Numerical Ship and Offshore Hydrodynamics

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Lecture - 13
Wave - 3

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Welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 13. Today we are going to discuss a very interesting concept which is called the distribution of the singularities and I will tell this is one of the most important aspect of this numerical ship and offshore hydrodynamics. So, before we start the boundary element method we have to understand this concept ok.

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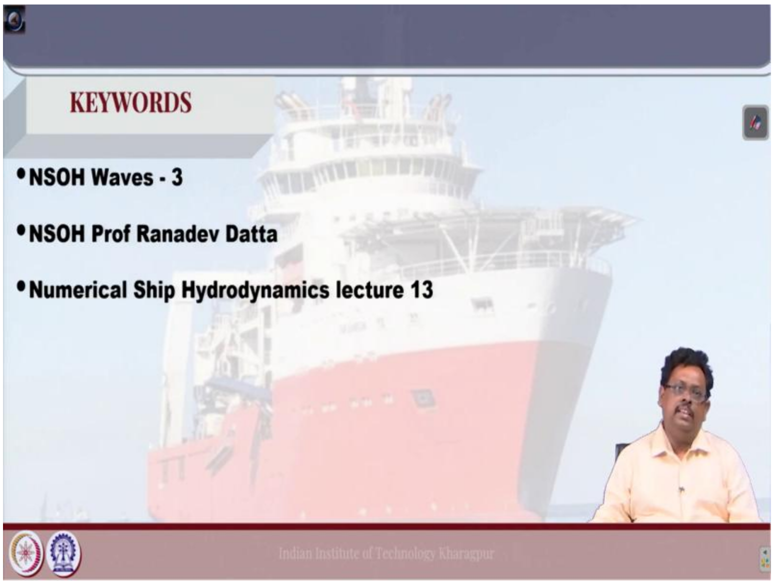

So, this is the keyword that we are going to use to get this lecture. Now let us comes back to this topic.

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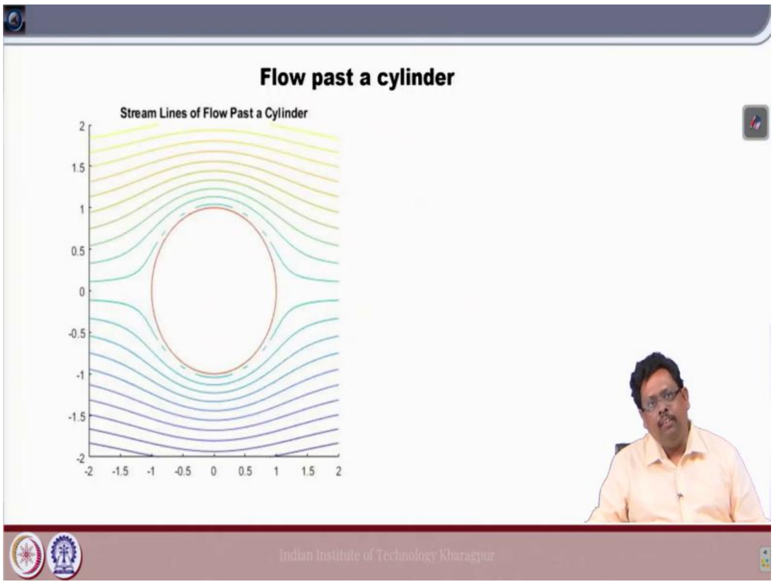

Now, it is very well known thing right it is a flow past a cylinder and you know in your basic hydrodynamics you have this two dimensional problem you solved it using potential theory if you are going to solve this problem how you solve this? Basically, you are really not putting any cylinder there right here you have the cylinder, but you really do not put any cylinder here physical cylinder what you do? You have replaced this cylinder by distribution of a dipole or doublet right. So, how we solve this problem? Let us have a look ok.

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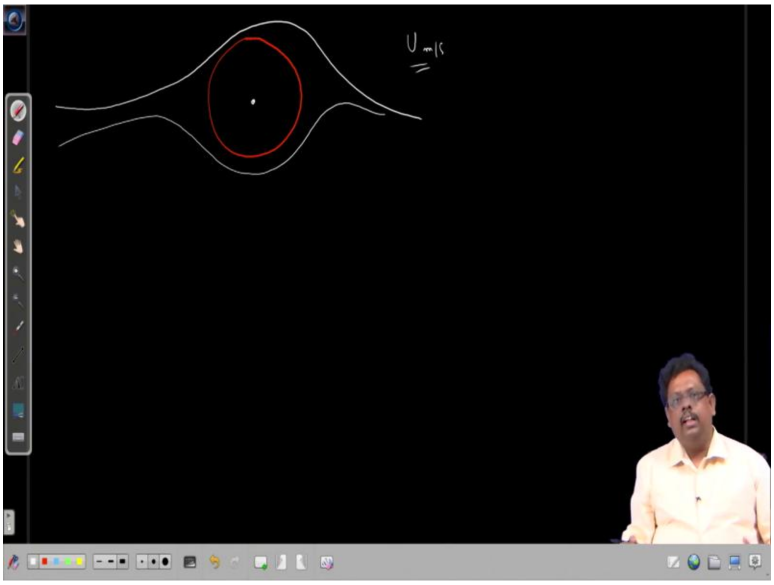

