Numerical Ship and Offshore Hydrodynamics
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Lecture - 54
Hydroelasticity (Contd.)

Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 54.

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Now, today we are going to discuss about the how I can solve the structural problem using the finite element ok. Though you know the finite element is a very vast subject and we cannot cover the entire thing, so, but the thing is required for to proceed and to solve this Hydroelasticity problem, that part we are going to definitely discuss today.

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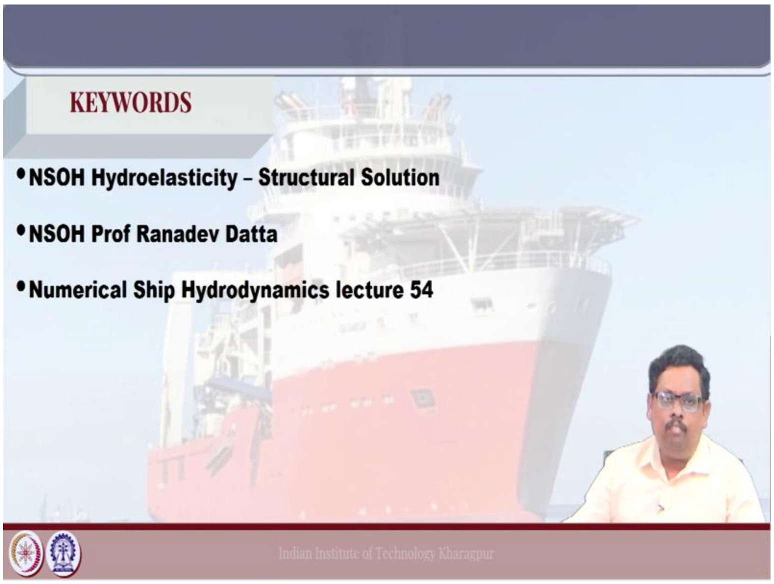

And this is the key word that you have to use to get this lecture ok. So, let us start.

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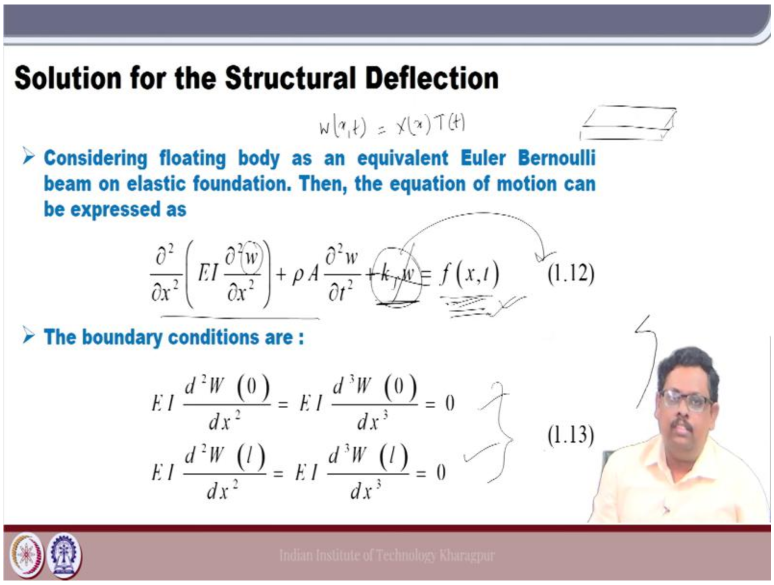

Now, we are going to use this Euler beam theory. And this equation is 1.12, just if you remember this connecting to the previous equation, it is actually the beam equation that we are going to solve. Now, here if you look at this equation 1.12, this term this the buoyancy term actually we can take in the right-hand side ok.

So, therefore, I mean if you do that then I just cross this thing and simply I can write this term equal to the $f(x,t)$ ok. Because I am taking this term in the external force because this is coming because of the buoyancy force ok. And therefore, since it is a free beam, you can assume that a ship you can consider the ship is a free beam which is resting on a spring. One can say like this buoyancy force can act as a spring. So, therefore, the shear force and the bending moment at the both the end is going to be 0.

So, this is the differential equation that we are going to solve and with respect to this boundary condition. Now, how I can solve this differential equation? So, there are so many ways. You can do that you can convert this partial differential equation. You can see here this w is the both the function of space; that means, x as well as the time t.

