

Numerical Ship and Offshore Hydrodynamics
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Lecture - 53
Hydroelasticity

Hello welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 53.

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Today we are again going to discuss about the ship Hydroelasticity. What we are going to do today is that we are going to discuss about the overall structure of the code like starting from the hydrodynamics and how I get the structural deflections.

We really do not discuss much detailed about the structure analysis. Remember that hydrodynamic solution as I said in the last class more or less similar to apart from the body boundary condition more or less similar to that what we discussed before for the rigid body case. However, in case of a structural solution we are using finite element method we are using the beam model like the details of this is this it should be a separate course.

But; however, whatever is necessary for this solving this particular problem we are going to discuss in the future class, but today we restrict ourselves for to demonstrate the overall pattern of the code, ok.

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KEYWORDS

- NSOH Hydroelasticity - 2
- NSOH Prof Ranadev Datta
- Numerical Ship Hydrodynamics lecture 53

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Now this is the key word that you have to use to get this lecture. Now let us start.

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Mathematical Formulation

Boundary Value Problem

$$\phi_t(\vec{X};t) = \phi_s(\vec{X};t) + \phi(\vec{X};t) \quad (1.1)$$
$$\nabla^2 \phi(\vec{X};t) = 0 \quad \text{on} \quad \vec{X} = (x, y, z) \in \Omega \quad (1.2)$$
$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0 \quad \text{on} \quad z = 0 \quad (1.3)$$
$$\phi, \phi_t \rightarrow 0 \quad \text{as} \quad R_{xy} \rightarrow \infty, \quad \text{on} \quad z = 0 \quad (1.4)$$
$$\phi, \phi_t \rightarrow 0 \quad \text{as} \quad t \rightarrow 0 \quad (1.5)$$

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Now, this is the equation the boundary value problem that we discussed yesterday last class that is the total potential should be split into two and all these things all these boundary conditions over here apart from the body boundary. Remember here you can see here we do not have the body boundary condition.

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Body Boundary condition

| Rigid | Flexible |
|---|--|
| $\frac{\partial \phi}{\partial n} = V(t) \bar{n} - \frac{\partial \phi_t}{\partial n} \quad \text{on } S_b$ | $\frac{\partial \phi}{\partial n} = V(\bar{X}, t) \bar{n} - \frac{\partial \phi_t}{\partial n} \quad \text{on } S_b$ |

(1.6)



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And that is what we discussed in the last class that is where there is a change.

Now, in case of a. Now in case of a rigid body this as I said it is only depending on the time the velocity not depending on the space; however, in case of a flexible structure it is dependent on the space also, ok.

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Integral Equation

$$\phi(p, t) = -\frac{1}{4\pi} \left\{ \iint_{S_b(t)} \sigma(q, t) G^0(p, q) dS + \int_0^t d\tau \left[\iint_{S_b(\tau)} \sigma(q, \tau) G_t^f(p, q; t - \tau) dS \right] \right\} \quad (1.7)$$

$$\frac{\partial \phi(p, t)}{\partial n_p} = -\frac{1}{4\pi} \left\{ \iint_{S_b(t)} \sigma(q, t) \frac{\partial G^0(p, q)}{\partial n_p} dS + \int_0^t d\tau \left[\iint_{S_b(\tau)} \sigma(q, \tau) \frac{\partial G_t^f(p, q; t - \tau)}{\partial n_p} dS \right] \right\} \quad (1.8)$$


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And this is the Green's function I mean that source potential or integral equation that we are going to use to get the solution for the ϕ , right. Now, here remember that this 1.7 and

in 1.8 this is only for the floating structure. Now in case of a forward moving body there should be another term should be with this which is about the water line that is absent here.

So, it is just another one term that is it; however, this is the main integral equation for the floating body. Let us discuss this whole exercise for this floating body problem only, ok. And then you need to differentiate with respect to the normal. So, from 1.7 you get 1.8. Now in 1.8 you have to solve for the sigma. Now once you solve for the sigma again you go back to this 1.7 and you can get the value for ϕ .

So, this is the basic structure of the hydrodynamic code and this is what we have discussed in fact, in last couple of weeks it is started from the frequency domain panel method, then we have strip theory, then we have time domain panel method in everywhere. Now we know this is how we obtain the value for the ϕ . Now here in case of the flexible structure.