So, now the problem is as follows, I have a cylinder like this and then we have some flow and we try to find out what is the what is the mathematical model for this that flow past a cylinder. Now we assume that there is a uniform flow and this it is U meter per second some flow is coming here and then we need to find out that what would be the flow pattern how we get the pressure etcetera etcetra right.

Well, now here because we are going to discuss some concept we really do not discuss the mathematics the important mathematics behind this and we assume that it is you know everybody.

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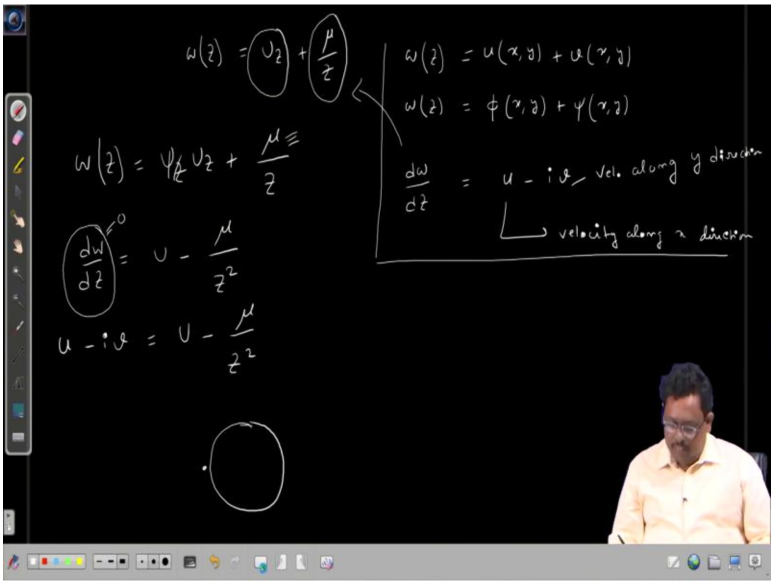

So, we start with a complex potential $W(Z) = U_z + \frac{\mu}{Z}$. Now this is how actually with this we can find out the flow past a two dimensional body in a in complex domain right. Now, what is this? This U_z is a complex potential for uniform flow and $\frac{\mu}{Z}$ is basically the complex potential for a doublet with a strength mu right. Now if it is you know unknown to somebody. So, just for them very briefly I will tell that any flow can be defined in terms of a complex potential and there is a one the real part which is u (x, y) and then we have a complex part let us say it is v (x, y).

Under the potential theory we can say that this W(Z) can be written as the velocity potential $\phi(x, y) + \psi(\mathrm{x}, \mathrm{y})$ plus a stream function $\psi(\mathrm{x}, \mathrm{y})$. Not only that if you differentiate with respect to Z and $\frac{\partial W}{\partial Z}$ becomes that u- iv where u is basically the velocity along x direction and v is nothing but the velocity along y direction. So, this concept is very well known. So, therefore, we are not really

discussing this concept let us try to find out using this concept how I can find the flow past a cylinder ok.

Now, if W(Z) equal to if W(Z) is equal to some U Z which is the velocity potential or complex potential for uniform flow is not subscript Z it is U into Z plus some $\frac{\mu}{Z}$. Now this mu basically nothing but the let us say strength of the dipole or doublet. Now if you do dW/dZ then you get it is nothing but U- $\frac{\mu}{Z^2}$. Now you know that dW this is nothing but your u -iv, the velocity along x direction and also the velocity along y direction.

So, this you can replace by u- iv is equal to this capital U which is the uniform stream minus $\frac{\mu}{Z^2}$. Now, let us try to find out the stagnation point. What is the stagnation point? At that suppose it is the cylindrical surface. So, where the velocity is equals to 0 ok. So, there is no velocity in u direction there is no velocity in the v direction. So, essentially that $\frac{\partial W}{\partial Z}$ should be equal to 0 right.