So, the basic idea is converting this equation the partial differential equation into the ordinary differential equation in terms of t. Now, there are many ways of doing it. The popular way of doing is that we can use the separation of variable like for example, you can simply take this W x, t equal to some function of x and then some function of t. And then you can substitute the whole thing here you can solve this problem.

Remember this, right hand side also we need to see how we can convert this function only in the function of time t. So, this is one way. So, there are many ways you can do. Actually, popularly you can say there is a two you can call the strong way I can solve the problem or sometime can say I can solve in a weak form ok.

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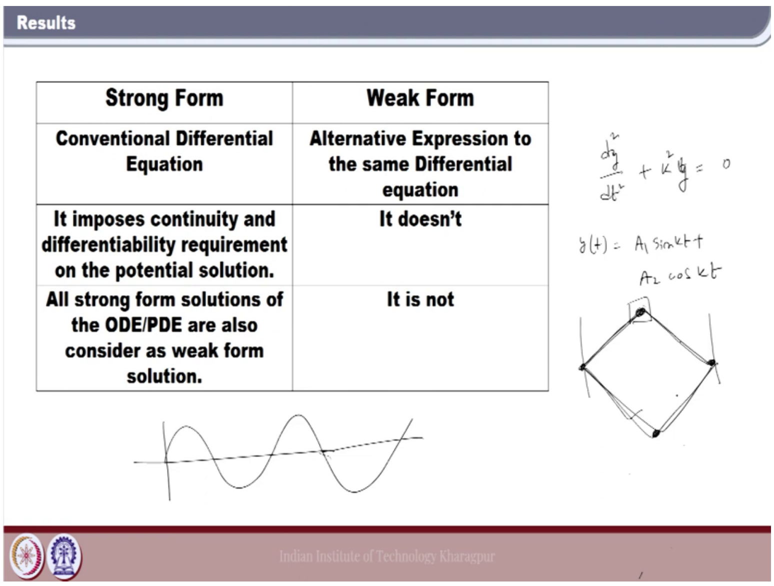

Strong Form	Weak Form
Conventional Differential Equation	Alternative Expression to the same Differential equation
It imposes continuity and differentiability requirement on the potential solution.	It doesn't
All strong form solutions of the ODE/PDE are also consider as weak form solution.	It is not

So, now let us see that what is the difference between a strong form solution and the weak form solution. Now, in strong forms form solution that the solution you are taking as it is like you are not making any changes to the differential equation. And also, the solution of this problem entirely that satisfy the requirement that differentiate requirement or continuity requirement everything.

For an example like the classical problem of string like let us take a second ordinary differential equation which is let us say d square y by dt square plus let us take k into t sorry k into y equal let us take it is 0 let us say. Now, I try to solve. Now you know that this the solution is you know it is some sin let us say k^2. So, $\sin(kt) + A_2 \cos(kt)$. So, this is the solution for this particular problem.

Now, here one can say this continuity, differentiability everything is preserved over here. Now, if you look at a string now this string actually realistic way, I can see that it can you know I can see this as my approximate solution. Now, if you look at the sin kt or cos kt, so, you can see the actual the solution is in this form right, it satisfies all the conditions, now it is a strong form solution.

Now, suppose I assume my if is a string. So, it is just connected here and then if I assume that string can actually you know oscillate in this fashion. Now, here it is a second order differential equation, but however, I can see this is a realistic solution, but here I can see that the second order differentiability is not actually maintained. Is it not?

So, physically I can see that that could be a solution, if you take a string and then if you oscillate it more likely it is going to look like this way right. So, let me just grab it and draw it somewhere here yeah.

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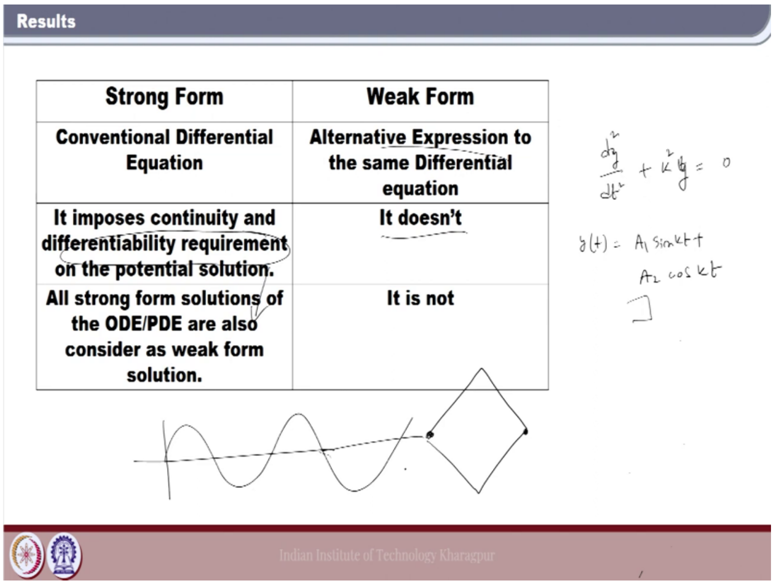

