

**Numerical Ship and Offshore Hydrodynamics**  
**Prof. Ranadev Datta**  
**Department of Ocean Engineering and Naval Architecture**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 37**  
**Strip Theory**

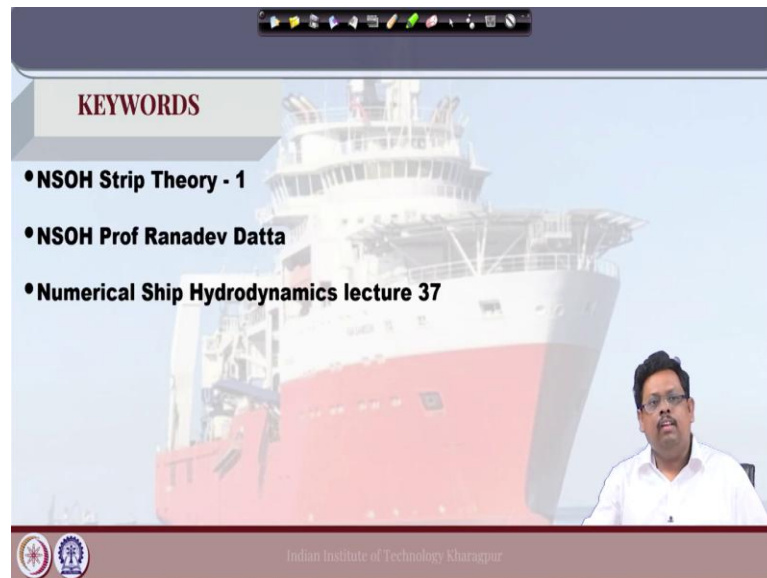
Hello welcome to Numerical Ship and Offshore Hydrodynamics, today we are going to discuss about the Strip Theory ok.

(Refer Slide Time: 00:21)



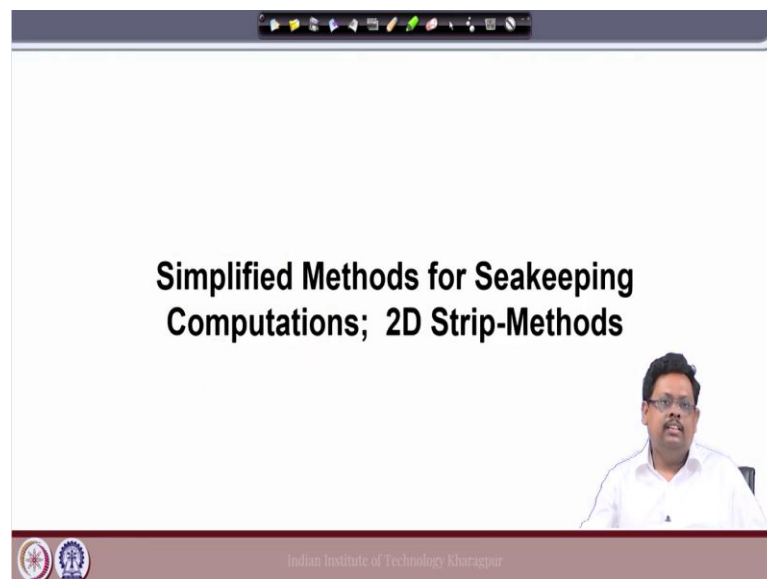
So, today there is an introductory class of the strip theory, today we are going to understand the basics of strip theory, what is that theory in is in very elementary level.

(Refer Slide Time: 00:36)



And these are the keywords that we have to use to get this lecture ok.

(Refer Slide Time: 00:41)



So, let us jump into this today's topic which is simplified methods; why is simplified? Because we can say it is a two-dimensional method to capture the three dimensional effect ok. And you know that these are one of the most popular method that we are going to use for in a industry we do all the sea keeping problems mostly people are using this some kind of strip theory ok; so, let us start.

(Refer Slide Time: 01:16)

The slide is titled "Sea-keeping, Ship motions and Sea-Loads". Below the title, it states: "From a hydrodynamics point of view, this is often referred to as:". A bulleted list follows:

- forward-speed sea-keeping problem
- forward-speed radiation-diffraction problem
- Neumann-Kelvin problem for oscillatory motions

In the bottom right corner of the slide, there is a small video inset showing a man with glasses and a white shirt speaking. At the bottom of the slide, there are two logos on the left and the text "Indian Institute of Technology Kharagpur" on the right.