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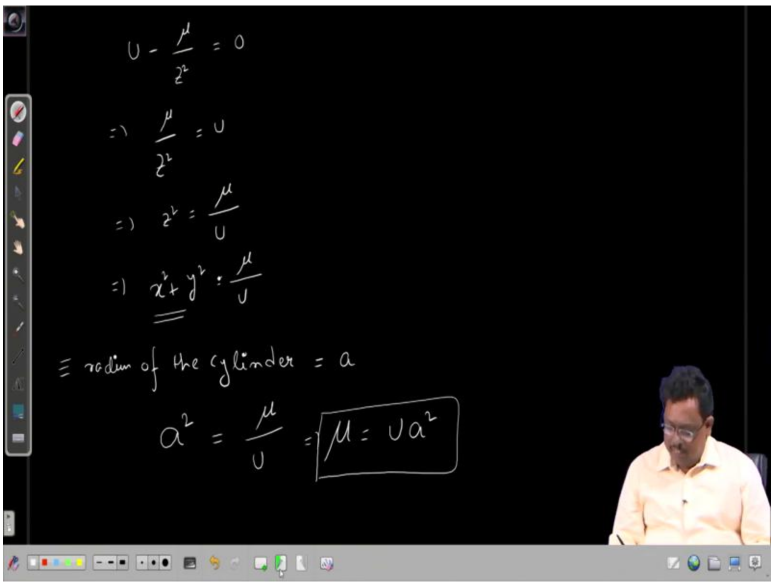

Now, if I do that if I make $\frac{\partial W}{\partial Z}$ equal to 0 essentially what we get is U - $\frac{\mu}{Z^2}$ =0. Now, if I solve this I will get $\frac{\mu}{Z^2}$ = U and then I get that $Z^2 = \frac{\mu}{U}$ and which implies now Z square is nothing but x a square plus y square equal to u by mu by u. Now this is nothing but the you know the circle then x square plus y square must be equal to the radius.

So, if the radius of the cylinder is a suppose we assume the radius of the cylinder this equals to a. So, I can find out that there is a relationship between the radius of cylinder and the strength right. So, it becomes $a^2 = \frac{\mu}{U}$ from here I can find out your mu equal to U into a square. Now, you see it is very important results. So, actually what we achieve from here? Let us see from the physical point of view.

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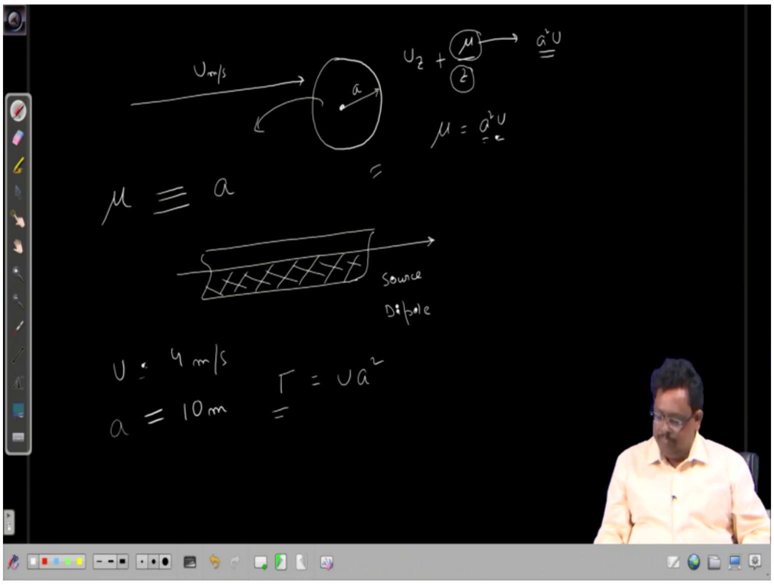

Now, I said I have a cylinder to and then I have a uniform flow which is U meter per second. Now if you give me the problem is that to find out that flow past this cylinder I have to put a velocity potential for the uniform flow which is U Z along with that I have to put a cylinder I mean a doublet or dipole with radius is mu by Z and then I found this mu is nothing but your a^2U.

So, what I get is that initially I have a cylinder right. Now I want to find out the flow past a cylinder. So, what I do? Actually I remove the cylinder I remove the physical cylinder instead I put at the centre a dipole or a doublet with a strength a square U and if when we put that I assume that it actually physically represent flow past a cylinder where the radius of the cylinder is a.

You see it means that if I try to find out flow past a cylinder of some radius a, b, c whatever then I have to first find out what is the strength of the doublet and you see that this strength of the doublet mu is somehow related to the geometric parameter which is the radius of cylinder. Now, in case of a cylinder it is very easy. Now suppose if I try to apply the same logic for a ship what is that?

Suppose instead of this cylinder I put a ship over here and I try to find out the pressure distribution along this hull right. So, idea is in case of a cylinder I put a doublet at the centroid of the cylinder or centre of the cylinder with a strength μ equal to a^2U where both is known to me.

So, if I try to find out the pressure field along this hull pressure field along this hull then what would be the distribution right? Is it a source or is it a dipole or is a combination of both? I do not know also I do not know what would be the strength of the source what would be the strength of the dipole see.

