Numerical Ship and Offshore Hydrodynamics
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Lecture - 30
IRF Based Solution

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. So, today, we are going to discuss on IRF Based Solution ok.

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Now, in IRF based solution if you remember in my last class we have derived one equation of motion, right. Now, these equation motion includes the effect which is called the memory effect. So, we have based on considering this memory effect we have come up a equation of motion. So, today we are going to discuss much on this equation of motion, ok.

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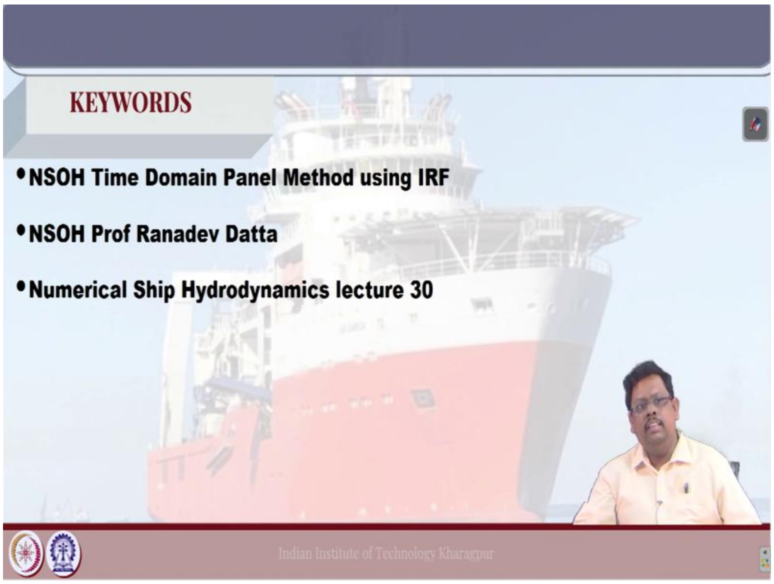

This is the keywords that we are going to use to get this lecture, fine. So, without much delay let us jump into the solution.

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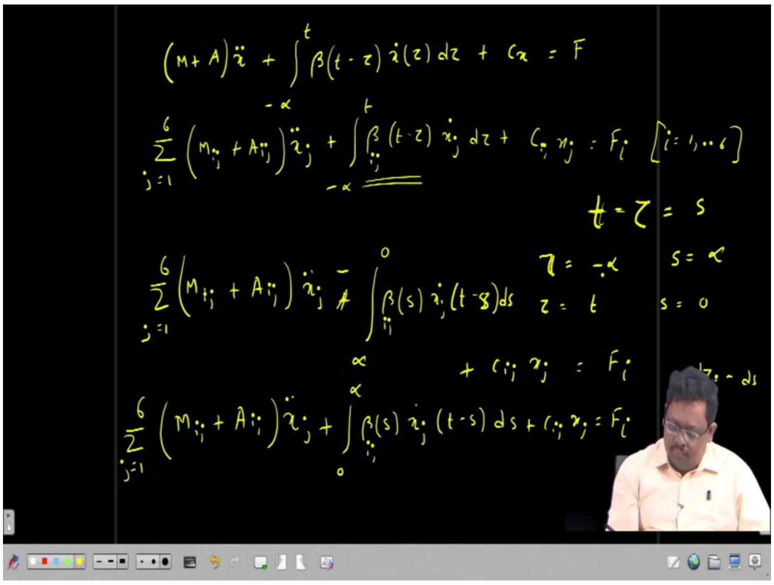

Now, if you remember that in last class we finally, we arrived a equation of motion which may be written in form of; now, again I am dropping all modes of motion considering a single degrees of freedom equation of motion. So, it should be $(M+A)\ddot{x} - \int_{-\infty}^{t} \beta(t-\tau)\dot{x}(\tau)dz + Cx$ = F. So, this is what we got, right.

Now, if we include here the six degrees of freedom equation of motion so, if I include all these degree so, then actually the same thing actually you can write in form of a summation. So, if we write this then it should be like this. So, $\sum_{j=1}^{6}(M_{ij}+A_{ij})\ddot{x}_j+\int_{-\infty}^{t}\beta_{ij}(t-\tau)\dot{x}_j dz+C_{ij}x_j=F_i$, where i also runs from 1 to 6. So, this is the we can say this is the complete equation of motion using impulse response function.

