**Numerical Ship and Offshore Hydrodynamics**
**Prof. Ranadev Datta**
**Department of Ocean Engineering and Naval Architecture**
**Indian Institute of Technology, Kharagpur**

**Lecture - 40**
**Strip Theory Part - 4**

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today, this is a lecture 40. And again we are continuing our discussion on the Strip Theory.

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And also we are still continuing about the basic mathematical formulation ok.

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So, this is the of course, this is the keyword that you have to use to get this lecture, ok.

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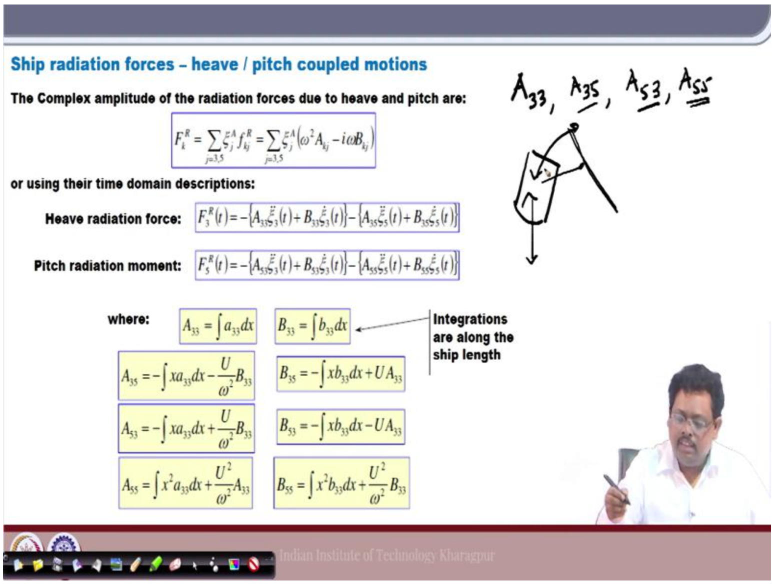


So, let us start. Now, in my previous lecture, if you remember that we have discussed how we could get the two dimensional added mass right. We did not discuss how we get the two dimensional added mass, but we discussed suppose I know the amplitude of the radiation force then how do I write the radiation force. Now, if you see over here in this slide. Now, here if you know that for the, now let us take a coupled heave and pitch equation of motion.

So, if you remember in case of a coupled heave and pitch equation of motion, you need the following added mass. So, in case of a you need the following added mass you need $A_{33}$ right. And then you need $A_{35}$, and then you need $A_{53}$ and then you need $A_{55}$. Now, for a ship motion you cannot avoid the coupling between heave and pitch.

However, if you need to remember that if you go for a strip theory and we are taking a only a section, then actually we do not get the pitch directly. So, we have to find out the pitch in terms of heave ok. And then also the added mass of $A_{53}$, $A_{35}$, $A_{53}$ everything we are going to get in terms of the heave ok. And of course, it is very obvious that suppose if you want to find a section something like this if this is your section and if you and this is your cg.

So, then if I know this oscillation around the heave direction, Now, if you take a moment about this axis also is a possibility or not possibility you can get the pitch also. So, this is the primary idea of getting all these modes ok. Now, and again also in the last class we discussed that how to get the radiation force if it is coupled with the other modes right. Now, I just briefly said because that is in the last class already we have discussed a lot.

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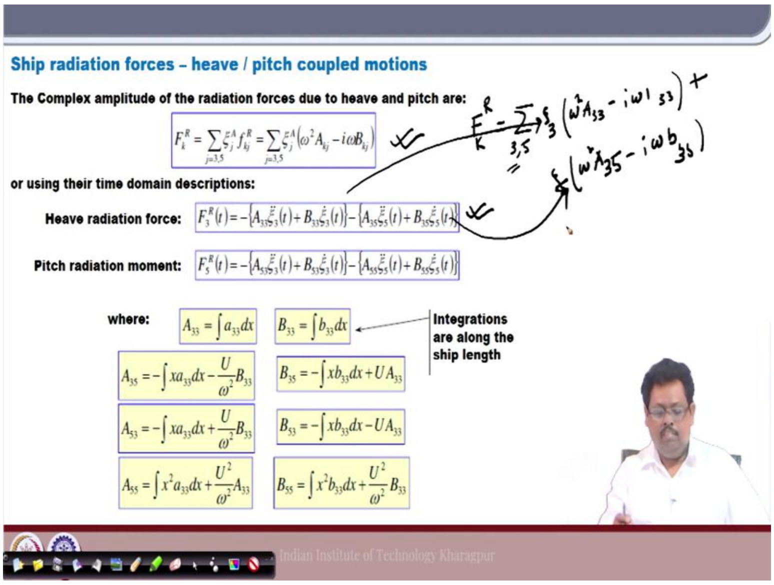


Like I said that if you want to find out the radiation force in some mode k right if I want to do this, then what you need to do that? You need to sum like so suppose I if I try to find out the radiation force on the heave mode, then we need to sum all six mode right and try to pick up that what would be the radiation force in the heave mode if I oscillate a body in the other modes right.

