Numerical Ship and Offshore Hydrodynamics
Prof. Ranadev Datta
Department of Ocean Engg and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 10
Hydrodynamics - 2

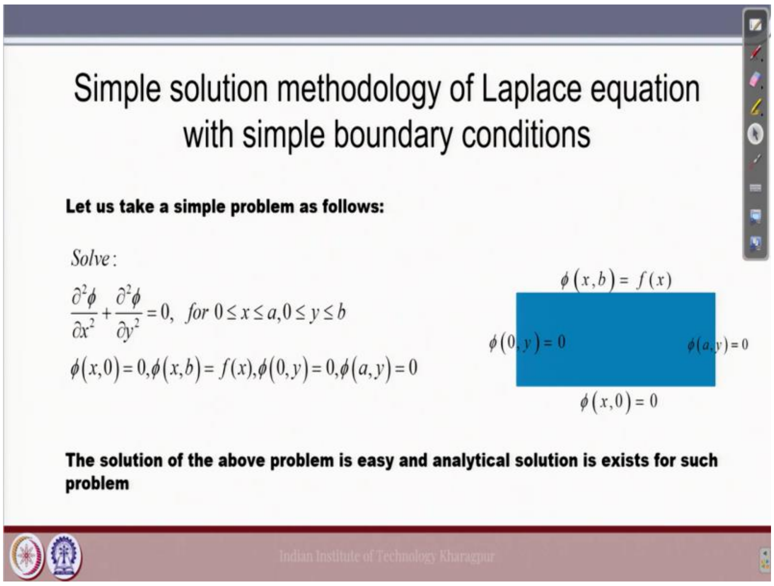
Hello, welcome to Numerical Ship and Offshore Hydrodynamic course. Today is the lecture 10 and today we are going to discuss we are continue to discussing the basic Hydrodynamics ok and this is the keywords that we are going to get to find out this lecture.

(Refer Slide Time: 00:20)

(Refer Slide Time: 00:26)

Now, in last class we have discussed about the boundary value problem that we are going to establish using the Laplace equation and some boundary conditions and from where how I get the pressure.

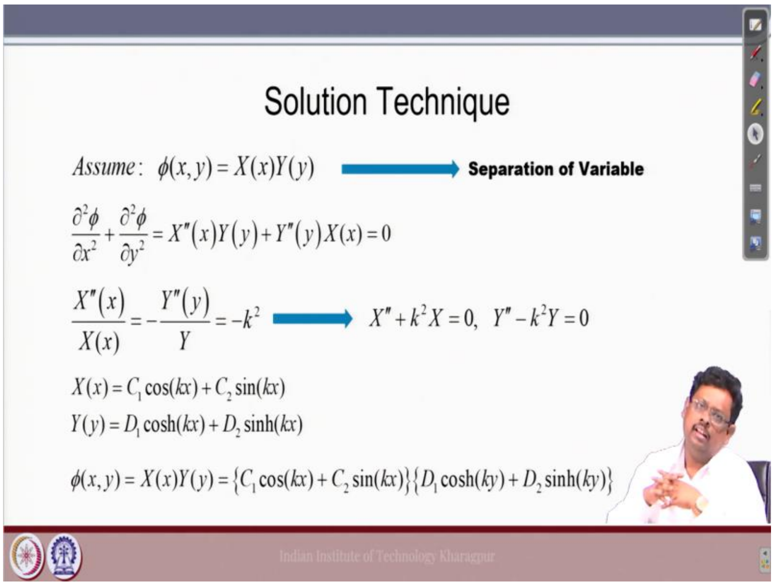
(Refer Slide Time: 00:34)



Now, let us take for a simple problem the solution also very simple ok. So, let us try one solution. Now, let us try to solve this Laplace equation ok, with this boundary conditions you can see that here the boundary conditions are not very complicated right.

Now, let us take this is your rectangular domain and this is your boundary conditions and your governing equation is basically the Laplace equation that is we are going to solve. Now, how to attack this problem ok.

(Refer Slide Time: 01:38)



So, let us see for this sort of problem many analytic solution exist ok. Now, assume that your ϕ is you know with this is called the separation of variable, this ϕ is function of x and y is a two dimensional problem and I separate this solution in X which is only the function of x and then Y is the only the function of y.

