit 2 is the location of maximum camber in 10th of chord from the leading edge.

4 that means, the location of the maximum camber is 40 percent of the chord length from the leading edge, here again for the next case NACA0012 because there is no camber we also have the second digit as 0; 3rd and 4th combined give the maximum thickness of the airfoil section in 100th of chord. That means, 3rd and 4th combined is 12. So, the maximum thickness is 12 percent of the chord length which is the same between the 1st and the 2nd foil sections that we have shown.

So, now what is the main difference between them? So, the second foil NACA0012 here is a symmetrical foil section and the first one is a cambered foil section. Symmetric foil section means the upper and the lower surface of the foil are symmetric about the chord line, this one the dotted line below the camber line is the chord line ok. So, the picture shown here is that of a cambered foil which is typical of this section NACA2412 which has a finite camber ok.

So, if we have a foil section NACA0012 it will be a symmetrical section. Similar to these 4 digit foils we have 5 digit and 6 digit series of NACA airfoils and the geometry depends again on the nomenclature for those kinds of foils.

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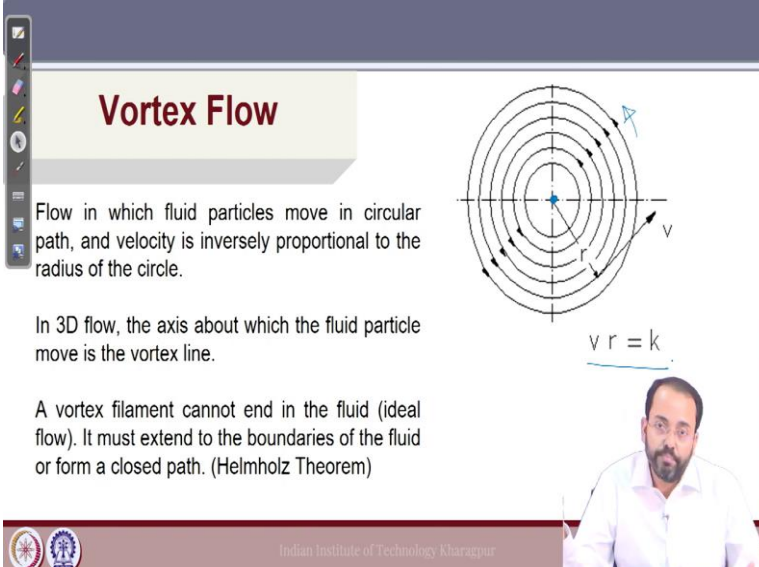


So, if we look at the lift characteristics, so let us say we have a symmetric foil and a cambered foil, this is a symmetrical airfoil and this is a cambered airfoil ok. What will be its lift characteristics based on the angle of attack. If we draw the lift coefficient curve C_L as a function of α for each of them, for the symmetric foil the pressure and suction sides or the top and bottom surfaces are evenly spaced with respect to the chord line. So, when the angle of attack is 0 a symmetrical foil will not generate any lift.

On the other hand a cambered foil as we had discussed earlier, because of the camber even at 0 angle of attack there will be a circulation around the foil we will discuss the concept of circulation soon. So, in that case a lift will be generated even at 0 angle of attack. So, for a cambered foil the lift coefficient diagram will be somewhat like this where at 0 angle of attack there will be no lift. And as the angle of attack increases initially it will be linear and then at a particular value of angle of attack the flow will separate over the foil and the foil will stall and the lift coefficient will drop.

The pattern is similar for the cambered airfoil only the difference is that at 0 angle of attack there will be a value of lift and C_L will be 0 at an angle of attack α_0 , at a value of negative angle of attack which is given as the no lift angle. That means, at 0 angle of attack because of the camber the cambered airfoil will generate lift. So, this is an example of the NACA2412 the cambered 1 and this is NACA0012. Just an example I have mentioned the 2 names to show the difference between these airfoil sections.

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Vortex Flow

Flow in which fluid particles move in circular path, and velocity is inversely proportional to the radius of the circle.

In 3D flow, the axis about which the fluid particle move is the vortex line.