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Pressure and Sectional Force Calculation

$$P(\vec{X}, t) = -\rho \left(\frac{\partial(\phi + \phi_1)}{\partial t} + \frac{1}{2} (\nabla\phi + \nabla\phi_1)^2 + gz \right) \quad (1.9)$$

$$\vec{F} = \iint_{S_0} P \vec{n} ds, \quad \vec{M} = \iint_{S_0} P (\vec{X} \times \vec{n}) ds \quad (1.10)$$

$$F = \sum_{i=1}^m p_i n_i d s_i$$

The slide also features a diagram of a ship hull cross-section and a grid of points representing a discretized surface. A small inset image shows a person speaking.

Once you get this value for ϕ , we have to get the pressure as well as the force. Now this force here this 1.10 this force is that we do not want because this is the force acted on the body and it is applied only for the rigid body scenario, right. In rigid body what is happening?

Suppose you have the ship over here and then you just discretize it number of panels. And then you find out the pressure at each panel and then you integrate this and then you can get the total force and this total force applied at the cg this is for the rigid body; however, in case of a flexible structure what is happening that this pressure now this force actually I really do not want at the centroid. What we need we need a pressure distribution or the force distribution in fact. So, what I do over here?

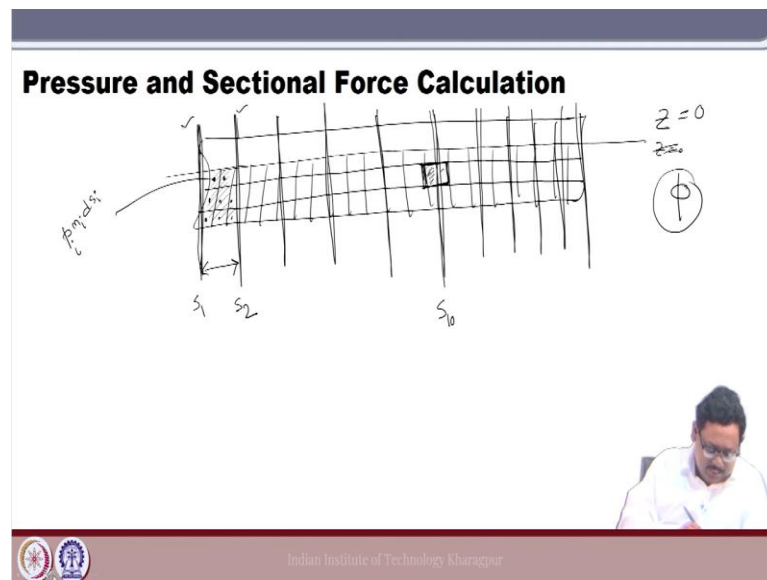
Now, in case of a rigid body we have the panels. And now I am happy with the getting the force here at the cg and then I solve the equation of motion and then I can get the value for the displacement velocity whatever that you want. Now here what we need to do is that I need the force it is the force per unit length. So, F per unit I mean length.

So; that means, I need the sectional force, why? Because this sectional force that is my input to my structural solver. Now see as similarly the moment I mean not movement here in fact very specific you know we have considered the all kind of modes here. So, explicitly we really do not want the moment equation here is simply the force equation and then the structural element to solve the structural problem to get the generalized the velocity and then displacement etcetera etcetera anyway.

So, now the problem is, how I can find out the sectional force? Right. I know that how can I find out the total force, right. I have this expression the pressure term over here. So, I integrate this pressure dot n and then I integrate over the body. So, if I write it in discretized form, it should be F is equal to summation i is equal to 1 to n. So, number of panels.

So, then each panel I am finding out what is the centroid that the pressure at the centroid of the panel and then I multiply by the normal at that particular panel and then I multiply by the area of this particular panel. Once I do that then if I add everything, I can get the total force, right, ok. But now in case of the structural you know the flexible structure I want that sectional force. Now let us see that how could I get the sectional force.

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Now, the first step what I need to do is, suppose I have a panel. I mean there is a many way for doing it, but that is what actually we do here normally. Now this is the Z equal to 0 line of course just a little bit bigger. So, that you can get it, ok. And then now let us first now, what I do is let us do the panelling first. So, I am doing the panelling. I am doing it for you know some arbitrary fashion and now let us draw some sections.

