Numerical Ship and Offshore Hydrodynamics
Prof. Ranadev Datta
Department of Ocean Engineering and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 31
IRF Based Solution - Part 2

Hello welcome to the Numerical Ship and Offshore Hydrodynamics. So, today we are going to discuss how numerically we can find out this $B(\tau)$ and the $A(\infty)$ ok.

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So, if you remember in the last class, we derived the equation of motion using impulse response function also the relationship between the $B(\tau)$ and the $b(\omega)$. So, today we are going to see that how numerically we can get the value for $B(\tau)$ if I know the frequency-based solution which is $b(\omega)$ ok.

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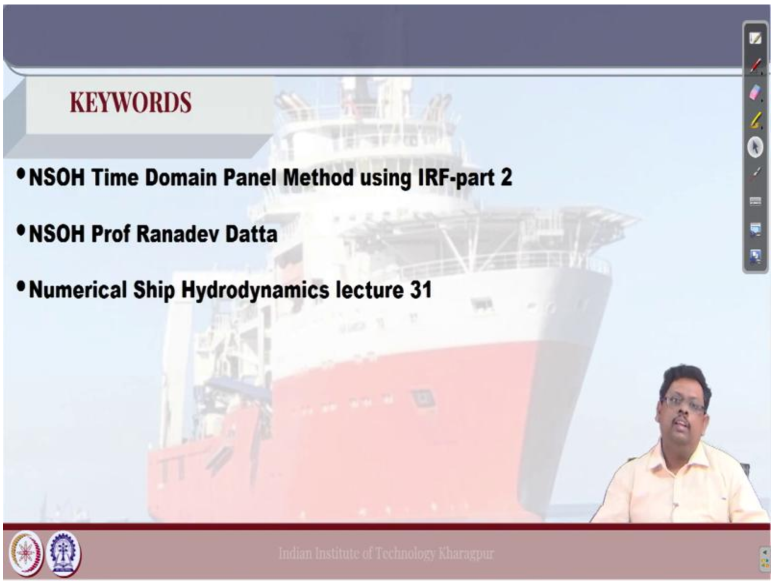

And this is something that you have to use to get this lecture ok.

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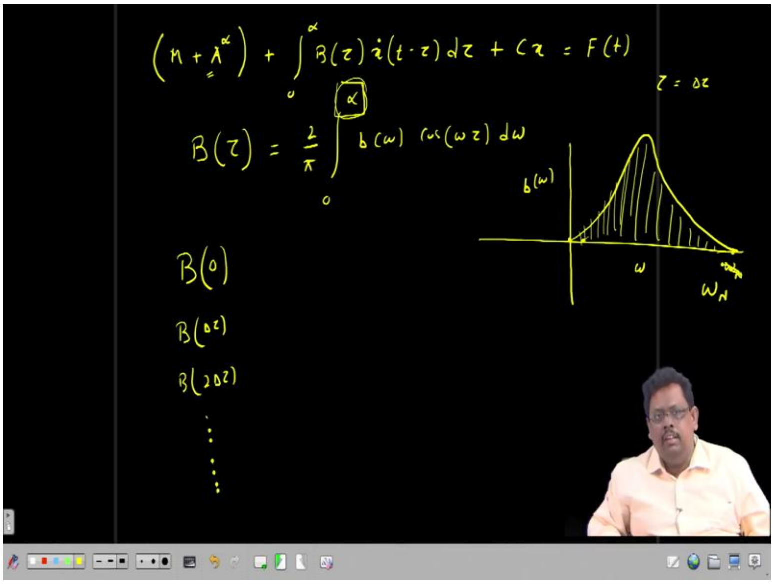

So, let us jump into the solution. Now here actually that is how we finished in last class. So, now, today we need to find out numerically how actually we can get this solution for B tau ok. Now here if you this integration it is ranging 0 to ∞; however, you know that if I plot this graph of $b(\omega)$ now it is if I plot $b(\omega)$ versus ω mostly the graph should look like it is very low frequency it is 0 and then you have high frequency and then I mean ideally it should be like this. The point is at some point this b goes to 0 right.

So, let us take at some ω_N I can really find that ok let me write in little bit bigger to understand it. So, at some ω_N this graph should go to 0 ok. So, having said that I have this understanding on this. So, this limit infinity when you calculate numerical it is not really till infinity right, but here remember that the tau also you know here is a vary over the tau. So, here I have to get $B(0)$ when $\tau = 0$ and if you increase this τ with some increment $\nabla\tau$ right.