A vortex filament cannot end in the fluid (ideal flow). It must extend to the boundaries of the fluid or form a closed path. (Helmholz Theorem)

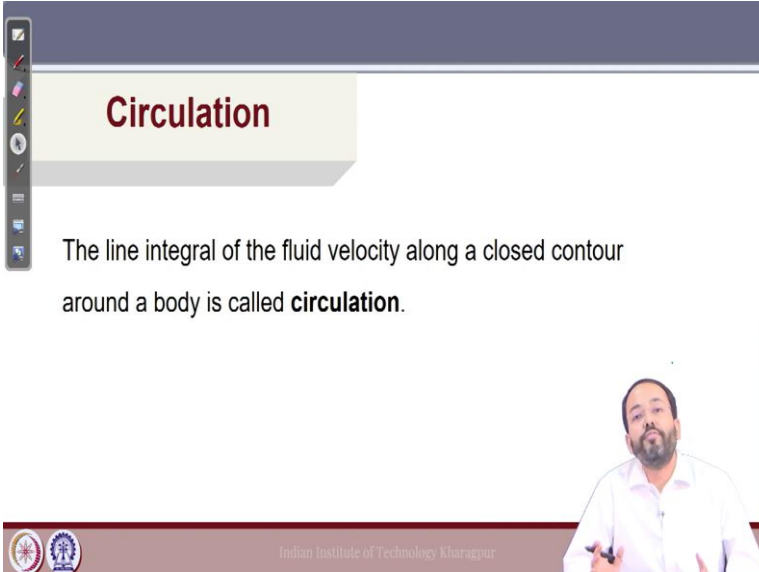
$vr = k$

The slide features a diagram of a vortex flow with concentric circular streamlines and arrows indicating the direction of flow. A central point is marked with a blue dot, and a radius r is shown from the center to a streamline. The velocity v is indicated by an arrow tangent to the streamline. Below the diagram, the equation $vr = k$ is written. In the bottom right corner, there is a small inset video of a man speaking.

Another very important aspect of fluid mechanics we need to discuss to understand the concept of foil flow another very important aspect of fluid mechanics we need to understand before going into the flow characteristics around airfoil is vortex flow. What is a vortex flow? A flow in which the fluid particles move in circular path ok and the velocity is inversely proportional to the radius of the circle. So, here we see the fluid particles they move in circular path about an axis and the velocity is inversely proportional to the radius, that is why we have can write it as $V \times r = k$.

In a 3D flow this fluid particles move around an axis which is called a vortex line and spatially we call it also vortex filaments in 3 dimensional flow about which the fluid particles move. And a vortex filament in an ideal flow condition cannot end in a fluid. So, it must either extend to the boundary or form a closed path this is the Helmholtz Theorem.

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Circulation

The line integral of the fluid velocity along a closed contour around a body is called **circulation**.

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The slide features a title bar at the top, a central title 'Circulation' in a light green box, and a definition below it. A video inset in the bottom right shows a man in a white shirt speaking. The footer contains the IIT Kharagpur logo and name.

The concept of circulation around a closed contour surrounding a body in a fluid flow is basically the line integral of the fluid velocity along that contour ok. This circulation around anybody is related to the lift force generated by that and we will try to understand this using Kutta Joukowski's Theorem which relates the circulation and lift around a body.

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Circulation & Lift **Kutta-Joukowski Theorem**

The flow around an airfoil can be represented in inviscid flow using a vortex sheet around its surface. For thin airfoil at an angle of attack, the vortex sheet is distributed along the mean camber line.

