Numerical Ship and Offshore Hydrodynamics
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Lecture - 17
Lower Order Panel Method (Contd.)

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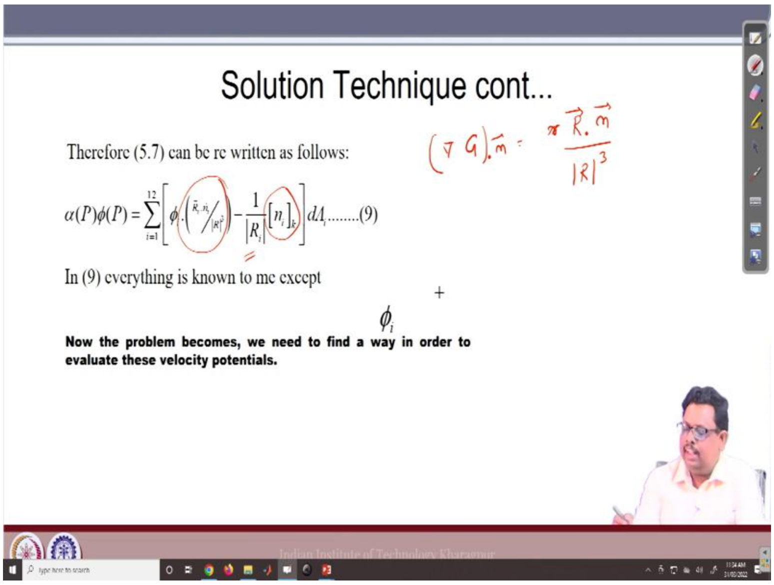

Hello. So, in the last class we are here. So, till this point we have found out the so, many things first one is that now I know the value of you know $(\nabla G).\vec{n}$ which is nothing but. So, here it is capital R ok. So, let me write in the way it is written $\frac{R.n}{|R|^3}$. So, this is my $(\nabla G).\vec{n}$. So, it is here and also I know that at n^{th} panel.

So, that normal is the component normal component is n k and also I know it is that my Green's function $\frac{1}{R_i}$. So, I know everything apart as I said I know everything apart from the value of ϕ_i ok. So, now, let us see that how I can get the value for ϕ_i.

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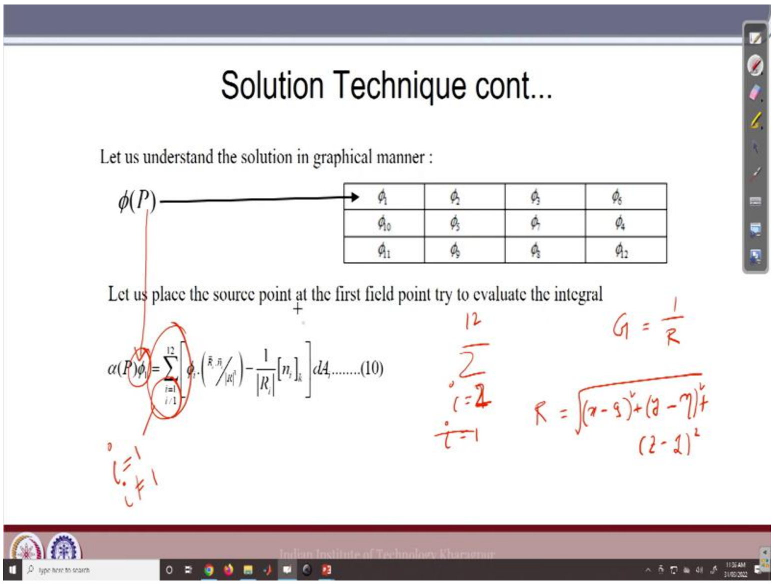

Now you see we understand that ϕ be the any point on the fluid domain. If ϕ be the any point on the fluid domain, then ϕ can be point on the surface also. So, this ϕ could be ϕ_1 also right. So, therefore, what I do here that I substitute that ϕ in ϕ_1. Now if I substitute this ϕ in ϕ_1 then what is going to happen? Now if you look at this equation number 10 you can see that I replace this ϕ as ϕ_1.

So, I writing this part as ϕ_1 and then what I do is here that I just I take that summation that now it is may be difficult for I do not know like I said that i equal to 1 and then i not equal to 1. So, it means that *i* this here now this *i* is runs from of course, *i* =1 to 12, but it will not run when *i* = 1. So, it actually started from *i* 2 to 12. Now why I said that at this particular moment that *i* is not equal to 1 the reason is that if you consider our Green's function G = $\frac{1}{R}$ and this R is nothing but that $\sqrt{(x-\xi)^2+(y-\eta)^2+(z-\zeta)^2}$. Now, if you put it ϕ =1 then what is going to happen?

