Numerical Ship and Offshore Hydrodynamics
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Lecture - 29
Cummins Equation

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Hello welcome to Numerical Ship and Offshore Hydrodynamics. So, today we have Lecture 29 and today we are going to discuss on the Cummins Equation which is popularly known as time domain panel method using impulse response function ok.

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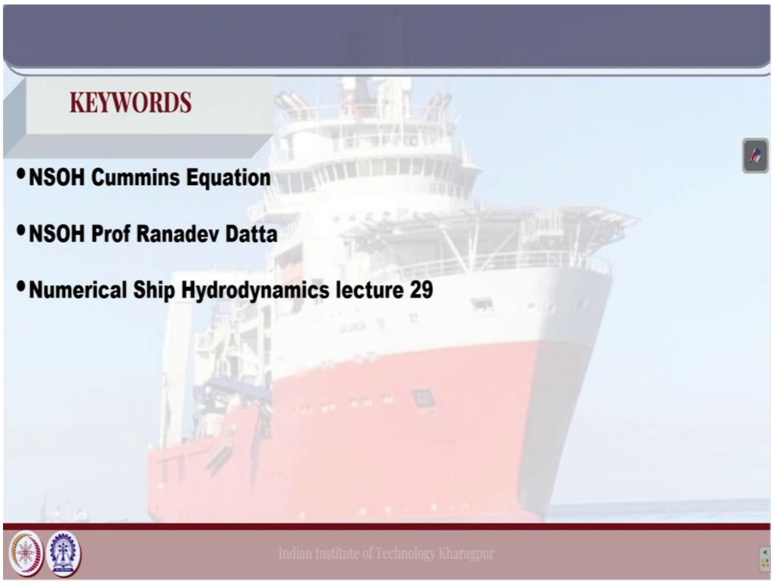

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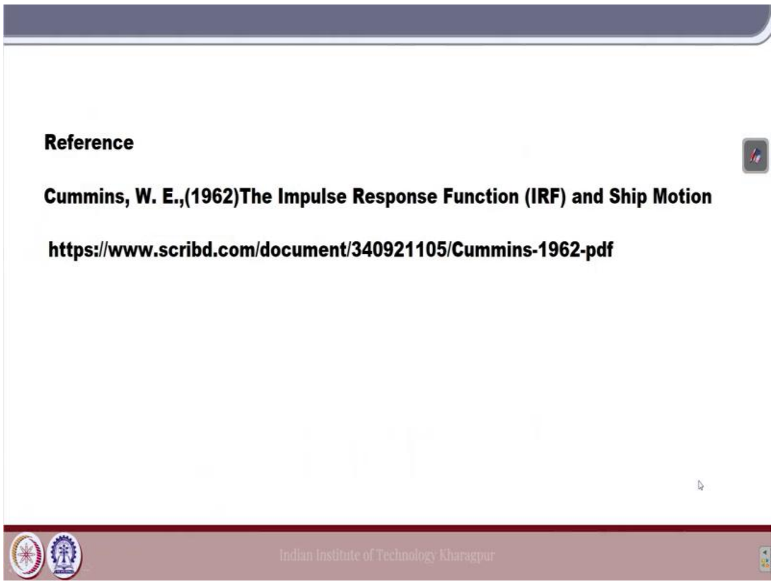

So, to get this lecture we have to use this keywords ok. So, let us go to the topic. Now today the all the discussion I am making from this paper by Cummins 1962 very famous paper and if you look at this below link you can use this link to download that paper ok.

Now in fact, this Cummins equations or the impulse response functions is you know very popular in the domain of electronics they are they use this some impulse response functions to get signals transfer functions. He first realized that concept that is there in the field of electronics should can be incorporated in the case of finding out the response of a ship also ok.

But; however, this paper mostly based on most of the thing based on the mathematical derivation to finding out the solution is consistence or uniqueness everything. But somehow lacking some physical explanation later on Ogilvie in 1864 I guess he actually in from physical aspect he discussed this Cummins equations and till that point. Nowadays it is a very standard equation or standard method to find out the motion of a vessel ok. Without further delay let us see some key aspect of it ok.