So, this is the section one let us take and then let us say this is another sections and then this is another sections. So, this is another sections this is another sections this is another sections this is another sections is another one. Now we have the we have lot of sections. Now what we have right now after solving ϕ after the solution of ϕ what we have is the what is the force value at that particular panel, how? Here the force value is nothing but my pressure at this particular.

Let us say p_i the pressure and then n_i the normal and then ds_i of course, we know that. So, what now we can do is I know that what are the panel between this and this section. So, let us say this is a sectional line number 1 this is sectional line number 2. Now within the sectional line number 1 and with the section line number 2 what I get is I can get that all the panel.

So, this is my overall the force between these two sections. And then what I can do is I can average out this force and I can distribute half here and half here. So, this is the idea.

Now here you can see there is a catch. What is the catch? Catch is that is why I made this type of panel. Now this is let us take this panel, I just make it little bit darker.

So, let us say this is the panel and then this is my the section line. Now you can see that some of the panel I mean sectional lines are here you know left-hand side some part of the panels in the left-hand side of the section let us take let us say 1, 2, 3, 4. Let us say S let us take S 10.

Some part of this panel is some part is the left-hand side of the section 10 and some part is the right-hand side of the section 10. Then how can I take the contribution of this panel. Now, this could be fairly complicated if I do like this way. So, normally we remove this possibility from the beginning. What we do here?

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Pressure and Sectional Force Calculation

$$F_i = \sum_{j=1}^8 \rho \cdot g \cdot h_j \cdot dS_j$$

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Let us see this is how actually one should do the panelling. Now you see that this is my geometric input and I have every right to do that as per my choice. So, this is some trick we are following.

Now, here again first I am write I am doing the section lines. So, now, this panelling I do later first I do the section line. Now once I do the section line, I make the panel such a way that you know this panel either start from the section line or it finishes at the section line. So, if I do this now you see this is my panelling. Now if I do this, I can avoid all sort of confusion.

Now, this is the section line and then all the panel are within the sections. So, now, I am ready to average out everything, right. So, now, you can see within let us say S_i and S_{i+1} . Now within this I have 1, 2, 3, 4, 5, 6, 7, 8. So, I have this eight panels. So, I can do the contribution using.

So, I just add this i is equal to 1 to 8 this all eight panels with p_i into you know d_i and then I can get the total force at let us call this F_i and then I average out this velocity by dividing by the length sectional length L and then I make half at this node and another half at this node.

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**Transforming Hydrodynamic Load to Structural Code
(Fluid - Structure Coupling)**

Sections for Structural Codes

Required to shift the hydrodynamic pressure from panel centroid to sections

Pressure Obtained from BEM code

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Now you see that is actually where we have done here. Now here it is very nicely again demonstrated to you. I have this is the section lines, right. So, these all are my section lines now. Now here this all this ϕ_i is distributed at the centroid at the panel. So, the question is if I have the centroid of the panel, it is more simplified thing, right.

It is more simplified because you can see within two section lines you have only one single panel, right anyway. So, if I have the value at centroid how I can shift this value to here and here? So, there is not there is no as such mathematical formula is available you can do as per you know your realistic thinking or common sense mostly.

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$$f(1) = \sum_{j=1}^m \frac{P^M(j)A_p(j)n(j)}{2W_p(j)} \quad \text{for } X(j)|_{Panel} < S_2 \quad (1.11)$$

$$f(n) = \sum_{j=1}^m \frac{P^M(j)A_p(j)n(j)}{2W_p(j)} \quad \text{for } X(j)|_{Panel} > S_{n-1}$$

$$f(i) = \sum_{j=1}^m \frac{P^M(j)A_p(j)n(j)}{2W_p(j)} \quad \text{for } \begin{cases} X(j)|_{Panel} < S_{i+1} \\ X(j)|_{Panel} > S_{i-1} \end{cases}$$

So, what we did here we use this formula. Now what we did this we take as I said I take the all the panel contribution between two sections, right. Now let us take for the first section. We have now in general what we have that we have between two section line there are lot of lot of panel is available.

But here in case of one only one side. So, in case of the first panel we are using only this panels, ok. Now in case of a section 2 or again in case of n^{th} panel I am using only this sort of this panels only these two panels that is what is written.