Now, you see I am actually in numerical ship hydrodynamics we are actually solving the similar type of problem, but now problem is more complex right. In case of a two dimensional cylinder I know everything I know the you know this abstract doublet the strength of this abstract doublet μ I know it is related to the geometry of the cylinder right. So, it is for me it is very easy, you can tell me that find out the you know let us say it is U equal to 4 meter per second ok.

And then you find out let us say the radius of the cylinder a and that equal to let us say some 10 meter and then you ask me find out I try to find out the pressure pattern along the cylinder I say it is very easy because my gamma is related to both. So, gamma is nothing but U into a square. So, if I put this strength then definitely I can get what you are asking for right.

So, this is the idea this is the basic idea of the distribution of the singularities why call this singularities? Because you can see here it is μ by Z. So, at Z equal to 0 this function is a singular because it functions become infinity right. So, here for this particular problem I distribute the singularity at centroid with a strength which is I relate this strength with my geometry.

So, given a geometry I understand from this particular problem given a geometry I really can find out a strength and then I can solve the problem. I only need to know two things like how I can distribute the source or distribute the singularities dipole and also second point is how to estimate the strength clear ok.

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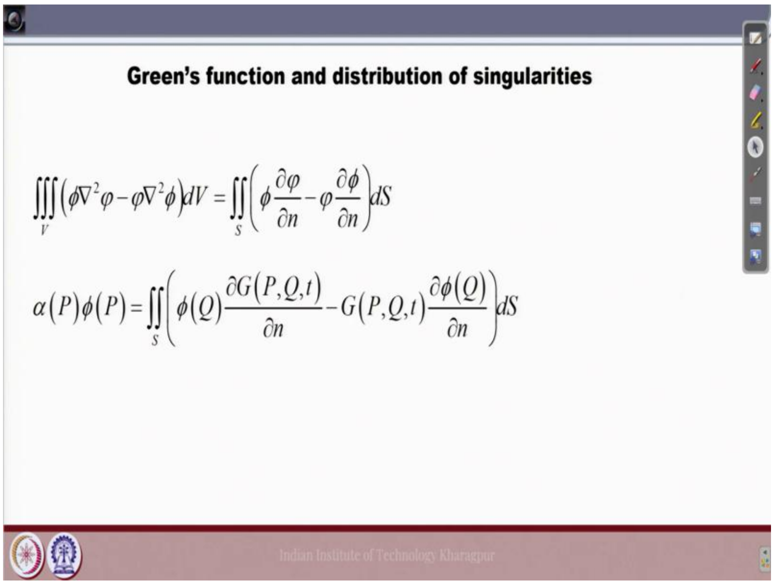

Getting this idea. So, getting this idea let us move on with the Green's function and then distribution of the singularities. Now, here we are going to use this integral I mean this Green's theorem it tells you let us say φ and ψ be any function. So, there are other mathematical definition for at this moment let us ignore it, here it says that this is a volume integral you can see from the left hand side you have a volume integral. So, volume integral φ into del square ψ. So, its a Laplace here right here this is a.

So, here this is a Laplace here right. So, φ I mean del square ψ is Laplacian, del square φ is a Laplacian rate and then this equals to the right hand side which is $\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}$. I am going to use this result ok I am going to use this result to find out something. Let us see.

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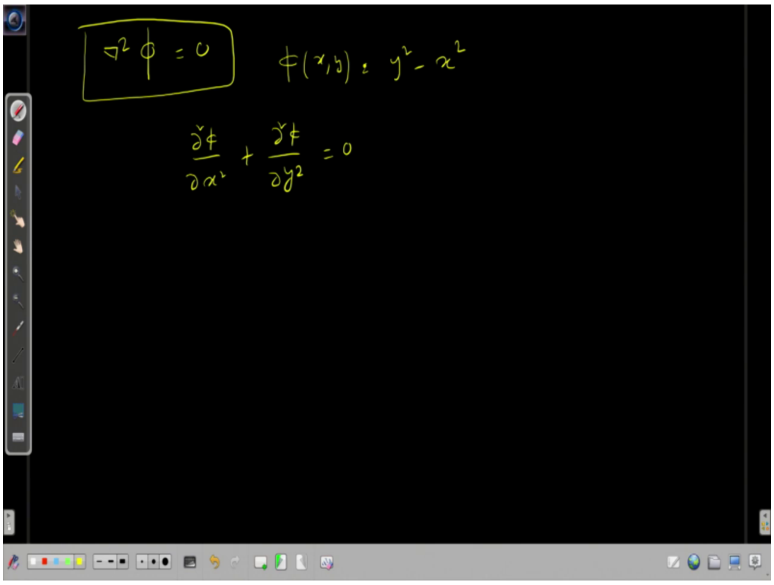

Now, before we proceed further try to find out a simple thing that the what is the meaning of a velocity potential? So, we can call the φ is a velocity potential if $\nabla^2\phi = 0$ right. This is how actually we say that whether this function is a velocity potential or not right.