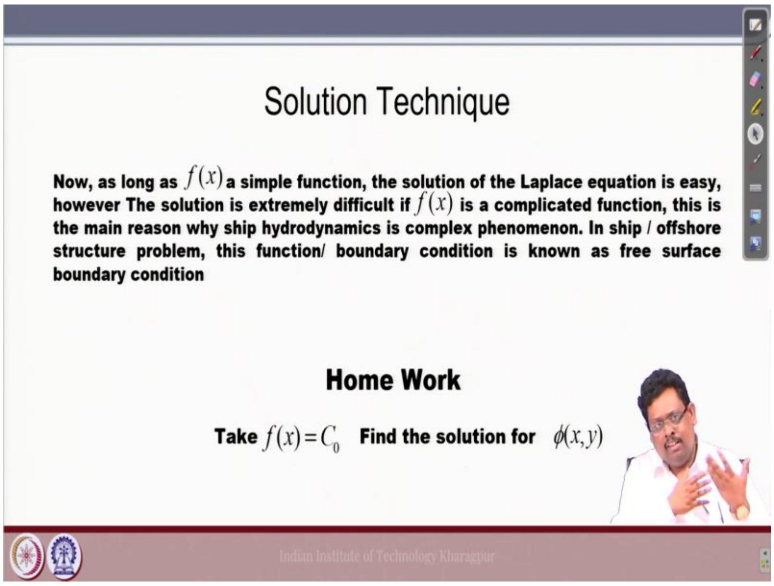
Now, if you substitute this in Laplace equation then then this will get this $X''\times Y + Y''\times X$. Now, if I rearrange this then we can get that X''/X, $Y''/Y = -k^2$. Now, here the tricky part is it should be $+k^2$ or $-k^2$. To see that if I go back here, I can see that horizontal direction is my X and then vertical direction is my Y.

So, I really do not need the sinusoidal solution in the direction of Y. So, therefore, you know that is how I can fix my k size is $-k^2$ Y. So, that you can get here two ordinary differential equation. One is $X'' + k^2 X = 0$ and second one is you know $Y'' - k^2 Y = 0$.

Now, here it is very well known that solution is basically a solution for this problem is harmonic and it is parabolic right. So, I can write the solution for X is $C_1 \cos(kx) + C_2 \sin(kx)$ and direction of Y is $D_1 \cosh(kx) + D_2 \sinh(kx)$. So, now, why it is that is why I see that this taking the k is you know typically based on the fact that I need the sinusoidal solution in the direction of X and I need that other solution is the direction of Y because Y D is not created in direction of Y it is only propagating in the horizontal direction right.

And, then you know that your solution is very simple $\phi(x,y) = X(x) \times Y(y)$. So, now, you substitute everything in this equation. So, this is basically your solution. Now, this C_1 C_2 D_1 D_2 is your constant and you have to find out this constant from your boundary conditions right.

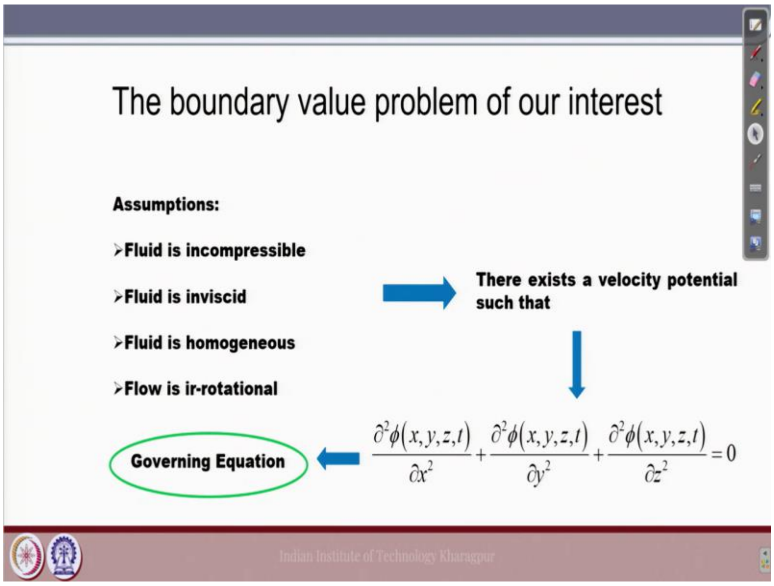
(Refer Slide Time: 04:14)



So, what we get overall? Now as long as this f(x) is simple function, the solution of the Laplace equation is easy; however, the solution is extremely difficult. If f(x) is a complicated function, now in context of our problem this f(x), I mean it is in first of all it is not the f(x) it should be f(x,t) because the wave is function of time as well as the function of space and this is complicated.