So, then I need to get $B(0)$ and then $B(\nabla\tau)$ then $B(2\nabla\tau)$ and so, on right. Now looking at this expression then what comes in our mind then because I know this $b(\omega)$ is this graph. So, therefore, if I divide this with number of sections and simply if I use the trapezoidal rule that will do for me and in fact, which is very true if you calculate for the you know $B(0)$.

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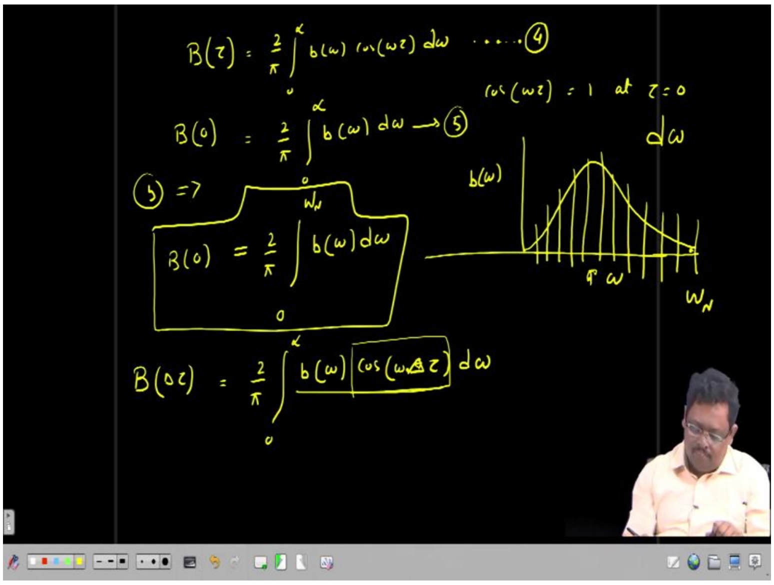

Now, why because now let me write again this $B(\tau) = \frac{2}{\pi}\int_0^\infty b(\omega)\cos(\omega\tau)d\omega$. Now, here if I know it is true for beta 0 why because if it takes $B(0) = \frac{2}{\pi}\int_0^\infty b(\omega)d\omega$ because $\cos(\omega\tau) = 1$ at $\tau = 0$. So, we know that now here it is nothing, but the area under the curve right because again if I draw this curve once again, this side if I call this $b(\omega)$ and this side if I call them ω and I know this is going to like this in some ω_N.

So, definitely it is a finite graph and this graph if I split in some regular interval right which we can call as $d\omega$ of course, in this regular interval if I cut this. So, eventually actually I can get a number a real number definitely because this is a finite. So, I need to modify or I can modify this equation let me give a number to this equation let us take ok I have 1 2 3 with continuation of the that thing we can we can keep it as an equation number 4 ok.

So, this equation number 4 and then it has let us take equation number 5. So, then now from this 5 actually I can say that this $B(0) = \frac{2}{\pi}$ and it is really not infinity right. So, it should be some 0 to some ω_N which is finite value then I need to calculate this $b(\omega)d\omega$. So, this is a standard trapezoidal will do this one right now what happened for $B(\nabla\tau)$? Let us see now if I try to do the same $B(\Delta\tau) = \frac{2}{\pi}\int_0^\infty b(\omega)\cos(\omega\Delta\tau)d\omega$.

Now, you know you can say that ok what is the big deal over here because in last thing I just integrate using trapezoidal. Now instead of $b(\omega)$ now I can integrate $b(\omega)\cos(\omega\Delta\tau)$ ok Δ is I just call ∇ ok. Now there is a problem. So, what is the problem? This is problem because of this inclusion of you know cos omega into delta tau why? Because if you see over here the only thing is changing over time is nothing, but the $\cos(\omega\Delta\tau)$.