Now, if you remember here we have tried to coupled heave and pitch. So, definitely the summation should be 3 and 5. And then we need to find out the about that if I oscillate the body let us take in the heave mode 3 and then what would be the my added mass in ok. Now, I just ok later on I drop this summation. So, it is $(\omega^2 A_{33} - i\omega B_{33})$.

And then we need to add like if I oscillate the body in case of a pitch mode; that means, in mode 5 then what is the added mass? So, $\omega^2 A$, I always try to find out 3, but I have oscillating body in the fifth mode $-i\omega$ and again I oscillate, the damping it is 3 and 5 right. So, this is what actually it is written over here and then. So, therefore, in case of a heave oscillation.

So, it should be $A_{33}\ddot{\zeta}_3(t) + B_{33}$. This is for this particular first case and then this is actually you are getting from the second case which is $A_{35}\ddot{\zeta}_5(t) + B_{33}\dot{\zeta}_5(t)$. So, how we can write, suppose we are getting the frequency domain solution how to write in the time domain that is also we have discussed in the last class. So, again we are not repeating it.

Now, you can see here similarly I can write for the pitch also right. So, similarly that here also it is summation about 3 and 5. So, first thing that I am oscillate the body in let us say in third mode and I try to get the radiation force in the fifth mode. So, then because of that this one is there right.

And, then similarly if I oscillate the body in the fifth mode and again I am trying to get the radiation force in the fifth mode. So, we sorry not this one sorry and this part is for that if I oscillate the body in the third mode and if I try to get the radiation force in the fifth mode. Similarly, if I oscillate the body in the fifth mode.

So, I am oscillating body this way; and then I am getting the radiation force also in the pitch mode and for that basically the second component is there ok. So, now, here if you remember I said the radiation force that I am getting from the strip theory.

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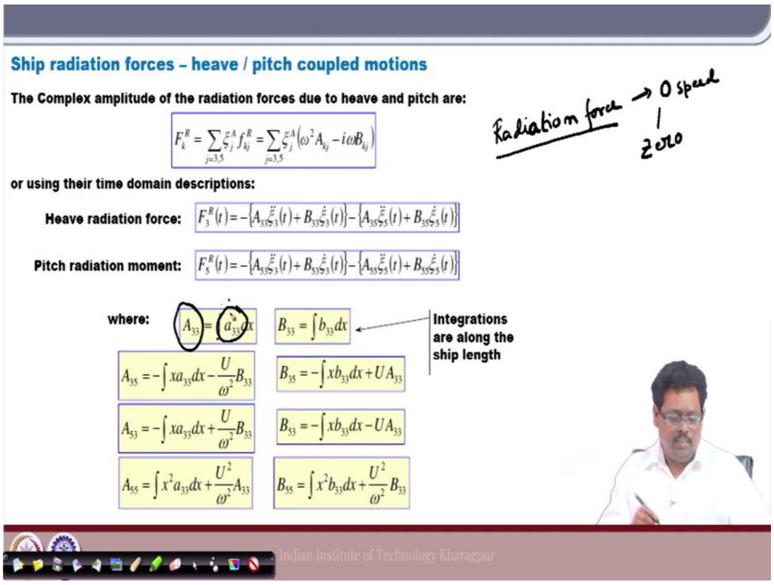


The radiation force this radiation force is actually in the zero speed right. This somebody can say 0. So, let me write zero speed. Now; however, how I get this use this I mean how I incorporate the forward speed into this equation that is the main challenge and I said that lot of theory are available. So, if he started with the Kevin Krakowski theory and then there is a there is a lot of people have most popular one is STF probably the is the Calvin’s and Talk and Faltinsen, I mean they have developed in 1970s is there is a very very popular paper.