So, therefore, you know such simple solution might not be useful when you solve typically a hydrodynamic problem that ship is moving in ocean and then then under the waves condition. Such a simple solution you know not possible right. So, that is why although the solution of Laplace equation is not that difficult you know that you can get it from here it is simple this separation of variable will do; however, because of this complex you know the boundaries right typical see hydrodynamics problem is not as simple as solving a Laplace equation with simplified boundary conditions right.

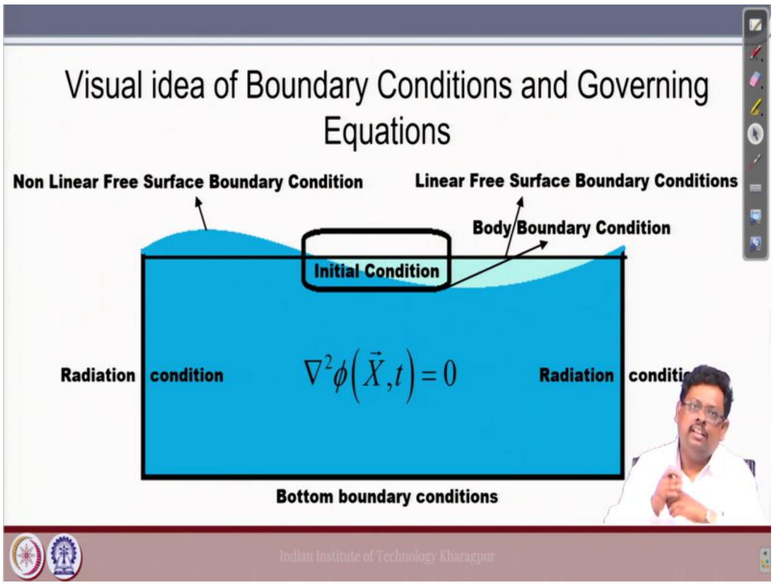
(Refer Slide Time: 05:52)



Now, I actually I am leaving one homework for you let us I mean just try to solve this. Next we are going to move that why I say it is complicated ok. Now, try to find out that what is the equation and what is the boundary condition in case of a ship hydrodynamics. Now, we have this assumption that fluid is incompressible fluid is inviscid fluid is homogeneous and the fluid is rotational right. Now, when we assume the fluid is inviscid then there I mean or the irrotational everything.

So, then we can define a velocity potential ϕwhich is the Laplace equation right ok. So, this Laplace equation we are going to take our governing equation is simple because in the last class also we have discussed the same; here also we are discussing the same. Now, let us see that what is the boundary condition.

(Refer Slide Time: 06:44)



Now, you can see this blue region is basically the fluid region where this $\nabla^2 \phi = 0$ exist right. The mass converse and then let us see the boundaries you can see here you have this I just make this boundary as a you know the black lines right. Now, let us find out that what are those boundaries. Now, this bottom part you can just see bottom we can call the bottom boundary surface or here we can apply the bottom boundary conditions.

Now, you can see these two part actually you can call a radiated, radiation condition applied because it is we can say it is a far field because we can assume the ocean if you look at this horizontal direction it is infinity. Then what happen to the far field that is also we need to apply some condition we called the radiation condition, and then this we can call this as a non-linear free surface boundary condition why?

Because I am assuming now this is the boundary of I mean we can call it is a free surface because this is the boundary we are having the air and then water, and here you remember this is the second interesting thing that actually we are different from any or any other field in case of mechanical we really do not deal with the free surface.