The lift per unit length (span) of the airfoil is given by
Kutta-Joukowski theorem

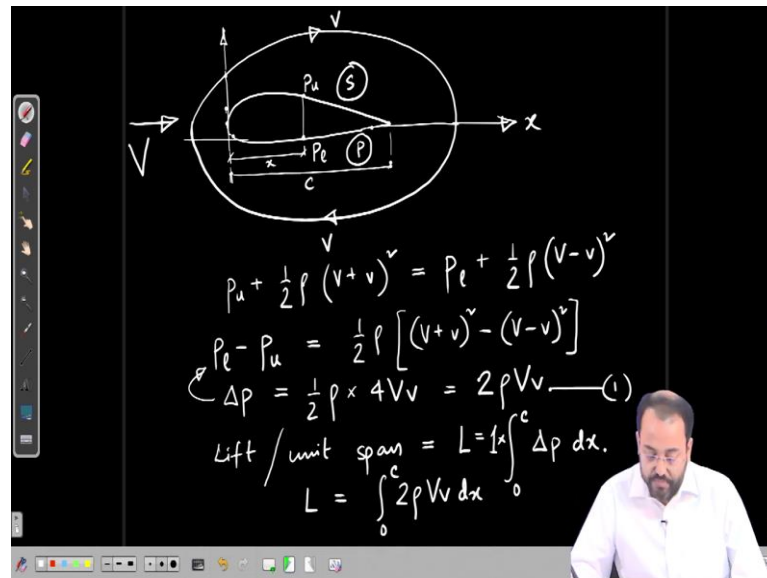
$$L = \rho \Gamma V$$

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So, if we think of a flow of an airfoil it can be represented in simple inviscid flow using a vortex sheet on the surface of the foil. So, if I distribute a set of vortices on the surface of the foil we can represent the flow past an airfoil. In a similar way for thin airfoil we can make an approximation and as we have seen the camber line for thin airfoil we can distribute the vortex sheet on the mean camber line and it can represent the flow using suitable boundary conditions we can represent the flow over a thin airfoil.

Now, the lift per unit span of an airfoil is given by Kutta Joukowski's theorem which is given by this equation. So, the lift $L = \rho \Gamma V$. We will try to use a very simple way of explaining we will try to use a very simple way of explanation of Kutta Joukowski's theorem using a 2D airfoil section.

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Suppose, we have a 2D airfoil section and I draw a close contour about that just drawing the system of axis. Let me take a small location at a distance x and let the length here the chord length is C ok and we have an inflow velocity V and the induced tangential velocities are v on the top and bottom. So, the circulation as we see is basically the line integral of the velocity around a closed contour about the airfoil. So, v is the tangential velocity induced due to the circulation ok.

So, let at any location x the pressure on the upper surface is P_u and on the lower surface of the foil is P_l . So, in this case using Bernoulli's equation we can write $P_u + 1/2\rho(V+v)^2$. So, the total velocity over the upper surface will be $(V + v)$ and the total velocity over the lower surface is $(V - v)$. So, using this Bernoulli's equation we can relate the pressure and velocity over the upper and lower surface.

Now, we can write $P_l - P_u$ remember that the bottom side the lower side here is the pressure side. So, this is the pressure side and this is the suction side. So obviously, P_l will be higher than P_u , so $P_l - P_u = 1/2 \rho((V+v)^2 - (V-v)^2)$. So, if the pressure jump is given by ΔP this $P_l - P_u$ is $1/2 \rho \times 4Vv = 2\rho Vv$. So, let us say this is equation 1.

So, this is the equation for the pressure difference at a point x from the leading edge between the lower and upper surface of the foil. Now, how is the lift defined? The lift per unit span is $L = \int_0^C \Delta p dx$. So, the integration of this pressure difference over the area, area

means length \times the width. So, here we are taking the unit span, so we just multiply by 1; so it will be $1 \times$ this if we take a unit span right. So, this can be written as $L = \int_0^c 2\rho V v dx$.

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$$\begin{aligned}
 L &= \int_0^c 2\rho V v dx. \\
 &= \rho V \int_0^c 2v dx \\
 &= \rho V \left[\int_0^c v dx + \int_c^0 -v dx \right] \\
 &\Rightarrow \boxed{L = \rho \Gamma V}
 \end{aligned}$$

Now, what is V ? Capital V is the incoming velocity into the foil. So, we can take ρV outside and do this integration $\rho V \int_0^c 2v dx$ and we can write it as. Now the v here is the tangential velocity induced due to the shape of the airfoil and the angle of attack at the given inflow V . So, what we have within this third bracket $\rho V \int_0^c v dx + \int_c^0 -v dx$ is basically the line integral of velocity on a closed contour around the foil. So, basically this is the circulation (Γ).