Now, why I called this impulse response function because here this function we can call the impulse function, ok. Now, this equation also we can little bit change the limit $-\infty$ to t. So, now, we can further modify this limit also by right changing the coordinate system like if I take let us take t minus just make it bigger t minus tau let us take s. Now, if you take like tau equal to $-\infty$ so, it becomes s becomes your ∞.

Now, if you take $\tau=t$ then your S becomes 0 right. So, therefore, if you do here we can check that this becomes j = 1 to 6, then $(M_{ij}+A_{ij})\ddot{x}_j$ plus here you can take that if you now change the limit from s to j so, therefore, we can get here it is at t = τ = $-\infty$, s = ∞, it is ∞ to 0 and then beta into $\beta(s)\dot{x}_j$ now it is become τ =(t – s). (t – s) here it is $(t-s)ds+C_{ij}x_j=F_i$, right.

So, and also here $d\tau=-ds$, so, therefore, here instead of plus sign we have the minus sign. Now, what I am going to do is I am going to shift the $-\infty$ to 0 to 0 to infinity. So, if I do that then minus sign become plus sign and then finally, we are having j = 1 to 6 into $(M_{ij}+A_{ij})\ddot{x}_j+\int_{0}^{\infty}\beta(s)\dot{x}_j$ ok, I just miss ij here.

So, I just do $\dot{x}_j(t-s)ds+C_{ij}x_j=F_i$, right, ok. So, after doing this finally, we are having a new equation I mean equation are same I just change the integration limit for my convenience only.

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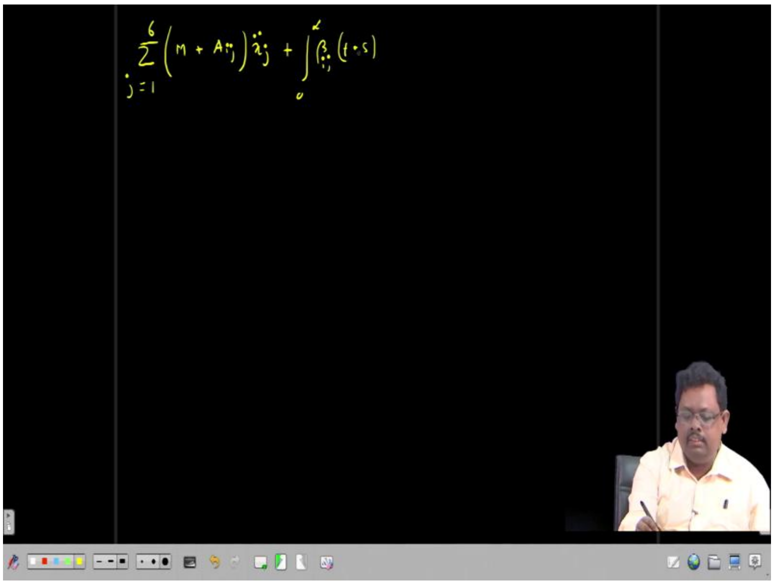

So, then finally, we can get this equation of motion is $\sum_{j=1}^{6} (M + A_{ij})\ddot{x}_j + \int_{+0}^{\infty} \beta_{ij}(t-s)$.

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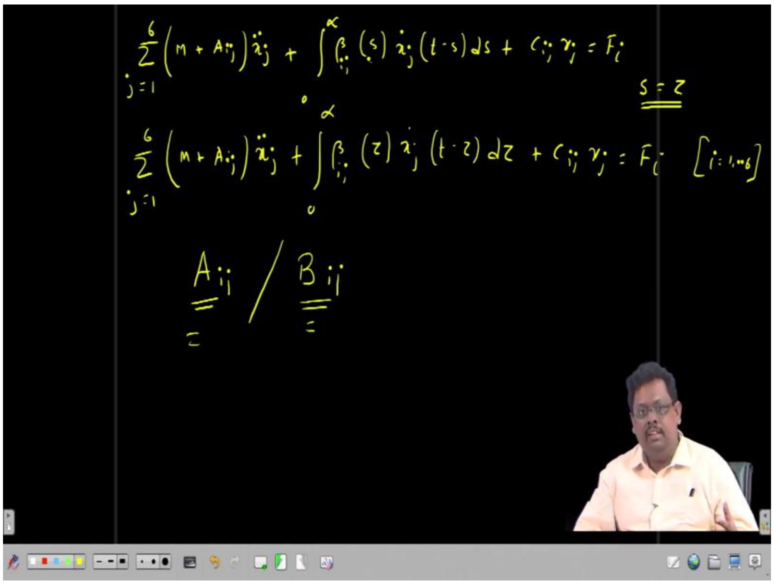

