Numerical Ship and Offshore Hydrodynamics
Prof. Ranadev Datta
Department of Ocean Engg and Naval Architecture
Indian Institute of Technology, Kharagpur

Lecture - 49
Non Linear Time Domain Panel Method

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 49.

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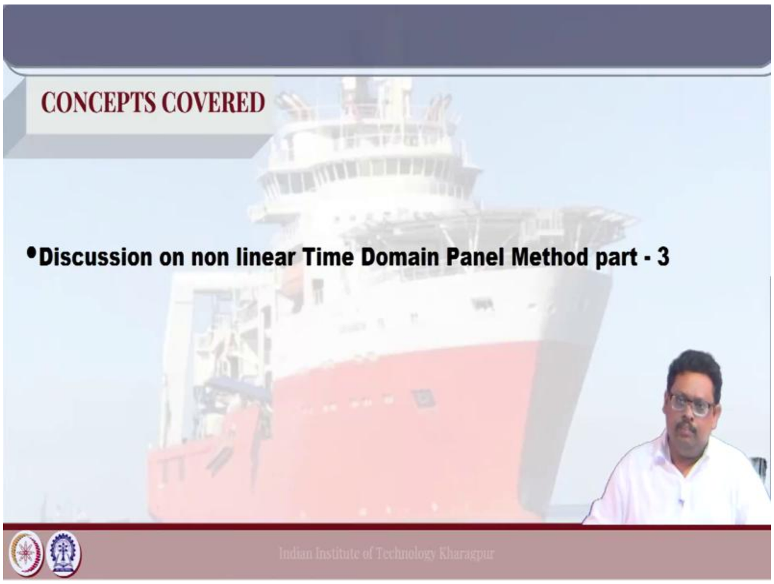

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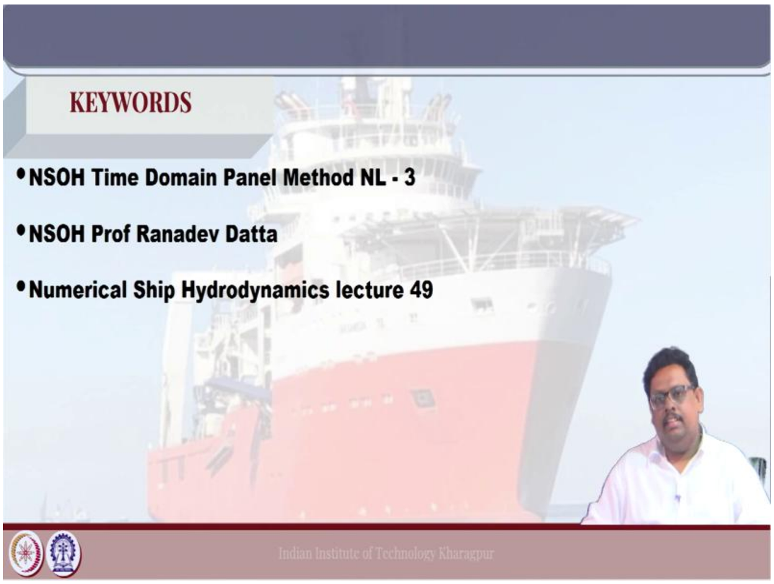

Today we are going to discuss on the Non Linear Time Domain Panel Method. It is a continuation of our previous discussion ok. And these are the keyword that we are going to use to get this lecture.