Hence, we can write lift around the foil is $L = \rho \Gamma V$. So, this is called Kutta Joukowski's Theorem which relates the lift generated by an airfoil to the circulation around the airfoil.

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Flow over a Finite Wing

The difference in pressure distribution causes 'Lift'

The diagram illustrates the flow over a finite wing. An arrow labeled 'Inflow' points towards the wing from the left. A yellow arrow labeled 'Lift' points upwards from the top surface of the wing. A red arrow labeled 'Low Pressure Side' points to the upper surface of the wing. A blue arrow labeled 'High Pressure Side' points to the lower surface of the wing. The wing is shown in a perspective view, and the flow is depicted as a stream of particles curving around the wing. A small inset shows a man speaking, with the text 'Flow over' and 'High Pressure Side' visible below him.

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Let us see how we represent the flow over a finite wing ok. Finite means it has a finite span length along the direction perpendicular to the inflow ok. So, it has a finite span. As we have seen due to the inflow over the wing there will be a high pressure side which is the bottom part of the wing and a low pressure side on the top. And the difference between them as again we have seen in the derivation the pressure difference between them gives rise to the lift force ok. So, this is the lift force perpendicular to the inflow direction.

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Flow over a Finite Wing

The difference in pressure distribution causes 'Lift'

This pressure difference causes the flow to curl along the two ends of the wing span.

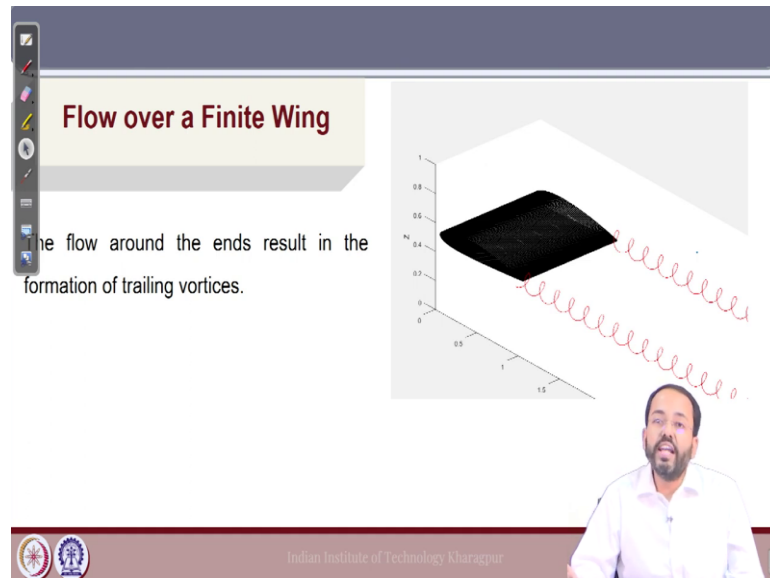
The diagram illustrates the flow over a finite wing, showing the flow curling at the ends. An arrow labeled 'Inflow' points towards the wing from the left. A red arrow labeled 'Low Pressure Side' points to the upper surface of the wing. A blue arrow labeled 'High Pressure Side' points to the lower surface of the wing. The wing is shown in a perspective view, and the flow is depicted as a stream of particles curving around the wing. A small inset shows a man speaking, with the text 'Flow over' and 'High Pressure Side' visible below him.

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And also due to the pressure difference this is very important for finite wings, due to the pressure difference between the 2 sides along the ends of the span the flow will also curl.

So, there will be a curling flow from the high pressure side to the lower pressure side at the 2 ends which is very important for finite wings.