Oh, sorry it is plus integration of 0 to ∞ it is β_{ij} now it is s $\dot{x}_j$ here we have $(t-s)ds + C_{ij}x_j = F_i$. Now, again people really do not want to see here s. So, again you just simply change s= τ and then we can get the same equation as $\sum_{j=1}^{6} (M + A_{ij})\ddot{x}_j + \int_{+0}^{\infty} \beta_{ij}$. Now, here it is $(\tau)\dot{x}_j(t-s)ds + C_{ij}x_j = F_i$ and here of course, i =

1 to 6. Now, this is we call the impulse response function equation of motion using impulse response function.

Now, if you look at the previous day lecture that this getting A_{ij} or getting B_{ij} these are really tough right because we can see in the our last class this expressions of A_{ij} the expression of β_{ij} is really very difficult right. But then what is the advantage of using this impulse response function equation of motion I mean how we can you know what is the advantage and like why we are doing it right we have the frequency domain solution we know how to solve it and then, what is the advantage of using this equation? Right.

So, before we discuss all these things, now let us try to find out how we can get these terms A_{ij} and then B_{ij} easily, right. Now, in order to do this to get this A_{ij} and then B_{ij} we need the solution in the frequency domain. Suppose, I have a frequency domain solver with me.

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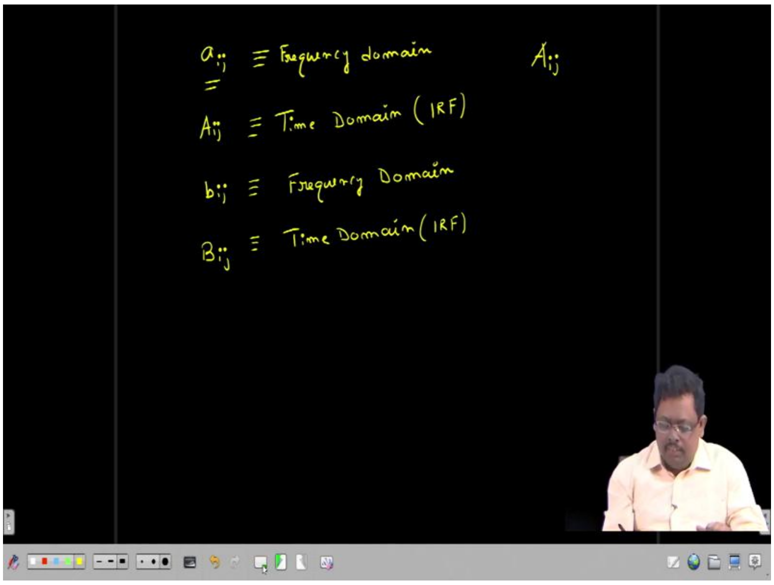

And, then I have some added mass component a_{ij} I mean now remember that when I call a_{ij} it is in frequency domain when you when you use the a_{ij} and when I use the impulse response function A_{ij} I use the A_{ij}. Now, this is it is for the frequency domain. And, then this A_{ij} we use for the time domain or we can call it is with the impulse response function. When I call the b_{ij} it refers again for frequency domain and when we call this or B_{ij} sometimes we called β .

So, to make this more consistent let us call not beta let us call B instead of β. So, when we call the B_{ij} it is again the referring to the time domain and we are trying to say it is based on the IRF based solution. So, we need to understand this nomenclature ok, fine.