So, mostly this all this lecture also mostly covered that that paper. So, I put all this reference at the end of this strip theory lecture. So, please follow those papers also. It is very interesting and you know it is if you understand that then probably its very good. However, in this course our primary focus is this theory is fine like how to implement a theory we are just taking the final results from those theories and we try to find out numerically how we can implement in our solution.

So, the primary objective of course, is the to try to understand the limitations how we are incorporate it and then how could we write a code of this particular method right. So, in view of that we really do not discuss about the theory as such here in this course of course, but the final result definitely we are going to see and we try to you know physically try to see that ok these are the results ok, and this is how we can incorporate the forward speed.

Now, here you can see in case of a heave mode. Now, this capital this A in the left hand side is basically the total added mass of the body for the forward speed. However, this small added mass are the added mass for the 0 speed ok; now you can see from this expression in case of a heave.

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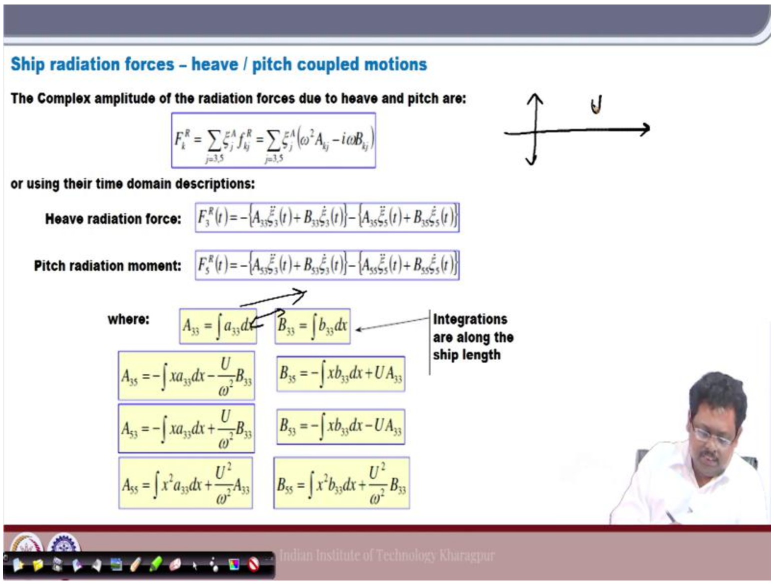


In case of a heave there is no effect on the forward speed as such so; that means, if the body is oscillating in this fashion now even if a moving with a constant velocity U. So, there is not much effect on the heave right. So, that means, that in case of a forward speed it is we are taking the added mass for the zero speed, ok

So, added mass value does not change with respect to the forward speed. However, if you look at the cross coupling term or if you look at the pitch mode there is a strong dependency on the forward speed ok. Now, why this strong dependency? As I said we really do not discuss about the theory, but just for the understanding.

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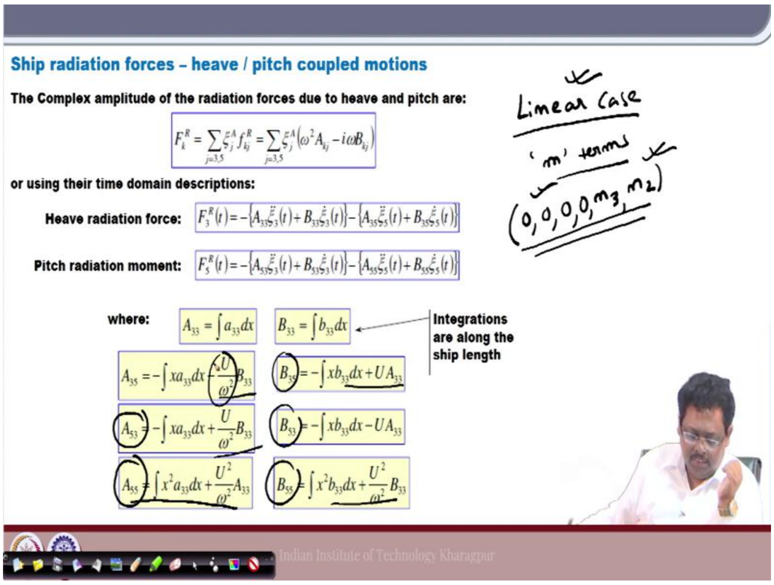


In case of a if you take a linear case. So, if you remember in our theory we have something called so, called m terms. And this m terms is related to the effect of the forward speed ok. So, in case of a in if you take a linear case if in linear regime this all this first four mode m terms are actually 0. So, therefore, in case of a surge, sway and then heave and the roll this four mode m terms is 0 and in case of the pitch, this m terms turns out to be the $n_3$ and in case of a yaw it turns out to be the $n_2$.