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Flow over a Finite Wing

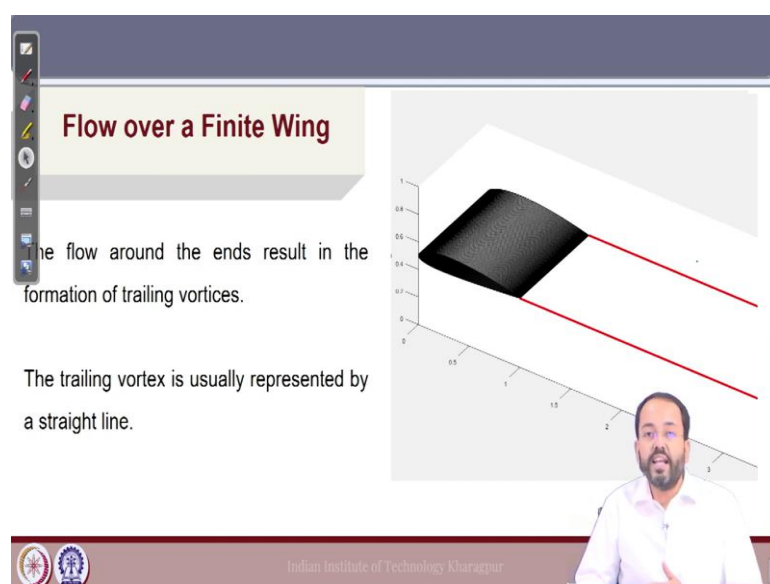
The flow around the ends result in the formation of trailing vortices.

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The slide features a 3D plot of a wing with red wavy lines representing trailing vortices. A presenter is visible in the bottom right corner.

So, this curling flow leads to the formation of trailing vortices. So, whenever a finite wing is facing an inflow or moves forward with a particular velocity at an angle of attack it generates trailing vortices due to the curling flow from the pressure side to the suction side at the two ends. So, these are visualized by the formation of trailing vortices.

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Flow over a Finite Wing

The flow around the ends result in the formation of trailing vortices.

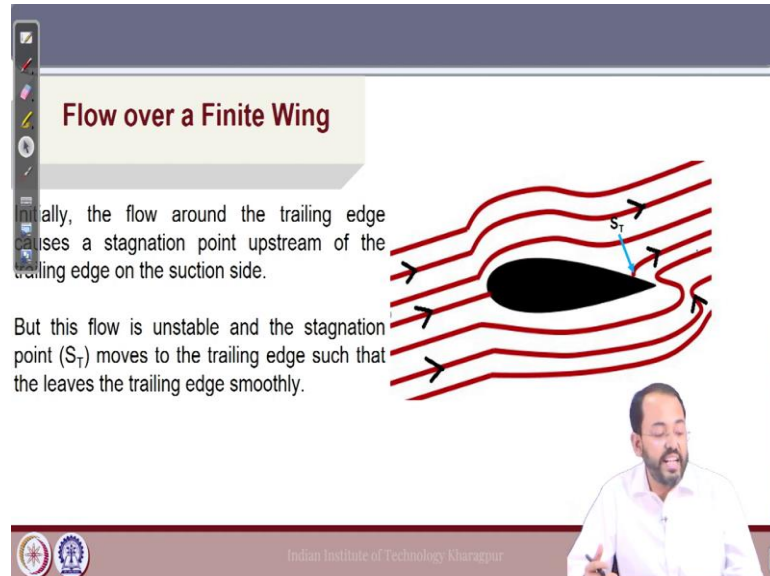
The trailing vortex is usually represented by a straight line.

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The slide features a 3D plot of a wing with a straight red line representing a trailing vortex. A presenter is visible in the bottom right corner.

So, the trailing vortex sheets are generally represented simply with straight lines from the two ends of span of a finite airfoil.

