Numerical Ship and Offshore Hydrodynamics
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Lecture - 56
Semi Analytic Method

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Hello, welcome to Numerical Ship and Offshore Hydrodynamics. Today is the lecture 56, today we are going to discuss on ship hydroelasticity using semi analytic method. So, till last class we discussed the hydroelasticity using a complicated bound integral equation method together with finite element.

However, this is enough motivation; this method gives enough motivation when actually you are dealing with some initial judgment on the behavior of the ship. For example, at initial design level suppose you want to find out what would be the bending moment is coming, what is the effect of the elasticity and how to compute the design load.

At basic initial level like in the basic design stage, so that time you really do not want to run a very complicated or very sophisticated some numerical tool. So, at that moment this Semi Analytic Method will help you a lot ok.

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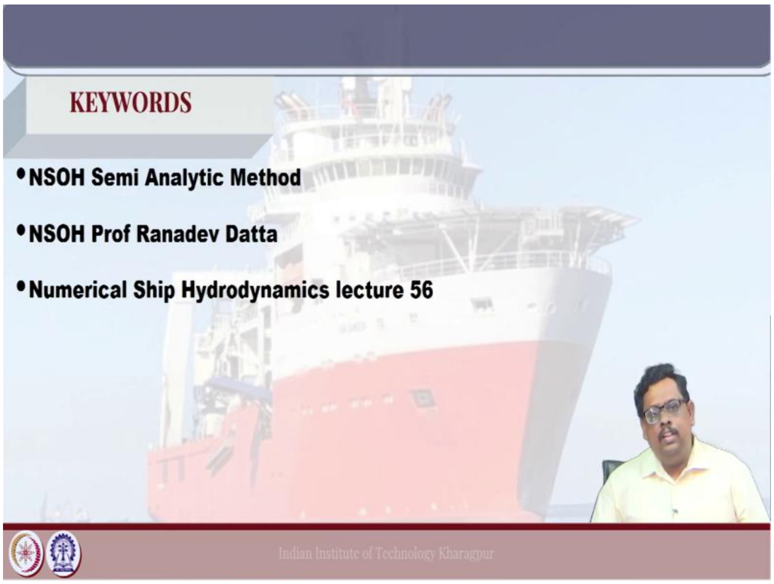

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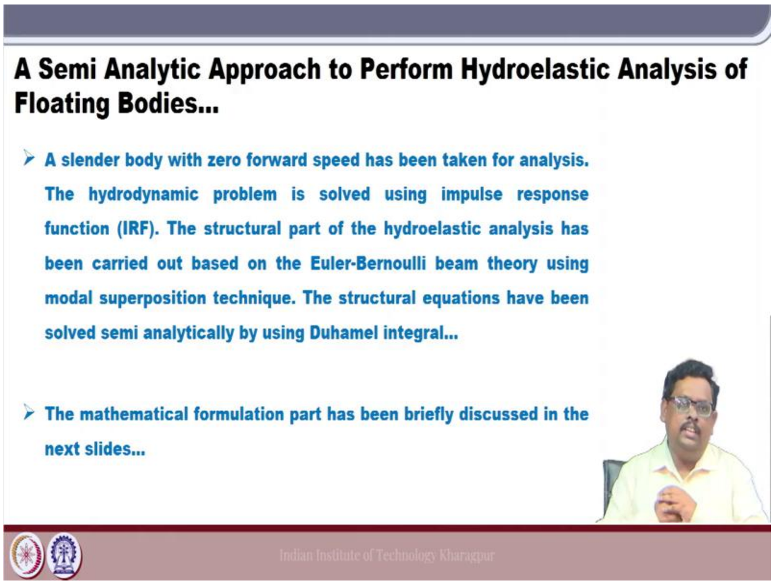

So, this is the keyword that you have to use to get this lecture ok. Let us now jump into the semi analytic method. Now here to do this what we are going to do is we are going to take a rectangular barge. Now this also applies for a ship shape body also theory because little bit more complicated so, but at the basic level let us drop the forward speed as well as the ship shape body. So, let us start with the rectangular barge.

Now, why I am telling it is rectangular barge the simplification is that the rectangular barge has a uniform beam right. So, therefore, assuming this rectangular barge as a beam will be much easier compared to the other ship shape body ok. So, here the idea is we are going to find out the pressure, using impulse response-based method.