So, therefore, I understand that forward speed effect actually coming because of the - I mean this effect mostly for the pitch and the sway. Now, how and this is very popular. If you look at any popular journal paper or any popular document you can see that this is the n terms for the linear case. So, that is why you know from here actually at least we can have an idea because the m terms is non zero for pitch.

So, therefore, when we calculate the $A_{53}$, $A_{55}$, $B_{55}$, $B_{53}$ and $B_{35}$ definitely there is a contribution from the forward speed ok. So; however, how these terms comes into picture. This only you can understand if you read this journal paper very carefully. And however, in this course actually we really do not discuss all these things ok.

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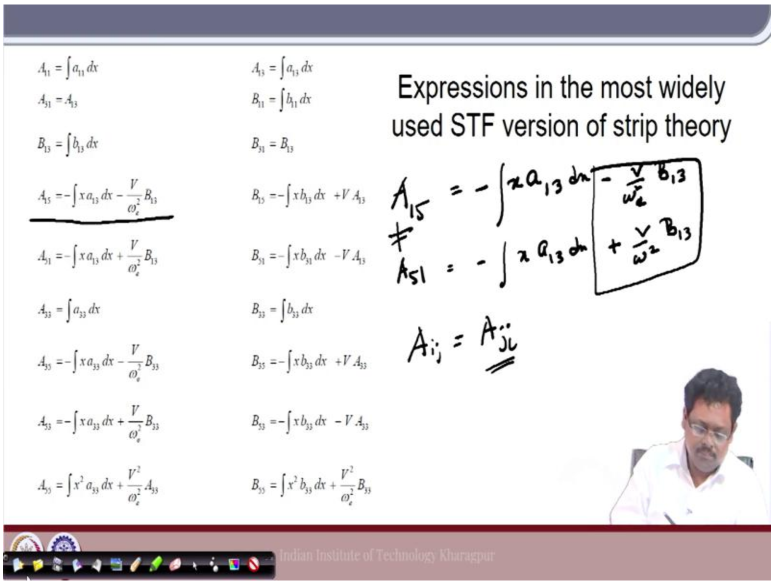


So, now this is actually the expression if you can see I just make a the list of the expression that is used in most of the popular strip theory code. So, here the thing is that if you have the two dimensional added mass this things definitely going to share. So, I think its little bit smaller, but still it is visible. So, in case of the vertical mode like surge and then heave and pitch if I coupled then these all the expression for all modes.

So, you can see here some interesting thing is that the forward speed effect if you look at this expression $A_{15}$, if you look at this expression $A_{15}$, I just writing over here a little bit bigger so that you can understand that, this is my point. These are the very you know it is you can see everywhere this expression there is nothing new I am writing over here.

So, it is just the interesting fact that I am going to discuss as follows. So, in case of $a_{13}$ you can see that the formula said it is $A_{15} = -\int x a_{13} dx - \frac{V}{\omega^2} B_{13}$. And also if I look the cross coupling term $A_{51}$, it is $A_{51} = -\int x a_{13} dx + \frac{V}{\omega^2} B_{13}$. It is not it is capital $B_{13}$ sorry. Now, you can see over here that in case of a forward speed $A_{15}$ is not equal to at $A_{51}$.

Now, as I said in case of a zero speed we always know that $A_{ij} = A_{ji}$, this is a very well known fact. So, that means, if I oscillate the body in the $j^{th}$ mode, if I try to calculate the force added mass in the $i^{th}$ mode, it is similar to if I oscillate the body in the $i^{th}$ mode and if I try to calculate the added mass in the $j^{th}$ mode that is actually for in case of a zero speed it is true, but in case of a forward speed it is not true. So, this is one interesting observation that one can get from this expression ok.