Suppose, if I given a function let us say φ x, y equal to you know y square minus x square and if you ask you if somebody asks you like whether it is a function of velocity potential or not. So, first thing what you do? You find out $\frac{\partial^2\phi}{\partial^2 x}+\frac{\partial^2\phi}{\partial^2 y}=0$ which is this equal to 0 or not in that case it is 0. So, you can call this function is a velocity potential fine.

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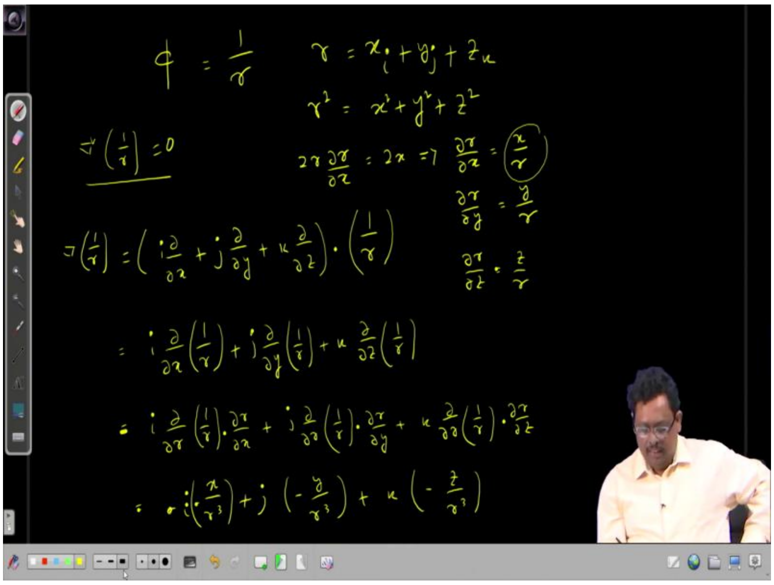

So, now first let us try to find out whether this φ 1/r. I mean this φ ok this φ this function is 1 upon r whether this function is a velocity potential or not ok and the definition of r is nothing but xi +yj+ zk and therefore, this r square equal to nothing but x square plus y square plus z square right.

So, now before we proceed further to check whether this del square into 1 upon r this equal to 0 or not ok let us try to find out little bit some manipulation. So, I just differentiate with respect to x we get $2r\frac{\partial r}{\partial x} = 2x$ which implies $\frac{\partial r}{\partial x} = \frac{x}{r}$. So, similarly I can find out $\frac{\partial r}{\partial y} = \frac{y}{r}$ and similarly $\frac{\partial r}{\partial z} = \frac{z}{r}$. We need to remember this results and then let us try to find out what is the grad of 1/r.

Now, I know the definition of grad is as follows it is $(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}).(1/r)$ right. So, I have $i\frac{\partial}{\partial x}\left[\frac{1}{r}\right] + j\frac{\partial}{\partial y}\left[\frac{1}{r}\right] + k\frac{\partial}{\partial z}\left[\frac{1}{r}\right]$ right. Now we know the chain rule. So, it should be $i\frac{\partial}{\partial r}\left[\frac{1}{r}\right].\frac{\partial r}{\partial x} + j\frac{\partial}{\partial r}\left[\frac{1}{r}\right].\frac{\partial r}{\partial y} + k\frac{\partial}{\partial r}\left[\frac{1}{r}\right].\frac{\partial r}{\partial z}$.

Now, if you substitute everything see you know del del r 1 upon is minus 1 upon r square at del r del x if I use the x by r. So, I will get it is minus you know

$i\left(-\frac{x}{r^3}\right)+j\left(-\frac{y}{r^3}\right)+i\left(-\frac{z}{r^3}\right)$ ok. So, I just make here also similar minus i. So, I just replace this as this way ok.

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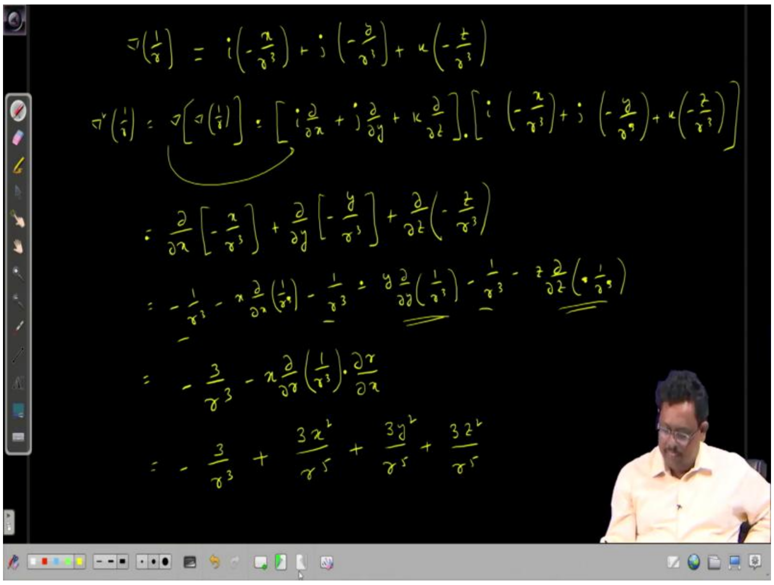