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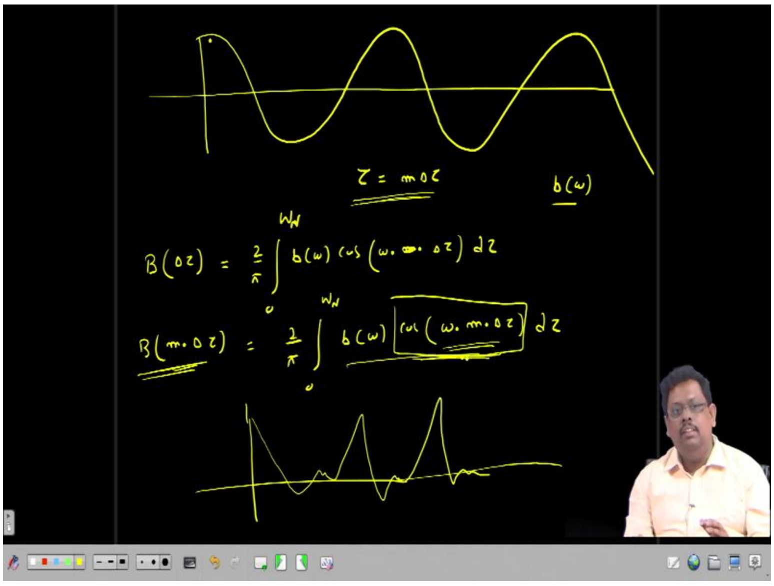

Now, people not you know we do not realize this the if I plot the graph of cos t or cos omega t it is a sinusoidal like this. So, now, the tau here this tau is nothing but some multiple of you know delta tau this side.

So, now, there is no problem with my $b(\omega)$ right. Now what is happening? This cos term since it is having $m\Delta\tau$. So, nature of this cos actually influences the solution. So, therefore, if I now try to calculate from this equation using the trapezoidal. So, then here it is $B(\Delta\tau)$ equal to you know $\frac{2}{\pi}$ into 0 to now it is some ω_N of course, now right.

And then we have $b(\omega)\cos(\omega m)$ ok now in case of a m=1 same $\Delta\tau \times d\omega$. So, this is the equation now if I try to do $B(m\Delta\tau) = \frac{2}{\pi}\int_0^{\omega_N} b(\omega)\cos(\omega m\Delta\tau)d\tau$. Now here clearly this depending on this $\omega m\Delta\tau$ this functional value. So, therefore, I can understand this is again turns out to be some kind of repetitive function it should I mean, it would be because it is I just put the functional value.

So, what we expect is, then this b this function this $B(\tau)$ function again should repeat the same thing you know something like this, again something like this I mean I am guessing because of this equation it should repeat right. Now before we do anything here let us try to understand the nature of the solution also.

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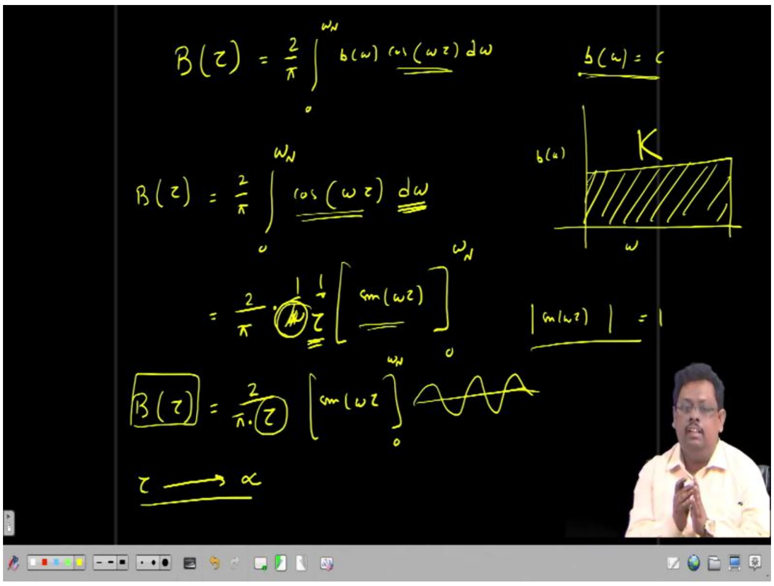

So, let us try to understand what would be the nature of the solution for $B(\tau)$ ok. Now I know that this is nothing, but $\frac{2}{\pi}$ now even if you take it is not ∞ some kind of ω_N ok into let us say $b(\omega)\cos(\omega\tau)d\omega$.

So, let us try to understand what would be the nature of this solution ok. Now the simple thing is that let us assume that b omega is something some constant c. This is a simple way of understanding what would be the nature of the solution $B(\tau)$ ok now let us assume it is nothing but $b(\omega)$ is a constant. So, what I am assuming? I am assuming that over this $b(\omega)$ and ω it is something a constant throughout ok. I know these are very vague assumptions.