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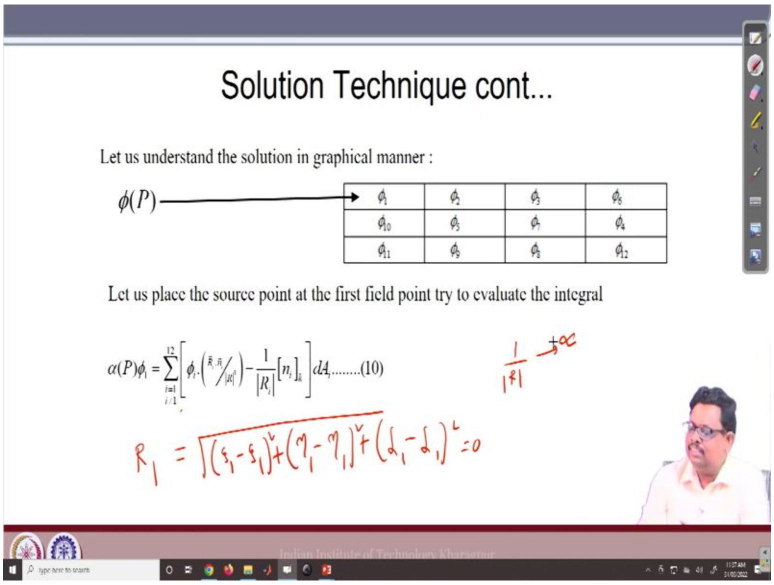

So, then in case of this the panel 1 what is going to happen as follows that you can have at this particular time the definition of R $_1$ let us say this is nothing but square root of $\sqrt{(\xi_1-\xi_1)^2+(\eta_1-\eta_1)^2+(\zeta_1-\zeta_1)^2}$. So, it is equals to 0. So, therefore, $\frac{1}{R_i}$ mean this one is goes to infinity. So, we can have a singularity.

So, therefore, we really cannot afford this practically speaking we actually we do not cut i = 1 normally in you know all the standard software they do not say that from i = 1 if the you can now here we have a name for this point P let us now use this names this point P is called the field point.

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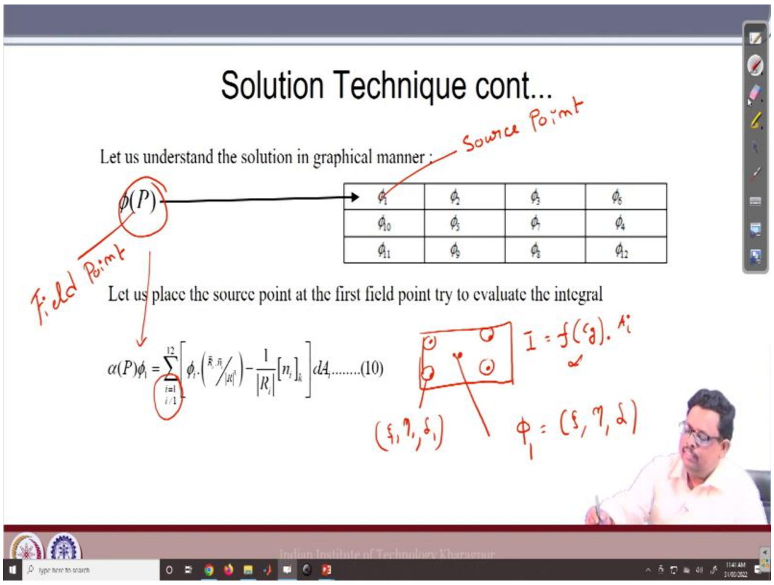

And this point on this body we call the source point. So, now, here normally when we call this summation for the first point ϕ_1 and we omitting that i equal to first panel because at that case what is happening field point is equals to the source point. Now in field point now here this is the critical issue that is coming from lower order panel method that eventually you are going to deal with how to come out from this sort of challenges.

Now, here I put ϕ_1 at the centroid. So, therefore, ϕ_p also actually at the centroid. So, therefore, now you are using the 1 point Gauss quadrature right. So, now, if you do this 1 point Gauss quadrature the meaning is this. So, for 1 point Gauss quadrature rule. So, we have this problem let us see what. Now what is 1 point Gauss quadrature rule?