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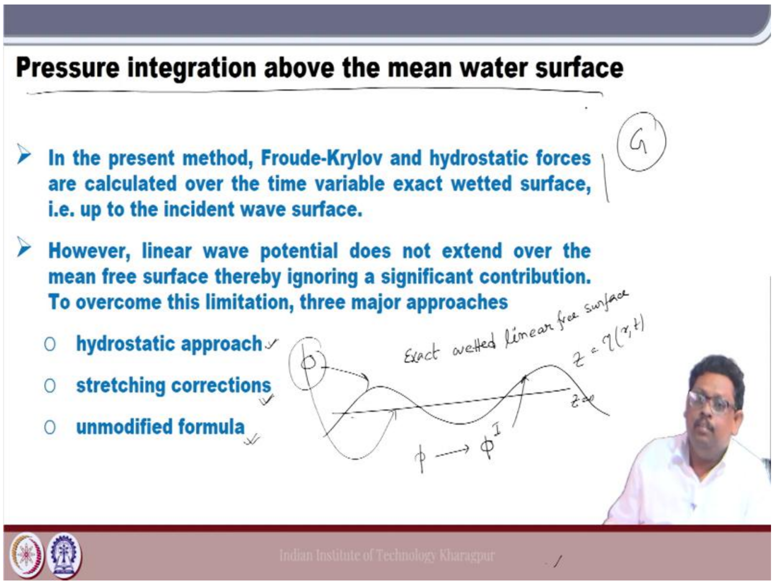

So, let us start. Now, if you remember that in the previous lecture we stop at a point that we need to find out that Froude-Krylov pressure, I mean the Froude-Krylov force over the non-linear wetted free surface. When we talked about the non-linear wetted free surface in last class we have discussed we are actually going to get through the incident wave potential.

The exact free surface, because of the incident wave potential. To be very frank if you want to modify further like even if the incident wave potential is not sinusoidal in general. It is not linear. Again under the assumptions of the linearity when you talk about the exact wetted free surface we are in fact, able to modify further the statement as follows. That sorry that it should be we can further modify the statement as follows.

It is exact wetted linear free surface ok. It is not exactly the actual question. Now, that is what is that is what actually it is written over here in the presence method. The Froude-Krylov hydrostatic forces are calculated over the time variable exact wetted surface. That is up to the incident wave surface ok.

Now, here also even if I do that we have a problem, because in under the assumptions of the linearity. We really under the assumptions of the linearity we really cannot go beyond Z = 0 ok. So, this you know we have repeatedly we told that the Greens function G, you can use the Greens function G only when you are using the linear free surface that is Z = 0.

Now, if you even take it is Z equal to you know $z=\eta\left(x,t\right)$ in this even if it is a linear free surface. Still you cannot apply the G. So, therefore, when we are actually calculating the ϕ it is not possible for us to calculate this ϕ on this surface. We can always get this you know phi on this surface. However, what we are going to argue is that we can calculate the pressure, because of the ϕ^I on this surface with some modification

Now, how we do that? There are three classical approaches we can discuss. One we call the hydrostatic approach, one we call the stretching corrections, one is called the unmodified formula. Now, in all like we need to understand how we can calculate the pressure above the mean free surface?

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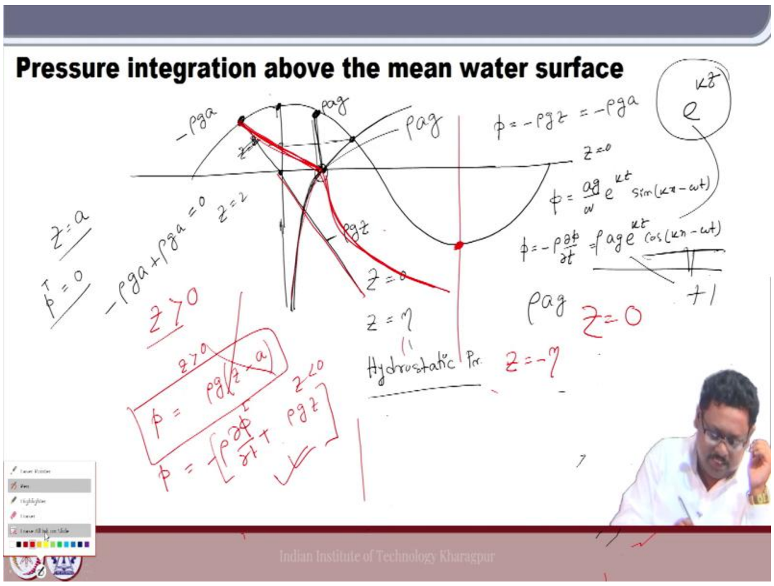

Let us now discuss. So, when we talked about the pressure integration above the mean water surface you know what I mean. Now, let us draw one picture I have this and now let us try to super impose I mean just draw a linear waves. So, you can call this is trough, this is crest, this is trough. So, now, here let us try to find out about the Froude-Krylov pressure. Now, you know that you have this expression $\phi = \frac{ag}{\omega} e^{kz} \sin(kx - \omega t)$.