So, the same picture let me draw over here. Now in realistically if you think that this is an approximate solution of the string problem. So, one can say it is valid. I mean its practical, but it does not maintain the all this you know differentiability requirements. So, it does not.

So, in weak form solution we can approximate some solution, but that does not you know maintain all the differential requirement of the solution right. So, in one way the all-strong form solution can be a weak form solution, but all the weak form solution cannot be a strong form solution ok.

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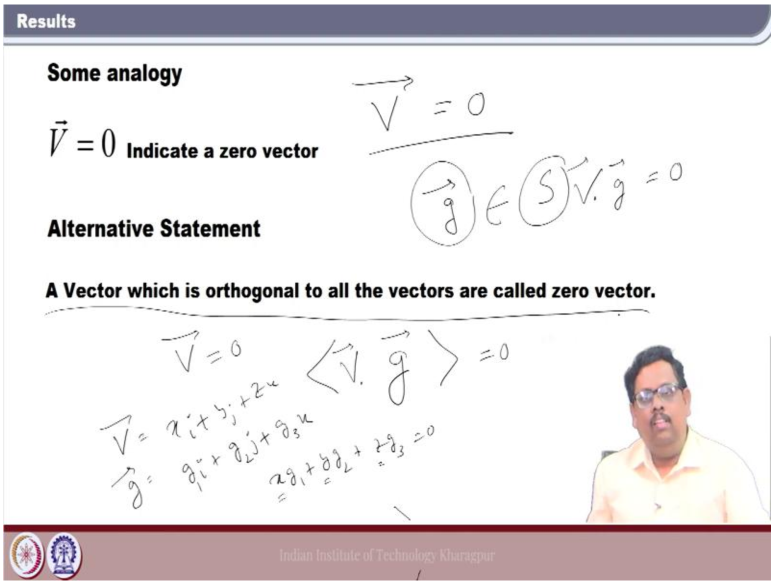

Now, let us see like some analogy like how we can see the solutions like. Now suppose one can see like the V =0 like say a vector V that vector V =0 ok, fine. Now, alternatively what I could say is like that a vector like assume that how I define a zero vector in alternative way. I can say that suppose g is become any vector apart from the zero vector any vector in a three-dimensional space or n dimensional space.

Now, here at this point I really do not want to confuse you with some space problem in n dimensional or whatever, but we considered a three-dimensional vector space. So, we have the component x and y and z ok. So, then I assume this a g vector is something like this. And then for all g belongs to this space let us say this vector space S that V dot g this dot product should be equal to 0.

So, it means that this vector V is orthogonal to any of the vector in the vector space then I can call this V is a zero vector. See idea is very simple. Idea is to tell something equal to 0, I just coming with some condition. Now, I say simple V = 0 I can see is a zero vector, but I am not saying that is a zero vector. What I am saying that I collect all the vector g which belongs to this three-dimensional space. So, g is the collection of all vectors, any vector that belong to this space the vector space.

Now, if it is so, then V the dot product of V with that g is always equal to 0; that means, that V is orthogonal to the all the vectors in that vector space. It means zero V is a zero vector right because it is not possible that you cannot find a vector Z apart from zero vector if you take a dot product with any other vector it becomes 0; think, like if I take V equal to some vector $x_i + y_j + z_k$.

Now, if you take any arbitrary g vector which is may be that that g vector may be equal to $g_1 i + g_2 j + g_3 k$. So, then dot product is $xg_1 + yg_2 + zg_3 = 0$. Now if these two are independent or whatever this is only possibility that x, y, z should be equal to 0. Otherwise, it cannot be 0 ok. So, this is one way I can say that V equal to 0, directly I say V is a zero vector. Alternatively, I can make another comment that a vector which is orthogonal to the all the vectors are called the zero vector.

