Marine Propulsion
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Lecture - 09
Propeller Theory VI

Welcome to the 9th lecture of Marine Propulsion we will continue with Propeller Theory.

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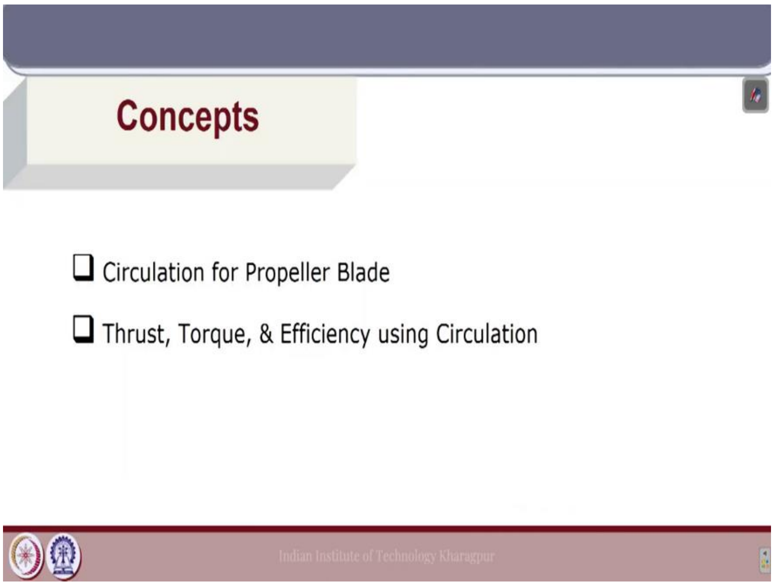

So, the key concepts which will be covered in today's class will be circulation for a propeller blade and how we relate that circulation to the thrust, torque, and efficiency of the blade sections. And in that way how we compute propeller performance using circulation theory.

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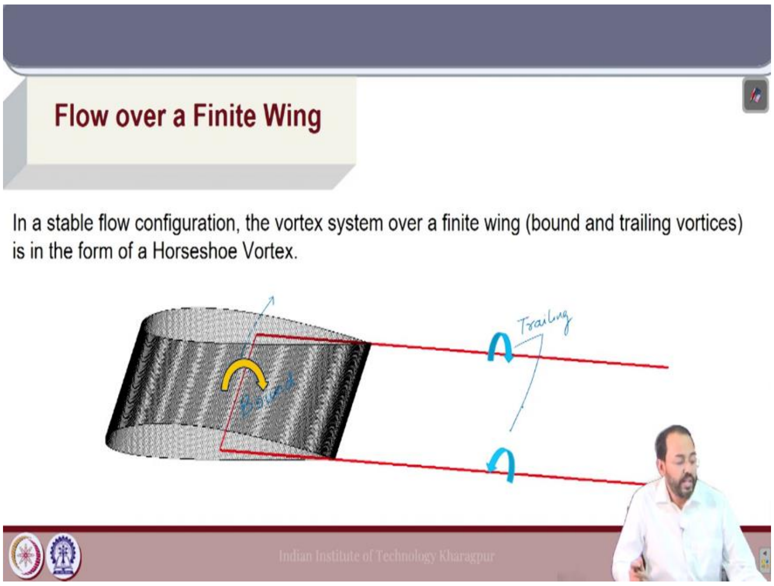

In the last class we have covered the key aspects of circulation related to a finite wing. So, from the concept of an airfoil we would now try to develop the concept for circulation around a propeller.

So, we have seen that the vortex pattern for a finite wing looks like a horseshoe. So, we have the finite wing vortex pattern is like a horseshoe vortex, where we have the bound vortex. Here, this one and we have two trailing vortices from the two ends of span. So, these are trailing vortex right. So, the combination of these give the horseshoe vortex pattern; why? Because initially we had the starting vortex, which gradually was shed downstream and after obtaining the stable flow configuration, we have this horseshoe vortex pattern ok.

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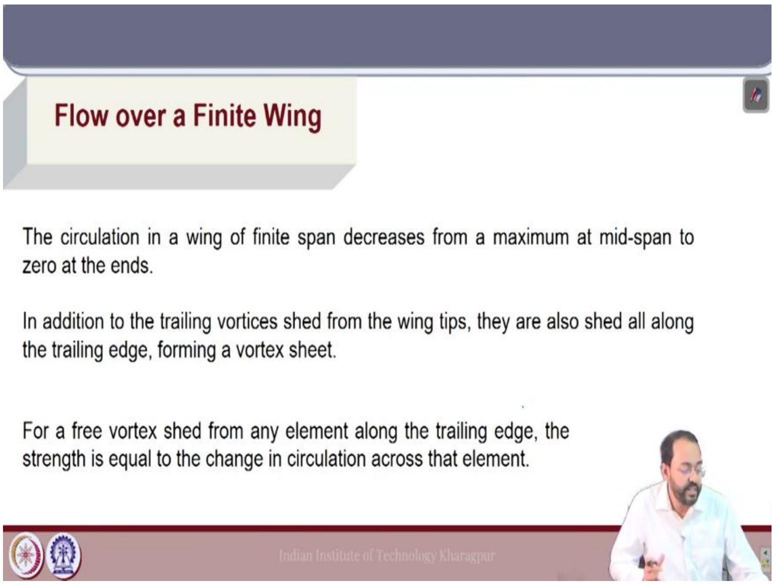

Now, if we try to look into the flow, the circulation distribution over the finite wing it is not constant. So, the circulation distribution on a finite wing is highest at the center of the span that is at mid span and gradually reduces to the value of 0 at the two ends. So, due to that additional trailing vertices will be shed throughout the trailing edge, because the two trailing vertices from the wing tips will be shed because the foil ends in the fluid at those points. And because of the circulation distribution which is not uniform, the trailing vertices will be shed throughout the trailing edge of the finite wing.