Now, pressure p is nothing, but $-\rho \frac{\partial \phi}{\partial t}$. Now, if you do minus to $\frac{\partial \phi}{\partial t}$ then ω will cancel out. Then it is minus and minus plus right. It should be $\frac{ag}{\omega} e^{kz} \cos(kx - \omega t)$ and of course, we have a ρ right? So, this is my ϕ. Now, if you look at it is driven to the e^{kz}. So, this pressure term is driven to f term called e^{kz}.

Now, z is negative below the free surface. It is z = 0. So, when z less than 0 and then this is the below, I mean the water and when z greater than 0 it is the above mean water surface. Now, if you look at it this expression the pressure expression actually driven by a exponential function. So, therefore, suppose this is my z = 0, this is the pressure term.

Now, if you put z = 0 and remember that at let us take that maximum pressure. So, $\cos(kx - \omega t)$ at crest it is value takes the value 1. So, this term goes to plus 1. So, eventually you have this term at z = 0 it is nothing but ρag. So, this point the value of this point is nothing but ρga ok.

Now, since this driven through e^{kz} at z equal to 0 e^{kz} equals to 1 right, because $e^0 = 1$. So, then value remain ρga. Now, here it should drastically reduce as a exponential function. So, this is the graph, below z = 0. Now, above z = 0. What would be the you know the pressure?

Now, here if you remember I said that there is a three method. When this unmodified formula if you use this it means that I really do not change this formula I use the same formula above the z = 0 also. So, therefore, if you go through this then this e^{kz} is now continuing above the mean free surface also and therefore, it goes like this way. Something like this.

Now, so, then in that case this is actually the pressure at free surface right fine. So, this is called the unmodified formula. Now, when you use this unmodified. So, there here actually it is not realistic. Why it is not realistic? Because since it is driven by e^{kz} then for this steep wave this value of e^{kz} at this particular point let us say it is let us it is z equal to some positive let us say even this some let us say 2 ok.

Let us take that z = 2 this point then value could be e square some large values it is. So, for the steep wave this really gives very unrealistic value. So, unmodified formula really does not help much. Then let us talk about the hydrostatic approach. Now, this hydrostatic approach actually little bit consistent as well. Why? Because the hydrostatic approach says that we are using the concept of linearity.

What is the concept of linearity? Concept of linearity say what is happening at z =0, the same should happen at $z = \eta$. Now, at z = 0 my pressure is ρga. So, at z = η also it should be you know ρga where a is nothing but my amplitude of the incident wave.

So, therefore, the pressure value what is here the same pressure value is over here and then now this is basically my graph for the incident wave potential pressure graph. Now, let us why it is consistent let me tell you. Now, if I look at the hydrostatic approach. So, now, if I try to apply the hydrostatic pressure over here. Now, at z = 0, the hydrostatic pressure is 0 and then it is linearly coming down right.

And with the formula ρgz right. Actually it is $-\rho gz$. So, when z equal to negative it become positive. So, that is why it is in this coordinate right. So, hydrostatic pressure $p = -\rho gz$. Now, in a below z = 0. It is negative. So, it is positive now what is above then what is the I mean what is the value at $z = \eta = a$? So, at z = a and a is positive right?

So, $-\rho gz$ will give you $-\rho ga$ right. So, then this is the value you get the hydrostatic pressure at this point. So, it is $-\rho ga$ right. Now, you see why it is consistent? Because at free surface the total pressure the total pressure should be equal to 0. That is our dynamic condition.