Now, normally this strip theory one can say that it is a forward speed sea keeping problem. So, today I mean from today onwards the rest of the discussions based on the method which has the ship is moving in a constant velocity ok; so, this is a forward speed sea keeping problem.

And also, sometimes it is referred as radiation diffraction problem, why? Because mostly you know as you know from our previous lecture that mostly we are going to deal with the two major forces one is the radiation force from there we are getting added mass and the damping.

And of course, another one is the diffraction force that is when ship is hitting, I mean the wave is hitting the ship and then the force because of that incident. Of course, we have another force which is very important we call it as a Froude Krylov force. But you know we really do not discuss on the Froude Krylov force much, because we can analytically find out the expression for the Froude Krylov force.

However, we need a proper boundary integral equation method or any other any other type of solutions to get this radiation diffraction force. So, mostly people when refer some sea keeping problem, they usually refers as radiation diffraction problem ok. So, that does not mean that restoring force or hydrostatic sorry or the Froude Krylov force does not have any influence on this code.

Of course, because frankly speaking the major part of the force coming from this Froude Krylov or hydrostatics in most many situations. There are situations where radiation diffraction force also very important, right? And in fact, why this is important because normally ship ranges the frequency in ocean it is about from 10 second to 20 second wave's right.

And therefore, and if you look at that in general the length of the vessel ocean going vessel is normally comes around someone 50 to 250 meter long. So, what is happening in some situation the length of the wave more or less equal to the length of the ship and in such situation you know that radiation diffraction becomes very important.

However, if you think of you know the low frequency region when for example, when actually we are going in the ocean and we are taking bath. And then if you remember we can simply go up and down with the waves. So, this is this zone is called the low frequency region where the length of our body is very very small compared to the length of the wave.

So, in that situation radiation diffraction effect really is not that influential; so, that time Froude krylov and the hydrostatic are important right. However, for the ocean, going vessel when these phenomena happen the  $\lambda_1$  falls into some kind of 0.75 to 1.5 in that region that the radiation diffractions also become very important the phase becomes very important.

So, when we are going to discuss slowly slowly all these aspect and sometimes this sea keeping problem also refers to the Neumann Kelvin's problem for the oscillatory motion ok. Now, this Neumann problem referring for the zero speed cases and; however, when we introduce the forward speed then Kelvin comes into picture ok.

(Refer Slide Time: 05:17)

The equation of motion:

$$(m+a)\ddot{z} + b\dot{z} + cz = F_a \cos(\omega t + \epsilon_{Fz})$$

radiation force  
(added mass force  
and damping force)

hydrostatic restoring force

wave-exciting force

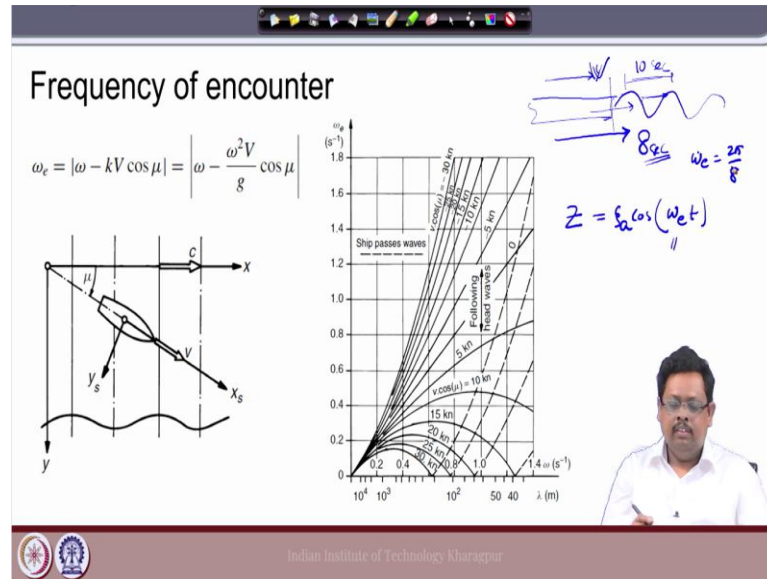
Indian Institute of Technology Kharagpur

Now, you know very well this is the equation of motion everywhere right. Now, here the first term is  $(m+a)\ddot{z}$  which is at the inertial part and then you have  $b\dot{z}$ , the radiation part and then the  $cz$  is the hydrostatic restoring part. And then right-hand side under the assumptions of the linearity we can assume there is a harmonic motion and then  $F$  is the amplitude of the harmonic motion right.