So, the vortex shed from the trailing edge is basically the strength of that vortex is equal to the change in circulation across the element. That is what will be visualized in the next figure.

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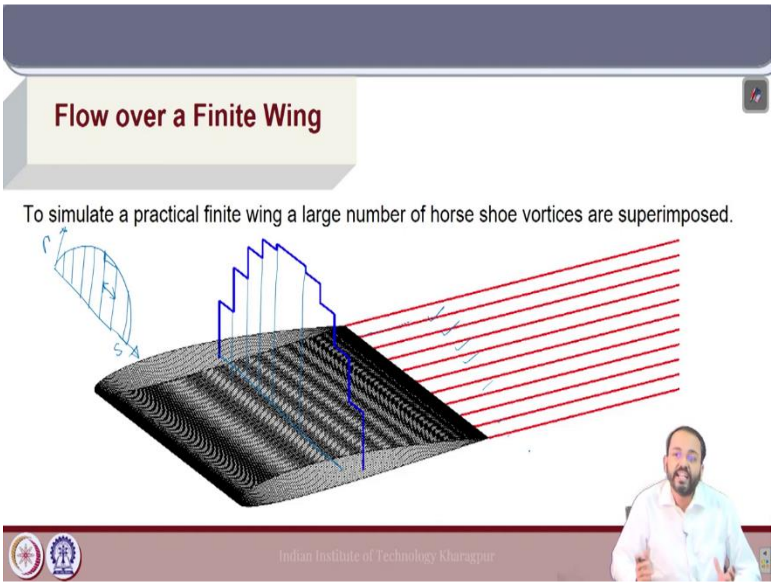

So, if we divide the finite wing into a number of elements, the circulation distribution is 0 at the ends and maximum at the center. So, if we divide this into a number of elements, this span then the circulation gradually increases towards the center of the span.

So, due to that the actual circulation distribution will be somewhat of an elliptical distribution, again depending on the foil geometry as well as the flow characteristics. And if we divide that into a number of elements there will be a change of circulation distribution across each element. So, across each element there will be a difference in the circulation distribution ok, this is the circulation and across the span S.

So, due to that change in circulation distribution from each element depending, how you resolve the number of depends on how we divide the span into a number of elements. So, from each of them, due to the change in circulation across the element a trailing vortex will be shed. So, these trailing vortex will be shed all through the trailing edge of the foil ok.

So, for this wing a number of horseshoe vortex are superimposed to show the entire trailing vortex sheet.

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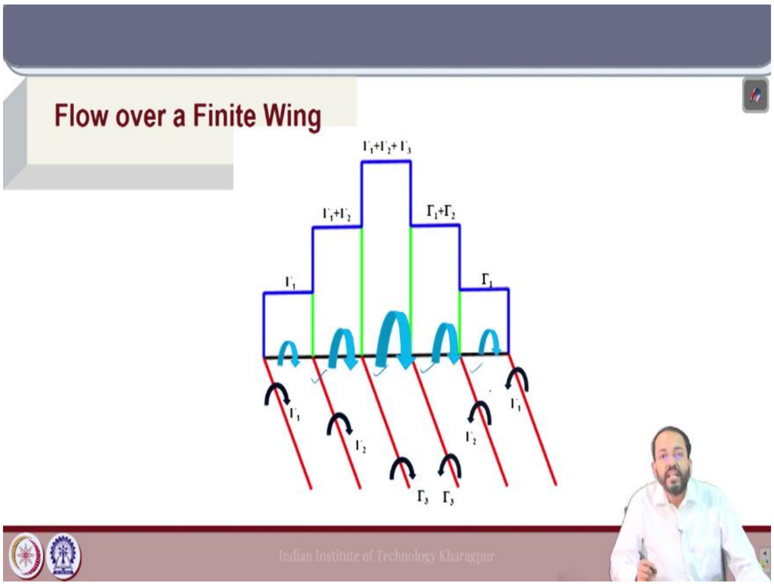

So, in the next figure we will see how it increases accordingly from one element to another. We have Γ_1, the circulation at the two ends due to the actual trailing vertices from the two ends. So, gradually as we go a very few number of elements is shown just to diagram show how the circulation increases towards the center of span.

So, gradually as we go towards the center the circulation is the highest and across each element there is a jump in the circulation value and due to that trailing vortex will be shed from the trailing edge of each of these elements ok.

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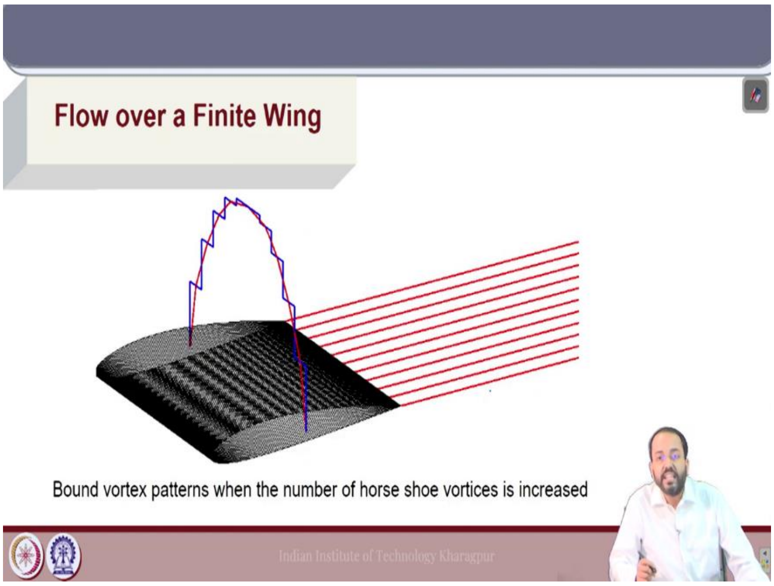

So, this is the trailing vortex sheet. So, when we increase the number of elements, we get a better estimate of the circulation distribution along the span.