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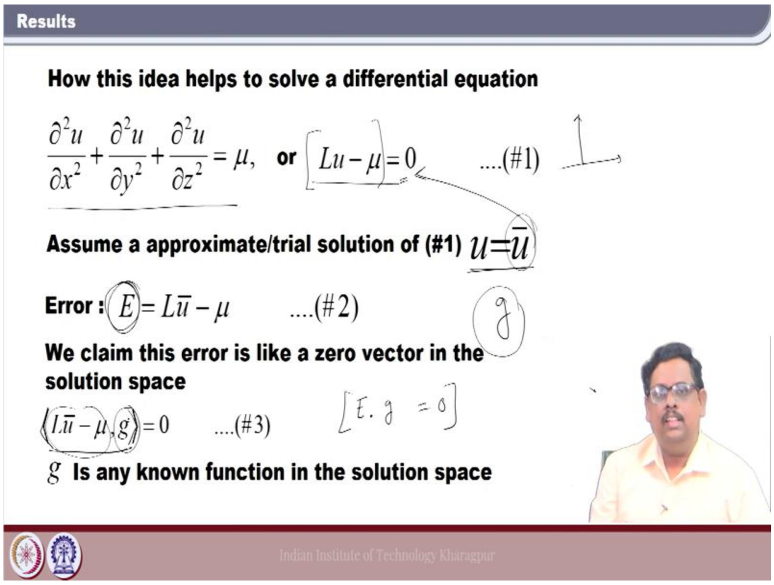

Now, let us see how I use this idea for this the finite element thing. Now, if you take this differential equation over here, so, I can write this equation in little bit composed way $Lu - \mu = 0$. Now, here this $Lu - \mu = 0$, a see one is saying that this is equal to 0 right, this is the normal way one could tell.

Now, let us consider this as a infinite dimensional space. Now again I dont want to confuse you as a infinite dimensional space and all, but let us solve this problem in some numerical or approximate way. So, I assume that $u = \bar{u}$ is the approximate solution. Now, approximate solution cannot be equal to the main solution. So, therefore, there must be some error.

And I can define this error as E. Now how the previous idea helps over here? I assume that this error E should be a zero vector. Now, I assume this g the vector any vector g or any function g which is the which belongs to the solution space; now this g could be the strong form solution ok.

Now, if g is belonging to the solution space, then it should be orthogonal this should to the error. So, the idea is I am simply telling that this error, now if it is really a vectorial operation, I could say that $E \bullet g$ that should be equal to 0 if it is a vector, but it is not it is a function. So, I am using this symbol this symbol defines that now this $E \bullet g$ means that E and g are orthogonal to each other normal to each other.

So, I can say that this error is orthogonal to the vector g, it means that this error does not belong to the space. Now, this is the basic key aspect of the finite element. I approximate a solution u, I try to calculate the error and then I multiply with some function g. We call this function as a test function, and this test function definitely be the solution of the problem and therefore, this should be orthogonal to the error function.

So, this is how actually we approach for the finite element and that is the basic of the finite element. It is a very vast and therefore, a lot of theories and lot of things are available. So, I am not going to go into deep into that. So, with this let us try to figure out how I can solve this structural problem ok.

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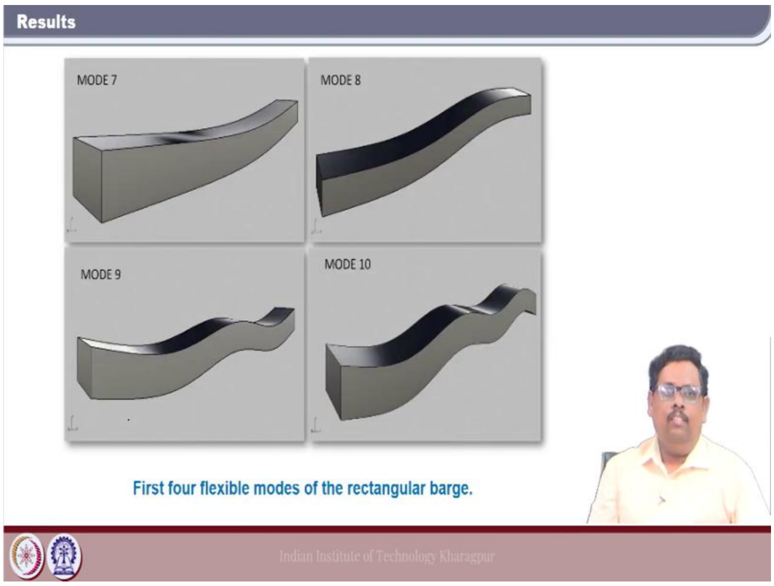

Now, here now this is something that solution space you can say like you call a test function. The test function could be anything like this right because you can see it is a mode ok. So, if a thing you know oscillate as I said that this weak solution may be some realistic. Now, if you think of a flexible structure it can oscillate in any, it could be oscillated like this way, it can oscillate like this way, this way or this way, it could bend like this.