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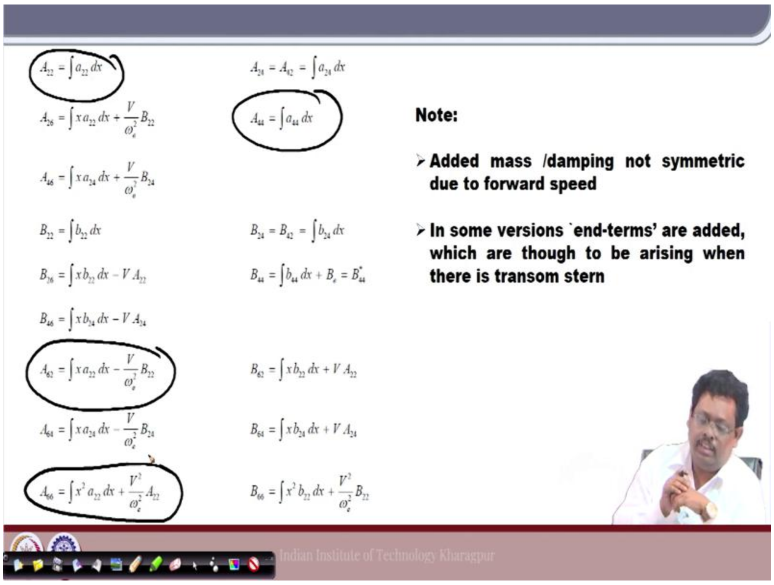


And this is for the coupled with roll the sway and yaw. And also here also you can see that there is a forward speed effect everywhere I mean in case of wherever is 6, wherever you have the yaw let us say for if you take this $A_{62}$ here, you can see there is a forward speed effect right.

Now, as such you can see that in $A_{22}$ actually there is no forward speed effect right; in $A_{44}$ there is the forward speed effect right, but whenever you can see that which involves the yaw. So, there you can have the forward speed effect. So, as I said that main reason is that m terms is for all first four mode is zero in case of a linear you know if you take the non-linear entire thing is different.

So, let us go with the linear. So, in so then the first four mode is zero and then you have the pitch and heave added mass ok. In this two added mass or damping coefficient you have the effect of the forward speed because m terms is not zero for this two particular mode, ok.

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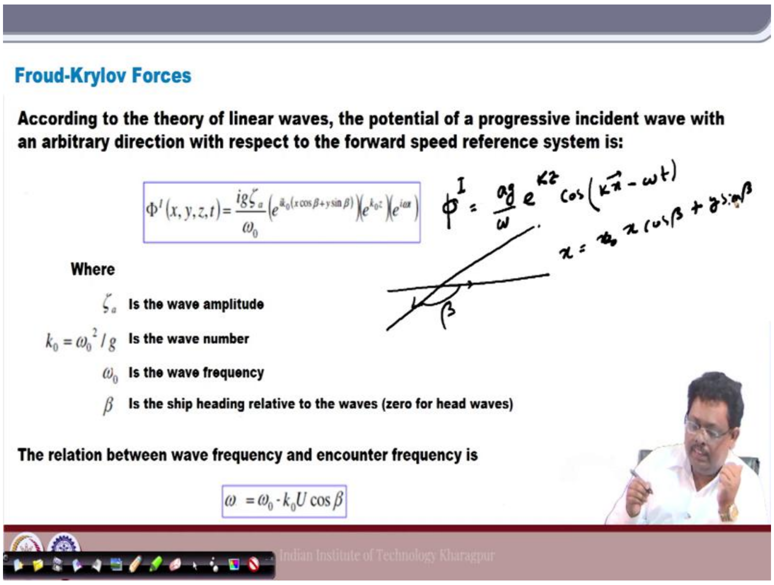


Now, so, this we talked about the radiation force. Now, still we have not you know discussed the we discussed a lot, like if I have this magnitude of the radiation force and then how we can incorporate the speed effect and we can get the added mass for the whole ship for all six different modes right that is what we discussed.

Definitely, in coming lectures we are definitely going to discuss how we are trying to get the zero speed added mass and damping and that is very important ok. And why it is so, important we will come to know when we discussed about the diffraction force. Now, in complex domain this is the expression for the phi. Now, in case of a time domain or if you take everything in the real line.

So, if you remember that now in case of a real line we know that expression for $\phi$ as in case of deep water, $\phi^I = \frac{ag}{\omega} e^{kz}\cos(\mathrm{kx} - \omega t)$. Now, here now I take that sinusoidal wave is coming and hitting you in 180 degree. Now, here you have suppose you have a heading angle $\beta$.

So, suppose you have an heading angle $\beta$. So, then definitely this x vector have two components. So, x we can say it is $x_0$ or sorry or it is $x\cos\beta + y\sin\beta$. So, I can make this x as this $x = x\cos\beta + y\sin\beta$.