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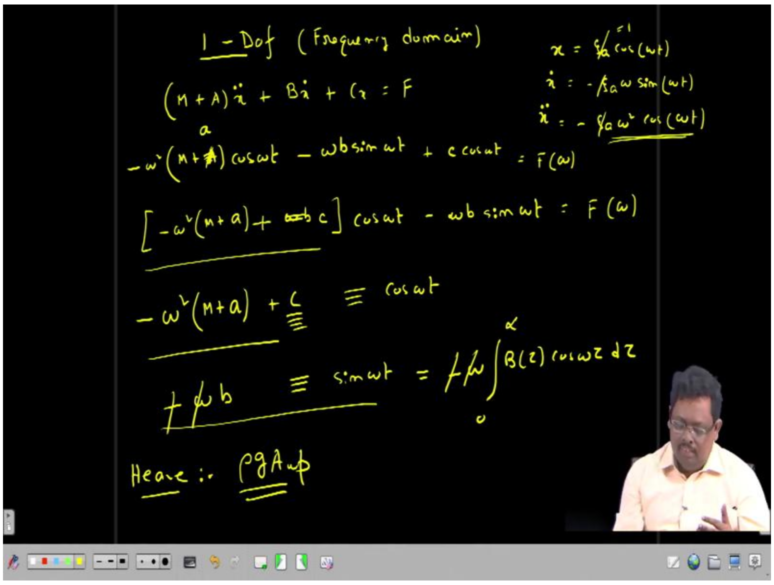

Now, once we understand this let us again take a single degree of freedom of equation of motion. So, I am taking one degree of freedom equation of motion. So, I just write 1 Dof one degree of freedom equation of motion in frequency domain. Then how you know it should look like? Now, if you know very well the general equation of motion is $(M + A)\ddot{x} +$ now here I just since the general equation of motion again I am using capital A, but eventually I make it a small when you use the frequency domain assumptions, ok.

So again, $B\dot{x} + Cx = F$. So, this is the standard format that we are using to get the equation of motion right that from our very first day we discussed this, right. Now, here I am assuming my the motion x is harmonic so, I can make this equal to some amplitude $\xi_a \cos(\omega t)$, ok.

Now, if I do that and then I can make that $\dot{x} = -\xi a \omega \sin(\omega t)$ and then this $\ddot{x}$ again it is $-\xi a \omega^2 \cos(\omega t)$. Now, without loss of generality let us drop this ξ_a and let us make it equal to 1.

I mean it is you know it will not harm in the final solution ok because here we are going to compare this frequency domain solution with the IRF solution. So, for the sake of generality or sake of simplicity I mean we just drop this ξ_a instead of ξ you can put 1. So, but if you want you can still you can keep ξ_a you can get the same equation does not matter.

Now, if I substitute this in this equation then what we get? So, now, this is my this is for my acceleration. So, definitely it is $-\omega^2(M+A)\cos(\omega t)$ right the first term and here if I substitute the x dot it is minus omega. Now, here I must use the small a, right. So, I just cut this and instead of writing the small a, because I am using the frequency domain.

So, instead of B also let me use the small b over here and then it is $\sin(\omega t)$ and then now instead of C I am using c into again $\cos(\omega t)$ and here I just I can write $F(\omega)$ just to say that it is in the frequency domain. So, now, I what I do is now I just compare I mean or not compare actually I am now taking this cos component together sin component together.

Then I can have here it is $-\omega^2 M$, now, I just I need to remember it should be small a right $[-\omega^2(M+a)+C]\cos(\omega t)-\omega b\sin(\omega t)=F(\omega)$, right. So, then I have this term just need to remember I have this term $-\omega^2(M+a)+C$ this is the term which is having with $\cos(\omega t)$ and I have $-\omega b$ and this is the term I have with my $\sin(\omega t)$. So, I need to remember this ok.

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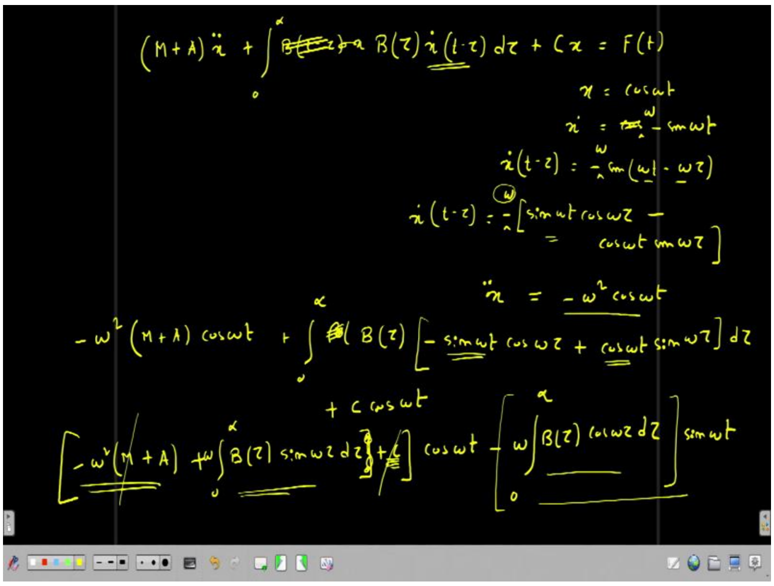