And then we are averaging out and we are distributing; however, in intermediate panels let us say for this particular section we are taking from here to here and then we are averaging out, ok. So, you see that is what we have done. In case of a in general section, section i in general section, section i what we do is, we take this range between i minus 1 to i plus 1 and then we are doing the averaging of this particular section.

So, that is why we are doing here. And in case of a first panel, I get this only for this panel, but I do not have this other side of the panel. So, therefore, I take the total contribution I average out and then I distributed half this side and I distributed half this side. So, this is how actually we are transferring my centroid the force at the panel centroid to the sections, ok. So, that is how we are going to write the sectional force, ok.

So, as I said that it is normally in a very frankly it is the pressure is continuously varying functions, right. So, therefore, this may be an approximation, but this is justified because I am assuming that variation of the pressure also not very radical. So, if it is not then this formula or this formulation definitely is going to work; however, may be for impact problem this might not work very nicely, ok.

So, this is about that how we are transferring the force or the pressure from the panel centroid to the section line.

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Solution for the Structural Deflection

➤ Considering floating body as an equivalent Euler Bernoulli beam on elastic foundation. Then, the equation of motion can be expressed as

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 w}{\partial x^2} \right) + \rho A \frac{\partial^2 w}{\partial t^2} + k_w w = f(x, t) \quad (1.12)$$

Handwritten annotations: "dynamic force + H^s" with an arrow pointing to $f(x, t)$; "hydrostatic force" with an arrow pointing to $k_w w$.

➤ The boundary conditions are :

$$EI \frac{d^2 W(0)}{dx^2} = EI \frac{d^3 W(0)}{dx^3} = 0 \quad \checkmark$$

$$EI \frac{d^2 W(l)}{dx^2} = EI \frac{d^3 W(l)}{dx^3} = 0 \quad \checkmark \quad (1.13)$$

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Now once I do that then I can do the structural (Refer Time: 17:52) and as I mentioned again I we are we are coming I mean I mean coming back in the future class may be next class about the details of how from how we get this and how we write that finite element formulation for this particular problem. Today we are just going to discuss everything in overall fashion.

So, this is my Euler beam equation. And in the right-hand side what you are getting that $f(x, t)$ this $f(x, t)$ actually that dynamic force that you are getting that sectional dynamic force that you are getting from your hydrodynamic code. There are many people who. So, they I mean here we can write that that only the dynamic force.

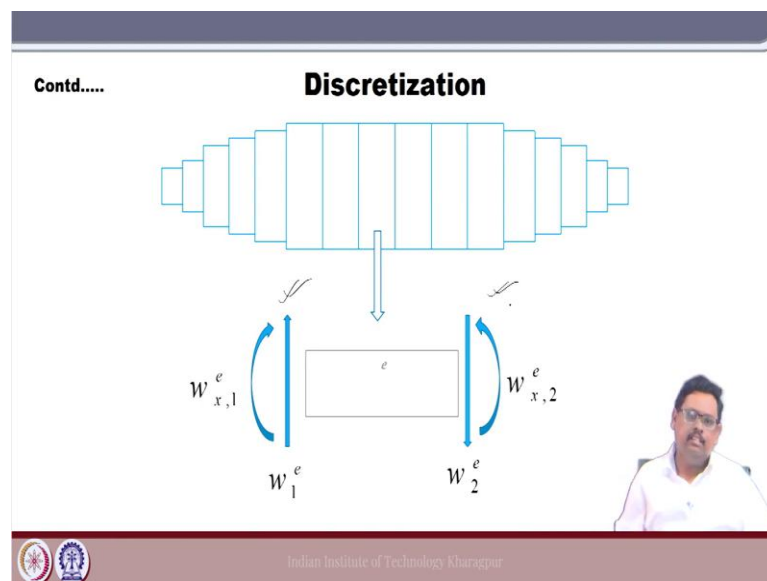
And in this the stiffness matrix here you can actually do the hydrostatic force. You know or maybe some people in fact, we also we use this hydrostatic force we do not (Refer

Time: 19:10) in into the into this stiffness matrix this you are doing this hydrodynamic plus hydrostatic force here as a external force. So, many people they use this hydrostatic force inside here and, but in our formulation, we have done this in the right-hand side, ok.