So, all could be the possible solution. Anyway so, let us now start that how actually I could solve this problem ok. So, now, remember that I put that stiffener part in the right-hand side under the f x, t.

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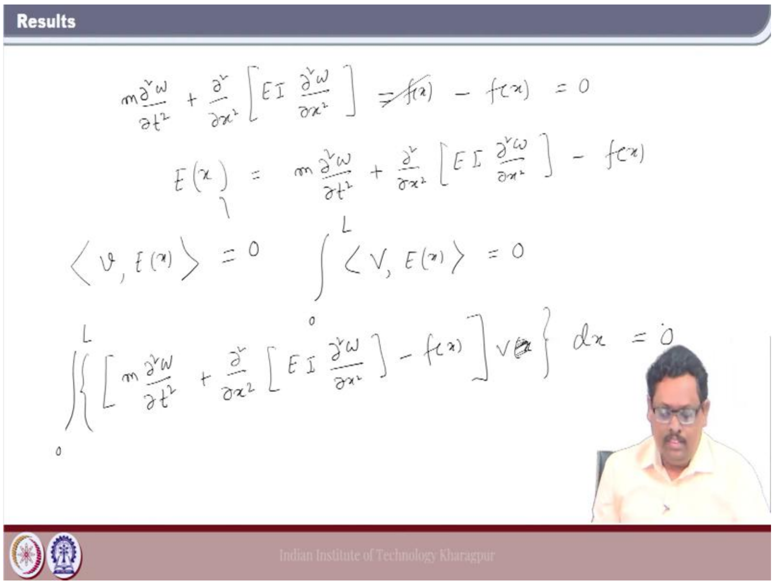

So, if I do that, so, I left with this following thing which is $m\frac{\partial^2 W}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 W}{\partial x^2}\right]$. This equals to f(x). So, I can call this is equals to f x. So, now, what I can do is I can make in the left-hand side. So, I write it is minus of f(x)=0. So, now, I can approach that a approximate solution of w and then I have to use some test function which actually is the solution of this space and then it should be orthogonal to each other right.

So, now, how I can set it? So, I can find out my error function you can say here E(x), let us say. It should be $m\frac{\partial^2 W}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 W}{\partial x^2}\right]-f(x)$. So, now, I have to define some test function v right this test function v with this error function E(x), it should hold some orthogonality property that should be equal to 0.

So, this is my idea. Now, how I define this? I define this way. If I integrate 0 to L then this V, E(x) this error and that should be equal to 0. So, this is my orthogonality condition. Now it is bit mathematical. So, but the application is pretty easy. So, it means that what I try to say I just multiply 0 to L this integration.

So, integration the whole error function. So, which is $m\frac{\partial^2 W}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 W}{\partial x^2}\right]-f(x)$. So, this is the whole thing. I multiply with this function V x or function of x and t, so, V and then the whole thing if I integrate along the length x that should be equal to 0. So, this is the basic idea of the whole thing ok. So, now in the next step, what I am going to do is I am going to simplify the thing ok. So, in next I am going to simplify the thing.

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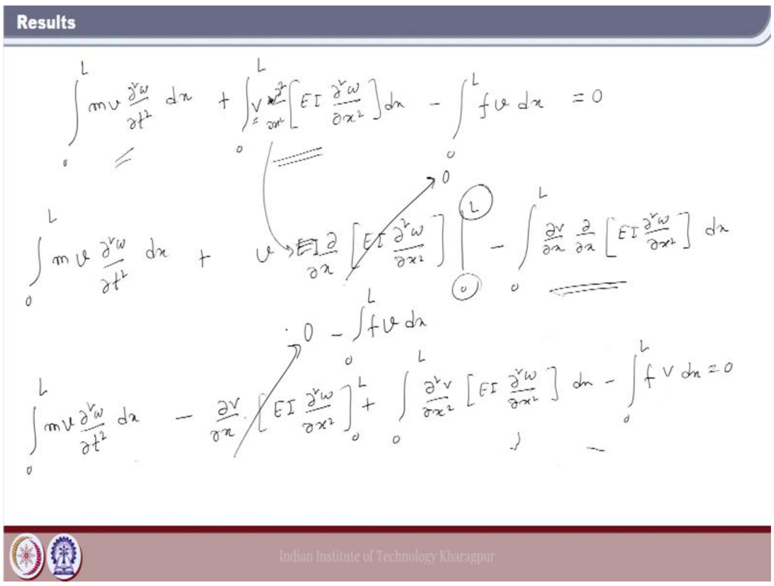