And then I have this impulse response function equation of motion which is M $(M + A)\ddot{x} + \int_{+0}^{\infty} B(\tau)\dot{x}_j (t - s) d(\tau) + Cx = F$. Now, I just use the B because you know $(t-\tau)$ no it is not $(t-\tau)$ in if you use the 0 to infinity it should be $B(\tau)\dot{x}_j(t-s)d(\tau)+Cx=F$. Now, here I can make F(t), not ω.

Again take the same solution we take let us say x = $\cos(\omega t)$, right. So, therefore, $\dot{x}$, I can take, it is $-\sin(\omega t)$. Now, here I have $\dot{x}\,(t-\tau)$ term. So, I am using $\dot{x}\,(t-\tau)$ definitely it is, $-\sin(\omega t - \omega\tau)$. So, therefore, I can again write $\dot{x}\,(t-\tau)$ $= sin(a-b)$.

So, it is $-\sin\omega t\cos\omega\tau$ and then $-\cos\omega t\sin\omega\tau$ right because I am using $sin(a-b)\sin a\cos b - \cos a\sin b$, alright. And, now nevertheless that x is nothing but equal to again it is cos t. So, $\ddot{x} = -\omega^2$ again it is $\cos(\omega t)$. So, now, I have everything with me. Now, I need to substitute the whole thing in this equation of motion, right.

Now, if I substitute whole thing into the equation of motion so, let us see that what actually we can get it is I know this, but a little bit mathematical jack, but at the end we will get very nice thing. Now, it is (M + A) it should be $-\omega^2\cos\omega t$. So, it is $-\omega^2\cos\omega t$. Now, plus 0 to ∞ we have ok all the time I am just writing beta just my habit. So, it should B(τ) not β, B(τ) whatever it is up to you.

Now, again it is $\dot{x}\,(t-\tau)$ I just substitute over here. So, what I get I include this minus sign this should be $\left[-\sin\omega t\cos\omega\tau + \cos\omega t\sin\omega\tau\right]d\tau$, right and then I have finally, I have the $c\cos\omega t$. Now, again here up to this point again I need to separate out my $\cos\omega t$ term and then $\sin\omega t$ term and then we need to compare.

So, now if I take that what is the related to the $\cos\omega t$ term so, here I can get $-\omega^2(M+a)$ and from this second term this is the term actually I am having with my cos. So, therefore, it is $\int_{+0}^{\infty} B(\tau)\sin\omega\tau d\tau$. So, this is the term I am and of course, I should not forget which plus C which is with my $\cos\omega t$ term and then what is left for the $\sin\omega t$ term. So, this is something it is with my $\sin\omega t$.

So, perhaps I am missing here some ω because if you differentiate this so, some ω will come out here. So, this ω will be here also and this ω will be here also. So, therefore, it is $-\omega\int_{+0}^{\infty} B(\tau)$ and then it is $\sin\omega t$ terms it is $\cos\omega\tau\ d\tau$. So, this is the term related to the and then this should be term related to $\sin\omega t$, right.

So, now, if you look at my previous, I have term this one and then if you look at the next time this $\sin\omega t$ term having with this term. Now, I need to; I need to compare this sin omega t term and the $\cos\omega t$ term. Now, if I now here now I know this. So, let us go back previously and then this sin omega t in this case it is minus now if you look at it is $-\omega B(\tau)$ into this term, right.

So, therefore, I just simply write it is this into $-\omega\int_{+0}^{\infty} B(\tau)$ and it is with $\cos\omega\tau\ \ d\tau$. Now, this is the term with related to the you know the damping coefficient right.