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Flow over a Finite Wing

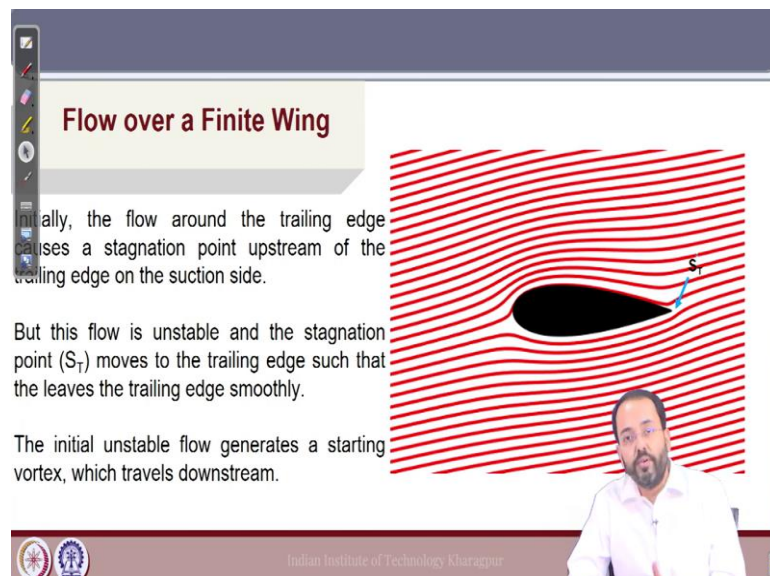
Initially, the flow around the trailing edge causes a stagnation point (S_T) upstream of the trailing edge on the suction side.

But this flow is unstable and the stagnation point (S_T) moves to the trailing edge such that the leaves the trailing edge smoothly.

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When we look over the flow over a foil section initially the flow due to the effect of the pressure side flow, the flow around the trailing edge it moves and creates a stagnation point slightly on the upstream of the trailing edge over the suction side. So, initially because of the pressure difference between the 2 sides, the flow moves around the trailing edge and the stagnation point is on the suction side.

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Flow over a Finite Wing

Initially, the flow around the trailing edge causes a stagnation point (S_T) upstream of the trailing edge on the suction side.

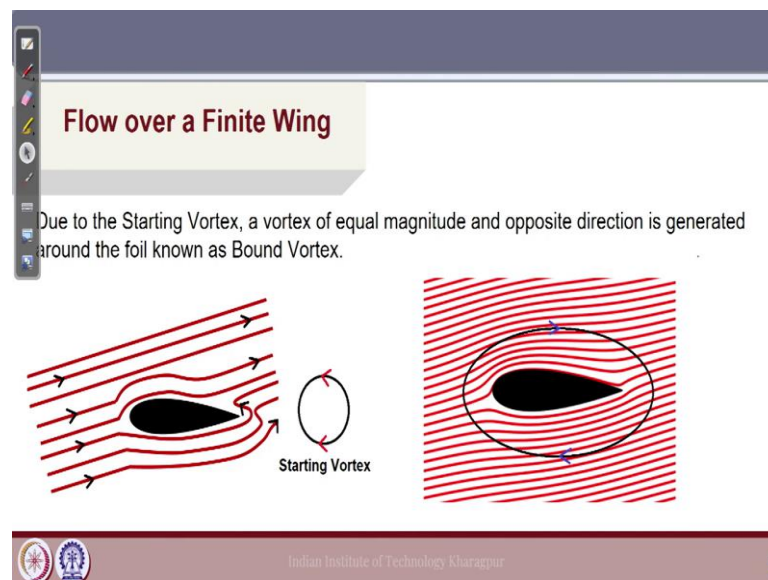
But this flow is unstable and the stagnation point (S_T) moves to the trailing edge such that the leaves the trailing edge smoothly.

The initial unstable flow generates a starting vortex, which travels downstream.

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But this is not stable gradually as the flow becomes stable the stagnation points shifts from that location to a location on the trailing edge. So, this initial unstable flow which later becomes stable that generates a starting vortex in the system which travels downstream ok.

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So, this starting vortex because of the shading of the starting vortex by the foil a vortex of equal magnitude, but obviously, in the opposite direction is generated around the foil which is called the bound vortex and this bound vortex around the foil that creates the lift around the foil that is how we explain the lift formation by a by an airfoil.

This is the starting vortex as the stagnation point moves to the trailing edge the starting vortex is shed downstream and a bound vortex is generated around the foil which is of opposite direction to the starting vortex. And this bound vortex is responsible for the lift generation of the foil.

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Kutta Condition

The circulation over an airfoil depends on its geometry angle of attack.