And also, we have discussed a lot that  $a\ddot{z} + b\dot{z}$  this is called the part of the radiation force. And then when actually write the equation of motion we simply write  $m\ddot{z}$  one side and you can take all other force in the opposite side; that means, the right hand side and then we are calculating the  $m\ddot{z}$  equal to the total hydrodynamic force.

In fact, total force because we take  $cz$  also in the right-hand side right and then we solve the equation of motion right. Now, here there is a change like you know this  $\omega$  is not really the  $\omega$  that is the wave frequency no, here this omega we referred as a frequency of encounter ok.

(Refer Slide Time: 06:48)



So, now, what happened when you look at this expression and this is very popular expression and largely used in the field of sea keeping. So, if you have done this course sea keeping; so, you are very aware of this expression anyway, here that idea is as follows; now, when ship is not moving right.

Now, if you look at this expression over here when the ship is not moving then this  $V=0$ ,  $V$  is referred for the velocity of the ship. Now, if ship is not moving then  $V = 0$ , then this encounter frequency  $\omega_e$  equal to the  $\omega$  right. So, that is why if I look at this previous slide, now if there is no velocity then  $\omega$  and  $\omega_e$  both are same right.

However, if  $V$  has some velocity; so, definitely this  $\omega - \frac{\omega^2 V}{g} \cos \mu$  this is the standard way of getting that encounter frequency very standard formula. Now, if it and also, we have discussed in the last class also how to achieve this; so, here  $\omega_e$  is different from  $\omega$ .

Now, what is  $\omega$  physically we explain that it is when you are on the board and now suppose the wave is approaching to you from the front. Now, if you are not moving then you can take this wave may be for each 10 second you are receiving this wave right. Now; however, if you move along these waves then to you it appears that you are receiving the waves in each 8 second right.

So, that is called the encounter frequency so; that means, when you do not have any velocity, you stand still over here and wave is slowly approaching. So, you can find out that two the time taken between two wave crest is coming to it is let us say 10 second. Now, if you move along this wave then to you it will appear that that you are receiving that if the wave crest like this wave is coming hitting the first crest is coming and hitting to you.

And then the again you can say the second wave is approaching coming and hitting to you. So, the time taken between this to crest is coming to you, the approaching to, is that 8 second; so, that is called the encounter frequency. And then the ship is excited on that encounter frequency ok; so, this underlined principle is as follows. Now, here now we here what we are going to do is suppose we have a ship here.

And this ship is moving as you say is some velocity  $V$  let us call some velocity  $V$  and approaching over here, and then let us say this is the wave is coming right. Now, if you stand still, then these two-wave crest let us take approaching here it will take 10 second ok. And suppose, but if you moving with some velocity then these two-wave crest maybe you know you can approach you can think this is the 8 second.

Then as a result what is happening that the ship let us say if the let us say the heave velocity the displacement  $z$ , then this  $z$  actually can we write in terms of a harmonic version  $\zeta a$ . And then  $\cos \omega_e t$  and this  $\omega_e$  is nothing but you know that  $\omega_e$  is nothing but  $\frac{2\pi}{8}$  not  $\frac{2\pi}{10}$  that is what I am talking about ok.

So, the ship is excited on encounter frequency it is not excited about the wave frequency. So, we have to now very you know you have to be very careful about that whether you are going to use the encounter frequency for the forward speed case. Let us say, in general you can write the code in terms of encounter frequency and then when  $V = 0$  definitely this  $\omega$  equal to  $\omega$  ok.