So, here the red line is shows the circulation distribution and the blue line is the elemental distribution depending on the resolution of the span into a number of elements ok. So, the closer we make these elements, the closer we get the value according to the actual circulation distribution. And we see that finally, the entire trailing edge will shed a vortex sheet depending on the strength of which depends on the airfoil geometry or the wing geometry basically and also on the angle of attack and the flow conditions.

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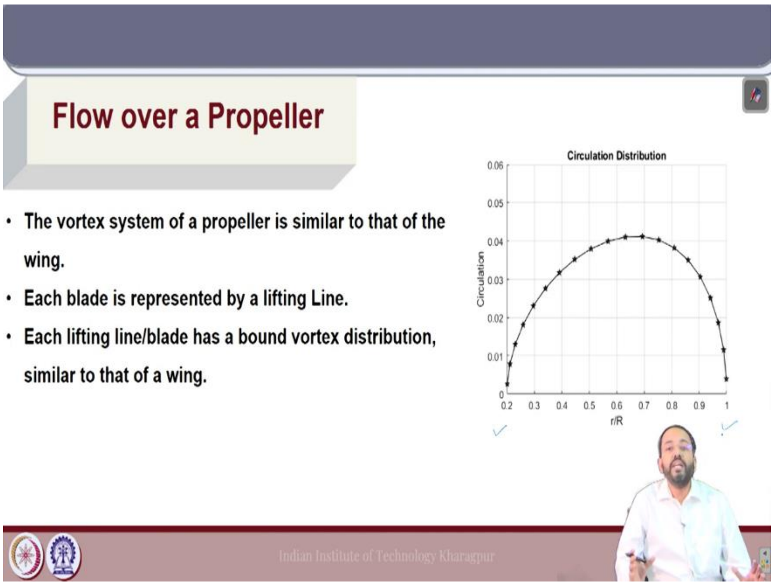

Now, we will move on from the wing to the propeller, this is very important because the entire discussion on wings and the airfoils was because we represent the propeller blade section is typically the shape of an airfoil. So, to understand the vortex pattern and the circulation around a propeller blade, it is imperative to understand the circulation distribution and the vortex patterns of wings. So now, what is the difference between a simple finite foil and the propeller blade? The propeller blade is of a specific geometry and each blade is rotating as well as moving forward.

So, here what we do is each blade is represented by a lifting line and the vortex system basically is represented by a bound vortex distribution similar to that of a wing, ok. So, the circulation distribution can be seen here in this graph. So, we have the circulation distribution at the from the blade root to the tip, blade root is at 0.2 r and tip at r = R, which is the radius of the propeller blade. So, this is the typical circulation distribution for propeller blade when we neglect the effect of the hub.

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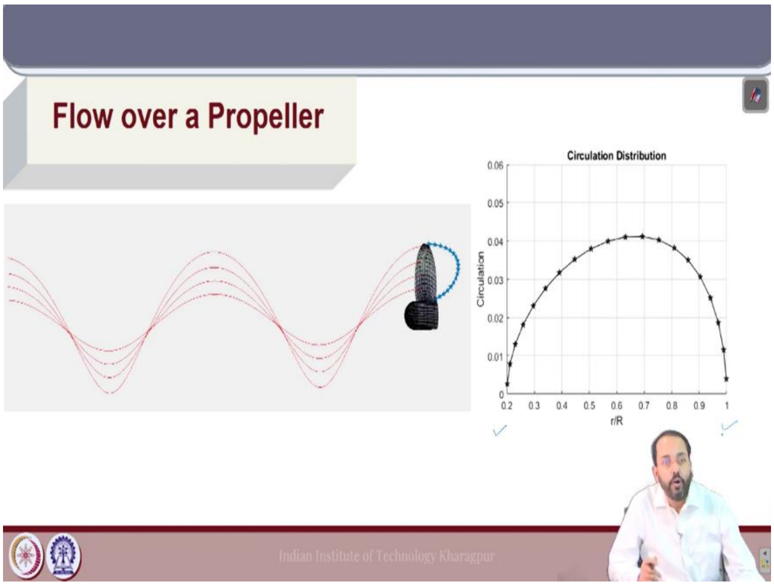

So, if we take care of the effect of the hub, we will have a slightly different distribution. Now, the next part is the vortex pattern; as the propeller blade rotates and also moves ahead. So, the vortex distribution, so the trailing edge vortex pattern of the propeller blade will be helicoidal in nature, because the path traced by the trailing edge of any blade as it moves ahead and also rotates is a helix, right. So, because of the helicoidal path the vortex sheet will be helicoidal.