Suppose you have a panel. So, suppose you have this panel ok and then here you know this is your centroid. So, what 1 point Gauss quadrature rule says that you take the functional value at the centroid and then you multiply this functional value with the area. Now in our case; in our case this functional value become infinity. So, we really cannot go with this 1 point Gauss.

However, if you do the you know that 4 point Gauss quad or 4 point Gauss quad in fact, then you can say some other four points. So, these are your some we can say this is your

ξ_1 , η_1 , ζ_1 and so, on. So, all these 4 point is the different than your centroid. So, your centroid that ϕ_1 the value of $\phi_1 = (\xi, \eta, \zeta)$.

And; however, this if you use the 4 point Gauss quadrature rule, the source point there are four source point in that case and their location is ξ_1, η_1, ζ_1, ξ_2, η_2, ζ_2 and so on. So, really you can have very large value of course, but it is not infinity. So, you can actually do the integration. So, anyway, but we are going with this 1 point Gauss quadrature rule.

Because you know the important part to understand the concept and let us not go to the integral complexity of course, once we understand the concept very clearly remaining thing is we can deal with later on. So, here that is the idea I am not considering when the source point equal to the field point ok. So, then I use the 1 point Gauss quadrature rule now it is very easy.

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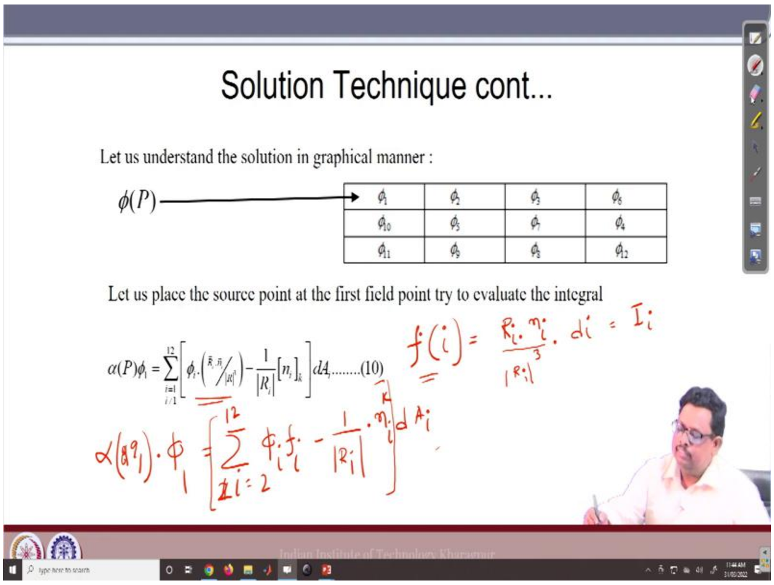

My functional value what is the functional value here? My functional value here is nothing, but f(i) equal to that what is the distance from the source point to the field point that is R_i, then I take a dot product with the normal to that particular point and I divided this by the distance cube I mean that mod of distance cube which is $(R_i)^{3/2}$ So, this is my functional value.

So, if I multiply by the area of this particular panel. So, I can get the integral value I of the i^{th} panel fine. So, therefore, you see I can write this as remember I am doing for the first panel. So, I can write is $\alpha\left(P\right)$ now I can write alpha is the first point. So, I can write at this you can write as a you know q_1 alpha at the first point and why q is meant for the source point 1 q_1 into ϕ_1 then this should be equals to $\sum_{i=2}^{12}$.

So, I can say that i = 2 to 12 and then I can write this as ϕ_i and I can just simply call as $f_i - \frac{1}{|R_i|}$ into now I can simply k, I just take the top. So, n ith panel the heave mode normal and multiply this by dA_i right. So, this way is fine. $\alpha(q_1)\phi_1 = \left[\sum_{i=2}^{12}\phi_i . f_i - \frac{1}{|R_i|}.n_i^k\right]dA_i$

So, now, I just writing in the summation form the whole thing right I write everything in the summation form.

Now, if I break this summation from ok if I break this I mean writing a_1, a_2, $a_3$etcetera then what I get?