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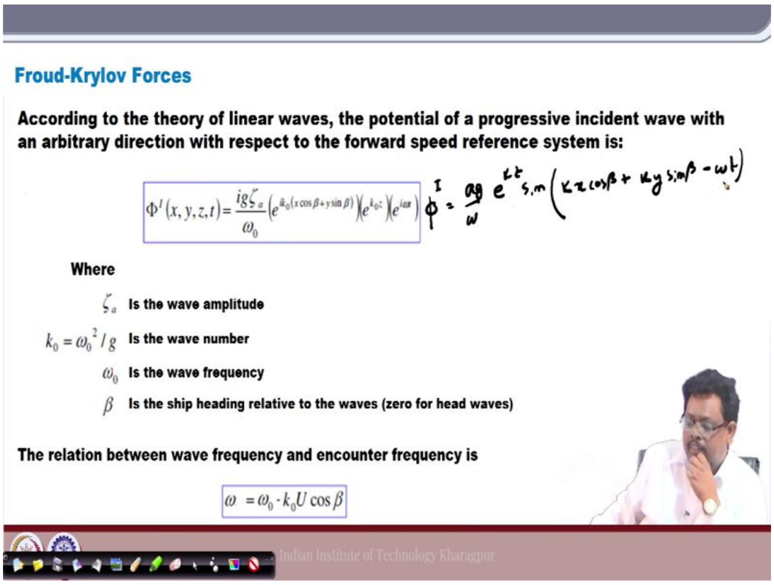


Now, if I do that even in case of a you know time domain in case if you use the real plane then it becomes $\phi^I = \dfrac{ag}{\omega} e^{kz} \cos(\mathrm{k}\, x \cos\beta + ky\sin\beta - \omega t)$. So, that would be the case. Now, here if you do in the complex plane.

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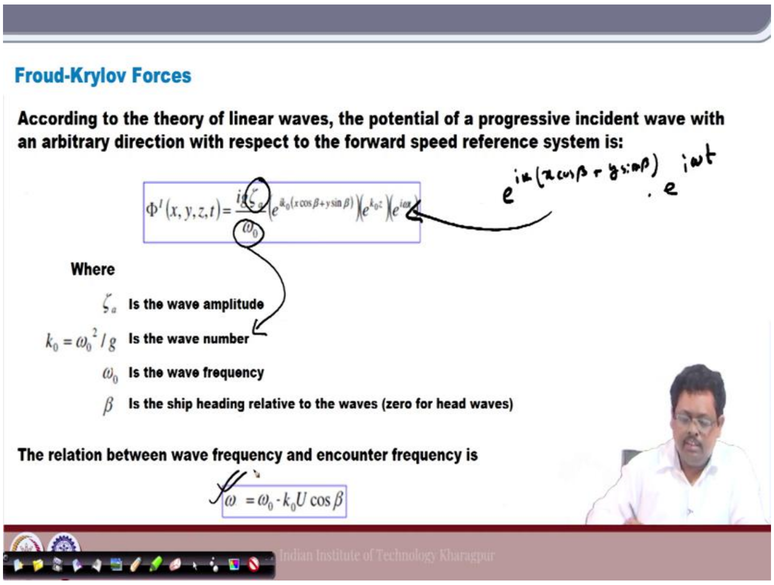


So, you know that if you if you want to do in the complex plane, so, then I write that expression as in complex plane as it is $e^{ik(x\cos\beta + y\sin\beta)}$. So, this is how I can write it. And then in case of a $\cos\omega t$, I can write $e^{i\omega t}$. So, that is what actually we have written over here. So, this is how we can write in frequency domain ok.

And then $\zeta$ of course, the amplitude of the wave. And here if you look at this $\omega_0$, this $\omega_0$ actually I use for the you know wave frequency, and then I use $\omega$ for the encounter frequency ok. So, now, in complex plane in frequency domain this is how actually we are going to write the expression for the $\phi$.