But this really helps you know very simple thing helps you to understand that what will be the nature of the solution something like this. Now you see now using this we have $\beta = \frac{2}{\pi}\int_0^{\omega_N} b(\omega)\cos(\omega\tau)d\omega$.

Now if we integrate this term then definitely you can find out if you integrate this thing definitely we have $\frac{2}{\pi}$ and again we have $\frac{1}{\omega}$ multiply this functional value right has 0 to ω_N this elementary I really does not I do not care about this ok I mean it is of course, it is a sine function, but I really I do not care what is here because I know is this the modulus of this function this equals to 1.

So, it is harmonically it oscillate between -1 to +1 most importantly I in the denominator I have this $\omega\tau$ very sorry because this integration is with respect to ω. So, I have $\frac{1}{\tau}$. Now most importantly what we have this we have in the denominator I have $\frac{1}{\tau}$ ok. Now you see I have $B(\tau) = \frac{2}{\pi\tau}$.

So, I have denominator a τ with some function I and this function oscillate within a 0 -1 to+ 1 because the sine function right. Now here when tau goes to infinity because you see like that is what I want to achieve right. I am trying to find out my $B(\tau)$ and then

τ starts from 0 then $\Delta\tau, 2\Delta\tau$, $3\Delta\tau$ in that way it is going $m\Delta\tau$ now m can be anything it could be infinity also I mean very large number also.

Now, in very large number I have the denominator is having a tau. So, now, you see that this $B(\tau)$ should go to 0 when τ tending to ∞ because the numerator the value of numerator always lying between -1 to +1. So, numerator does not go to infinity, but the denominator is going to ∞ when τ tending to ∞ right. You know before understanding or doing the whole thing we need to understand this very basic thing so, that actually that helps you to do this numerical thing approximating the things right.

So, here now here you see that at $\tau = 0$ I do not have this cos term. So, then it is integration from 0 to ω. So, now, if we integrate this function, it is some finite number right we can call some finite value k right and when tau goes to infinity and then $B(\tau)$ comes down to 0.

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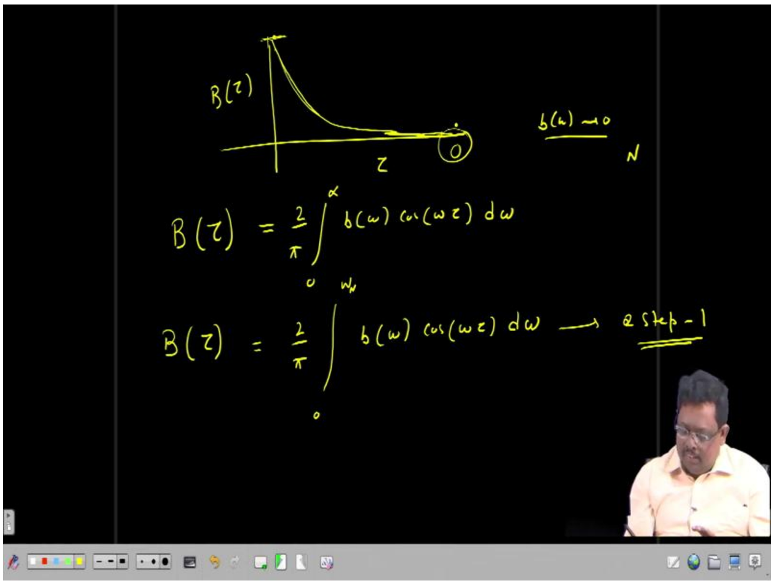

So, we have to understand and we need to keep these things in our mind that the therefore, this nature of $B(\tau)$ versus τ should be with some constant value because at $\tau = 0$ I am doing the integration in the area under the curve right.

So, it must have some values and then it definitely somehow it should go to 0. It may not be the nature of the curve it could be anything, but at least for larger tau it should go to 0 that is what. Now coming back to this what we are discussing here, now if we do this then this and you just put the functional values in trapezoidal rule, we really do not get this trend right.

So, then how we can solve it? Right. So, mostly we can solve this by some kind of semi analytical method normally we prefer to solve this equation ok. So, here what we do as follows. Now I have this $B(\tau)$ let me write again this equation. This equal $\frac{2}{\pi}\int_0^{\infty} b(\omega)\cos(\omega\tau)d\omega$. So, this is the basic equation that actually we started from there we come up with the idea is since the graph of $b(\omega)$ is a damping curve which is at very low frequency and very high frequency it is approaches to 0.