In fact, in aerospace also we do not deal with the free surface only; only in our case when you do the you know ship waves interaction right, in this particular free structure interaction we have to deal with the free surface which is the surface in the top side is air and the bottom side is water right. You think that in aerodynamics you really do not have these cases right in mechanical you do not have these cases right.

Typically for offshore structure for civil engineers they are encountering this problem and also we are. Now, why I call this is a non-linear free surface because this wave is non-linear in nature right; however, we can make you know little bit approximation and what are the approximation definitely we are going to discuss in coming classes, and we can approximate this non-linear free surface to the linear free surface boundary condition, that what is this condition, and what is non-linear free surface condition, what is linear free surface condition, definitely we are going to discuss in future classes.

And, one more very important boundary condition we have not yet discussed which is called the body boundary condition. Now, what is the body boundary condition body boundary condition says that under this waves when the waves hitting the structure you know you can see my initial days this first class I think I showed that video when this body start oscillating and because body start oscillating with there is some kind of waves is generated and also when the incident wave or the ocean waves hit the structure, the structure also start oscillating.

Then what are the kinematics is there, what is the condition. So, that condition is known as body boundary condition. Now, you know the condition is very easy like you know what is the condition that body remain on the ocean. If you think you know what is the condition that body cannot leave the ocean what is the condition.

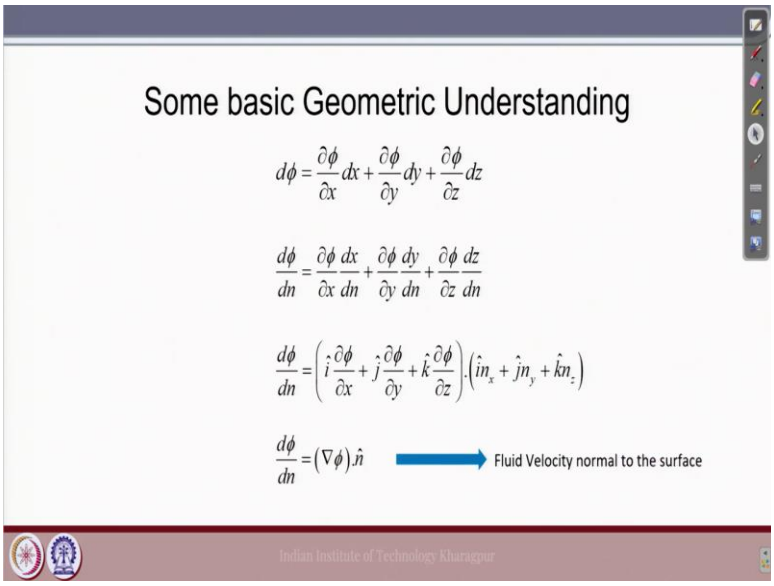
Now, you see here I just show from here that it is the pain and this my hand is with me. Now, now suppose if I you know move my hand up and down then what is the condition then pain also remain here. The condition is the normal velocity of my hand should be equal to the normal velocity of the pain right, if that velocity is different and let us see what is happening.

Now, let us see the velocity of the pain and velocity of my hand has different velocity with different normal velocity then what happened, let us say the velocity of normal velocity of the pain is more than the normal velocity of the hand. Then, what is happening then you can see that my pain is coming out of my hand right.

So, ship really cannot go air right it is not like a ping pong ball it is going up and down I mean that is we are going to play in when you go to the beach you play with the ball right, it is ship is not like this. So, it means that normal velocity of the water particle associate with the means you know touched with the ship both are having the same velocity and that condition we called the body boundary condition ok.

And finally, we have the initial condition initial condition is basically where there is no waves. So, at time t = 0. We assume there is no disturbance then ship is very much you know happy with in the static condition then weight is balanced by the buoyancy. So, that condition is called the initial condition ok. Now, before I move into the next topic some geometric understand let us try to understand ok.

(Refer Slide Time: 12:33)



Now, if you know that this is the well known when this dϕ = (∂ϕ/∂x).dx + (∂ϕ/∂y).dy + (∂ϕ/∂z).dz like, if ϕ is a perfect differential then you know this is the condition and of course, like from mathematics if you have studied mathematic little bit. So, we can find out that velocity potential is a perfect differential ok.