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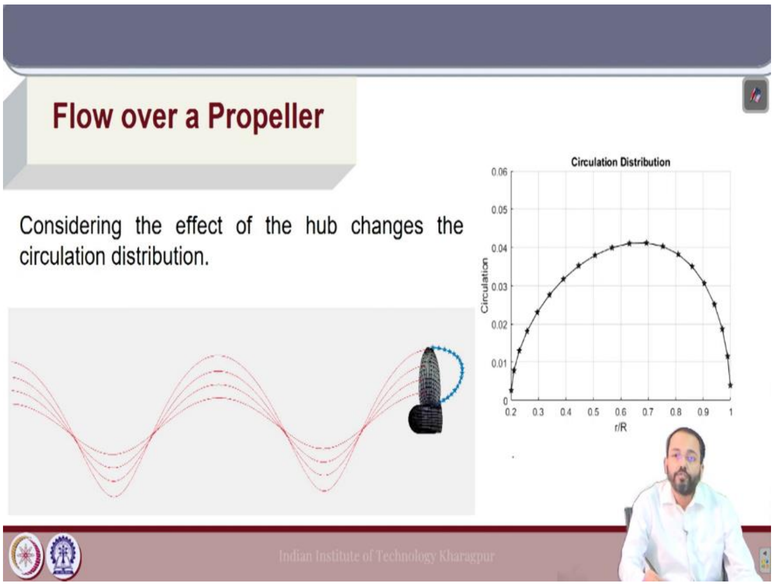

Now, if we consider the effect of the hub, then the circulation distribution will be slightly different. So, this is the original circulation distribution from the root of the blade to the tip.

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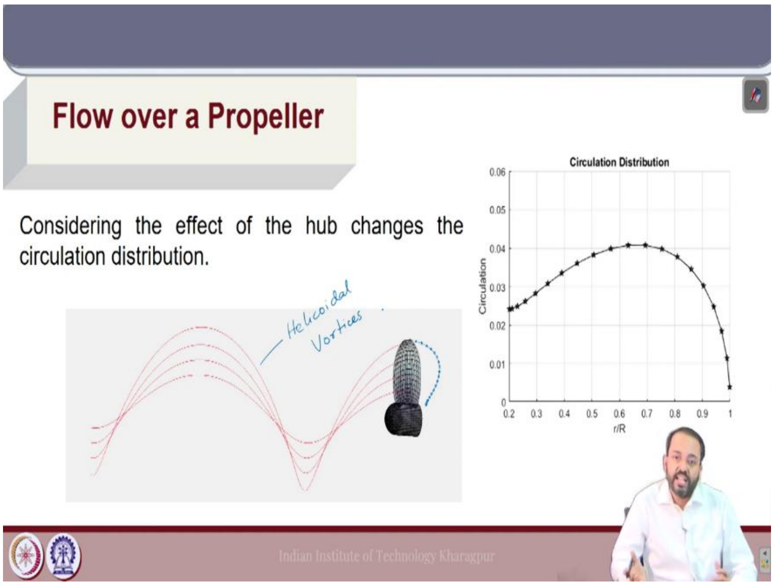

If we consider the effect of the hub the circulation will not be 0, at r/R of 0.2 here we take that the hub radius we take it as 20 percent of the propeller radius. So, it will have a finite value, because now considering the hub that part of the blade does not end in fluid.

So, it will have a finite value of circulation which also slightly changes the trailing edge vortex pattern behind the propeller blade. It is still helicoidal, but the values will be slightly different ok. So, this is the vortex sheet generated by the propeller in the week. So, these are helicoidal vortices, which are shed by the propeller blade downstream as it moves ahead.

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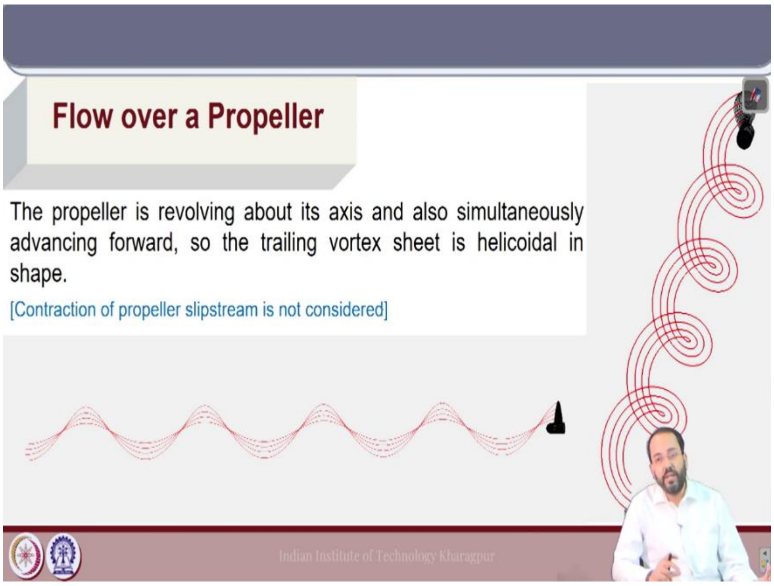

This is a representation of the vortex sheet, trailing edge vortex sheet behind the propeller and it should be noted that we have not considered the contraction of propeller slip stream in this case.

In a realistic scenario, there will be contraction effects in the propeller slipstream and so the vortex pattern will also be modified as it moves downstream of the propeller ok.