So, definitely it should be 0 to with some ∞ right sorry not infinity with some value right and then it is $b(\omega)\cos(\omega\tau)d\omega$. So, this is my step 1 realizing that the graph of $b(\omega)$ cannot be infinitely long it should be it should truncate I should truncate at some point where that value of $b(\omega)$ is very close to 0. So, at that end I must truncate this value and then can split this integration over this right.

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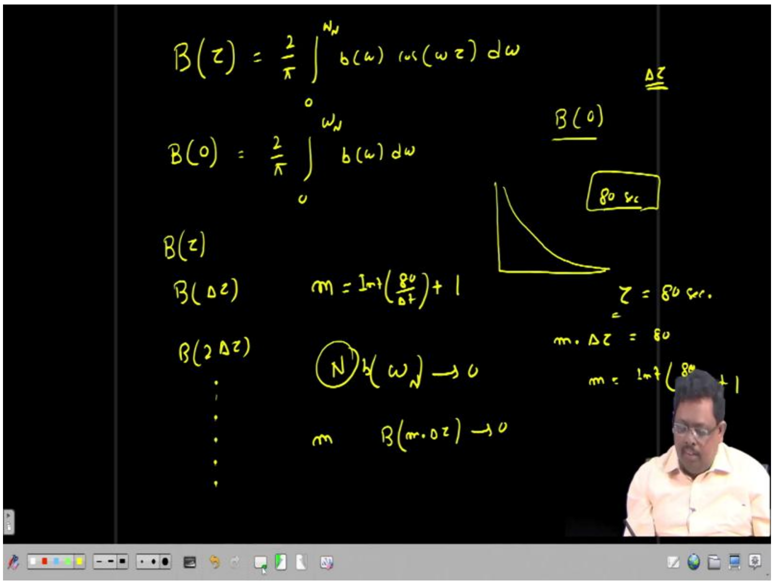

So, this is my second step. So, writing down the second step it is $B(\tau)=\frac{2}{\pi}\times 0$ to some $\omega_N \times b_\omega \times \cos(\omega\tau)d\omega$ then what we do? Then separately we calculate at $\tau=0$. So, I calculate $B(0)$ which is $\frac{2}{\pi}\int_0^{\omega_N} b(\omega)d\omega$.

So, this we called and we can get this $B(0)$ right then I need to calculate $B(\tau)$ right. Now here we need to understand we understand already we have understood one thing like we have understood 1 thing like this tau definitely goes to 0 at some value of τ.

But what is that value that we really no we have that value actually we only get through experience. Now normally we have seen that you know if you take for 80 second, then after 80 second it really goes to 0. So, trust me it is sufficient to calculate this $B(\tau)$ for 80 second ok. Now then second thing that. So, therefore, I understand that tau should be with 80 second now then the second questions come what should be my $\Delta\tau$ because this is something we really need to do right.

Because it is $B(0)$. So, the next step it should be $B(\Delta\tau)$ then $B(2\Delta\tau)$ and so, on right. So, then from here we need to find out what is the value of you know τ should be 80 seconds. So, definitely depending on $\Delta\tau$. So, this solution also very easy you can call some small into $\Delta\tau$ that should be 80 seconds. So, from there I can get some idea about that m equal to some integral value of ok let me write this $\frac{80}{\Delta t+1}$ right.

So, we understand that my m is the integer. So, it should be some integral value. So, I divide $\frac{80}{\Delta t}$ it could be real it could be integer I do not I mean sometimes it is a divisible by 80, it is integer otherwise is a fraction.

So, then just to make that it I can take the minimum that value I just simply choose my m in that way. So, the second thing also we have completed. So, the first thing we have to find out what is my N so, that your ω_N goes to I mean $b(\omega_N)=0$ this is my first job and second job I need to calculate this m such that your that capital $B(m\Delta\tau)$ that goes to 0 with that m.

So, these two things it is based on our experience we can set ok. So, two things are over. So, therefore, now again let us try to attack the problem.