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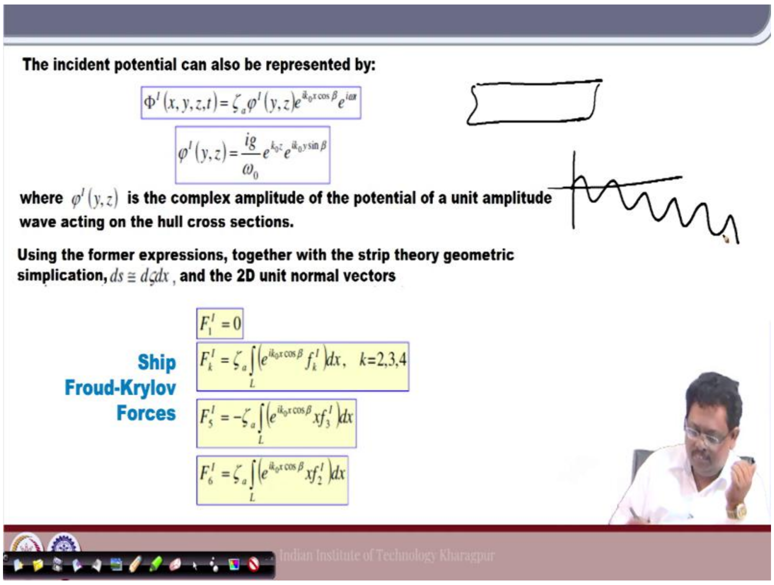


So, now, once I know how to write the $\phi$. So, definitely I know how we can write, you know the force right? So, a force is nothing but the pressure multiplied by the normal right. So, now, again we just try to relate with the our learning from the real line that that is a one way type solution when we will learn. Now, here since I need to find out a strip theory.

So, what I am going to do is. So, suppose I have a strip. Now, if you remember I said that you know this forward speed effect actually we are trying to get in exciting force where we are taking the effect of the angle heading angle right. And that is not true for radiation because in radiation there is no the wave is coming right; we are oscillating the body in the still water.

So, many people have some doubts like they say that I mean you know if you look at intuitively people will try to think that like in the mode of heave and in case of a roll in case of a pitch, only in these three mode we have a sinusoidal force. However, we do not have sinusoidal force in the sway in case of a sway yaw and also in the surge because there is no restoration right.

So, I mean, but, however it is sinusoidal in all modes right. So, one has to you know throughout that concept that these three modes have a restoration definitely. If you considered the radiation for radiation force is in all modes. So, it can oscillate. So, when wave is hitting the body. So, body can oscillate in all 6 degrees of freedom motion.

So, it can oscillate in heave mode it can oscillate in the surge mode also it can oscillate in the sway also, because this wave is hitting in some arbitrary angle right. So, if the wave hits in some arbitrary angle. So, therefore, this body can oscillate in all different mode right. So, this radiation force we have in all mode this that Froud Krylov force also we have in all modes and which likely to be sinusoidal right.

However, that three mode which is the which is surge, which is sway which is yaw we do not have a restoration. So, if you do not take care. So, you know sometimes if you get a time signal plot specially you can see that the surge may be oscillating and then it is go like this way, because it does not have any restoration I mean if you do not take care of this. So, it will like to be like as it goes it slowly coming down.

So, those are very nice thing like when you write the code you will realize all such things. And then that actually sometimes will helpful to write code because the understanding will develop otherwise in intuitively we can think that ok there is no restoration in case of a surge. So, therefore, the motion is not harmonic right. Anyways let us coming back to this here.

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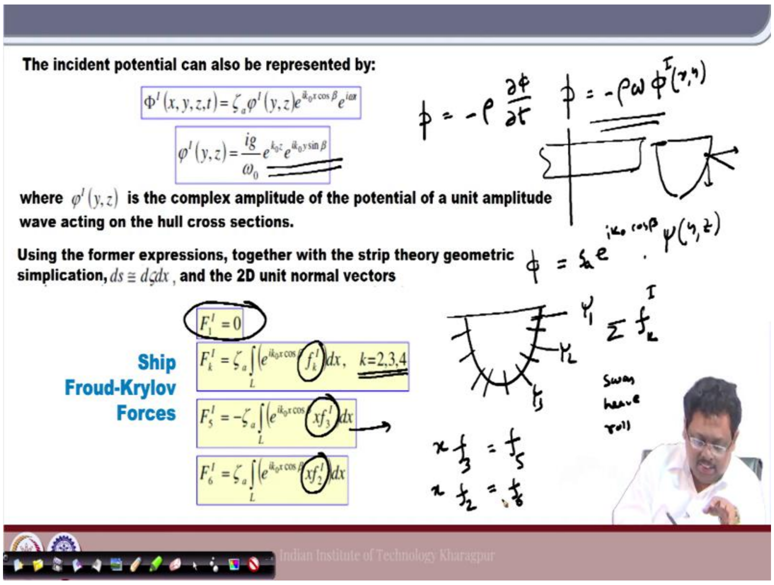


Now, if you remember that how I get the pressure the expression for the pressure $p = -\rho \dfrac{\partial \phi}{\partial t}$ right. Now, here everything I write in case of a frequency. So, it is this replaced by $e^{i\omega t}$ of course, right and so therefore, if you write the pressure here it is it comes at $-\rho\omega$ right, because there is a $e^{i\omega t}$ is there.