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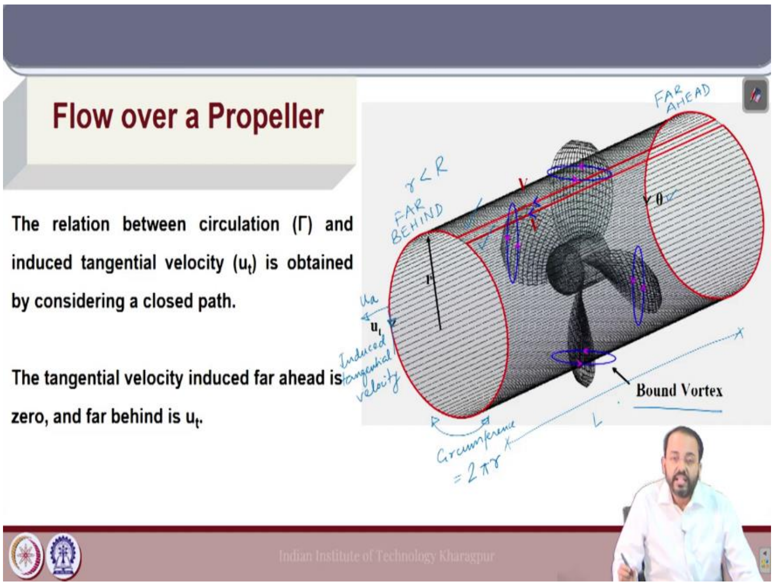

Now, let us try to understand the relation between the circulation and the induced velocity for a propeller blade, because the induced velocities impact the resultant velocity distribution on the propeller blade and finally, the thrust, torque and efficiency of the propeller ok.

So, in this case we have taken a domain which is cylindrical around the propeller blade of a radius r, which is less than the propeller radius R ok. So, in that case this cylinder intersects all the propeller blades at a radius r right and just like the Kutta-Joukowskis theorem we will try to get the circulation on this domain. So, basically what is circulation it is the line integral of velocity around these contours ok.

So, in this cylinder we will relate the induced velocities to the circulation distribution and see how they are related. So, this is far ahead and this is far behind the propeller, for ship we prefer to use the word Aston. Now far ahead the induced velocity in the tangential direction was 0, there was no induced velocity right and far behind we have the vortices shed by the propeller. So, the trailing vortices shed from the edge of the propeller blades, they will induce a velocity downstream and due to that we take that the tangential velocity which is induced, this is the induced tangential velocity u_t for downstream ok.

And on top of that there will be an induced axial velocity. So, there will also be an induced axial velocity u_a ok, in the axial direction. Now, if we take a closed contour on the cylinder around the propeller blade on this cylinder intersects each propeller blade and on that propeller blade we have the vortices which are the bound vortices on the propeller blade. So, each propeller blade will have a bound vortex as we have seen for the finite foil and we have the trailing vortices which lead to the induced velocities.

Now, we take two lines on the cylinders, these two red lines this one and this one these two red lines which are very close at infinitesimal distance from each other and try to unwrap the cylinder and see what is the circulation in that contour ok.

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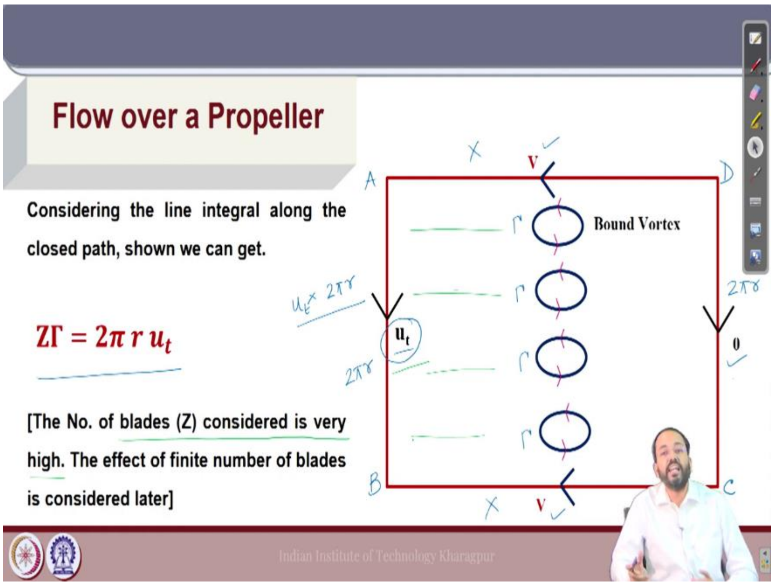

If we do that what will we see? If we unwrap the cylinder now about the two lines then we will have this length if we go back again to this. This circumference at the end, this circumference entire is 2 π r as the radius is r for the cylinder and on the other side we will have a length of the domain which can be of any length let us say L.

Now, if we unwrap this cylinder, it will be in a plan form rectangle and on that we will have the circumference opened at the two ends, because the cylinder radius was r right. What is the velocity? The tangential velocity u_t will be along the tangent in this line and far ahead the velocity was 0 ok, and the velocities in the axial direction are V ok, which are the axial velocities.

Now, if we take a line integral of the velocity around this contour, we will get the circulation. What is the line integral? Basically line integral will be if we start from any point A, B, C, D from A to B the line integral will be $u_t \times 2 \pi r$ right. Now, from B to C it will be velocity V×length, from C to D the velocity is 0, so it will be 0. And from D to A it will be again V integrated over a length L, but this circulation is based on line integral in the direction sense. So, when going from D to A as we have seen earlier the line integral will be in the opposite sense as the line integral from B to C. So, these two arms will cancel out each other ok.

If we do the line integral around the closed curve, because the velocity is same and also the length is same, but in the we will integrate in the two reverse direction from B to C, because it is around the closed curve ok. So, finally, the line integral value will be nothing but $u_t \times 2\pi r$. This is what is given in this particular equation. Now, what is that equal to? That is equal to the Γ, the circulation around each blade.

Now, Z is the blade number, the propeller has Z number of blades. So, the entire circulation created by Z number of bound vortices is Z times Γ right, $Z\Gamma = 2\pi r u_t$. So, this is the relation between the circulation and the tangential velocity induced far downstream for a propeller ok. Now, we will use this concept to relate the circulation to the blade, thrust, torque and efficiency.