Now, we have already discussed about the impulse response-based method initially. So, again we are using the same technique to find out the sectional force and then we are going to use some modal superposition technique, to find out the structural deflection to convert the you know PDE that Partial Differential Equation into the ordinary differential equation and then we are going to solve it.

Now, let us see although we can call it the semi analytic method, but still it involves lot of mathematics of and lot of numerical technique also, some basic numerical techniques still it is required to solve this problem ok.

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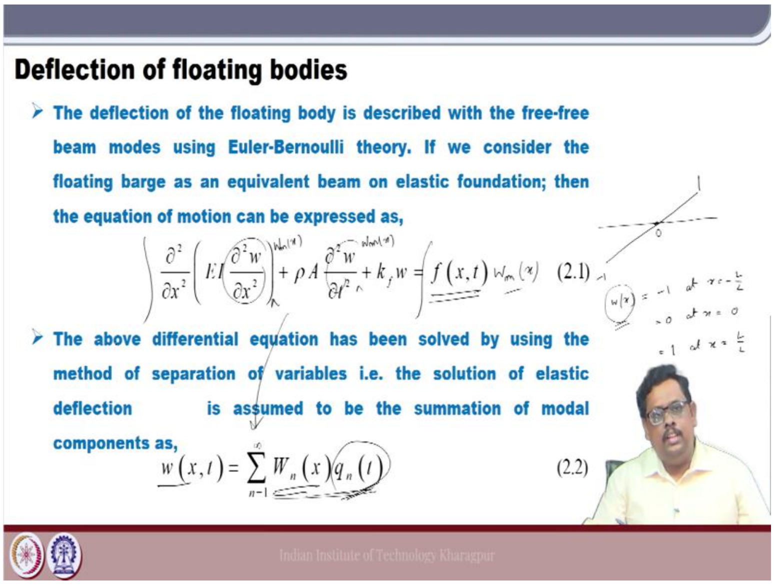

Let us start. Now here this is how we are going to discretize the barge. So, we are assuming again it is a Euler beam and this is the governing differential equation. Now here to solve this we use the deflection the $W(x,t)$, now we have split into two parts. Now, one part is the time dependent which is $q_n(t)$ here.

And second part is the space dependent which is called the $W_n(x)$ and we call them the mode shapes. Normally, it is popularly it is known as mode shape. Now what is this mode shape? Now if you do the vibration analysis of course, you know what is mode shape, but now for the beginners.

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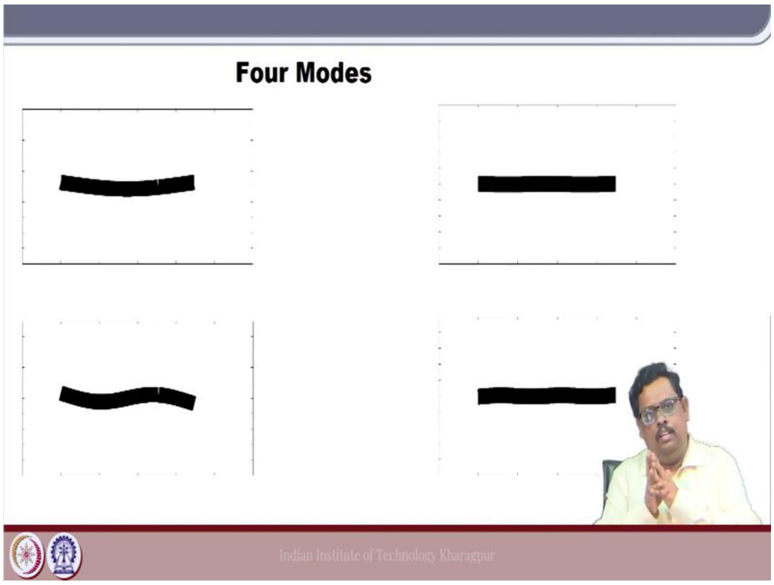

Now, let us see that how the structure can be bent if you apply some load on it. So, then structure can oscillate in this fashion right and also the structure can oscillate in this fashion as well right and also if you look at it, the structure can be oscillated in this fashion also. Now, structure can be oscillated in you know this way also.