So, I get that grad of 1/r is equal to $i\left(-\frac{x}{r^3}\right)+j\left(-\frac{y}{r^3}\right)+i\left(-\frac{z}{r^3}\right)$. So, $\nabla^2\left(\frac{1}{r}\right)$ nothing but grad of 1 upon r. So, it is again I just apply over here let us quickly apply this it is $i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}$. So, these are definition of this and now I just find out is nothing but dot product $i\left(-\frac{x}{r^3}\right)+j\left(-\frac{y}{r^3}\right)+i\left(-\frac{z}{r^3}\right)$.

So, if I use the dot product it is simple. So, then I get it is $\frac{\partial}{\partial x}\left[-\frac{x}{r^3}\right]+\frac{\partial}{\partial y}\left[-\frac{y}{r^3}\right]+\frac{\partial}{\partial z}\left[-\frac{z}{r^3}\right]$. Now, you know this how to do this. So, it is u v function if you can break it. So, del del x so a its become like $-\frac{1}{r^3}-x\frac{\partial}{\partial x}\left[\frac{1}{r^3}\right]-\frac{1}{r^3}-y\frac{\partial}{\partial y}\left[\frac{1}{r^3}\right]-\frac{1}{r^3}-z\frac{\partial}{\partial z}\left[\frac{1}{r^3}\right]$.

So, this is something that you are going to see and then now if you do this, I will get here minus 3 by r cube coming from here and if you do here. So, again I know that I should write x into del del r of I just doing for 1 ok it is 1 upon r cube into del r del x. So, I just perform this. So, I just get $-\frac{3}{r^3}+\frac{3x^2}{r^5}$

Similarly, if I perform this I will get plus $\frac{3y^2}{r^5}$ and if I perform this I will get $\frac{3z^2}{r^5}$. So, you can do by your own you can check that.

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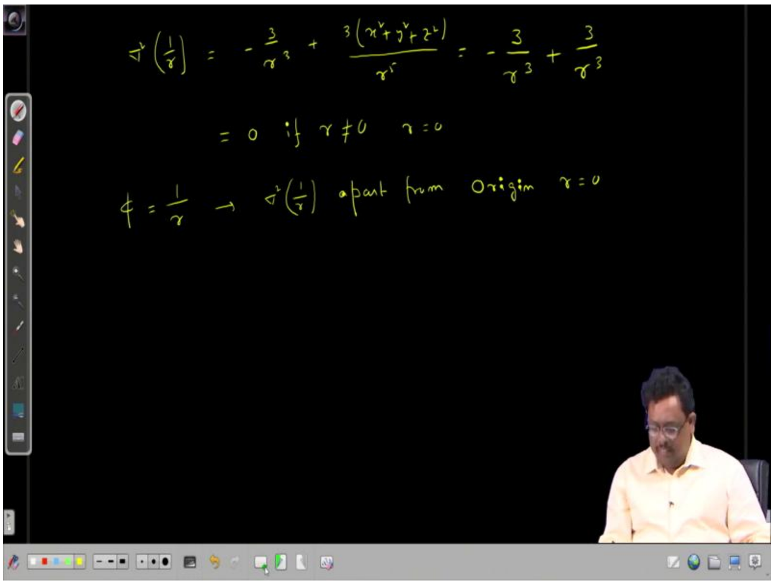

And then finally, interestingly what I get is del square 1 upon r this equals to minus 3 by r cube and then plus if I take common 3 into x square plus y square plus z square divided by r to the power 5 now this equals to r square. So, I will get a nice fine minus 3 by r cube plus 3 by r cube. Now, you see the interesting part is this equals to 0 if r not equal to 0 else when r equal to 0 this function goes to infinity.

So, its a singular function. So, what I get? I get a nice thing that this function $\phi=\frac{1}{r}$ it satisfy $\nabla^2\frac{1}{r}$ everywhere apart from origin; that means, at r equal to 0. So, we can say that this is basically a Laplacian everywhere apart from the origin and then it basically its at that particular point of time it is a singular function ok. Now, this is a very powerful function for us why it is so, powerful let us try to figure out.