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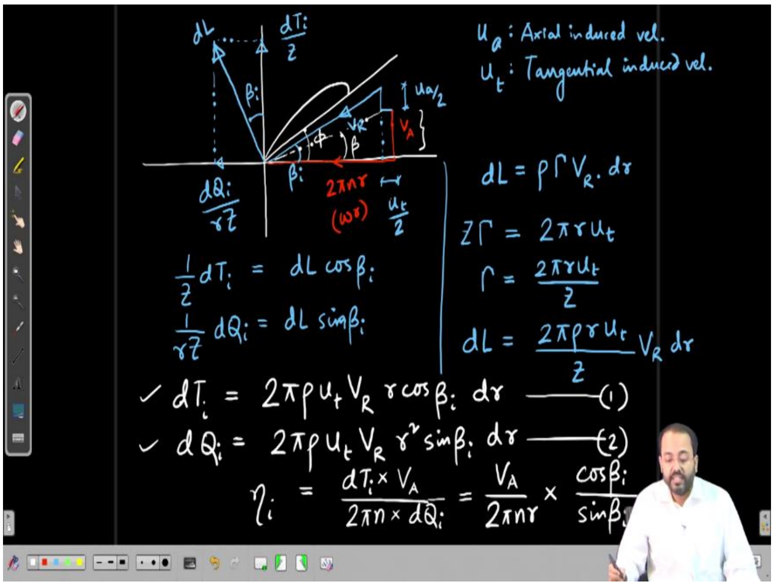

So, we will draw the blade element diagram again and the blade is having a phase pitch angle of ϕ, first the axial and the tangential velocity due to rotation will be shown. So, 2πnr is nothing but (ω r), where ω = 2 π n, n is a rotational speed.

Now, we will have to draw the induced velocities, as before the induced velocities they result in the reduction of the tangential velocity component. So, this we have written as a'× 2 πnr here, we have found out the relation with the tangential induced velocity and the circulation. So, we will write it as u_t and because it is at the propeller plane we will write it as $u_t/2$. So, we assume that at the propeller plane both the induced velocities axial as well as tangential are half that of the values induced for downstream.

And here similarly, this will be $u_a/2$ ok. So, u_a is the axial induced velocity and u_t is the tangential induced velocity far downstream. So, at the propeller plane we are taking $u_a/2$ and $u_t/2$. So, finally, this will be V_R and the angle it makes with the base is β_i the hydrodynamic inflow angle right ok. Let us assume for a simplistic case we neglect drag; we are only taking the lift force for an idealistic case. So, this will be dL which is inclined at β_i with the vertical.

And since we are neglecting the drag directly from the components of lift we can get dT_i/Z we name it as dT_i because this is for the ideal case we are neglecting drag and dQ_i/rZ ok. Just like before we have drawn the lift and the thrust and torque by neglecting drag. Now, from Kutta-Joukowskis theorem what is dL? Lift is given by $\rho\times\Gamma\times V$. Here, what is the final inflow velocity V_R to the propeller blade section at a radius r and over a strip of thickness dr. So, this is the dL or the lift generated by that section related to the circulation right.

Now, $dT_i/Z = dL \cos(\beta_i)$ and $dQ_i/rZ = dL \sin(\beta_i)$ because we have neglected the drag ok. Now, what is Γ? We have just calculated the Γ for a propeller, the circulation will be $2\pi r u_t$ for Z number of blades. So, $Z\Gamma = 2\pi r u_t$. So, $\Gamma = (2\pi r u_t)/Z$ this gives our $dL=((2\pi\rho r u_t)/Z)V_R dr$. So, we can write the sectional thrust and torque as $dT_i = dL\cos(\beta_i)Z$, Z will cancel out.

So, $dT_i = 2\pi r u_t V_R r \cos(\beta_i) dr$ and $dQ_i = 2\pi r u_t V_R r^2 \sin(\beta_i) \times dr$ ok. These are the equations for the sectional thrust and torque ok. Now, the efficiency of the blade element η_i can be given by $dT_i \times V_A$, where V_A is the velocity of advance by $2\pi n\times dQ_i$. So, we can write it as $V_A/2\pi n r \times dT_i/dQ_i$, we will only have $\cos(\beta_i)/\sin(\beta_i)$ and r we have taken here this will be $\cos(\beta_i)/\sin(\beta_i)$.

Now, what is $V_A/2\pi n r$? $V_A / 2\pi n r$ is the initial $\tan(\beta_i)$, this was our β before we have considered the induced velocities ok.

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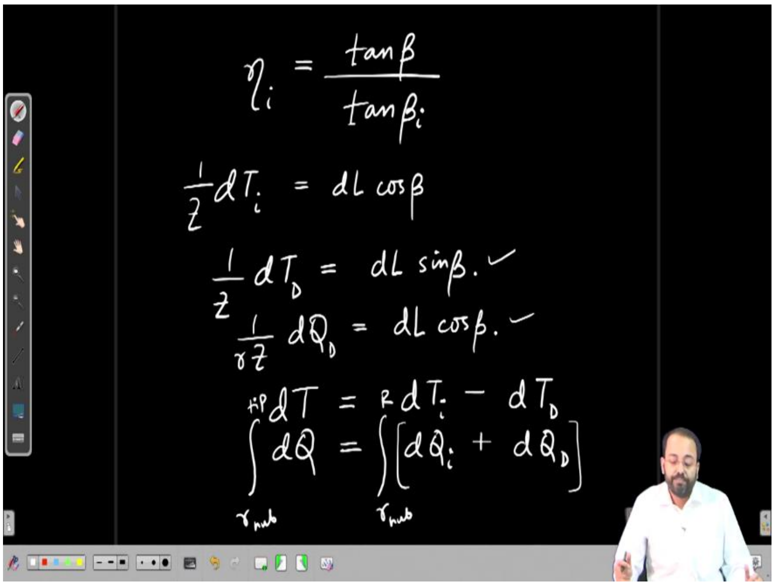

So, η_i = tan β / tan(β_i). So, η_i will be tan β / tan(β_i). Now, remember one thing in the thrust we have only considered the ideal component ok. Similarly, which is the part dLcosβ right, dT_i 1/Z right.