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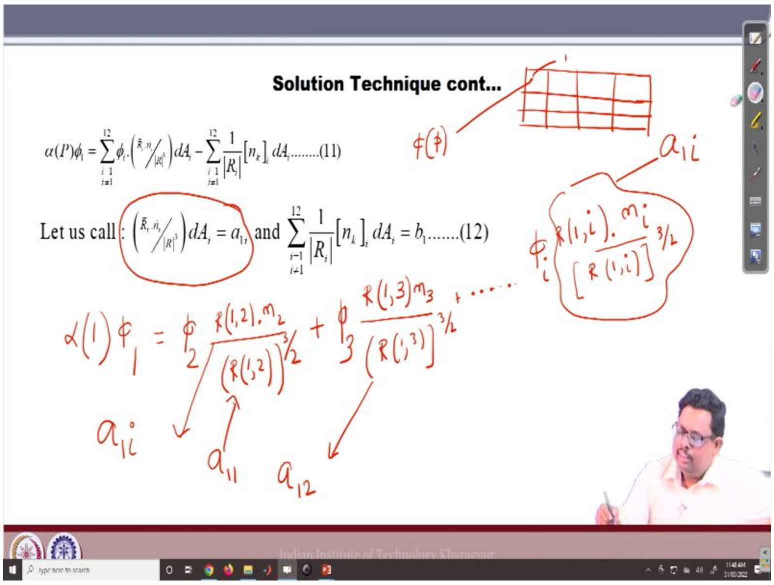

So, this is the written in the summation form right. Now if you write it in basically now you call this value as a_{1i} .Now what is the meaning of a 1i? So, here that each point you need to understand ok. So, let me explain again that what is called a 1i here. Now, I am dealing with here just let me draw this it is the 12 panels. Now I am going with this the first panel right.

So, I put my source point which is $\phi(p)$ at the first point remember. So, since I put it in the first point. So, now, if I break this equation, how will I write? Let me do that. So, I can take α and instead of the first panel I just writing α_1 and then I write it is ϕ_1 then what I get? Now, I get it is now it is going with ϕ_2. So, I just write ϕ_2 and then I said R and then I write it is distance between the first panel to the second panel. So, first panel to the second panel.

So, I just write R 1, 2 multiply it is it should be normal at the second point. So, it is n 2. So, it is normal to the second panel divided by now this R it is again the distance from first panel to second panel now it is mod right. If I take out the mod it should be 3 by 2. Now this whole thing you can what you can see this whole part? You see that everything based on the first panel.

Now similarly if I write for the third panel. So, it is ϕ_3 and then we have R which is distance from the first panel to third panel and then you have multiply the normal for the third panel and divided by it is the distance from first panel to the third panel and then whole to the power 3 by 2. Now, dot dot dot what happened to the like some arbitrary ith panel? What will happen? It should be the ϕ_i and then R is from the first panel to the ith panel the distance multiply by the normal to the i^{th} panel right and then divided by R 1 panel to the i^{th} panel and then to the power to the power 3 by 2.

$$\alpha(1)\phi_1 = \phi_2 \frac{R(1,2).n_2}{(R(1,2))^{3/2}} + \phi_3 \frac{R(1,3).n_3}{(R(1,3))^{3/2}} + ... + \phi_i \frac{R(1,\mathrm{i}).n_i}{(R(1,\mathrm{i}))^{3/2}}$$

Now you see everything here basically the common part is 1. Now, if I write this coefficient this coefficient I can write some coefficient with a common part is 1 and of course, this varying component is i. So, I can call as a 1 i right. So, if I call this a 1 i. So, definitely a_{11} is nothing but this component and this component I can call as a_{12} and then this component this component I can call as a 1i right. So, this is the whole thing actually explain in very short way we can call this as a_{1i}.

So, now we understand that what is the you know notation I am using here when I call this integration I can call as a_{1i} right ok now see the second one. Now in case of a second one you know that here now in case of a first one why I called a 1i? This one because I have one unknown quantity which is ϕ_i, right.

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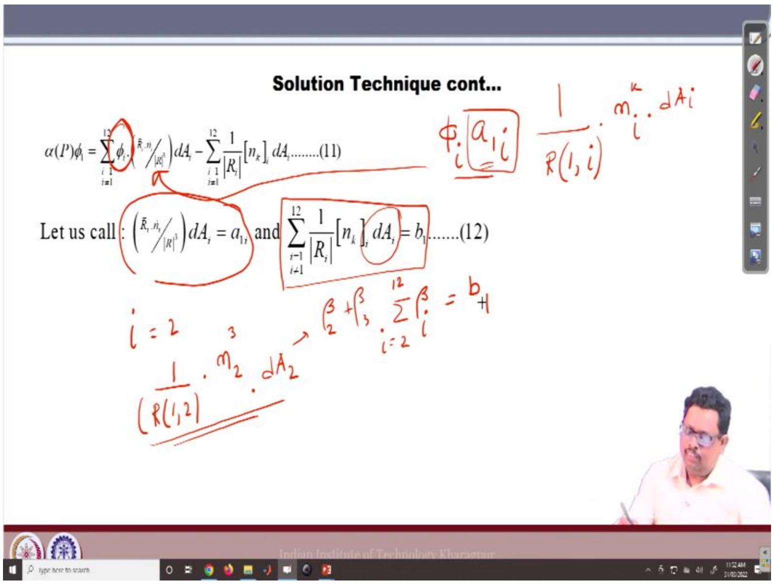