(Refer Slide Time: 11:32)

**Flow physics**

$$\phi^t = (-Vx + \phi^s) + (\phi^w + \phi^l)$$

$\phi^t \Rightarrow$  the potential of the total flow  
 $Vx \Rightarrow$  the potential of the (downstream) uniform flow with ship speed  $V$   
 $\phi^s \Rightarrow$  the potential of the steady flow disturbance  
 $\phi^w \Rightarrow$  the remaining unsteady potential  
 $(-Vx + \phi^s) \Rightarrow$  steady (time-independent) flow  
 $(\phi^w + \phi^l) \Rightarrow$  periodic flow due to the effect of the sea-waves

Handwritten notes:  $U/V$ ,  $C > V$ ,  $C < V$ ,  $C = V$

Now, because of this forward speed then you can see the flow physics is now you know not same as the floating body. Now, what is the changes over here the changes this inclusion of this inclusion of this term; now, you can see that this term is absent when you do not have the forward speed right.

Now, you see that time we are only having two component; one is the radiation component, and one is the diffraction component. Now, because that that ship is now moving forward, now because the ship is now moving forward is going some velocity  $U$  or let us say some velocity  $V$ .

Because of that we have two things, one is the one is that there is a force because of that you can assume that there is a uniform flow with the ship with the velocity  $V$ . Let us say, then again you have to take two cases right; one is that there is a uniform flow let us say current kind of thing, and then this ship encountering this uniform flow when it moves with the velocity.

As you as if, now when you solve the radiation problem if you remember there what we what are doing is we are having a calm water and then we are oscillating a body and because of that we are getting some radiation force. Now, here what we are doing we have a calm water and then we are moving the velocity with the  $V$ ; so, because of that you have this first term. Now, what is happening once you have it, then actually you can have some kind of radiated wave at the downstream also right.



And this is also you can call the radiation radiated wave because the ship is moving forward. Now, you know if you some of you if you have there in the river side if you take a some boat, and then if you are just travelling along the boat then you can see that at the downstream we can have some kind of waves right. Now, you see that then we have; so, it means that if you have a ship and if it is travelling in forward with a velocity  $V$  and then we that generate lot of waves at the downstream.

Now, these waves some waves have the velocity or you can say cellular it is  $C$  which is you know greater than  $V$  or let us take I mean it is not say some  $C < V$  and possible there are some wave  $C$  their velocity is equals to  $V$  ok. So, in such case in this third type case actually you can have some potential of the steady flow disturbance.

And this is coming with there are we can call typical wave resistance problem, here this term is coming it is a time independent flow we can call that the steady waves ok. Why it is steady? Because, these waves are and the velocity of the ship both are same ok now, you see like so; so, now, I am moving the ship, I am moving this ship and then downstream we can create lot of waves, some of the waves is traveling with the ship ok.

So, therefore, we can consider the relative velocity is 0; so, therefore, we can consider this is a time independent disturbance and this is the term we can refer this as  $\phi^s$  ok fine. And of course, the remaining the unsteady potential  $\phi^w$  we can say typically it is as same as my zero-speed case.

So, when the body is oscillating, we can have  $\phi^w$  and of course,  $\phi^l$  is the instant wave potential. So, these two term; so, these two term is very much similar to my zero speed thing and then and this extra term this basically comes from the forward speed ok.

(Refer Slide Time: 16:27)

Free-surface conditions

$$\eta_t(x, y, t) = \phi_z(x, y, \eta; t) + (U - \phi_x)\eta_x$$

Forward-speed-dependent term

$$\phi_t(x, y, \eta; t) = -g\eta(x, y, t) + U\phi_x$$

Forward-speed-dependent term

this makes the theoretical treatment of a forward-speed ship motion problem very different then the counterpart of offshore structures problems with zero speed

Indian Institute of Technology Kharagpur

So, now, in the last class also we discussed this is the expression for the free surface boundary condition. So, the first term this first term refers to the kinematic free surface condition and the second terms refers for the dynamic free surface boundary condition. Now, here also it is little bit different from the zero-speed case right, because of the forward speed we are having this extra term in the kinematic free surface boundary condition right.

Similarly, for the forward speed you have one extra term in the dynamic free surface boundary condition right. Now, you see that this makes this theoretical development forward ship motion is in a very different from the development for the zero-speed case like mid type theory whatever we have discussed in our initial day class right.

So, here because of this the total this forward speed problem is very much different from the zero-speed case. Because, we need to consider the kinetic free surface condition which includes some kind of the effect of the forward speed, we need to consider the dynamic free surface condition which includes some effect for the forward speed and so on right ok.