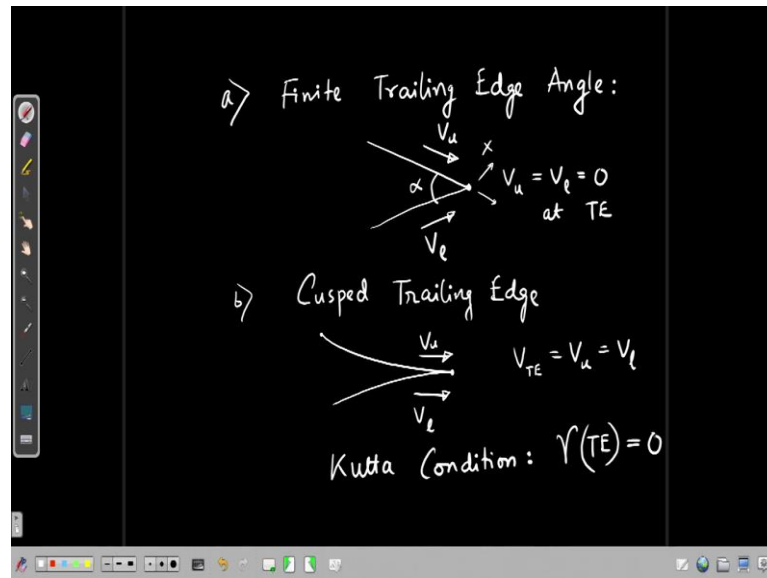
At a given angle of attack is such that the flow leaves the trailing edge smoothly.

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Now, the circulation over an airfoil depends on its geometry as well as angle of attack. We have seen that a symmetric foil generates lift only at a finite value of angle of attack at positive angles of attack it generates lift and that depends on the airfoil geometry.

But for a cambered foil there is a circulation around the foil because of its geometry and even at 0 angle of attack it generates lift. So, the circulation generated over the airfoil depends on both its geometry as well as the angle of attack and at a given angle of attack the flow adjusts itself in such a way that it leaves the trailing edge smoothly. So, the circulation develops in a certain way at a particular angle of attack, such that the flow leaves the trailing edge smoothly.

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Actually this can be simply shown using 2 type of trailing edge. So, first one let us assume a finite trailing edge angle, let us say the foil has a finite trailing edge angle α . So, there is a velocity coming from the top surface V_u and there is a velocity from the lower surface V_l let us say at the trailing edge ok. Now here at this point we cannot have 2 different velocities at a particular point. So, if there is a finite trailing edge angle then V at this particular point that velocity has to be 0.

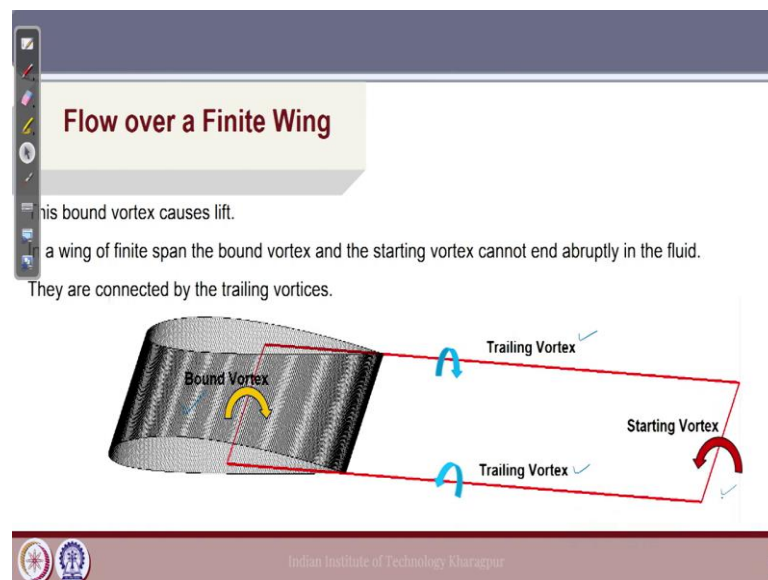
So, $V_u = V_l = 0$ at the trailing edge at TE, because at a particular point we cannot have 2 different velocities in 2 different directions. So, this is not possible that is why for finite trailing edge angle we will get a condition where the velocity at this trailing edge should be 0 ok and another condition depending on the design we can have a cusped trailing edge. Let us say that the trailing edge is cusped.