Now, if it is so, then what is dϕ/dn. So, here we can see it is I can write in this form dϕ/dn = (∂ϕ/∂x).(dx/dn) + (∂ϕ/∂y).(dy/dn) and (∂ϕ/∂z).(dz/dn). Now, if it is so then actually what we could do is I can write this in a vector form. How? Now, we can see that in the next slide we can see that I can write (∂ϕ/∂x) as i (∂ϕ/∂x) because this is the component along the x direction then j (∂ϕ/∂x) and then k (∂ϕ/∂z).

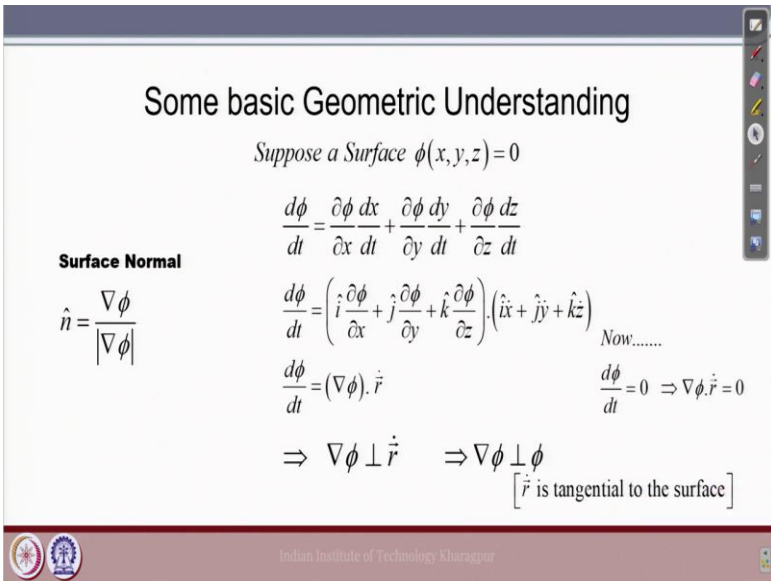
So, I can write in vector form right because (∂ϕ/∂x) is the direction of i, (∂ϕ/∂y) is the direction of j, and (∂ϕ/∂z), is the direction of k, and similarly I can write the (∂x/∂n) is =

n_x and ($\partial y/\partial n$) = n_y and ($\partial z/\partial n$) = n_z. So, that is also I can write in a vector from n_x in the i_{th} component, n_y in the z_{th} component, and then n_z is the k_{th} component.

So, if it is so, basically that ($\partial\phi/\partial n$) is nothing but the ($\nabla\phi$).n. So, this is also you know from the vector understanding of the vector also we can get this like velocity or a.b means what the component of a along b right. So, in similar way ($\nabla\phi$).n is basically the component of the velocity in the normal direction right.

So, now on when I say the component of the velocity in some direction or in the normal direction, normally you are going to use ($\nabla\phi$).n. So, it is understand that component of the velocity along the direction of n ok. So, this is one thing that you have to understand I mean I know that you know all these things known to you, but still I just discussed this ok. So, that is what I said the fluid velocity normal to the surface.

(Refer Slide Time: 15:34)



Next one, suppose some more basic understanding suppose a surface ϕ. So, ϕ is a surface (x, y, z) = 0. Assume, that this is the surface. In fact, you can add t also over here that $\phi(x, y, z, t) = 0$ ok, I mean reality that is what the case ok. Now, if it is so, then $d\phi/dt$ = I can again in similar line I can write it is $\partial\phi/\partial x \times dx/dt + \partial\phi/\partial y \times dy/dt + \partial\phi/\partial z \times dz$ because I know that $d\phi = \partial\phi/\partial x\, dx + \partial\phi/\partial y\, dy + \partial\phi/\partial z\, dz$ right, so, this one.

So, in similar line I instead of n I just use the divide it. Now, again I can write in vector form vector dot product as follows like $d\phi/dt = i\, \partial\phi/\partial x + j\, \partial\phi/\partial y + k\, \partial\phi/\partial y$ and then this dx/dt I can write as a x dy/dt I can write as a y. and dz/dt I can write as a z . right.