So, let me do that. So, 0 to L and then I just now if you see this the last term, so, the first V should multiply with this term. So, it is a function of t ok. So, then just write it out m into V and then it is $\frac{\partial^2 W}{\partial t^2}dx$, see. So, this is the term actually I am separate out and then I am doing this next part. So, I just split again it is 0 to L.

Now, this V should be multiplied with $EI\frac{\partial^2 W}{\partial x^2}$ right. I think this is it yes and then multiplied by the dx and then minus 0 to L f into v into dx that should be equal to 0 right. So, I am not doing anything with this. So, I am just using try to solve this by integration by parts. So, I am writing it is as it is 0 to L into $mV\frac{\partial^2 W}{\partial t^2}dx$.

Now, if I do integration by parts, so, I will get v into E I just second sorry I missed this one that is what I am thinking. So, I just missed this part here $\frac{\partial^2}{\partial x^2}V$, yes. So, then it should be now it is v multiply the integration of this. So, it should be $\frac{\partial}{\partial x}\left[EI\frac{\partial^2 W}{\partial x^2}\right]$ you put 0 to L and then integration by parts. So, therefore, it is 0 to L.

Now differentiation of V is $\frac{\partial V}{\partial x}$ and then integration of this part. So, it is $\frac{\partial}{\partial x}\left[EI\frac{\partial^2 W}{\partial x^2}dx\right]$. Now, why it is because you see it is integration it is $\frac{\partial^2}{\partial x^2}$. So, that is why it becomes $\frac{\partial}{\partial x}$ right because of the integration. And then minus it is 0 to L f into v into dx. Now, if you apply the boundary condition the shear force. So, del square by del x is 0 at both the end l as well as the 0. So, this goes to 0.

So, I left with 0 to L $mV\frac{\partial^2 W}{\partial t^2}dx$. So, this is the first term and then I have this one. Now again if you do integration by parts then you will get minus it is $\frac{\partial V}{\partial x}$, and integration of the second part which is $EI\frac{\partial^2 W}{\partial x^2}$, it is 0 to L and then again plus. Now it is the differentiation. So, it is 0 to L $\frac{\partial^2 V}{\partial x^2}$ and then it is integration.

So, it is $EI\frac{\partial^2 W}{\partial x^2}dx$. And then it is finally, minus 0 to L f into V into dx that should be equal to 0. Now, again if I apply the boundary conditions, so, this goes to 0 right.

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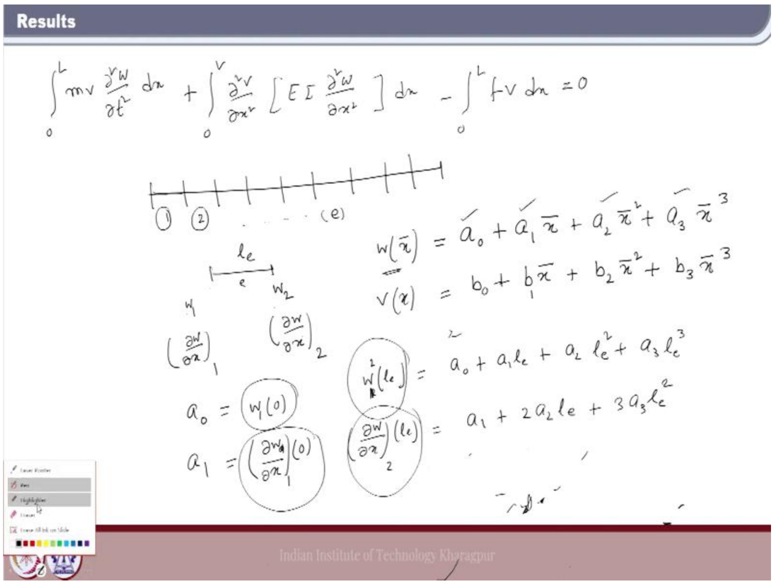

Now, if this goes to 0 then finally, you have 0 to L. It is and then this goes to 0. So, this term. So, plus 0 to L and $\frac{\partial^2 V}{\partial x^2}$ then $EI\frac{\partial^2 W}{\partial x^2}dx$ yeah and minus 0 to l f into v into dx that is equal to 0.