So, in this way we can have a velocity V_u and V_l over the upper and lower surface. So, in this condition there can be a finite velocity at the trailing edge. So, $V_{TE} = V_u = V_l$, because at the trailing edge we can have a velocity here, but that velocity should be equal. So, at the trailing edge for because if there is a velocity jump that will also indicate a pressure jump and that cannot be possible in the free stream flow.

So, for that reason in this particular case there can be a trailing edge velocity, but it has to be equal ok. So, the condition which has to be met for a foil the condition which has to be met for circulation at the trailing edge is called the Kutta condition, which says that the

value of the circulation (Γ) at the trailing edge should be 0. So, this Kutta condition has to be satisfied at the trailing edge of the airfoil ok. That means this is what is meant in this line that at a given angle of attack the circulation around the foil is developed such that the flow leaves the trailing edge smoothly.

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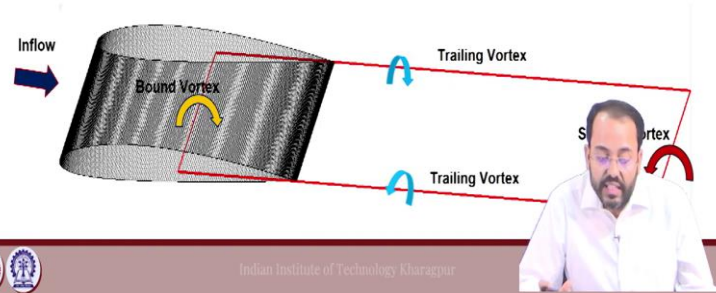
Now, we can see that this bound vortex is causing the lift and we have a, we have the trailing vortex sheets from the 2 ends of the foil and there is also a starting vortex. So, for a wing of finite span the bound vortex and the starting vortex cannot end abruptly as we have seen in the fluid. So, they are connected by the trailing vortex and this is how the vortex system appears for a wing.

So, for a finite wing we have a bound vortex and at from the 2 ends we have trailing vortex and it is connected to the starting vortex which was shed initially from the wing. So, this representation gives the vortex structure of a wing.

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Flow over a Finite Wing

This trailing vortices do not produce any lift and is aligned parallel to the inflow velocity.
Bound vortex is aligned perpendicular to the inflow velocity and is at a fixed location in the flow.



The diagram illustrates the flow over a finite wing. A blue arrow labeled 'Inflow' points from the left towards the wing. A yellow curved arrow on the wing's surface is labeled 'Bound Vortex'. Two blue curved arrows extending from the wing's trailing edge are labeled 'Trailing Vortex'. A red curved arrow at the end of the wing is labeled 'Starting Vortex'. The Indian Institute of Technology Kharagpur logo and name are visible at the bottom of the slide.

Now, one has to remember that the trailing vortices do not produce any lift, but they are aligned parallel to the inflow velocity. But the bound vortex which is aligned perpendicular to the inflow is actually producing the lift.

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Flow over a Finite Wing

In course of time, the starting vortex moves far downstream.
The vortex system now is in the shape of a horse shoe and is called as a Horse Shoe Vortex.



The diagram shows the wing from the previous slide. The 'Starting Vortex' (red arrow) has moved far downstream, forming a large loop that encircles the wing and the trailing vortices, creating a 'Horse Shoe Vortex' shape. The Indian Institute of Technology Kharagpur logo and name are visible at the bottom of the slide.

So, in course of time what happens? The starting vortex move much further downstream and in a realistic flow due to the effects of viscosity is gradually dissipates. So finally, the vortex system around an finite airfoil looks like a Horse Shoe structure. So, this horseshoe

vortex structure is shown here which consists of the bound vortex and the 2 trailing vortices from the 2 ends of span of the finite airfoil ok.

So, this will be all for today's class, in the next class we will relate this vortex structure to the development of circulation around a propeller blade section, because the propeller blade sections are generally airfoil sections. So, understanding the flow over an airfoil is important to understand the circulation and lift generated by the propeller blade, so which we will continue with in the next class.

Thank you