Now, my static pressure gives my the value ρag. The dynamic Froude-Krylov pressure gives the value $+\rho ga$ and if you add these 2 you will get 0. So, it is somehow it is consistent at wave crest. So, people are; so, this is the hydrostatic approach. So, what is hydrostatic approach? When you have; when you have; so, now, hydrostatic as let me now just draw is that the total pressure just change the color.

So, hydrostatic approach is above mean free level the total pressure. So, hydrostatic approach cannot distinguish between the what is the static pressure, what is the dynamic pressure? This simply tells you what is the total pressure together with hydrostatic and the dynamic? So, when. So, therefore, when z is greater than 0 it follow this line right. So; that means, when z greater than 0 that pressure is follow rho into z into.

Now, we can call it is you can say it is z minus a. Something like this right. So, this is when z greater than this thing. Now, you can see it is very it is actually consistent also right. So, and then after that when z less than 0, the value that of the value p should be $-\rho \frac{\partial \phi^I}{\partial t}$ which is the this graph and then together with $+\rho gz$ which is the this part.

So, this is the hydrostatic approach. We have two component; one is above mean free surface, one is below mean free surface right. And when is above mean free surface it follow this straight line. $-\rho$ started with $-\rho gz$ and an end with ρga here.

And after that it if you add this two then actually the; so, that total pressure graph look like somewhat like this. So, this is the non-linear pressure plot ok for the non-linear Froude-Krylov force Froude-Krylov pressure. So, here when you do the non-linearity you really cannot split out the static component and the dynamic component above mean free surface, but the below mean free surface you can of course do that.

And now the stretching algorithm is it is just extension of these things ok. Here it if you do this now in case of a bottom actually now when in case of a trough. In case of a trough we really do not do this. We really do not use this idea that what is happening at z equal to what is happening at z = 0 same thing happening at $z = -\eta$.

Here we are assuming that what is happening at z = 0. The same thing happening at $z = \eta$. So, that is why in the free surface also we are taking the value ρga. However, in hydrostatic approach when you below z = 0 that time where we do not use this upper part. We do not use it we can only use the lower part

Now, here is the difference between the this hydrostatic approach and the stretching algorithm. In case of a stretching algorithm we are assuming what is happening at z = 0. The same thing happening at $z = -\eta$ also. So, that if you apply this stretching algorithm then it is consistent at crest. It is consistent at trough.

Now when I say that consistent the crest means that at crest that at free surface the total pressure equal to 0 and here also if you use the stretching algorithm. Similarly at this point also it is 0. Why? Because we are assuming now let me discuss why it is 0 here also which is not for the hydrostatic case.

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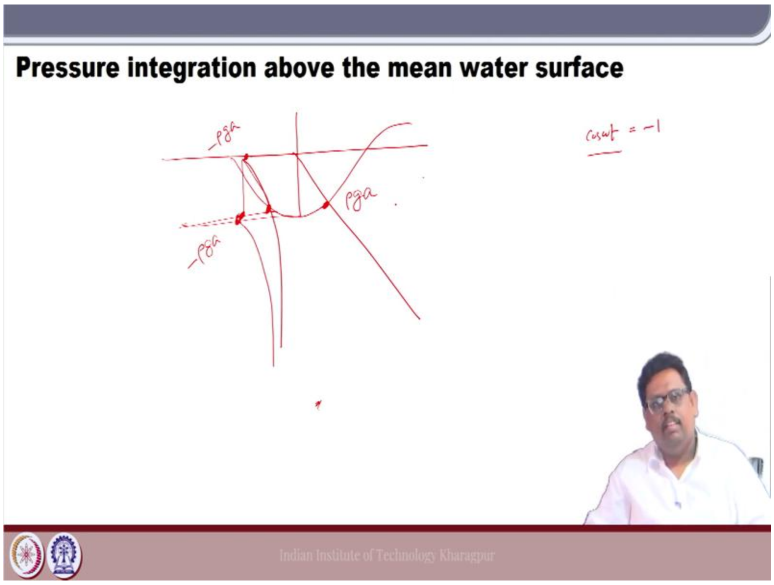