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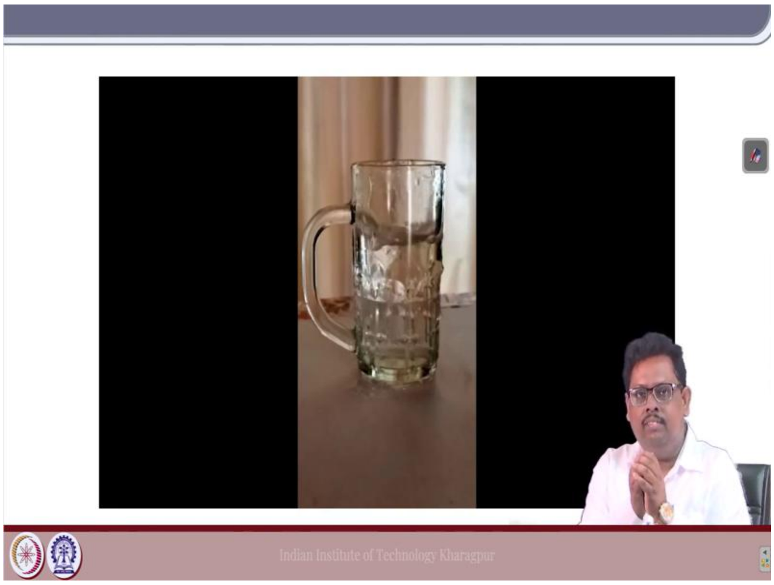

So, now you can see this is a glass and the water and let us see that what is happening if I throw some you know either stone or it is not possible or some ice cube here. It is very nice thing to see now this is coming and then you can see that there is a excitation in the water surface.

Now thing is that remember I throw the ice cube at t equal to 0 let us assume. Now you can see that even if actually now the ice is not moving inside the water it is cool and calm, but if you look at the free surface the free surface actually still you can see a lot of waves are there.

Now, this is a very important you know these are very important understanding that you have to very clear about this. Now the difference between the water in I mean below the free surface and that the water on the free surface the difference is in below that water. If you move a body or if you move a particle we have discussed in infinite fluid domain it then the moment you stop the body then surrounded particle they do not oscillate they stop; however, that is not the same in case of a free surface.

Now, I just throw the ice at t = 0, but I can see the wave is still continue in quite some time. So, it means that it has some kind of you know the what do you say that effect of throwing the ice into the water at t = 0. But still it has some effect at after sometimes it is really do not die down. Now you can see here you know even if after quite of some time it is still oscillating right. Now since you know slowly slowly this you know ice cube is coming out on the free surface.

So, this ice also starts oscillating with respect to this body, but you can see it very slowly slowly actually it is dying down. So, it means that wave it created at t equal to 0 and then this wave will continue for quite some time. So, then you know this is something he can relate this with kind of a memory effect. Now what is that memory effect let us now try to see this and then let us try to discuss the same ok.

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So, remember this is the paper that this is a link you are going to get this paper. Now let us go back to the physical explanation of this phenomenon.

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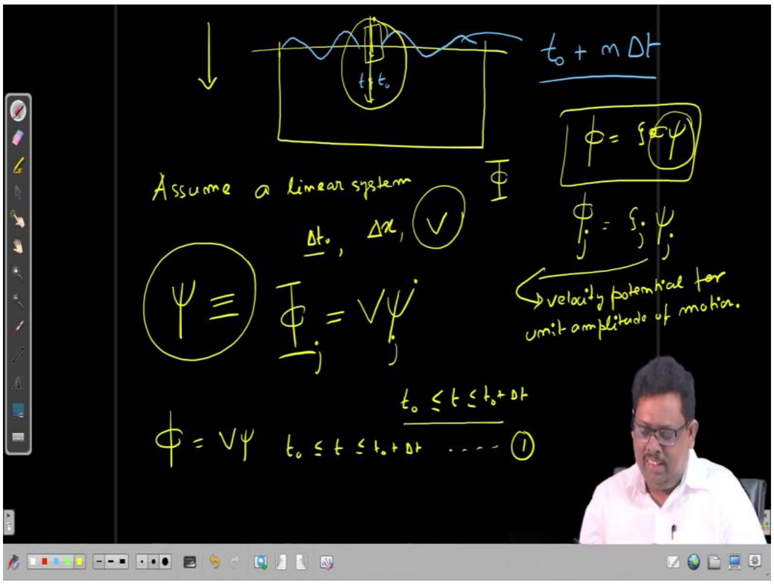