Now, this is the final formulation. So, now, we have to find out the value for this. And now this finite element concept we can use. Now what I do is, now, if we assume this is a beam element, so, I can similar to the panel. Now you know the panel method. So, you do not have difficulty understanding this. So, assume these are the elements. This could be element number 1, this could be element number 2 assume that there is a element number E.

Now if you take this element E ok, then actually I can allow is two degrees of freedom. One is you can say here I can say it is a w 1. So, it is a displacement and also, I can call the slope $\left(\frac{\partial w}{\partial x}\right)_1$ and then I have another element here you can call w 2 the node and also, I can call this $\left(\frac{\partial w}{\partial x}\right)_2$ ok. So, I have four degrees of freedom right. Now if I have four degrees of freedom; that means, I have four unknowns, is it not?

Now, this is one that then how I can write that now still now I did not write the approximate solution. So, how I can approximate the w? Now since I have the four unknowns w_1, w_2. so, I need four equations right. So, therefore, the function actually I need to write the approximate function it should have it should have the four unknowns. So, therefore, it should be cubic polynomial right.

So, I just write this is in terms of $a_0 + a_1 \bar{x}$ a 0, now this $\bar{x}$ is the approximate solution and then $A_2 (\bar{x})^2 + A_3 (\bar{x})^3$. Now, similarly I can write this another function also V x remember I am just not writing that t term though it is there. Now V x equal to again it is you can call b_0 plus because I am integrating along the length. So, it is $\bar{bx} b_1 \bar{x} + b_2 (\bar{x})^2 + b_3 (\bar{x})^3$.

Ok. So, now, the thing is that I have this four. So, now, let me just very quickly just say that I can express like if I put 0 over here. So, here 0 if I put $\bar{x} = 0$. So, I can use it a_0 is equals to you know w_1 right w_1. So, and then you can say similarly if I differentiate this with respect to x, so, I know that my a 1 should be equal to $\frac{\partial w_1}{\partial x}$ or you can say $\left(\frac{\partial w_1}{\partial x_1}\right)_w = 0$. So, w_1 it is this right.

And then if I take this now if the length I assume this length of this element is le, so, similarly I can write that the other condition also I can write it is $w_2 le$ or $w_1 le =$ $a_0 + a_1 le + a_2 le^2 + a_3 le^3$ right. And then I can write for the $\left(\frac{\partial w}{\partial x}\right)$ at.

Now, at le it is 0.2. So, it is 2 over here and it is le. So, it should be $a_1 + 2a_2 le + 3a_3 le^2$ right, fine. Now, you see now from here actually what you need to do is you need to write the expression w x not in terms of a_0, a_1, a_2 and a_3. You need to write the whole expression in terms of w 10 and ∂ x ∂ w ∂x_1 at element 10 then $w_2 le$ and ∂ w and ∂x_2 at le.

So, this is how actually you need to write the whole expression ok. Now this is actually very much possible. You need to do a little bit of exercise and then you can find out that normally if you do this exercise, so, I can I can just say that how it should look like.

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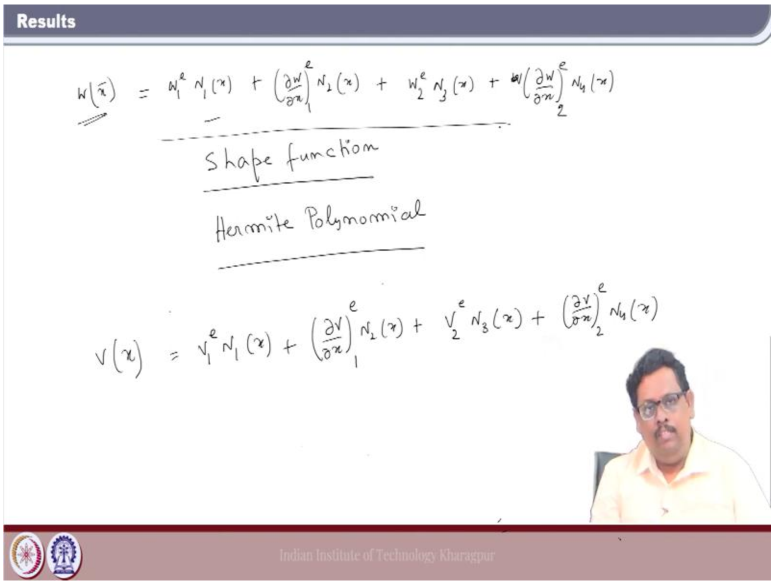