Now, here if I draw the crest. If I draw the crest. Now, here the now if you consider if you consider the hydrostatic approach since it is z less than 0. So, at z = 0 at crest it is $-\rho ga$. Why it is $-\rho ga$? Because in this at the crest that $\cos(\omega t) = -1$, that is why. In crest $\cos(\omega t) = +1$ and in that in case of a trough the $\cos(\omega t) = -1$. So, now, it is as it is goes like exponentially down and then in the hydrostatic also it is the linear like this.

Now, if you see that at this point the value is ρag for hydrostatics. Now, in case of a now we are not using the stretching algorithm. We are using this graph and it is decay exponentially. So, at this point this value is less than $-\rho ga$. So, therefore, we have a anomaly in case of hydrostatic approach, but we ignore this anomaly, because it is a single point.

However, in case of a stretching algorithm if I use that what is happening at z = 0. Same thing is happening at $z = \eta$. So, then actually this becomes the plot of the dynamic pressure. So, then at this particular point that value should be $-\rho ga$ and; so, therefore, the if you do the addition it becomes 0.

Now, at this point I must tell you one thing that whether you are using the hydrostatic approach, whether you using the unmodified formula, whether using you are using at the stretching algorithm nothing is you know very much justified by the fine you know sophisticated mathematics.

It is simply some kind of a technique that actually we are using to get somehow the non-linear portion of the Froude-Krylov force. Now, if somebody tells now is it is mathematically sound, it is mathematically well justified, it is all these I mean all these things actually the answer is it is not ok. However, as an engineer we are trying to modify the reality try to replicate the physics with simple mathematics.

So, all these three method is somewhat not very sophisticated in the mathematical point of view, but it is very practical especially the hydrostatic approach ok.

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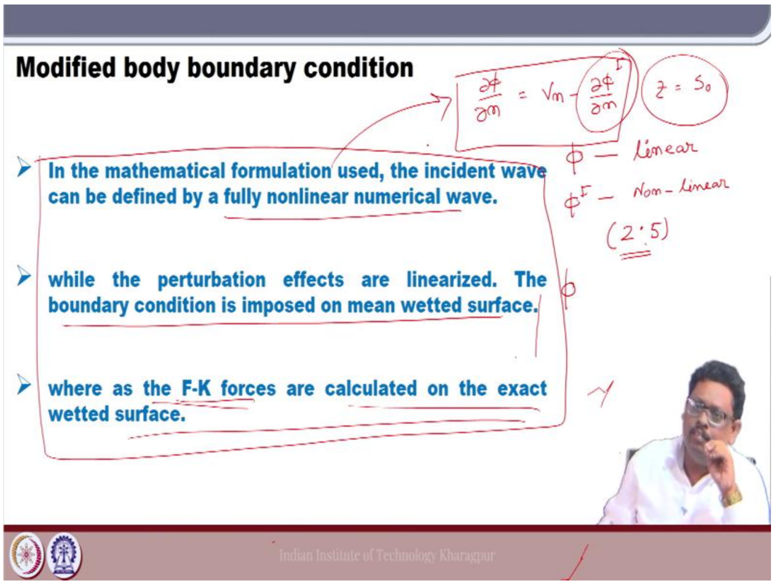

Now, here let us do one more thing that in this non-linear formulation. If you remember what is my boundary condition? This boundary condition is which is $\frac{\partial \phi}{\partial n} = V_n - \frac{\partial \phi^I}{\partial n}$ and this apply at z = S_0; that means, the mean wetted body right?

Now, if it is so, now, I am using a non-linear approach. Now, here initially if you remember that our level 1, level 2, level 3, level 4 approach. If you see our previous lecture, in level 2 approach what we say that we say that we do phi linear and then ϕ^I , it is non-linear. That is what we discussed right. Now, here let us now introduce some something not level 2, not level 3. So, you can call this a level 2.5.