And now you know this is the boundary condition shear force and bending moment 0 because you are assuming this ship as a free beam. So, therefore, shear force and bending moment should be 0 at the both ends, right. Now this is a very well this anybody said that popular and well-known boundary conditions this thing.

So, it is not much discussion is required here anyway, but I missed one point here when actually we are using this normal yeah, this normal this normal is a generalized normal, ok. Now what is the expression for the generalized normal? Definitely we are going to discuss in the future classes, ok. Right now, today is just I am going to discuss the overall pattern of the code, ok, fine.

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Now, now this is how actually the discretization do. It is the it is my beam model and then I just make that sections section lines and then we can have the element and then this each of element have two degrees of freedom, right. So, we have that it can one is the rotation another is the translation.

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➤ Here, $[M]^e$, $[K]^e$, and $[F]^e$ are mass matrix, stiffness matrix and force vector for an element 'e'

$$M_{ij}^e = \int_0^{l_e} N_i N_j dx \quad \checkmark \quad (1.14)$$

$$K_{ij}^e = \int_0^{l_e} \left(\frac{\partial^2 N_i}{\partial x^2} \frac{\partial^2 N_j}{\partial x^2} - g B N_i N_j \right) dx \quad \checkmark \quad (1.15)$$

$$F_{ij}^e = \int_0^{l_e} f N_i dx \quad \checkmark \quad (1.16)$$

➤ Combining these elemental equations with consideration to the nodal connectivity of different elements, Adding a Rayleigh damping term, we get the following system of global finite element equations.

$$[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = \{F\} \quad (1.17)$$


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And now this is how we can write the generalized mass matrix, stiffness matrix and also the force matrix.

Now, each detail as I said we are going to discuss in the next class and then finally, we can write the global equation of motion which is 1.17. And then once I write this then we are using some Newmark beta technique to get the sectional velocity sectional displacement and the acceleration, ok.

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➤ The quantities at t_{i+1} are to be evaluated on the basis of known values of x_i , \dot{x}_i and \ddot{x}_i at t_i

➤ Compute

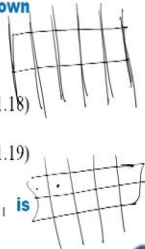

$$\ddot{x}_{i+1} = \ddot{x}_i + [(1 - \delta) \ddot{x}_i + \delta \ddot{x}_{i+1}] \Delta t \quad (1.18)$$

$$x_{i+1} = x_i + \dot{x}_i \Delta t + \left(\frac{1}{2} - \beta \right) \ddot{x}_i (\Delta t)^2 + \beta \ddot{x}_{i+1} (\Delta t)^2 \quad (1.19)$$

➤ Based on the values of \dot{x}_{i+1} and x_{i+1} in step (2), a new \ddot{x}_{i+1} is computed

$$\ddot{x}_{i+1} = \frac{F_{i+1}(t)}{m} - \frac{C}{M} \dot{x}_{i+1} - \frac{K}{M} x_{i+1} \quad (1.20)$$

➤ Steps (2) and (3) are repeated, beginning with a newly computed value of \ddot{x}_{i+1} , until satisfactory convergence is attained.

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Now, this also the Newmark beta again we are going to discuss in the coming classes. Today point is not how I obtain this; this is definitely it is much this methodology definitely we are going to discuss in the next classes. All the details about what is delta? What is beta? What is Newmark beta method? Etcetera etcetera we are going to discuss next class.

But today objective is supposed I am getting this velocity this displacement this acceleration I get it at the sections. So, now, I am writing a beam the point is after solving this equation 1.18 and 1.19 right now you know just leave it like this. I am solving this and then I am getting all this displacement the velocity at the sections.

However, in order to run the panel method code, I need all this. In fact, this normal velocity at the centroid at the panel. So, now, the problem is actually the opposite. Now first we understand that I need the you know the force at the sections; however, I have the force at the centroid panel centroid. Now I have the opposite problem. I have the velocity and the displacement at the sections. Now I need to transfer this to the panel centroid.

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The slide is titled "Transforming Sectional Velocity to BEM code". It features a diagram of a blue panel with several vertical lines representing sections. A pink horizontal bar is shown on top of the panel, with arrows pointing to it from the text "Pressure Obtained from BEM code". To the right, a graph shows a velocity profile V versus position Sec . On the left, a circular callout contains the mathematical expression $\frac{\partial \phi}{\partial n} = V(\vec{X}, t) \cdot \vec{n} = \frac{\partial \phi}{\partial n}$ on S_b . Text next to it says "Required to shift the sectional velocity from sections to panel centroid". The slide also includes the logos of Indian Institute of Technology Kharagpur and a small inset photo of a man in the bottom right corner.