So, now what actually I showed you that a body that or a thing actually suppose this is let us say surface of a ,I mean this is this some tank and then you have some water over here ok. And then if you throw some object then what is happening that you can get some kind of wave. Now what is happening that even if you throw this stone let us say at $t=t_0$, ok and then this waves is continue with quite some time, it is may be some $(t_0 + n\Delta t)$.

So, it is a quite some time it start oscillation right. Now keeping that in mind and we call basically that as a memory effect. Why I am calling this memory effect let us try to understand it. Now keeping mind this fact ok now assume a system a linear system, now assume a linear system ok and assume that this the this the fluid is in visit and the fluid is irrotational homogeneous in incompressible everything.

So, that we can define a potential function we can see the capital ϕ let us say is a potential function. The total potential that when you throw the ice cube or throw a stone inside the water let assume that ok. So, I am doing in with respect to the potential theory ok.

Now if you assume this now let us take at very tiny time let us say Δt_0, okay, then because of this if I throw the stone or ice inside the water what I am essentially doing I am displacing the water particle from it is calm steady mean position right. Now when I do that let us say that I am doing some impulsive displacement which is Δx and then this velocity at the velocity actually I am throwing the stone or the ice cube in this water let us say this a constant velocity V ok.

Now, you know needless to say that this disturbance in the water depending on the velocity. If I put in the in a more velocity right then actually I can get the displacement more, if I throw with the less velocity definitely I can get the less displacement right.

So therefore, let us assume a another velocity potential ψ if you remember that always we define a velocity potential equal to the velocity potential or normalized velocity potential multiplied by the some magnitude. Now that is what we have that is what we have done in case of a you know one mid based solution, remember that time I define this $\psi = V\psi$.

Where this ψ is basically the velocity potential for the unit amplitude of the motion right, this is well known that we have done in our previous lecture. Please get this previous lecture where actually I have defined that my velocity potential $\phi_j = V_j \psi_j$. Where I define this ψ_j is nothing but the velocity potential for unit amplitude of the motion.

Now, having said that we know this concept, so definitely this ξ we can call this a normalized velocity potential ok. So therefore, if it is a normalized velocity potential then this total velocity potential ϕ can be equal to this v which is the magnitude at which I am throwing the object multiply by this normalized velocity potential ξ right. Now if you have the six mode then we can say that $V_j = \phi_j = v.\psi_j$. But when it is when it is when this your time is $t_0 \le t \le t_0 + \Delta t$.

So, I am taking a very small interval from t_0 to $t_0 + \Delta t$ and in this small interval actually I am throwing the stone or anything any object right and then actually I get some disturbance and this disturbance I am telling this disturbance is proportional to the velocity. So, if you add more velocity the disturbance is more. So, then and if ψ_j be the you can say the velocity potential for unit amplitude of the velocity, then and if ϕ_j be the total velocity potential then $\phi_j = v.\psi_j$ right.

So, then actually I come up with a equation $v.\psi$ or let us take a single degrees of freedom, let us not take this all the multi degrees of freedom motion let us go for a single. So, that is why I can drop the index j simply I can write phi equal to $v.\psi$ where at which time when $t_0 \le t \le t_0 + \Delta t$. So, this is let us take this is my equation number 1, ok clear.

Now, the fun part is that when I show you the video even if you drop it, but then you can see the wave is continue.