Now level 2.5 what we are doing now what we are trying to do here. It is very interesting thing we really do not want to disturb the framework of the linearity. We really do not want to disturb the framework of the linearity; however, we try to impose some kind of non-linearity in this equation. So, that we can get some non-linear effect in the component of the ϕ also, ok clear.

So, here the idea is we do not want to disturb the framework of the linearity. So, that means, that we are exactly we are doing z = 0 we apply over here S_0, however, here we try to impose some kind of non-linearity into the second term. Now, how it is possible? Again I am telling you that there is no much standard sophisticated mathematics behind this sort of approximation or ideas. However, you know it is coming little bit of some sense of physics. We try to do some analysis.

And try to see that if these effects are really effective or not ok. Now, here if you look at this line that what is said that. In the mathematical formulation the incident wave can be defined by a fully non-linear numerical wave which is which is absolutely the best thing that one should do, but here in this formulation we did not do that. While the perturbation effects are linearized the boundary condition imposed on the mean wetted surface. That is what we are doing. Like we impose the boundary condition at the mean wetted surface fine.

And here that F-K force are calculated on the exact wetted surface. Now, we just discuss that how we can calculate the exact wetted surface right. So, this is also this is actually our idea. Now, here we add one more thing these all are this is these all things are based on your level to computation. Here we just want to add one more thing which is, I try to modify the boundary condition to incorporate some kind of non-linearity. Let us see how we do that.

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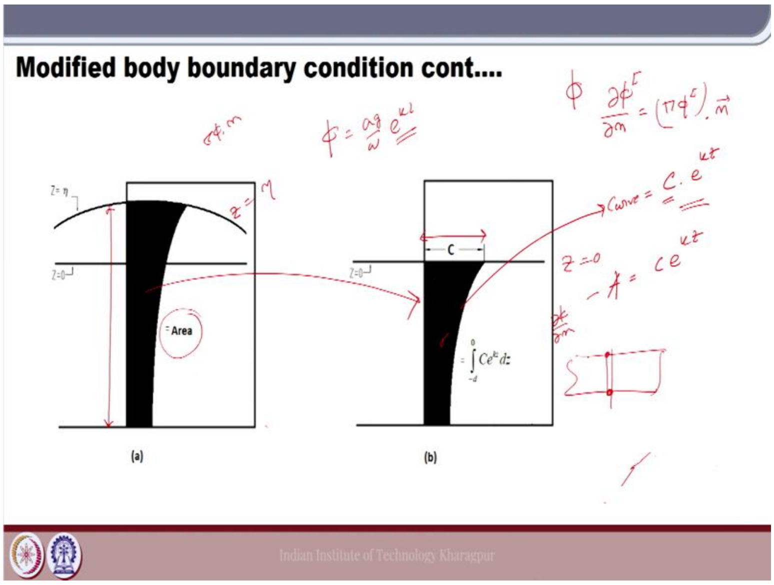

Now, you see now we are applying the boundary condition at z = 0 ok. Now suppose I have the wave ok. So, this is the pressure plot. Let us say unmodified or modified whatever. Now, what I am doing over here I try to calculate I have this value for ϕ. Now, remember that $\frac{\partial \phi^I}{\partial n} = \left(\nabla \phi^I\right).\vec{n}$. So, let us say this $\left(\nabla \phi^I\right).\vec{n}$ actually I plotted over here.

Now, what is the $\left(\nabla \phi^I\right)$? $\left(\nabla \phi^I\right)$ is nothing but the velocity potential dot n in the normal direction right. So, you can see that it is some kind of a push right. So, if I; so, physically what we assume that it could be some kind of a push we are pushing something. So, we are getting this much of area. And we are doing it at free surface. So, I am doing it at $z = \eta$. So, I get this area.