Now, how we do that? So, now, this is the problem as I said. Now each panel I have to apply this boundary condition right, $V(\vec{X}, t)$. So, this velocity I must know at the panel centroid; however, I know this value actually at the sections. So, therefore, how do I shift this velocity from the sections to the panel centroid?

Now, you see here again I would like to say that there is not much very fixed method to do that, ok. I mean you can do many ways like you can do in fact if you ask a random person like you can ask ok, let us do it for some curve fitting. So, curve fitting means suppose I have here the velocity I just try to figure out how the velocity distribution going over here.

So, this section this point, may be this section it is this, this is the velocity. I mean what I try to say is that suppose I make a graph and then you know here is the velocity term and according to the section along the sections what are the velocity. So, I have some point like this let us take and then I can fit a curve.

It could be to the quadratic curve it, it can be some cubic curve I mean its it become a curve fitting problem then see you know you can do that and, but there is you know you cannot defend it physically. I mean how why you are using some kind of quadratic curve, why you are using some kind of cubic curve its more or less become a machine learning type problem like I have lot of data's.

I found out then I can do this; however, if I try to approach from the fundamental understanding of the physics, I would like to think that this velocity or the displacement I can assume that is varying also you know almost like a straight line it there is no average variation along the horizontal axis.

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(1.21)
$$\begin{aligned} V(i)_{Station} &= \text{Nodal Velocity}(2k-1) \quad \text{for } \{x_1(k)\}_{Element} = X(i)_{Station} \\ V(i)_{Station} &= \text{Nodal Velocity}(2k+1) \quad \text{for } \{x_2(k)\}_{Element} = X(i)_{Station} \end{aligned}$$

(1.22)
$$\begin{aligned} V(i)_{Station} &= \text{Interpolation} \begin{cases} x_1(k)_{Element} \rightarrow \text{Nodal Velocity}(2k-1) \\ x_2(k)_{Element} \rightarrow \text{Nodal Velocity}(2k+1) \\ X(i)_{Station} \rightarrow \text{Unknown} \end{cases} \\ V(j)_{Panel} &= \text{Interpolation} \begin{cases} V(i)_{Station} \rightarrow X(i)_{Station} \\ V(i+1)_{Station} \rightarrow X(i+1)_{Station} \\ X(j)_{Panel} \rightarrow \text{Unknown} \end{cases} \end{aligned}$$

The diagram shows a grid of points with a velocity profile curve passing through them. The curve is labeled with $\frac{\partial \phi}{\partial m} = v_m(r,t) = \frac{\partial \psi}{\partial m}$.

If you do that then that is what actually we were doing it we are doing the linear interpolation, ok. We have reason for that like if you look at that if I get time, I show you some results where you can see that when you plot the velocity or other quantity along the length then you can see that it is not very rapidly varying along the length.

So, let us make it simple. Let us not make very complicated like if you take cubic line curve or fit some nerves something just fit a linear curve. So, that is what we have done in the this is what actually is written over here like an interpolation I have these sections, I have another section, I know what is the velocity over here, I know the what is the velocity over here.

Now, I have panel here, I know the velocity here, I know the velocity here. I do the linear interpolation from here to here and then I find out in this x location what would be the velocity, ok. And this velocity again we fit into the body boundary condition which is

$$\frac{\partial \phi}{\partial n} = V_n(x, t) - \frac{\partial \phi'}{\partial n}.$$

So, in this equation again we put this and again we are getting the ϕ at the next time steps and again I get the force at the centroid I change this force from centroid to the sections and then I solve this structural problem using finite element and then I get the generalized velocity and displacement at the nodes I mean in the sections and then I do the linear interpolation and change from sectional to the panel.

And this is how actually we have coupled the structural fem code with the boundary element code, ok. So, today let us stop here in next class we are going to discuss because there is not much discussion left for the hydrodynamic solver. However, how I solve the structural problem, what is Newmark beta method, how I use the finite element to get into this stiffness matrix or mass matrix those we are going to discuss from the next class.

Thank you.