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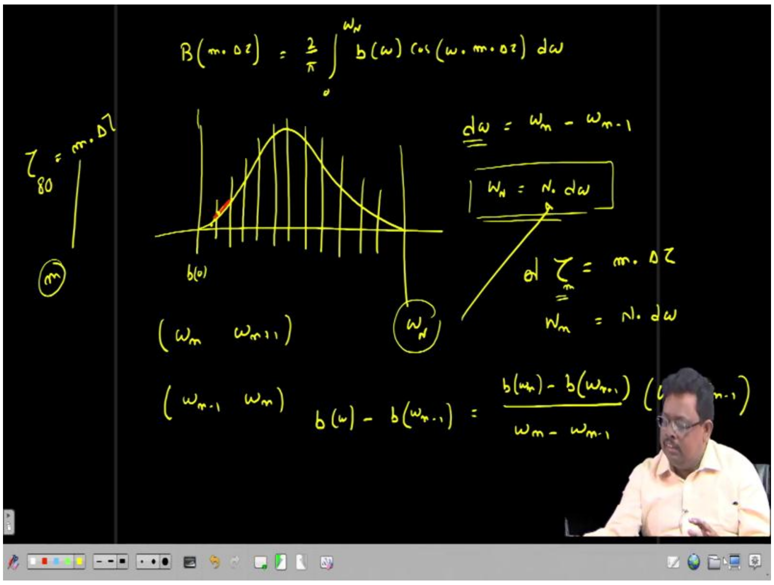

Again, we have B into now I can write $m\Delta\tau$ that should be is equal to $\frac{2}{\pi}$ and 0 to ω_N. Now this ω_N also we need to write somewhere let me do that it is $\int_0^{\infty} b(\omega)\cos(\omega m\Delta\tau)d\omega$ right.

Similarly, now let us try to look into the graph of $b(\omega)$ and it is of course, this ω_N depending on how you are numerically defining the $b(\omega)$ ok. Now assume that this is your graph for $b(\omega)$ and this is your value at ω_N here. Now you are discretising it is now it is of course; it is $B(0)$ and then you are discretising with equal interval. Now if you discretising with the equal interval and then we can define your you know $d\omega = \omega_n - \omega_{n-1}$ that you can define.

So, therefore, here you can write your $\omega_N = N \times d\omega$ right fine. So, we also understand that because you see this integration ultimately this integration sign, I need to replace by some kind of a summation sign right. So, therefore, I have to you know we have to represent this ω in form of this N into $d\omega$ right ok. So, now, I know that my $d\tau$ or I know my $\tau = m \times \Delta\tau$ and I also know my ω or you can say that τ m where the τ goes to 0 and ω_N this equals to some $N \times d\omega$.

So, these two things I know. Now my next job how I can define my $b(\omega)$ between two intervals. Now here the catch now we can do that you know with respect to you know we can fit a curve we need to fit a curve definitely right because these are the value I know right. So, then how the b omega varies between these two lines that we need to understand how $b(\omega)$ varies between these two lines right that we need to understand right.

Now, let us go with the simplest way we assume that it will go here you know very linearly right. So, therefore, in between let us take some interval is $(\omega_n \times \omega_{n+1})$ anything. So, this is the interval you can take or maybe you can take some ω_{n-1} or ω_n. Now I mean it is the way you feel comfortable you can take this and then you can assume within this interval that you're this variation of $b(\omega)$ is a straight line right.

So, therefore, we can assume this $b(\omega)$ it is a straight line. So, how we can write a straight line. So, $b(\omega)$ minus you can take this value it is b let us say ω_{n-1} should be equal to $b(\omega_n) - b(\omega_{n-1})$ and that is how you define a straight line right. It is $(\omega_n - \omega_{n-1}) \times (\omega - \omega_{n-1})$ that is how we can define my straight line right. So, now, I understand now actually now I am ready to attack this equation.

So, what we do here? First, I split it and I define this value of $d\omega = \omega_n - \omega_{n-1}$ and we define this $\frac{\omega_n \times n}{d\omega}$ again I understand that the τ the maximum value of τ it should be $m\Delta\tau$ right.

So, with understanding we understand here that τ it is we need to till get the for 80 second normally we are taking let us take 80 second and if it does not work then you can take a 100 second it is up to you it is up to the experience how much tau you should take but this should be equal to some $m\Delta\tau$ and this from here actually, I can get the value for m ok.

So, then the last thing what we did today is how I can define the $b(\omega)$ between the interval ω_n and ω_{n-1}. I assume a straight-line approximation right. So, this is something that actually we have to do before we solve this integration in using helping the semi analytic method ok.

So, now I am ready to do the integration. So, in the next class we are going to see how in with as I said is a semi analytic approach, we find out the value for $B(\tau)$ each $\Delta \tau$ ok. So, today we can stop here and in the next class we are really try to find out how we can approximate the $B(\tau)$ using the semi analytic methods right ok.

Thank you.