Now this is something called the mode shapes or the how a structure can oscillate or and can deflection. Now, if you remember the rigid mode then the you can we know that we have some 6 degrees of freedom. The structure only can oscillate, let us say in the surge mode or sway or heave right or in case of pitch, the roll and then yaw.

Now in case of a flexible structure it can oscillate in infinite different way. So, so this is the difference major difference between the rigid body and the flexible body. Now, in rigid body we know very well it is finite number of modes it can oscillate, but in case of a flexible body it can oscillate infinite way. Now it is good for us that though it can oscillate in infinite number of modes; however, only first few modes are actually making some impact.

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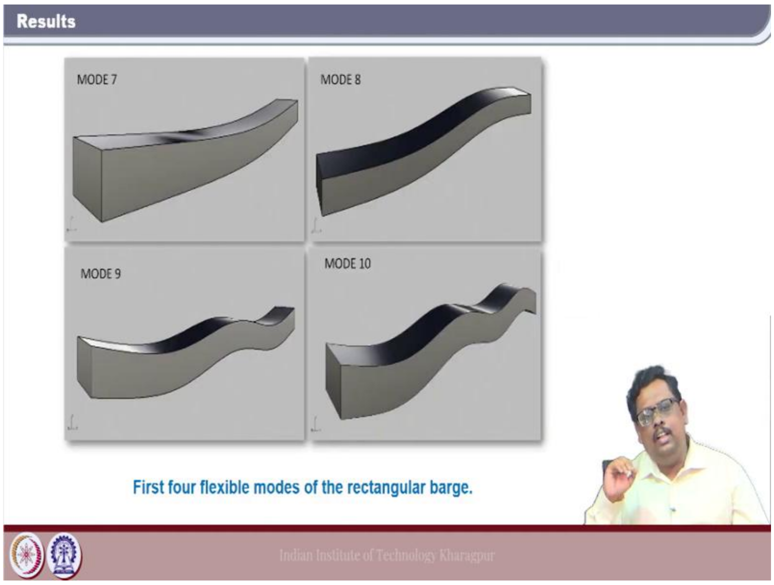

So, therefore, actually we can take let us say the first four modes as your I mean as your mode shape and you can ignore the remaining one and we can say that, the impact of the other modes is not influential, so I can go ahead with these four modes. Normally, not exactly not four it is better to take at least first six of the flexible modes, apart from the first six rigid mode it is better to take first six oscillatory mode ok.

So, now what is going to use is like here we have this mode shape, now we have this equation. So, we know the equation of the $W_n(x)$. So, what we know as follows that we know the how the structure can oscillate or how I get the structural deflection along the horizontal axis. Now this is the same thing also we know in case of rigid body also there is not much difference in this idea, like in case of a rigid mode also we know that a structure can oscillate in six different ways right.

We can assume the structure can oscillate in this way or this way or this way, so we know that in which six way the structure can oscillate. Then what is unknown to us is that, what is the amplitude of the oscillation is it not? So, you see that here also in case of a rigid mode also we can use the same you know ideology or same kind of definition, that I know that how my body is going to oscillate, but I really do not know what is the amplitude of the oscillation.

For example, for in case of a heave I can simply take it is the mode shape $W_n(x)$ we can say simply 1. Why? Because it is a horizontal line right and then if I understand that amplitude is the what is my amplitude of the heave right. So, so see this is the idealization of how I can see the heave mode in terms of this $W_n(x) \times q_n(t)$, is it not?

Similarly, the pitch also we can think of it is may be that θ, it is a angle, so it can oscillate in some fashion, but I really do not know what is the amplitude of the angular deflection. So, similarly also in that case the $W_n(x)$ in case of a in case of a pitch we can see that we can write in case of a pitch maybe it is here let us say it is minus.

Let us take draw this diagram, it may be -1 here it may be +1 here and it may be 0 to 1. So, we can define this w(x) which is you know -1, let us say $x = -\frac{1}{2}$ which is 0 equal to at x equal to 0 and it should be 1 at you know $x = \frac{1}{2}$, based on this condition I can construct the value of w(x) right.

So, and then we can define my pitch is this $W(x) \times q_n(t)$ right. So, I just what it what would be the mode shape for the pitch mode I will give that work to you ok. So now, let us see that how we can use this same technique for the flexible structure right.