(Refer Slide Time: 18:06)

Applicable body boundary condition

radiation potential

$$\frac{\partial \phi_j}{\partial n} = i\omega_\infty n_j + U m_j$$

generalized normal

the 'm' terms

$\phi_j \equiv$  radiated potential for  $j^{th}$  mode, Amp. of oscillation = 1

$m_1, m_2, m_3 = \vec{m} = -(\vec{n} \cdot \vec{V}) \vec{W}$

$m_4, m_5, m_6 = -(\vec{n} \cdot \vec{V})(\vec{x} \times \vec{W})$

velocity of the steady flow with reference to the moving reference

existence of these m terms makes the problem very complex !

Indian Institute of Technology Kharagpur

Now, here it is the radiation from; now, you know that now we are not talking about the steady part let us talk about the unsteady part ok. So, now, you know that in unsteady part we have two component right; one we defined as radiation potential  $\phi^R$ , and then another potential defined as the diffractive potential  $\phi^D$ . And then another component normally we do not speak much, because we have the analytical solution for this particular thing which is the incident wave potential  $\phi^I$ .

So, now by this time everybody should be very much aware of all this terminology right; so, here look at this radiation and the boundary condition ok. Now, here this what all I am trying to do is like I try to tell you that how this forward speed problem that mathematical development is different compared to the zero-speed case ok. Now, here if you remember the all the terminology that we discussed in when we develop the potential flow theory, this  $\phi_j$  stands for the radiation potential for the  $j^{th}$  mode right.

This  $\phi_j$ , it is nothing but the radiated potential for  $j^{th}$  mode; so, it means that I am oscillating the body in the  $j^{th}$  mode. And if you remember I am oscillating the body, this, what is the amplitude of the oscillation? The amplitude of oscillation if you remember I use this amplitude of the oscillation is nothing but unit amplitude 1.

(Refer Slide Time: 20:33)

Applicable body boundary condition

radiation potential

$$\frac{\partial \phi_j}{\partial n} = i\omega_s n_j + U m_j$$

generalized normal

the 'm' terms

$$m_1, m_2, m_3 = \vec{m} = -(\vec{n} \cdot \vec{V})$$

$$m_4, m_5, m_6 = -(\vec{n} \times \vec{V}) \cdot (\vec{x} \times \vec{W})$$

velocity of the steady flow with reference to the moving reference

existence of these m terms makes the problem very complex!

Indian Institute of Technology Kharagpur

Now, once we do this then my radiation boundary condition which is  $\frac{\partial \phi}{\partial n} = V.n$  right?

Now, what I assume? I assume by this forcing the amplitude of the oscillatory motion of the body. Now, this is how we do the radiation force I have a still water and then in some amplitude I am oscillating the body and then this amplitude I call this is 1 right. So, this; so, I can assume my  $x$  is nothing but some 1 into let us take  $e^{i\omega t}$  in a complex domain or if you take the real it is the  $\cos \omega t$ , right?

And then  $\dot{x} = i\omega e^{i\omega t}$  and which is refers for my velocity  $V$ , we have discussed it we have discussed a lot of this. So, I am just repeating the same thing again and in that case,  $V.n = i\omega n_j$ . Now; so, in case of forward speed, as I say it should be with respect to the encounter frequency not the wave frequency  $\omega$  that we discussed initially right.

That even if this wave is coming in frequency  $\omega$ , but I am receiving because I am moving, I am receiving with some different frequency which is  $\omega_e$ . So, therefore, I must oscillate that frequency that I am receiving. So, therefore, instead of  $\omega$  we have the  $i.\omega_e.n_j$  right? Now, this is fine till this point there is no.

Now for the forward speed problem we have one more additional term which is  $U m_j$  and you know it is a very classical m terms. So, we can call so, called m terms and it is very popular for you know people are using it very popular just saying that so called m terms

ok. Now, these terms is coming because of the forward speed right; now, you can see here even this  $U$  is 0, then we do not have this term. Now, what is this  $m$  terms physical means that it is the; it is the radiation potential which is coming due to the forward speed right.

Now, you see normally what is happening here; now, in zero-speed case the ship is just like oscillating this way. Now, in case of a forward speed ship is moving forward as soon I mean as well as it oscillates right. Now, when the ship moves forward as well as they oscillate, then for the first part when it oscillate the first term is there; so, it is  $\omega n_j$ .