Similarly, we will have a component from the drag ok, if we remove the ideal assumption. So, if we consider the drag, we can have another part dT_D for which 1/Z will be dL sinβ and similarly for torque 1/rZ dQ_D from the drag part will be dL cosβ. So basically, we will have the dT =dT_i - dT_D for the blade element right. And similarly for the torque dQ will be, because dLsinβ is negative for thrust and for torque dLcosβ will be added. So, dQ the ideal value plus component from the drag ok.

Now, for either thrust or torque if we want to calculate the total propeller thrust or torque we can as before integrate these values. For example, if I show for the torque we can integrate these values from the hub to the tip ok, to get the total propeller thrust and torque using circulation theory.

$$\int_{r_{hub}}^{tip} dQ = \int_{r_{hub}}^{R} [dQ_i + dQ_D]$$

Now, there is one assumption here which is very important the number of blades is considered very high; that means, infinite number of blades in the actual sense here, because we have considered the induced velocity u_t, uniformly in the downstream of the propeller. But actually, in the real case there are Z number of blades, each of them will shed a trailing vortex sheet. So, these trailing vortex sheets will actually lead to the induced velocities which should not be continuous because the vortex sheets are discrete from each of these blades.

So, that consideration of finite number of blades effect of that will be considered and we will see the corrections in the next class.

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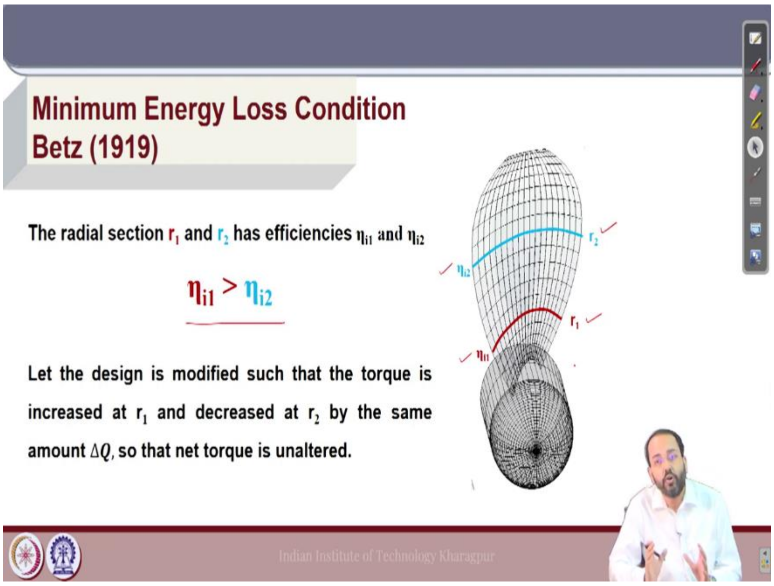

Now, one important aspect for a propeller is the minimum energy loss condition proposed by Betz. Let us take 2 radial locations of the propeller blade at two sections at radius r_1 and r_2, where the ideal efficiency are η_{i1} and η_{i2}. And we assume that we have taken the sections in such a way that the ideal efficiency at η_i, η_1 at the location r_1 is greater than at location 2. So, η_{i1} is greater than η_{i2}.

Now, what we will do? We modify the design, by let us say we change the distribution of pitch in such a way that the torque is increased at r_1 and decreased at r_2 by the same amount ΔQ ok. So, at r_1 we increase the sectional torque by changing the design and r_2 we decrease it, but both by the same amount. So, that the net torque is same.

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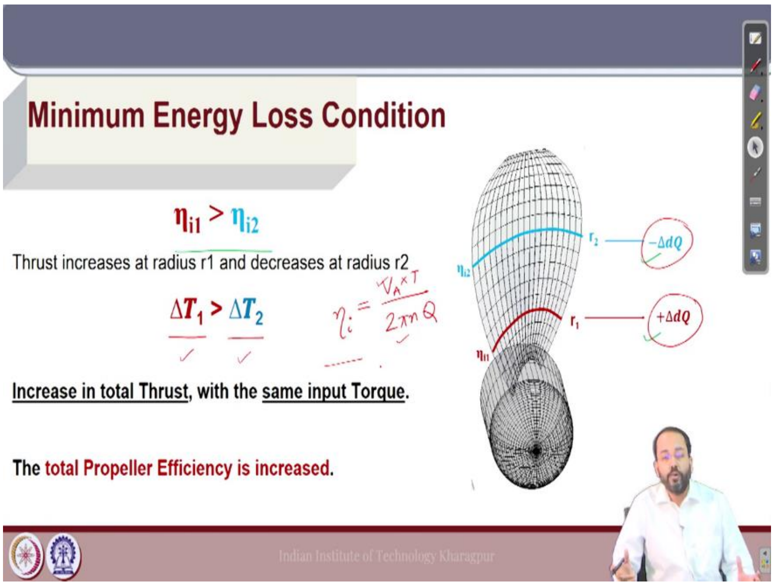

What we will see? Here the torque change at r_1 is by +Δ Q and r_2 by -ΔQ.