So, therefore, this first coefficient I can write as ϕ_1 or ϕ_i into a_{1i} that is how I can write this term right. I have to use this a_i this coefficient because this part is unknown to me. If this part is known to me then everything comes as a number right; however, here since this is unknown to me I can only have this part; however, the second part all the component are known to me I know the value of R_i. R_i is nothing but the distance from panel 1 to the i^{th} panel. So, I know what is 1 upon R_i and also I know in this panel i what is the normal component of the k^{th} mode that also I know.

Let us say heave. So, it is i^{th} panel heave mode normal. So, that also known to me and also I know what is the area of that particular panel. So, that is also known to me. So, all these components are known to me now here this is nothing but your addition. So, that what am I doing? I am adding all the numbers is it not? Because you see here for the let us say for the first panel ok I mean sorry its runs from 2 to 12.

So, when let us say it is i is equal to 2. So, what is the value? Value is 1 divided by distance between the first panel to the second panel. So, this is known to me and then multiply by the normal comma let us say heave. So, heave component for the second panel that is also known to me and then I have to multiply this with the area of the second panel that is also known to me.

So, this value gives me some number. So, I can call this number is let us say my β_2. So, for in case of a third panel I can get this everything is again is known to me and I can call that as β_3. So, 1 if I now if I keep adding everything. So, therefore, if I keep adding everything. So, i is equal to 2 to 12 β_i and I call this as b again it is 1 why this 1? Because this is I am doing it when I put my field point at the first panel.

So, in all equations, when I put my point in that field point into the first panel, everything is respect to the first panel. So, that is why I called this as b_1. Now I hope now after explaining this you know this equation 12 is very much clear to you. So, only thing I think I missed when I discussed the coefficient I miss this we need to multiplied by the panel area also right ok. So, now I understand that what is this right all these things.

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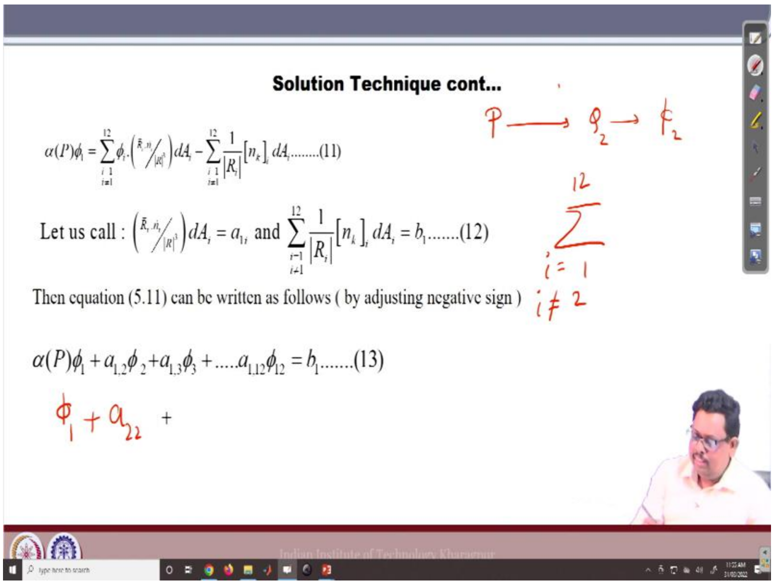

So, then this equation this form this $\alpha(P)$ ... this one this equation 11. Now if you adjust the sign now these things if you take this sign inside the integral component inside the b only take it ok. Now and also this inside the a_{1i} you can adjust the minus sign because I need everything in classical way like $a_{1.1} + a_{2.2} + a_{3.3}$ ...is a linear equations. So, everything is the additive term right.

So, therefore, now I know you see that with respect to the first panel I am able to write one algebraic equation with 12 number of variable which is the variables are ϕ_1 ϕ_2 ϕ_3 .... ϕ_{12} right. Now if I practice this with the second panel now in the next case what I can do is I take this P point that field point into the you know Q 2; that means, where I define this ϕ_2.