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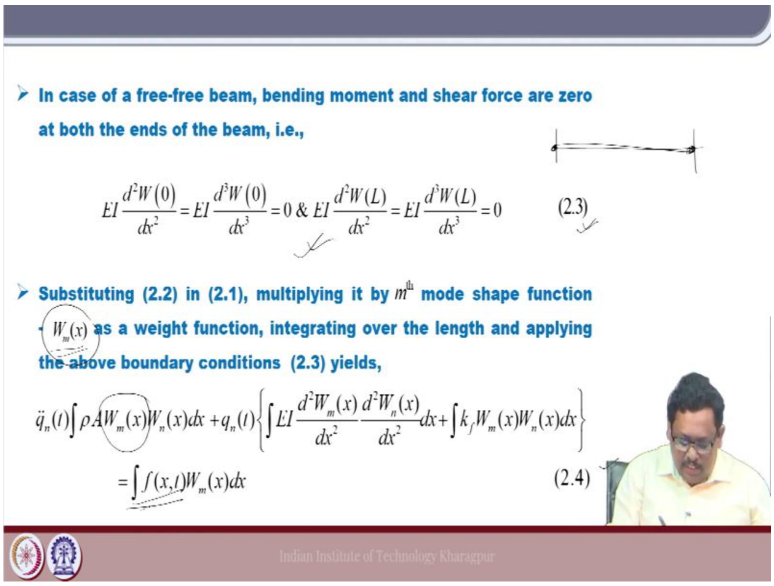

Now, where let us say we are using these four modes and then we are assuming that to be a free free beam. So, therefore, the bending moment and the shear force at the corner is equals to 0. So, if this is the body. So, at this point the shear force and bending moment is 0 and at this point also shear force and bending moment should be equal to 0. Now, if I apply this boundary condition ok.

So, then we can actually find out what is the mode shapes right. Now anyways now we know that what is the definition of modes as I say that for example, for pitch or heave I can I know what would be my mode shape. Similarly, for other flexible mode also we know what would be the mode shapes ok anyways. So now, what we are going to do is we define my deflection the total deflection which is $W(x)$ is nothing but the mode shape multiplied by the amplitude ok.

So, we substitute that here and then we integrate along l. So, then actually we can get this equation 2.4. Now, this is very standard way of approaching this problem and if you have these basic structural dynamics, you know that, this is how actually we can convert the partial differential equation into the ordinary differential equation. What we do is like we assume a particular mode which is W_n. So, then I can write this expression this W_n substitute this into this differential equation.

And then what I do is, I multiply another arbitrary mode shape $W_m(x)$. So, that I multiply here and then I use this boundary condition and then I integrate from 0 to l. So, therefore, in the right-hand side also in this equation in this equation this right-hand side also I multiply by any arbitrary mode $W_m(x)$ and then actually I integrate it.

In the left-hand side also, I substitute this in this equation and then actually I multiply by this $W_m(x)$ throughout, I multiply this $W_m(x)$ throughout and then I integrate it from 0 to l. So, this is the very standard idea how we apply over here to convert this equation into the ordinary differential equation. Now, here this is all about the structural part. However, that most important thing is how I can write here in the right-hand side this $f(x,t)$.

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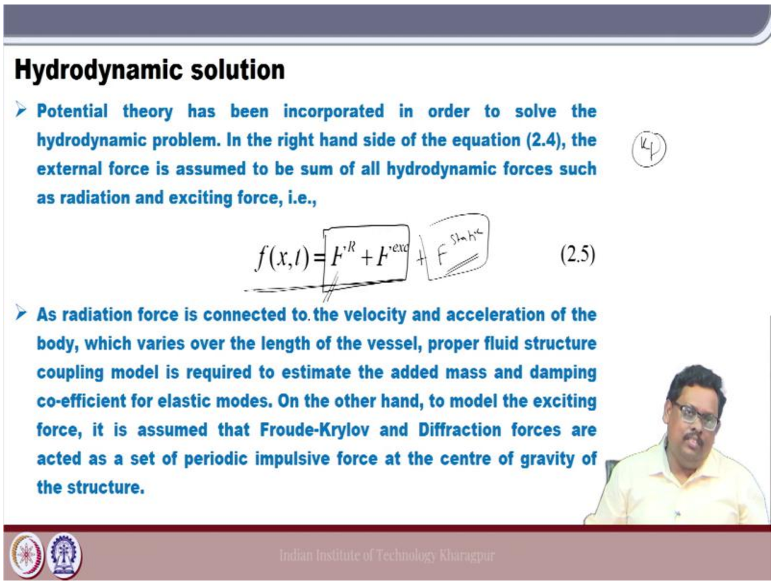

And this $f(x,t)$ actually you know it is a combination of the radiation force F^R and the exciting force F^{exc}. Now here unlike the previous one when I derive the finite element formulation that restoring force actually, we use as an external force and keep it as the right-hand side.