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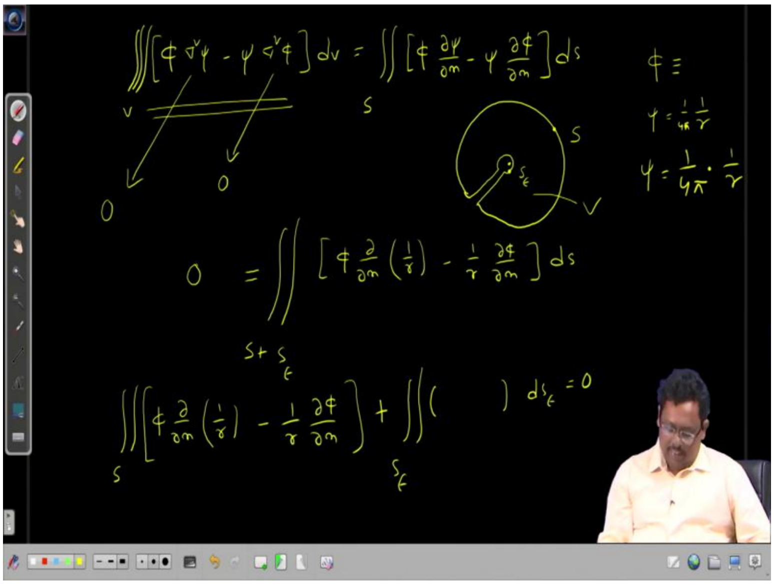

Now, again as I said I am going to use that particular boundary you know boundary integral method like that greens identity which is if I apply over a $\iiint\left[\phi\nabla^2\psi-\psi\nabla^2\phi\right]\partial v$. Now it become a surface integral as you know it is $\iint_s\left[\phi\frac{\partial\psi}{\partial n}-\psi\frac{\partial\phi}{\partial n}\right]\partial s$. So, this is my result.

Now, what I do is, let us find out a domain a circular domain like this and let us take a origin over here and then I just cut this origin with a small circle and then I make this two surface one is. So, this is my volume v, this is my volume v and this surface outer surface call it s and inner surface let us call this a s eψlon which is very very tiny very small surface right which basically only capturing the just deleting that singular point.

Now, if I take the φ is basically my velocity potential and if I take ψ equal to that function 1 upon r. So, then ψ also satisfy the Laplace equation because I am cut out this one I cut. So, in this domain. So, this entire thing this del square φ should be go to 0 and then del square ψ also 0. So, entire left hand side is 0.

So, it is 0 and then in the right hand side what we have is that we have a $\iint_s\left[\phi\frac{\partial}{\partial n}\left[\frac{1}{r}\right]-\left[\frac{1}{r}\right]\frac{\partial\phi}{\partial n}\right]\partial s$. In fact, I can call d s plus d eψlon right and then what I could get is I can find out this s into φ del del n of 1 upon r ok. Let us take instead of φ let us say 1 upon 4 pi r ok. So, I just take ψ equal to 1 upon 4 pi into this one ok.

So, this minus 1 upon r del φ del n what I do is. So, this s I split s equal I mean s plus s eψlon. So, then I just add another one it is on s eψlon and the same thing right. So, d s eψlon that should be equal to 0 right. So, I take this in the right hand side.

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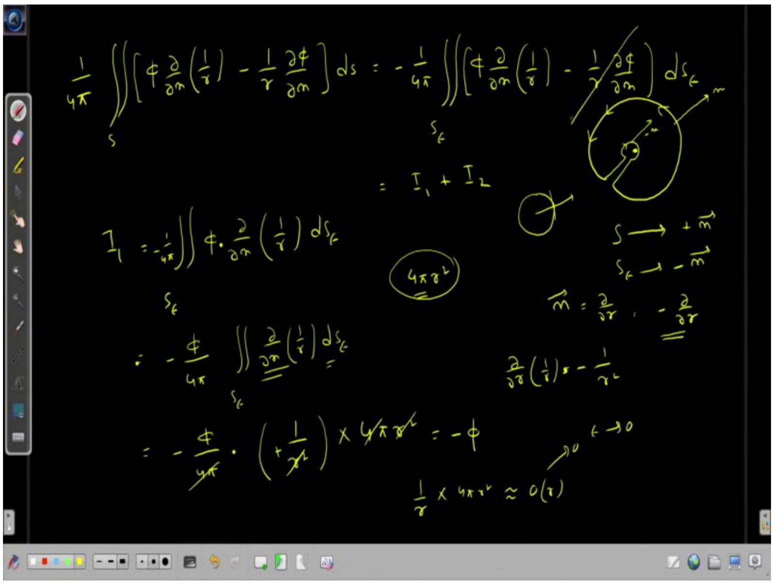

So, then I get s φ into del del n of 1 upon r. So, I just write 1 upon 4 pi here because I just take size 1 upon 4 pi r right minus 1 upon r del φ del n d s is equal to minus 1 upon 4 pi. Now I have over this s eψlon I take this right hand side this φ into del del n of 1 upon r minus 1 upon r into del φ del n into d s eψlon.