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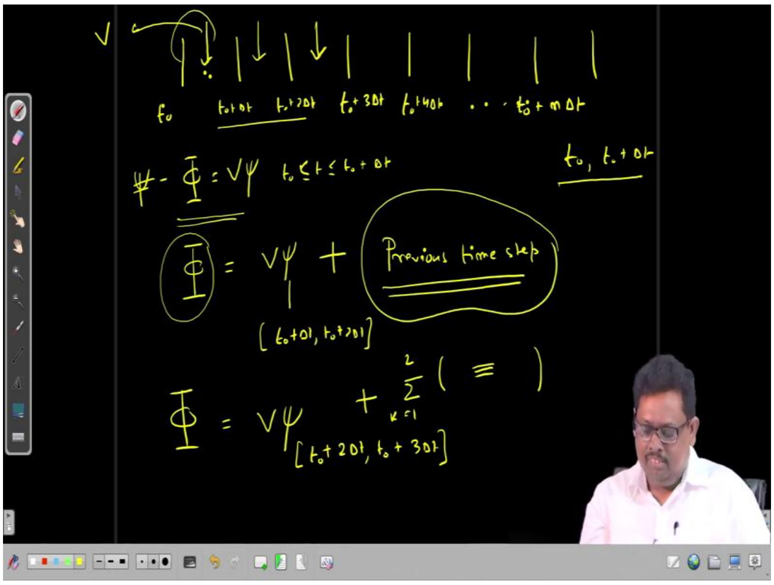

Now, if you so let us take assume that at each let us now take a block here you know let us take some blocks ok. Now let us take it is t_0 now it is definitely $t_0 + \Delta t$ then it is $t_0 + 2\Delta t$, then this block is $t_0 + 3\Delta t$ then maybe it is $t_0 + 4\Delta t$ and then so on. It is let us say you know $t_0 + n\Delta t$ let us take.

Now, what is happening, suppose I am throwing this velocity V now this velocity V, I am throwing throughout, ok. So, then here definitely I can get my ψ sorry my $\phi = v.\psi$ and that is I am doing in between $t_0 \le t \le t_0 + \Delta t$.

Now let us let us assume that again I am throwing a the body or at any object in between the time $t_0 + t_0\Delta t$. Now when I am doing it what is going to happen? See here that definitely there is another I can see that in this time frame also I have let us take this velocity potential ϕ this equal to not only this $v.\psi$ for that particular interval of time. So, you can say this ψ of it is now it is $t_0 + t_0\Delta t$, $t_0 + 2\Delta t$ right definitely this is like this.

However still this velocity potential is not complete, why? Because I have throw a stone or the same body just you know in the time interval of t_0, $t_0 + \Delta t$. Now if you are referring to my video now you can now understand that wave has effect here also and that effect is called the memory effect ok. Now thing is that I throw again a stone at t_0, $t_0 + \Delta t$ in that time frame.

However that wave is not is the only soul wave, because this also you know this total potential is not because of I throw a stone in the time interval $t_0 + \Delta t$ comma $t_0 + 2\Delta t$, also it has a effect the wave that when I throw a stone in t_0, $t_0 + \Delta t$. That means, here I have 2 effects one is some effect it is this is the primary effect plus some effect which is coming from the previous time step.

Now, how to measure this one right how to measure this one? And similarly if I now let us for the this in this case I can write my total potential ϕ = V in some ψ in this time interval $t_0 + 2\Delta t$, $t_0 + 3\Delta t$ right. So but this is not the total potential right, but this plus I have to add like some k = 1 to 2 because this is my previous to situation I can throw that stone. So, that effect also I need to consider right.

Now, let us try to find out how actually I can get the second part of it. So, first part is clear to me each time I am throwing with a constant velocity vv and because of that I am getting some kind of disturbance and I am calling it that is the phi. I mean the velocity potential is V multiplied by that ψ_j over ψ nothing but the velocity potential of the normalized velocity potential or I can say velocity potential for unit amplitude of the velocity right. But then what is the other part?