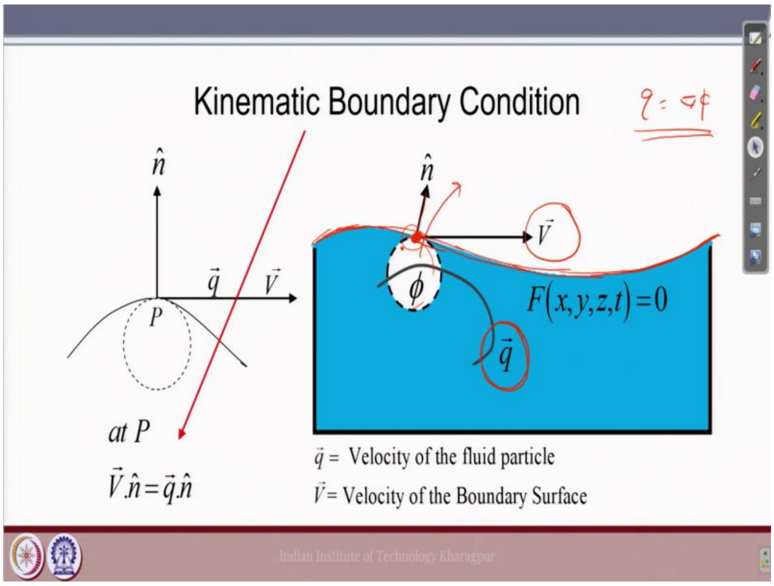
Now, you see what is this the left hand side is nothing but the $\nabla\phi$ and the right hand side is nothing but the velocity which is r So, I understand that $d\phi/dt$ is nothing but $\nabla \phi$. r So, now, r is the velocity. So, what is the meaning is that velocity along the tangential direction right. Now, if you look at the first line when I say that $\phi(x, y, z) = 0$ you can write $\phi(x, y, z, t) = 0$ ok, I mean ϕ can be function of t also and if it is so, then $d\phi/dt$ definitely going to be 0.

So, in the left hand side I called $d\phi/dt = 0$ right. So, here that $d\phi/dt = 0$ we know. So, therefore, I can see that $\nabla\phi.r = 0$. Now, you see from the vector what we get from here it mean that gradient $\nabla \phi$ is a vector and r is a velocity vector their product is 0. So, therefore, $\nabla \phi$ definitely perpendicular to the r . right. So, this is the take.

So, I understand that $\nabla \phi$ is perpendicular to the r. So, if it is so, we can see here suppose $\phi(x, y, z) = 0$ if this is the surface then definitely I can say that $\nabla \phi$ is perpendicular and also $\nabla \phi$ is the normal to that surface and therefore, I can say that the surface normal n can be defined as $\nabla \phi$ divided by modulus of $\nabla \phi$ ok fine.

So, these two geometric understanding if we have and then we can actually try to you know find out what is the kinematic body boundary condition.

(Refer Slide Time: 19:03)



And, this is very important concept in our numerical ship hydrodynamic it is not numerical ship hydrodynamic, it is this is the you know if you consider wave dynamics then also understanding this kinetic boundary condition is very important. Now, here what is we can see here like this F(x, y, z, t) is basically the similar to my ϕ here ϕ(x, y, z) =0. So, instead of ϕ I am writing that F(x, y, z, t) = 0. Now, this is basically the surface.

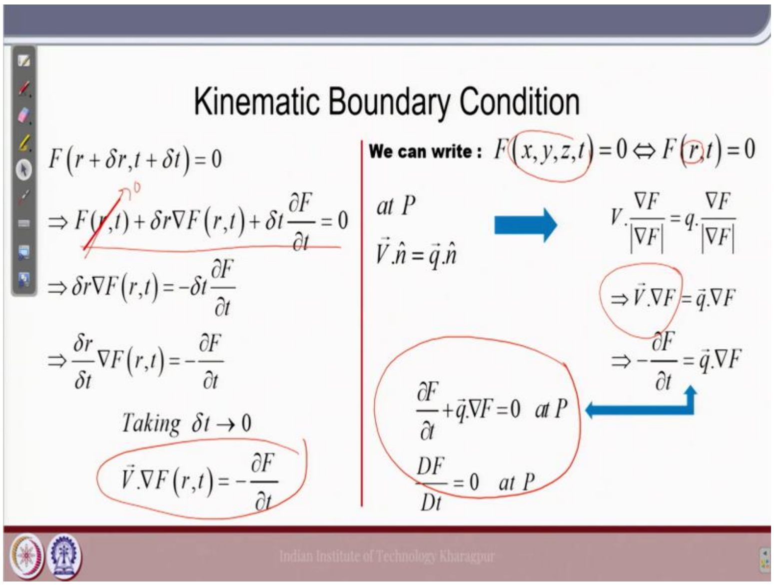
So, this basically this surface I can call that equation is F(x, y, z, t) right and then this surface has a velocity V let us assume this surface has a velocity V ok. At some point of time here, at some point of time. So, this is the direction of the normal n. Now, this white one is nothing but the fluid particle. Now, assume that this fluid particle also having a velocity q ok. So, and also this also you can see this q = nothing but gradient of ϕ ok under this all these condition that we have discussed.