Then how I write the equation? So, now, in this case this integration equation should be i=1 to 12 and then $i \neq 2$. See I am telling you again it is very simply simplest way I am doing it ok. So, in reality really we do not ignore the second panel ok, but to get the concept and suppose let us again in you know we try to write a MATLAB code for this particular problem also that time we are going with this maybe it is not close to the reality may be the solution may not be as good as expected, but; however, it will give you some kind of feeling how I write a panel method code ok.

So, now, here I write $i \neq 2$ and I do everything. So, then actually here we can get a second equation as follows some in ϕ_1 now everything with respect to 2. So, a 22 and in case of a 22 ignore it. So, therefore, what will happen that in case of here I just simply replace by you know it is here.

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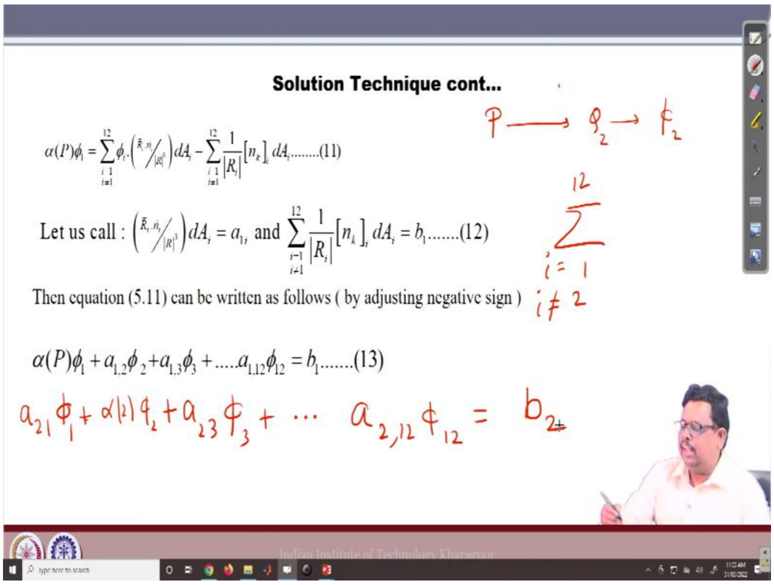

I write a $\phi_2 a_{12}.\phi_1 + \alpha(2)\phi_2 + a_{23}\phi_3$ in that way $a_{1,12}.\phi_{12}$ and this equals to your b $_2$. So, I write for 3, I write for a 4, I write for 5, I write for 6 right ok. So, now see now you see that how I convert that integral equation into the set of algebraic equation right.

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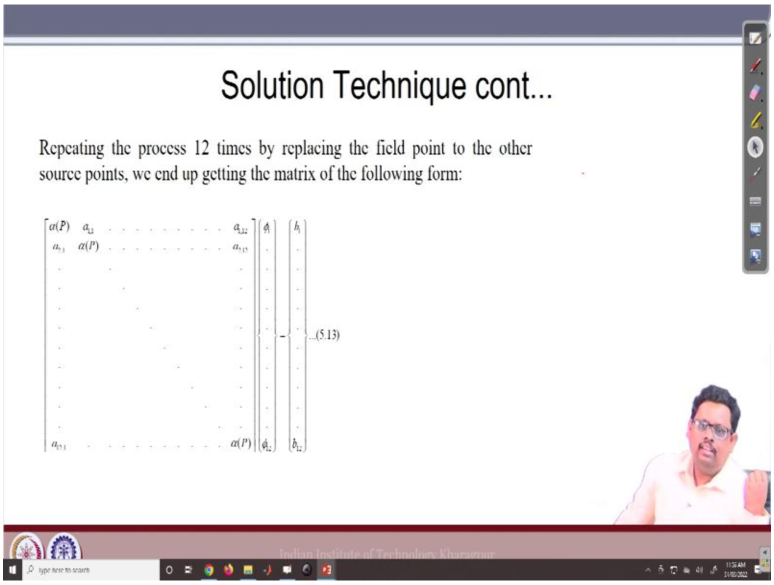

Now if I do this then finally, I am getting this systematic linear equation right. Now you know it is in MATLAB its a 1 liner you use a function the solve you can solve this equation and you can get all this value for $\phi_1\phi_2\phi_3$ and ϕ_{12} right. So, this is the way you know I will tell you rather I would like to tell you that this is the crude way or simplest way to find out the solution for the radiation problem. However, if you make this panel really is finer or smaller then I hope you will get a more or less reasonable answer for this ok fine.