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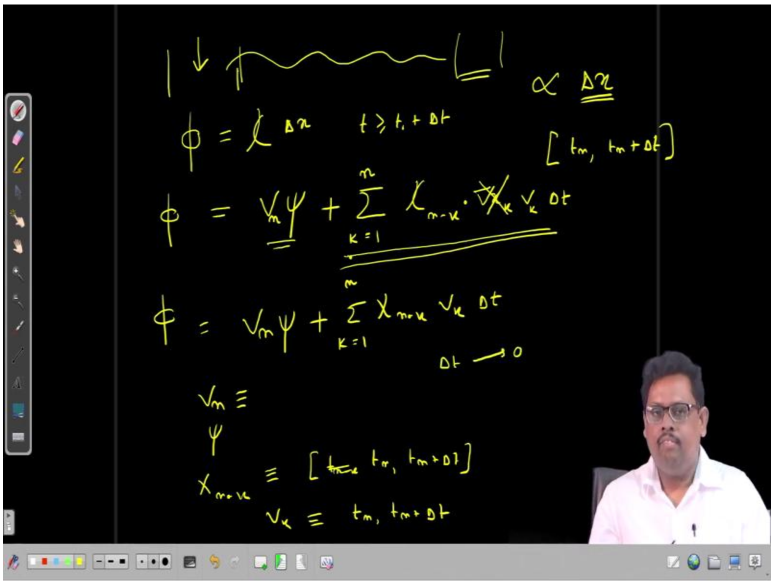

Now, if you think carefully like when actually you have more effect, like if I start you know here I am just excite that body then you know how far you can expect that effect will go right. Now if you look at this my previous video it last for some 5, 6 second I guess. Now when you think that it could last not five six seconds it probably it will end maybe around some 10 11 second or maybe some 15 second. How?

So, the only answer is if I that impulsive displacement is more right. So that means, if I do more impulsive displacement then my memory effect will more. So, then actually in the second part I can assume the ξ, since it is varies us with the you know impulsive displacement which is Δx. So therefore, again I can define a normalized velocity potential X in that case also and I can simply say this ϕ is nothing, but $X\Delta x$ and when this effects last for when t > $t_0 + \Delta t$ let us say, okay?.

So, now actually I am ready to write a general expression. So, then I can write my ϕ you know now let us take a uniform let us take not uniform let us take a you know general time interval at t_n, $t_n + \Delta t$. Let us take a general interval t_n, $t_n + \Delta t$, then now I am interested to write a velocity complete velocity potential because of that. Now if I write this how do I write it?

So, definitely I can write that you know I am throwing some let us say some V at let us take that nth time step and then I multiply with the ψ. So, this is the first thing I would do and then plus I can take the effect it is coming from the big I mean from the start. So, I can write this summation which is $k = 1$ to the n actually the present time-step right.

So, I have to count all the effect of because of this and this nothing but I can write this X we can call it as a (n – k) into multiply by the some impulsive displacement at $dx_k \Delta x_k$ something like this right. And again this now I know that now the displacement equal to velocity multiplied by the time.

So, definitely I can change this to this into some velocity into $V_k \Delta t$ ok. So, this is how actually I can write it ok, so that means, here these things I call it is a memory effect and these things I call that when actually I throw the stone that disturbance is coming from this particular you know I mean expression ok.

So, now this is may be it is let me write it finally, it should be $V_n.\psi$. So, see I am dropping the index ok. So, I am assuming it is a single degree of freedom motion. So therefore I am dropping this j^{th} in this otherwise it should be the j ok k = 1 to n and ($X_{n-k} V_k \Delta t$) ok. Now what is what? So, you can define what is V_n? V_n is nothing but the velocity at the n^{th} time step ξ is the velocity normalized velocity potential for this.

And then what is the X_{n-k}? X_{n-k} is nothing but the velocity potential in for the time interval it is you know t_n minus sorry it is $t_n, t_n + \Delta t$ and then V_k is basically the velocity at in the interval t_n , $t_n + \Delta t$. So, this is the - this is how actually I define the notation is given. So, V_n is the velocity at nth time step X_{n-k} is nothing but the velocity at t_n, $t_n + \Delta t$ times step and V_k is nothing but the velocity at t_n, $t_n + \Delta t$ time instant. Now if you take Δt goes to 0.