Now, what is the condition that this fluid particle will stay here with the in the boundary surface f. So, this question is very clear that what is the condition that this fluid particle stay here. So, this fluid particle is not coming out of this boundary surface a what is the condition. So, we discussed just right now like that how this pain is sticking with my hand. The idea is basically the normal velocity; the normal velocity should be of the boundary surface should be equal to the normal velocity of the particles right.

So, this is what actually discussed you know a few moments back when I say that pain should stick on my hand, if the velocity of my hand the normal velocity of my hand equal to the normal velocity of the pain right. The similar argument here I am just writing this this picture over here. So, this is the point P and here you have this q is the velocity of the fluid particle V is the velocity of the surface.

So, therefore, at this particular point the normal velocity of the boundary surface should be equal to the normal velocity of the velocity particle right. So, this is clear, now if it is clear then rest part is basically algebra ok just clear this yeah ok.

(Refer Slide Time: 22:41)



Now, if this is clear the remaining part basically the algebra. Now, again before we really try to solve this problem let us do one more thing ok. Now, this is we are writing that F(x, y, z, t) = 0 or just you can write that sake of writing little bit smartly that x y z we write in terms of r ok fine. Now, if it is so and then actually I just do a Taylor series expansion.

Now, I am just try to find out what happened with F (r + δ r, t + δ t) ok. Now, here this F of r + δ r if I use the Taylor series expansion it becomes F (r, t) + δ r × ∇ (r, t) + δ t (∂F/∂t). So, this is a; this is a very standard this is very standard right this is very standard. Now, here this part goes to 0 right. Now, if this part goes to 0 and then we can write that δ r × ∇ F = minus δ t (∂ F / ∂ t). So, then if I divide it the throughout the δ t.

So, I get $(\delta r / \delta t) \nabla F$ = minus $\partial F \partial t$ and if it is so we can find out that you know V dot ∇F = minus $\partial F \partial t$. So, now let us understand this concept ok. So, this is something actually we are going to use to find out the kinematic body boundary condition.

Now, you see at P as I said V. n is = q. n right. So, normal velocity of the boundary surface should be equal to normal velocity of the water particle and also I know that n is nothing but ∇F by modulus of ∇F that is also we know. So, therefore, I can replace V = ∇F by modulus of ∇F = q . ∇F by modulus of ∇ n right.

And, then we have V . ∇F = q . ∇F fine, because this modulus of ∇V is nothing but a scalar term and both side will be cancelled out. Now, from here; now from here we are actually using this result that V dot ∇F actually I am going to replace this V . ∇F ok. Why? Let see that what we get if I do this.

Now, once I do that ok let me take this somewhere else. Now here you can see that this V . ∇F. Now, actually I replaced by minus $\partial F / \partial t$. So, what we achieve by doing this, if I do that finally, we will get this equation that $\partial F \partial t$ + q dot $\nabla F = 0$ at P and this is very well known and everybody knows what is this equation right, this equation is very very well known to everybody and this equation is nothing but the material derivative = 0.

Now, this is the kinematic body boundary condition. Now, we said if F is the boundary surface then that the water particle should be sticking to that surface if and only if the material derivative $DF/Dt = 0$ ok. So, with this understanding let us stop today ok and in the next class we are going to discuss the more concept on the hydrodynamics.

Thank you very much.