So, this $\bar{w}x$ it should look like some w_1^e with multiplying with some function N x plus $\left(\frac{\partial w}{\partial x}\right)_1 x + w_2^e$ into some N 3 x plus you know sorry it is $\left(\frac{\partial w}{\partial x}\right)_2$ this element at for element t into N 4 into x. Now, this is actually you should try a little bit and you can write this, you can eliminate this a_0, a_1, a_2 and a_3 in terms of this w_1 and then $\partial w\ \partial x_1$ ok.

I forgot this element for element t of course, w t and $\partial x\ \ \partial w\ \ \partial x$ for the second note for the element t with respect to this. And this N_1, N_2, N_3 this is called something called the shape function, this is something called the shape function ok. And something called it is a Hermite polynomial as well ok.

And similarly, I can write for the V(x) also. V(x) also I can write in this form. We can this $V_1^e N_1(x) + \left(\frac{\partial V}{\partial x}\right)$ for element 1 I mean for the node 1 element e which is then N_2 (x)+ V_2 for element e into N_3 (x)+ $\frac{\partial V}{\partial x}$ for element e the second node into N_4 (x).

So, now this is how actually I write the approximate solution.

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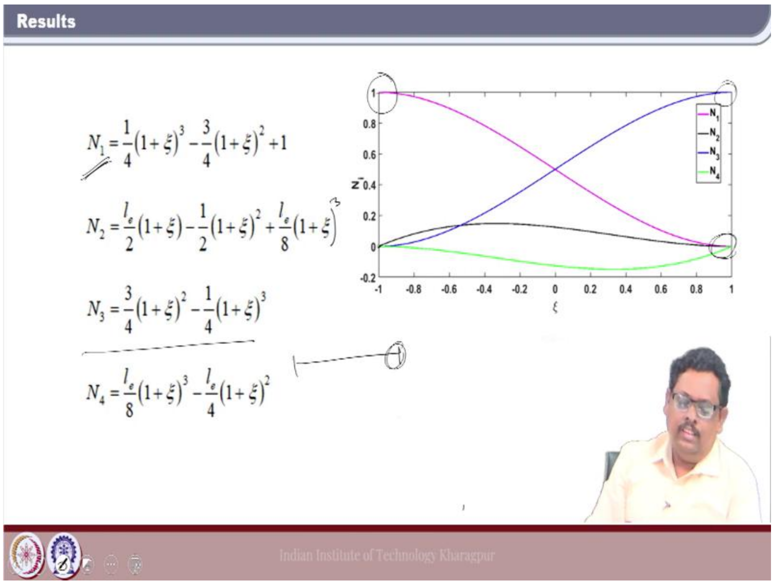

Now let us see that how actually if I solve this if you solve this the whole exercise so, you can get this shape functions this $N_1, \mathrm{N}_2, \mathrm{N}_3, N_4$. Now this xi is nothing but the approximate which I called as x bar. So, here it is written as in xi form, but anyway. So, this ok, this bracket is missing over here yeah. So, ok now, if I if you do this then actually you get this solution $N_1, \mathrm{N}_2, \mathrm{N}_3, N_4$. Now, this is how actually the shape function.

Now you see the important something is very important over you can see that if you look at the you know node 1 ok only you can see that only the shape function 1 actually is 1 and remaining values are 0. So, therefore, in element 1 you can see that the deflection has the maximum impact right.

Now, similarly you can see that this N_3 it is 0 over here when xi is equal to 0 and then it is it is here and xi= le. Now, if I look at this N_3 then you can see that for the for this beam and here actually that the this N_3 has the maximum impact and remaining terms are 0. All other terms are you can see that it has no effect, the slope has no effect in the both the end right.

The slope is 0 and also here you can see that here also the slopes are 0 which satisfy the shear force and bending moment. So, anyway so let us stop here. Now what we stop here that where I can write the approximate solution in terms of the shape function and the nodal variable which is w_1 ∂w ∂x w_2 and ∂w ∂x node 2.

Now, in the next class we will try to figure out that how this will help me to solve the structural problem. From here how we can write the element matrix and then how we can write the global matrix and how I can solve. All these things we are going to discuss in the next class ok.

Thank you.