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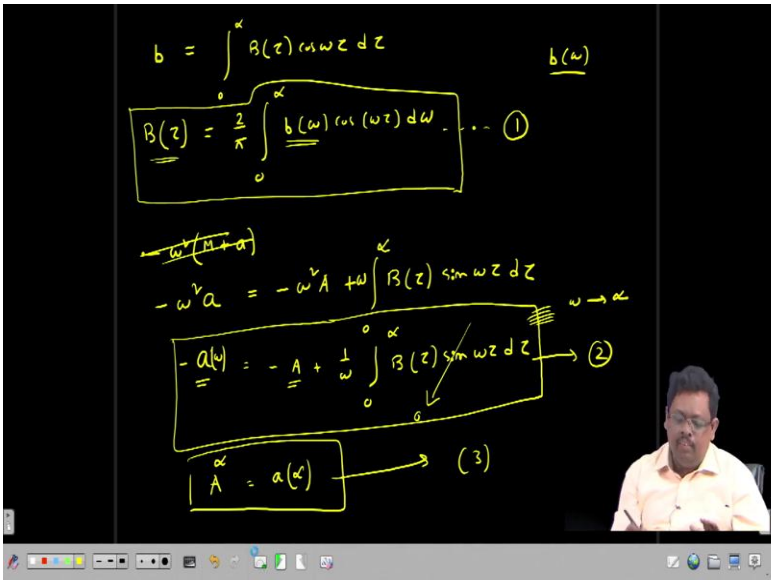

So, now here so, I get one relationship which is small b is equal to now, again if I now here if I just cut out this ω this ω this and this. So, then finally, we have b = 0 to ∞, $B(\tau)\cos\omega\tau\ d\tau$. Now, here I know that if I use this Fourier transformation so,

$$B(\tau)=\frac{2}{\pi}\int_{0}^{\infty} B(\tau)\cos\omega\tau d\tau.$$

Now, you see this is the equation after everything I am getting. So, this is the equation which actually gives you a nice thing a relationship between the frequency domain damping co-efficient v omega with the time domain damping co-efficient $B(\tau)$. Now, therefore, at least from this point I understand it is very difficult to find out the term $B(\tau)$ from that classical that equation that actually we have derived in last class from there getting $B(\tau)$ is really difficult.

However, if you have the knowledge of frequency domain that the damping co-efficient $b\omega$. So, you can using this $b\omega$ you can really get the value for $B(\tau)$, right. So, therefore, from IRF actually one has to understand that you know to get the solution with respect to IRF I must have the frequency domain solution, right. If I do not know the knowledge of $b\omega$ I really cannot use let us say give a equation name let us say 1. So, I cannot really use the equation 1 to get the value for the $B(\tau)$, right.

So, essentially this is now the question comes right like what I asked before that what is the point? I have the solution for $b\omega$ anyway to solving this frequency domain panel method, then why you are doing such a mathematical juggler and come up with something that it turns out to be if I do not have the knowledge of this, then I cannot get the other one. And, then what is the difference? Nothing, it is simply Fourier transformation, right at the end.

Then what is the cache, then why peoples are doing this and using this very commonly and you can see that there lot of work available. For example, right now I can give you a one reference which is ORCAFLEX where we can use this you know IRF based solution to find out many things. Now, definitely it has some use, right.

So, and we are going to of course, we are going to discuss that you know knowing that without the help of $b\omega$ I mean the frequency domain panel method we really cannot do anything within this IRF based solution, but still why we are doing it, right.

Anyway, we discussion that we can discuss the next class, but today let us complete the whole thing. And, then we need to compare the $\cos\omega t$ as well. Now, if I compare the $\cos\omega t$ now if you from here I can get $-\omega^2(M+A)$. So, let me write it. So, it is $-\omega^2(M+A)$ this is I am getting from the frequency domain ok. It should be small a all the time I am making this mistake very sorry, ok.

So, it is $-\omega^2(M+a)$ and of course, you mean probably we do not; we do not write this you know we do not; we do not use this c because anyway because this c is or this c is nothing but the you know the restoring coefficient which is same. If I use the time domain panel method or using IRF or we are using the frequency domain panel method, this c is always a geometric property.

For example, in case of a heave in case of a heave in whether irrespective of what the method you are using it is definitely it is ρg into water plane area we know that. So, therefore, that is why I am not writing the c. In fact, we do not need to write the M also because the mass also the same. So, therefore, here actually really we can ignore this term the M also, right.