And then with this we can only have the $\phi^I$ which is the independent of the t. So, $p = -\rho\omega\phi^I(x, y)$. Now, if you try to get it for a section. So, even that actually we can further simplify the thing right because what is doing in strip theory, we are having this ship and we are taking you know cutting a strip and now we are doing everything in the y z plane right for a fixed x.

So, therefore, I can write this whole $\phi$, $\phi = \zeta_a e^{ik_0 \cos\beta}\psi(\text{y,z})$. And then actually this is the part which is independent of the you know if you select some x. So, it so this part actually you can really cut out. And then remaining thing we can write as $\phi = \zeta_a e^{ik_0 \cos\beta}\psi(\text{y,z})$.

And so therefore, this $\psi(\text{y,z})$ is only that y and z part is here right. So, therefore, now if I try to find out the force then force definitely this pressure should be multiplied by the normal. So, therefore, now in case of this normal we have. So, this is the normal we have two component; one is y and one is z. So, there is no component in the x direction.

So, this part is goes to 0 right. And then you have. So, then in other part no I . So, so again actually what I do is now here if you look at these three. So, this is the exciting force the sectional exciting force. So, how I get the sectional exciting force? So, I do this and then I split in some segment numerical I try to get it and in the midpoint here. So, I midpoint I get the value for this $\psi$ that is a $\psi_1$, I take $\psi_2$, here it is $\psi_3$ and so on.

And then simply I add this value. So, then this value actually I called this as $f_k$ is a $k^{th}$ mode for the incident wave ok. Now, actually we can have a several section of how we can get this exciting force right, maybe later stage or we can do that because it is so elementary we have already done for the real plane when you do the WAMIT type calculations already you know how to do this.

Now, here the thing that I want to say is now if I it is for the 2, 3, and 4 so; that means, in case of a sway, and then heave, and then roll we do that. Now, once we now it is for the pitch how we do that if I know the sectional Froud Krylov of force $f_3$ and if you multiplied with the moment x, then I can get the this is nothing but the sectional Froud Krylov force in the fifth mode.

And similarly if I know the sectional Froud Krylov force in the second mode and again if you multiply it by the x then I can get the sectional Froud Krylov force for the sway yaw I mean yeah movement.

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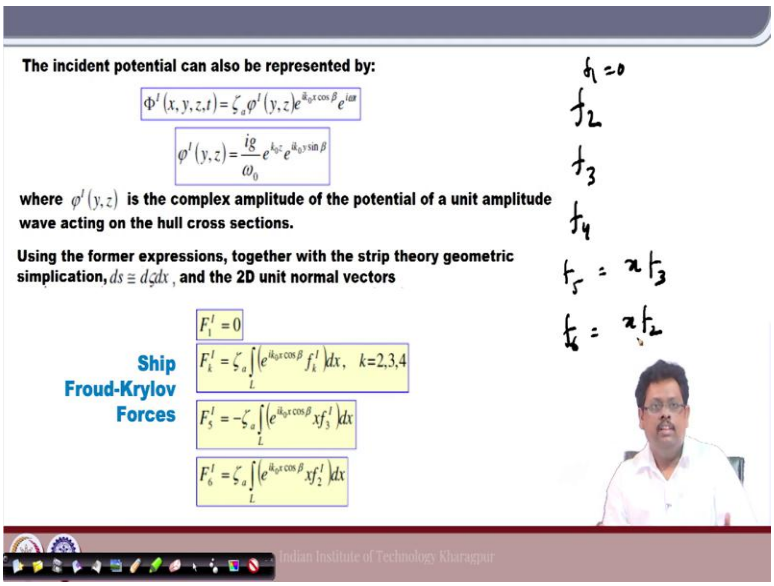

So, in this case now you see that it is sufficient to get the sectional force for the sway mode and for the heave mode and for the roll right, because $f_1 = 0$, and $f_5$ is nothing but $xf_3$. And $f_6$ is nothing but x into $f_2$ right ok, but then how we can get the two dimensional Froud Krylov force, and how we get the two dimensional diffraction force also that definitely we are going to discuss we are going to discuss in the next class ok.

So, for today we stop here.

Thank you.