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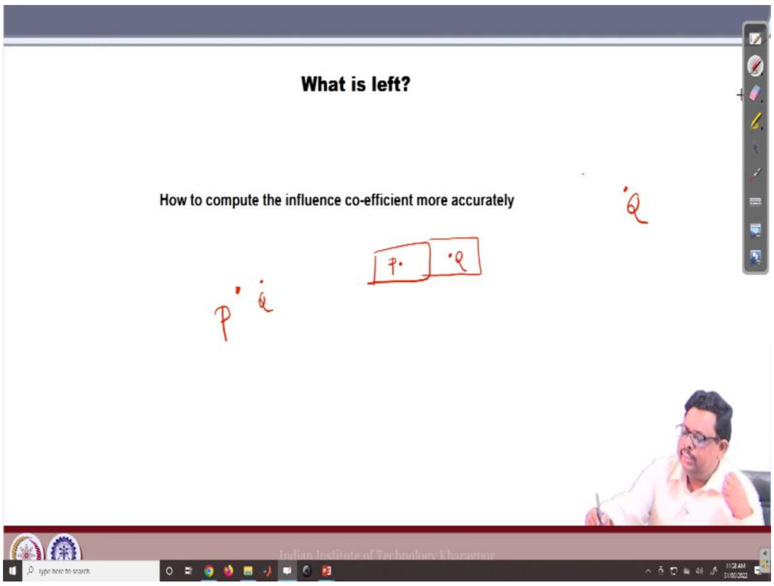

So, now; however, you know in today we really do not discuss how to compute the influence coefficient more accurately ok. As I said that we are going with the 1 point Gauss quadrature rule mostly we are going for 2 point Gauss quadrature rule or 3 point Gauss quadrature rule or 4 point Gauss quadrature rule depending on the situation how the source point and the field point are you know the orientation of these two point. For example, like if your field point is here this P and the source point is really away Q then we can use you know 2 point Gauss quadrature sometime it is 1 point Gauss quadrature also.

However, you know in case of a the P and Q are very close to each other for example, suppose this is your that point P you are putting over here and if you are trying to find out for your neighbouring panel. So, Q is here. So, P and Q is really small. So, in that case you know we normally we do not use that 1 point Gauss quadrature rule or 2 point Gauss quadrature rule depending on the situation we try to we are using normally 3 point Gauss quadrature rule or 4 point Gauss quadrature rule ok.

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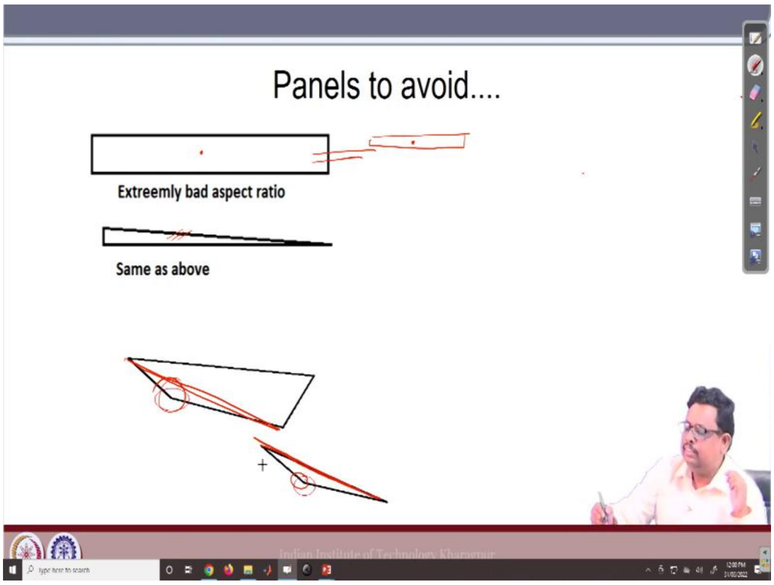

So, those discussions definitely we will discuss in coming days. Now here something that we need to take care like this is something called the aspect ratios. Now, in order to get good results your panel should be moderately good. Now however, you can see that these panels are not good panels because here you can see for this one this aspect ratio is too bad right and here also the aspect ratio is too bad. So, in case of this sort of panel normally here you can now we are assuming this ϕ is constant over at the centroid.

Now, if it is too elongated it is too elongated this type of panel, then we can assume that this approximation of ϕ is like constant over a panel it might not work very well and if you take this one the angle is so, large sometimes what is happening even its a quadrilateral panel even this is a quadrilateral panel sometimes you know in code numerically, you can it is as sense as a triangular panel something like this.