What I assume that we are mapping this area into the linear. So, what is happening here I am getting z = 0. So, what I am doing that this area and this area should be same. Now, if this area is the same then what would be the value for c? I am assuming this pressure graph you know this $\phi = \frac{ag}{\omega} e^{kz}$. It is defined by e^{kz}. So, therefore, this curve also can be written as let us say this curve I write this $curve = c.e^{kz}$.

Now, if I know the value of c; if I know the value of c, so, then if I integrate this value. So, this area and this area should be the same. That is my idea. So, then what actually I am equating? I am equating the total push by the ϕ^I should be same. So, that push I distributed over another hypothetical curve which is $c.e^{kz}$. So, total react $\frac{\partial \phi}{\partial n}$ is now it is changing to this.

So, the only objective how I get this value c. If I know the c. So, then I know this curve let us call this $curve = c.e^{kz}$. And this is nothing, but the curve of my modified $\frac{\partial \phi}{\partial n}$ ok. So, then since from then from this curve I can get that value of $\frac{\partial \phi}{\partial n}$ at any point right? So, that is what actually our idea. Now, to do that what we are going to do is that we are equating the area.

So, now, I first I calculate that ($\nabla \phi . \hat{n}$) over this length. I integrated from here to here. So, I get this area and then I equate this area with this. So, that is what I do.

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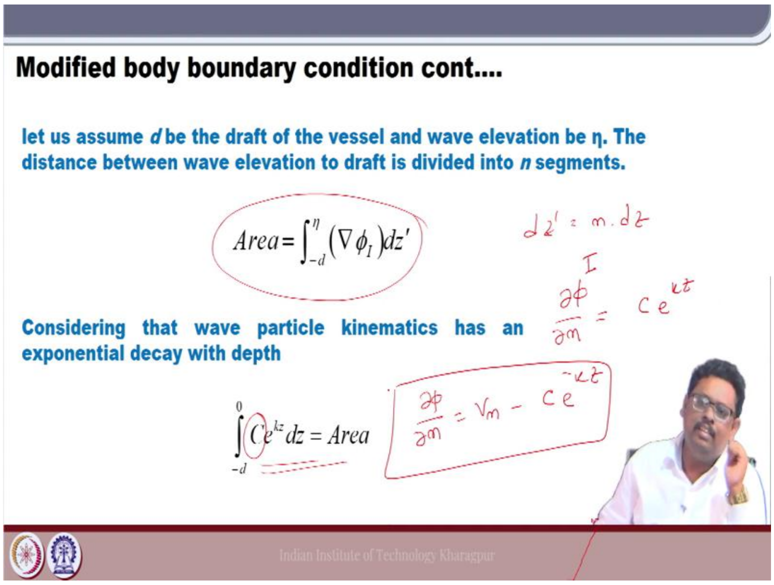

So, initially from -d to η, which is nothing but if you look at this say this is your minus the depth -d to η. We integrate this value $(\nabla \phi).\vec{n}$ ok. So, this dz' in this dz', it is basically $(\nabla\phi).\vec{n}$. So, $dz' = n.dz$. So, we are getting this area and this area I am equating with this graph. So, I know the value of k, I know the value of z.

So, now, it is -d to 0 and then -d to 0 is nothing but my what is d? d basically in the ship, this keel point is nothing but my d. So, now, I integrate -d to the free surface z = 0. So, when you do this we equate this I get the value of c. Now, once I get the value of c. So, now, I know my $\nabla\phi$, $\frac{\partial\phi}{\partial n}$ follow the $curve = c.e^{kz}$.

So, therefore, this boundary sorry my $\frac{\partial\phi^{I}}{\partial n}$. So, now, this $\frac{\partial\phi}{\partial n}$ now I can write as v $\frac{\partial\phi}{\partial n} = V_n - c.e^{kz}$. So, this is now my modified body boundary condition ok. So, this is how actually we are incorporating the some kind of non-linearity in the body boundary condition. So, that I get some non-linear value for the ϕ ok.

So, in future lecture we are going to see that whether it really helps or not that definitely we are going to discuss from our next classes ok. So, today we are going to stop here.

Thank you.