Now, because of this forward speed then also you can have some kind of waves, right? And that actually takes care by this  $m$  terms. And this course is pretty complex we are not really going into the details of this mathematics, but the expression is given here right? And it is actually for the purely non-linear case. In fact, for the linear situation we have much easier solution for the  $m$  term; so, but this physics entirely change the whole you know phenomena and it is become much more complex compared to the zero speed.

Now it is I it is written here also this exists in  $m$  terms make the problem even more complex right ok. Now, here this now referring this  $n$  is nothing but the normal and then this  $w$  is velocity of the steady flow with reference to the moving frame. So, therefore, one has to remember that again this  $m$  terms occurs if you are writing this whole exercise in terms of a moving reference frame ok, that what we discussed in the last class in fixed reference frame we do not have the  $m$  terms ok.

So, now, you see like here so many things actually we have to consider right, when you write this equation this radiation problem. Then you have to see that whether writing in the in case of a fixed reference frame or moving reference frame in, if you write a fixed reference frame then you can have one set of equation, if you are writing in a moving reference frame then you have one another set of equation; so, many complicated complication is coming right.

Anyways my job is not to tell you that how complex it is this our job is ok, I understand this is complex physics flow physics complex, but even taking care of everything how could I get a solution. So, and this formulation is valid for I think the last 50 years everybody know about the theory, but; however, still writing a code is a different task

right. So, here our main focus how do I write this whole phenomena in a code and I can get some realistic solution ok.

(Refer Slide Time: 26:12)

The above differences makes the problem of forward speed seakeeping very different and far more complex to solve compared to the zero speed problem.

In many simple solutions, one simply 'assumes' or finds the most complex quantities from graphs and charts, or simple formulae

**Example:**

- Get added mass, damping from series of plots
- Get expressions of exciting moment from some simplifications (mostly assuming waves are long compared to body)

Indian Institute of Technology Kharagpur

So, now, as you say this above difference make the problem of the forward speed sea keeping very different in compared to the zero-speed problem right. Now, in many cases that, we can see most difficult phenomena here to get the radiation potential; so, get the diffraction potential right; so, these are the very one of the most difficult you know parameters that we want to compute.

However, as I written over here that the many simple solutions and one can simply assumes or find the most complex quantities in terms of a graph or chart or simple formula yeah.

Those all are exist right this the examples is given here for example, added mass right. So, it is coming from the if you want to solve the added mass then you have to deal with all these radiation component you have to you know try to find out the velocity of you know this water plane and then we need to find out this  $m$  terms everything to get this added mass right.

However, there exist some I mean normally in industry, because these are. So, complex people have come up with some kind of empirical formulas right and then this added mass this damping coefficient that can be obtained by series of plots and graphs are

available right. Definitely we are going to discuss that aspect also compared to that this conventional way of getting this added mass damping right the theoretical numerical development, apart from that you have to remember that is also important right.

And also, people even this exciting force which is or exciting moment which is also difficult because it is not only Froude Krylov. Because, the diffraction forces involved over here and finding the diffraction force also not very trivial if you try to solve numerically. And then people are making lot of simplifications to get this exciting force as well.

Now, when you are dealing with these things definitely these two aspect also we are going to discuss, these two things that how from the chart we are getting this damping and added mass coefficient and how do I write this in exciting force and moment assuming some kind of simple physics ok. And mostly you know, but we have to understand like we will discuss everything, but again we need to understand that where I should apply this and where I should not apply all these things right.

Now, here it is written that mostly we assume for the long wave situation; that means, the wavelength is much more higher compared to the ship length. So, in that situation one can apply this, but we I mean you know theoretically we say that we should apply in this situation, but random practice in all frequency people are doing it right. And anyway so, but then sometimes they get some reliable result sometime you know result may not be that reliable, but anyways.

So, today actually we stop over here; so, we discussed very introductory level that what is the strip theory we did not touch much mathematics here I mean normally we do not we do not use much mathematics throughout this formulation of strip theory these are well established and, but you should know some basics. So, we discussed that and we discussed that how to get numerical code how to write a numerical code for this particular problem ok; so, we will see in the next class.

Thank you.