So, getting the normal in this case is sometimes not you know sometimes not coming very accurately. So, better try to avoid such type of panelling and similar for here also I mean maybe if it is you can approximate by some kind of a straight line not a triangle even ok. So, these are the; these are the some bad panels and one needs to avoid such kind of panels.

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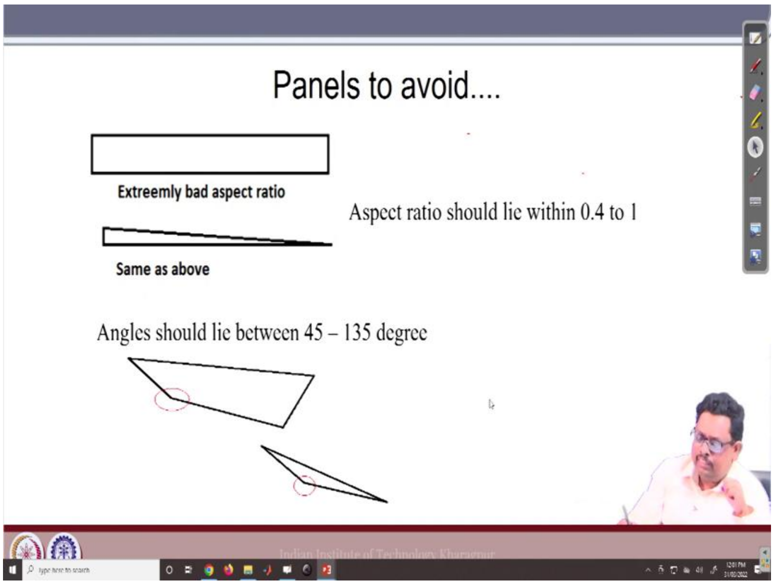

So, then now there are you know this is a thumb rule like the aspect ratio should lie within 0.4 to 1 and then that angle should lie within the 45 to 135 degree. So, these are the thumb rules that if you follow this then you will get better results ok.

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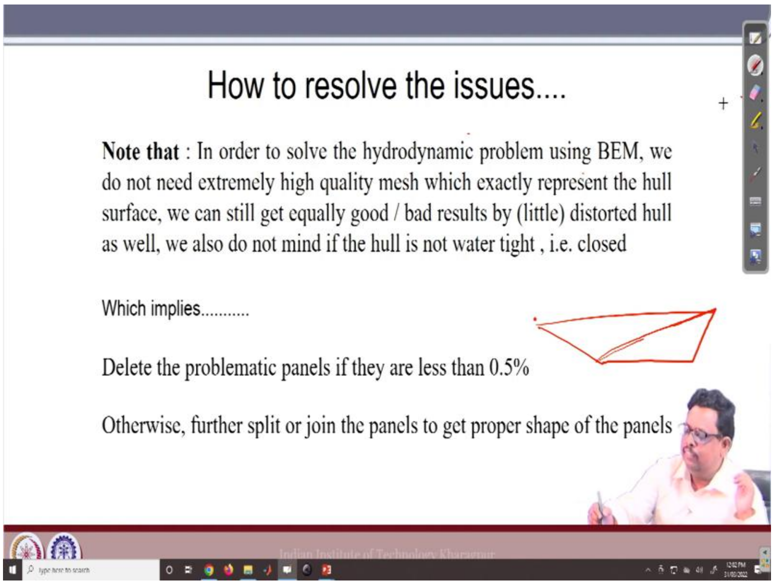

Now suppose in your panelling suppose you are having these issues then how to deal with? Ok. Now, I will tell you that here if you look the basic idea is I am replacing the body by distribution of the source and dipole. So, therefore, actually there is no body is there it is replaced by source and dipole right. So, therefore, we really do not need that exact definition of the hull surface.

So, some small distortion we can accept right. So, therefore, this is you know I can like is from my personal experience I can tell you like if such bad panels are you know less than 0.5 percent like out of 1000 panel suppose 1 panel is like that. So, what we can do is, you can ignore this panel or you can delete that panel or you can again re-mesh it like a bad panel can be you know replaced by 2 good panel like for two good panel means like suppose this panel is bad panel let us say.

But if you join, this becomes 2 triangular panels and we can say that 2 panels are good panels, right. So, that is what I said that you can split it further and so, that you can get better panelling ok. So, this is all for this day and we will start the frequency domain panel method or that how to calculate the coefficients and all everything is more accurately that is going to discuss from the next class onwards ok.

Thank you very much.