Because of this exercise what will happen? The thrust will also change at the two radial locations r_1 and r_2, it will increase at the radius r_1 and decrease at r_2 just like the change in torque. But because the efficiency is higher at radius r_1 as per the input we have chosen the thrust increase at radius r_1 ΔT_1 will be more than the decrease of thrust at radius r_2, that we can get from the equation of efficiency which is given by $\eta_i = V_A \times T / 2 \pi n Q$.

Here V_A and 2 π n these are constants, we have not changed them, we have just changed the geometry so that the thrust and torque of the sections are altered. So, from this equation, we can see that if we keep the torque equally different, if we change them by the same amount the thrust will change by a larger amount at a place where efficiency is higher. So, effectively what we will get? When we integrate now the thrust to get the total thrust, the total thrust will increase because ΔT_1 the increment at r_1 is higher than the decrease at r_2.

So, when we have the total thrust in the final case for the modified propeller geometry we will have the total thrust will be more than the initial case. But the total torque is same, because it has been increased by the same amount and decreased by the same amount at the two different radius values. So, the final thrust will increase for the propeller and the torque will be same, which from the efficiency equation for the entire propeller we will see that the total propeller efficiency is increased ok.

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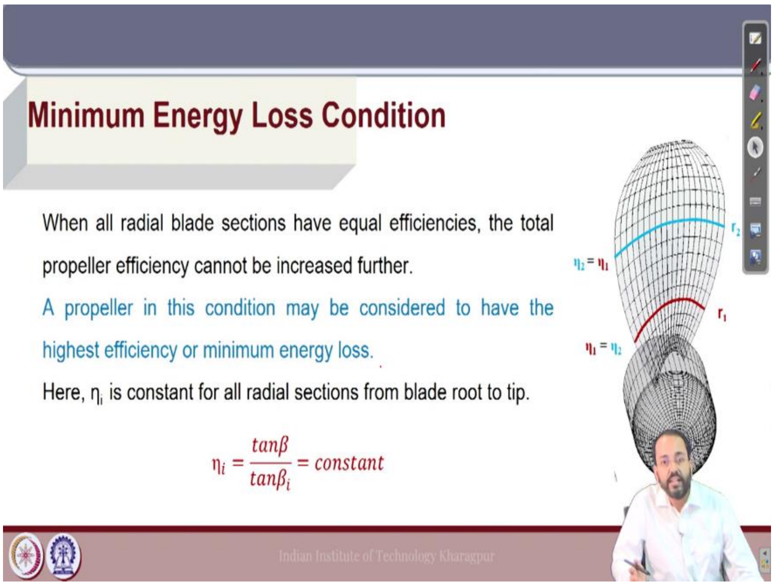

Now, what do we understand from this? So, whenever two blade sections have different efficiency, we can modify the sectional characteristics, the pitch distribution to change the efficiency in such a way that the thrust will increase, but the torque will remain same in a way so that the final efficiency will increase. So, we can continue doing this exercise till all the sections will have the same efficiency.

So, this is the condition where if all the propeller sections have the same efficiency, right from the blade root to the tip, that propeller will have the highest efficiency in the ideal case which is also called the minimum energy loss condition or the Betz case, which was defined by Betz. Now, in that case η_i, which is the ideal efficiency given by $\tan\beta/\tan\beta_i$ will be constant across the radius of the propeller blade. So, this will be the part of circulation theory for propeller blades, we will continue with some examples and other theories of propeller action in the next class.

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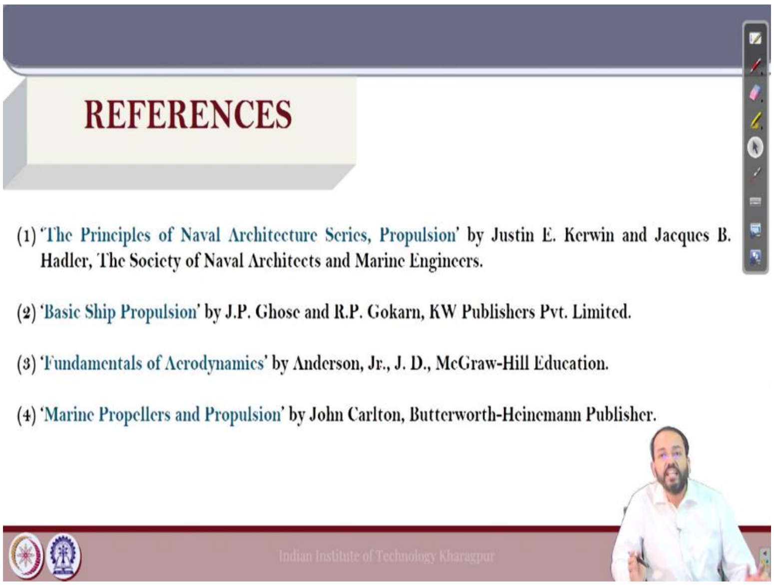

Before that let us look into some references which will be helpful to understand different aspects of propeller theory, for the part of propellers basic naval architecture Principles of Naval Architecture Series, Basic Ship Propulsion book and Marine Propellers Propulsion book can be referred. For the airfoil theory part where we discuss of the for the airfoil theory part regarding the circulation and lift for airfoil and the flow patterns and vortices this book Fundamentals of Aerodynamics will be useful to understand the basics ok.

Thank you.