Now, remember my domain is circular domain here this one, one important thing to notice if the outer the normal is positive here n. So, it should be the here it should be the minus n y because you can see here the direction got changed. So, here it is coming here and then here it is anticlockwise then here it is clockwise. So, the direction of the n should change. So, if the s in the surface of f which is the positive normal the same normal at eψlon become the negative normal. So, that one thing we need to remember.

Now, here we can say it is basically I 1 plus I 2. Now this I 1 equal to integration of s eψlon φ into del del n of 1 upon r. So, minus 1 upon 4 pi is there of course, d s eψlon. Now since this eψlon is very very small we can assume the whole contribution lying on the centroid. So, I can think that φ is a constant. So, I take out the φ outside. So, I become φ by 4 pi and then this s infinity simply del del n of 1 upon r into d eψlon.

Now, in case of this small circle the area is nothing but surface area is 4 π into r square right and then del del n of 1 upon r. So, now, it is a circle. So, my normal definitely for outer side if it is del del r, then for the inner circle it is minus del del r right. It is very easy because see this is the radial direction definitely normal to the surface right. So, this inner circle it is minus del del r.

So, I replace this equal to minus 5/4 π now here it is minus del del r. So, it is minus of now del del r of 1 upon r equal to minus 1 upon r square. So, this minus and this minus we cancelled out. So, it is become 1 upon r square and then I µltiply by the surface area which is 4 π r square. So, this r square r square cancelled out 4 π 4 π cancelled out this equals to minus of φ.

Now, this second term I 2 eventually go to 0 why? Because again the linear sense you can check that I have 1 upon r this is my integral if I µltiply by 4 π r square. So, then actually it becomes order of r now this goes to 0 as eψlon goes to 0. So, this is my argument. So, I can omit this I can ignore this I can make 0.

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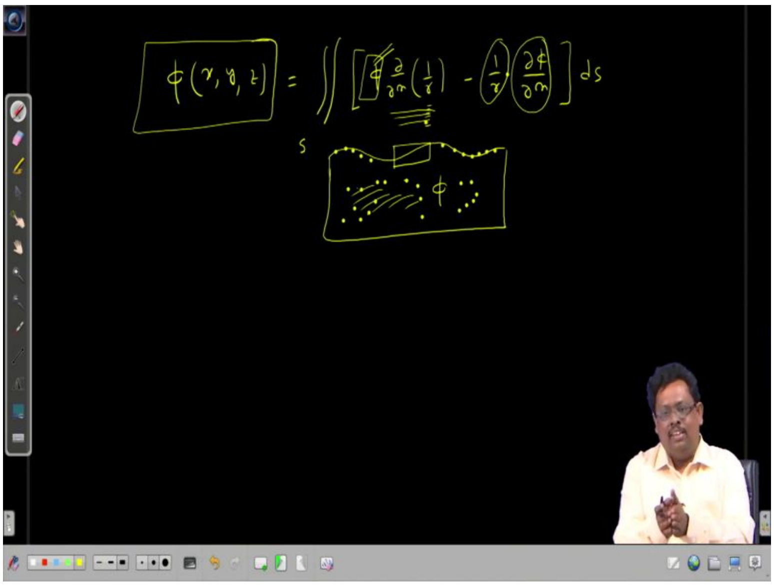

And then I get a very nice expression and my nice expression is as follows I can get this φ at any point x, y, z if I distribute the singularity over the surface and if I know that what is the normal of the singular function 1 upon r minus 1 upon r and what is the normal velocity of this particle at this surface.

Now, we see I can find out the velocity potential for wave velocity potential for anything ok at any point in the fluid domain if I know the velocity potential at this surface and the normal potential also at that surface. See its a very interesting result you see I just take a domain this is my fluid domain as you know that I draw it and then you have a surface and you have a body over here now everything. So, here entire domain is φ.

So, I can find out pressure at any point inside this fluid domain, if I know the velocity potential lying on the surface and the normal velocity potential lying on the surface with the help of the singular function 1 upon r and one more thing you see here that this 1 upon r it is classically known as source and sorry 1 upon I classically known as source. So, del del n 1 upon r is known as dipole.

So, we can say this is a source dipole distribution. See coming back to again this the problem that is related to the flow past a cylinder there you put a dipole at the centroid. Now here we have to distribute the source we have to distribute the dipole over the surface. So, therefore, I can see that this φ could be think of a strength of the dipole and then del φ del n can be thought of the strength of the source see.

So, you see everything is inter related for that two dimensional flow past a cylinder everything is fine, but here also the same concept I am applying to solve the three dimensional problem ok. So, now, comes back here. So, this is basically the classical boundary integral equation that we are going to solve in coming days ok. So, with this.

Thank you.