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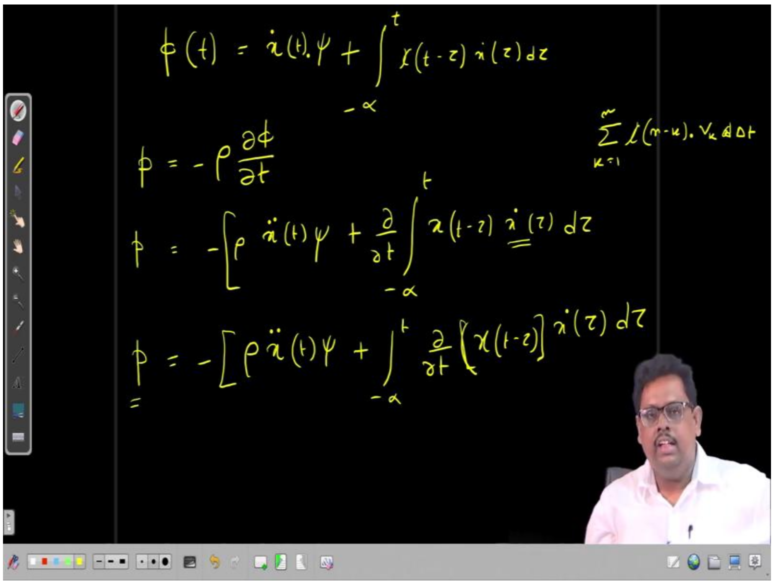

So, now let me write in the differential form. So, if you write in the differential form I can get that ϕ_t which is the present time step is nothing but the velocity at the present time step multiplied by the ψ plus the memory effect. Now if you remember for the discrete case the memory effect start from k =1 right. So, let me write over here that memory effect is summation k = 1 to the previous time step n which is $X_{n-k}V_k\Delta t$ right.

If you remember this now I can write the integral form it is minus infinity, because when you do the integration when it started is $-\infty$ to the present time step n which is t. So, then I replace that X it should be $-\tau$, τ stand for the k ok and then it is $\dot{x}(\tau)$ and integration definitely with that respect to the $(\tau)d\tau$. So, this is how I can write the expression for the velocity potential ϕ. Now when I write the expression for the velocity potential ϕ then how I get the pressure. So, pressure p definitely it is minus $-\rho.\frac{\partial\phi}{\partial t}$ ok.

So, now I have to differentiate the whole thing with respect to t. So, I can get my pressure equal to $-\rho\ddot{x}(\tau)\psi + \frac{\partial}{\partial t}\int_{-\infty}^{t} x(t-\tau)\dot{x}(\tau)d\tau$. Now here τ is not the function of t.

So therefore, further I can write the pressure p = $-\rho\ddot{x}(\tau)\psi + \int_{-\infty}^{t}\frac{\partial}{\partial t}x(t-\tau)\dot{x}(\tau)d\tau$. fine.

So, now if this is the pressure then how I can get the you know the force?

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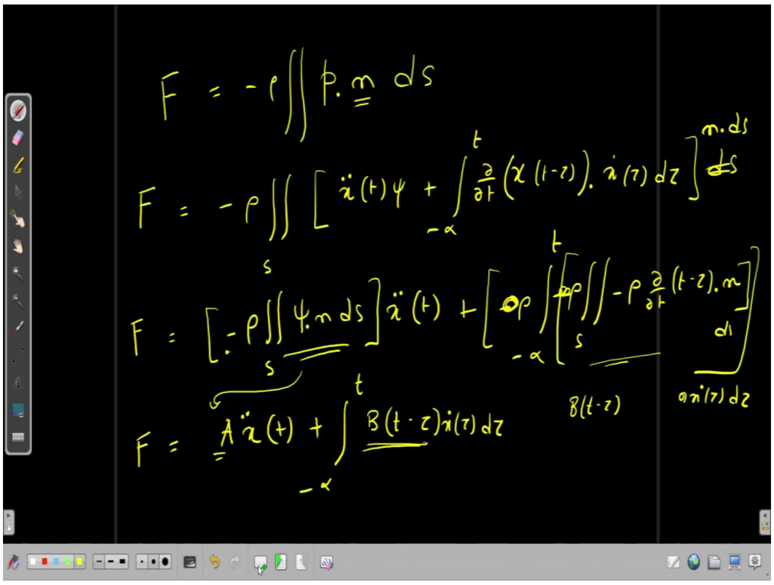

So, now when you look at the force the force is nothing but the pressure multiplied by the normal direction n right. So now, if I try to find out the force in this case. So, it is nothing but $-\rho$. So, integration over the S and I have to substitute my that pressure which is $\ddot{x}(\tau)\psi + \int_{-\infty}^{t} \frac{\partial}{\partial t} x(t-\tau)\dot{x}(\tau)d\tau$ right and then whole ds.