So, from the frequency domain finally, the only the $-\omega^2 a$ it should be enough. Now, let us see what I write if I do this for impulse response function. Now, here I understand that I can ignore this mass I can ignore this c also, but remaining thing I need to write. So, therefore, if I write this it should be $-\omega^2 A$ if you see that, right.

So, this term and also I need to write this. So, therefore, I must write plus I just copy from here 0 to ∞, $B(\tau)\sin\omega t d\tau$ ok fine, ok and here probably I need to have one ω right because this second term coming from this velocity. So, I have this ω. So, definitely I can have omega over here. So, therefore, this term I must have a ω. So, I just divide this. So, I will get a equal to $-A+\frac{1}{\omega}\int_0^\infty B(\tau)\sin\omega t d\tau$.

So, this again the second equation here again we can see this second equation having the relationship between this a frequency domain added mass with the time domain added mass A ok. So, ok so, here minus A. So, with this equation let us call 2. Now, from equation 1 and from the equation 2, I understand very well that I can get the $B(\tau)$ which is the time domain damping co-efficient and also the A which is the time domain added mass quantity through the equation 1 and 2, right.

So, again the question remain that why we are doing it right because anyway to get these two things we need to; we need to have this you know the frequency domain results, right. So, since we have the frequency domain result with us why where again we need to do all these things? Now, here I mean we definitely going to answer this in the next class of course, and of course, next class we are going to discuss that how actually we can numerically find out the $B(\tau)$ and then capital A.

But here I can modify this equation 2 very quickly because this is the equation it is actually this equation is for all frequency it is possible. So, so, when omega tending to infinity then also you can have this equation is hold very well now if you put ω tending to ∞. So, this second term this goes to 0 and then we are you know finding out this A is nothing but A^{∞}, right.

So, this equation 3 actually you are going to use and that is why in impulse response function this A is not frequency dependent we call that infinite frequency domain infinite frequency. So, now, if I able now I am able to write the equation of motion using impulse response function, so, how it looks like.

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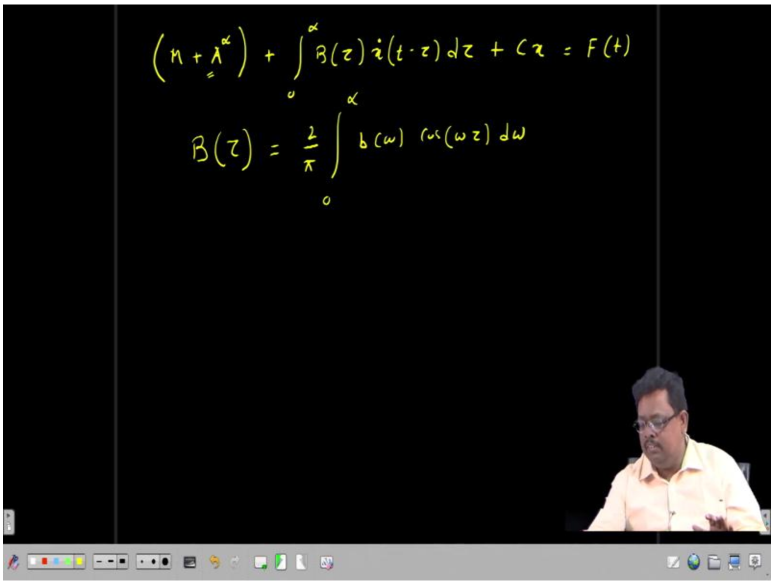

So, now I just I understand this added mass it is A^{∞}, right that is what we find out from the relationship. So, now, $(M+A)+\int_0^{\infty} B(\tau)\dot{x}(t-\tau)d\tau + Cx = F(t)$ and I can call that F(t), right. When this a infinity understand that infinite domain added mass and also I can get this $B(\tau)$ from the relationship $\frac{2}{\pi}\int_0^{\infty} b(\tau)\cos(\omega\tau)d\omega$ from this equation.

So, this completes the solution using impulse response function, ok. So, today we are going to discuss this much and from the next class we are going to how we can solve this numerical I can get this $B(\tau)$ this a ∞ etcetera that definitely we are going to discuss in the next class ok.

Thank you.