So, now I just need to rearrange this little bit ok. So, let me rearrange as follows this is now let us arrange it $-\rho$ over the s ok and then I just here it should be (n.ds) ok. So, then it is $\psi.nds$, I just multiplied $\ddot{x}(\tau)$ and for the second part what I do is I do it is plus now it is minus over here.

So, I just make it plus and then I just make minus rho into ok let us make it $-\infty$ to t it is $-\rho$ and then I just do the surface integral over s it is minus rho then it is $\frac{\partial}{\partial \tau}$ it is $(t-\tau)n$ right and if whole in ds and then the everything here should be multiplied by the it is $\dot{x}(\tau)d\tau$.

Now, why I am doing it I am doing it because I need to write this term of a added mass multiplied by the acceleration. Now here this term actually I can called as a added mass a ok and similarly if you look at this term the term inside this surface integral I can call this as a you know the damping term, now here I can call this as a you know $B(t-\tau)$ the whole thing ok. So therefore, I can write for the second term is $-\infty$ to t it is $B(t-\tau)\dot{x}(\tau)d\tau$ ok. So therefore, this force I am writing in terms of added mass and the damping.

This term is related to the damping and this term is related to the added mass ok. So, therefore you see this is force is called the radiation force.

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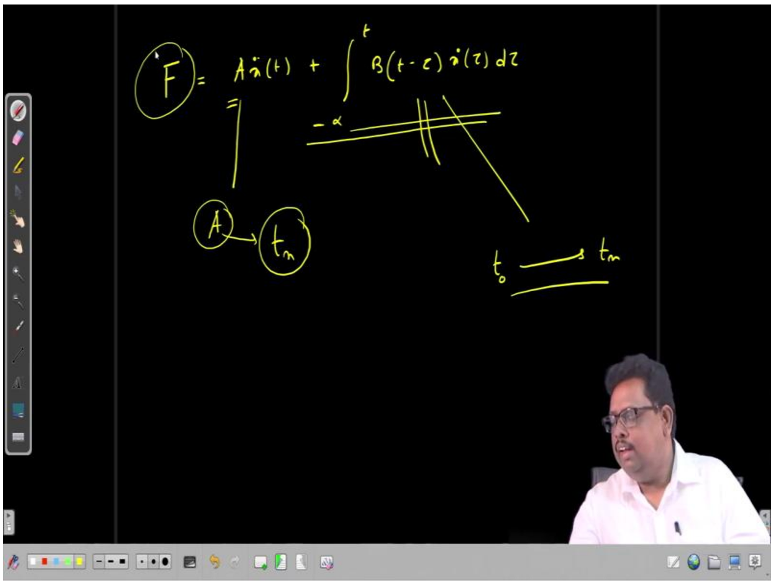

So therefore finally, I could now able to write my radiation force F in term of $A\ddot{x}(t)$ + a memory function which is $\int_{-\infty}^{t} B(t-\tau)\ddot{x}(\tau)d\tau$. Now unlike in general in frequency domain we do not have this memory effect. So, physically speaking this added mass is coming due to the fact that I am throwing the body at each instant of time t and the second component is coming because of the I am throwing this stone at let us say you know t 0 time.

So, consider the memory effect when actually I try to evaluate the motion at the body at some particular time t_n right. So, this these things we call as a memory effect remember I am telling again, here this part let us consider this part this is the added mass this is because actually I am throwing the stone at the time t_n and this will actually the cumulative effect when actually I start throwing the stone from t_0 till the time t_n. So, it is taking all this memory effect.

And then if I add together I get the complete radiation force ok. So, today let us discuss this I stop here, now this is the link and you can get this paper through this link. However, this physical explanation not given in this paper of course, but actually you can get the mathematical insight like if this equation is consistent or not and how to find out the motion how to get the body everything is written in this paper ok. So therefore let us stop today.

Thank you very much.