But in this formulation actually we are using it in inside the stiffness matrix; that means, in this k_f actually we are writing the hydrostatic component ok. So, this is so, but it is not necessary like you can still ignore it and you can still write here as F^{static} also you can do that. So, it is not that strict restriction that you have to write everything in the stiffness matrix that is in the k_f only, not necessary that hydrostatic part actually you can incorporate in the right-hand side and you can treat it as an external force.

Now, here the more important part is, how actually I write this radiation force and the exciting force in semi analytic measure. Now here now thing is for the radiation force actually this is not that complicated it is very well defined. In fact, like we do not have to use much imagination to find out how I write that radiation component.

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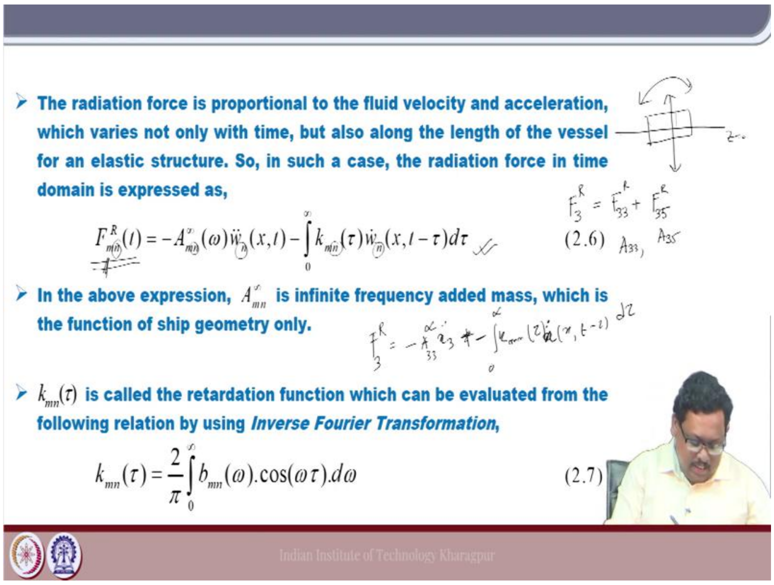

Now, radiation component you know very well that when actually you oscillate the body you are oscillating in the still water and then you can oscillate in all six modes for the rigid body right. Remember that how we obtain the radiation force in case of a rigid body, we have a body in still water.

So, of course, it is z = 0 and then we oscillate in six different ways. We oscillate in heave I mean which mode actually I am interested, so we are interested in the heave and pitch. So, therefore, we oscillate in the heave mode and then we can we sometimes we oscillate in the pitch direction also. But here when you have the infinite way you can oscillate the body. So, therefore, this radiation should be you know combination of all this infinite mode right.

Now, if you remember again for the strip theory when actually we try to figure out what is my radiation force or we can write F^R or let us say it is in the let us say in 3rd mode F_3^R. So, then we have to add; the radiation force right for the 3rd mode, when we oscillate the body in the 3rd mode plus radiation force in the 3rd mode, when I oscillate the body in the 5th mode right

So, in that way we have this added mass A33 and then we add the added mass A35 and so on right. So, similarly here also when you find out the radiation for the mth mode, then I can oscillate it on you know all nth mode we can do the oscillation right. So, therefore, that is how we can write this equation.

Now as you know it is similar to your rigid body, in case of a rigid body how we write this radiation force? If you remember, this radiation force $F_3^R = -A_{33}^{\infty}\ddot{x}_3 - \int_0^{\infty} k_{mn}(\tau)\dot{x}(x, t-\tau)d\tau$.

However, in case for a the you know the flexible body, you have infinite number of modes, so therefore, it is very you know we have to understand that it is not that trivial or not even the rigid body also not trivial.

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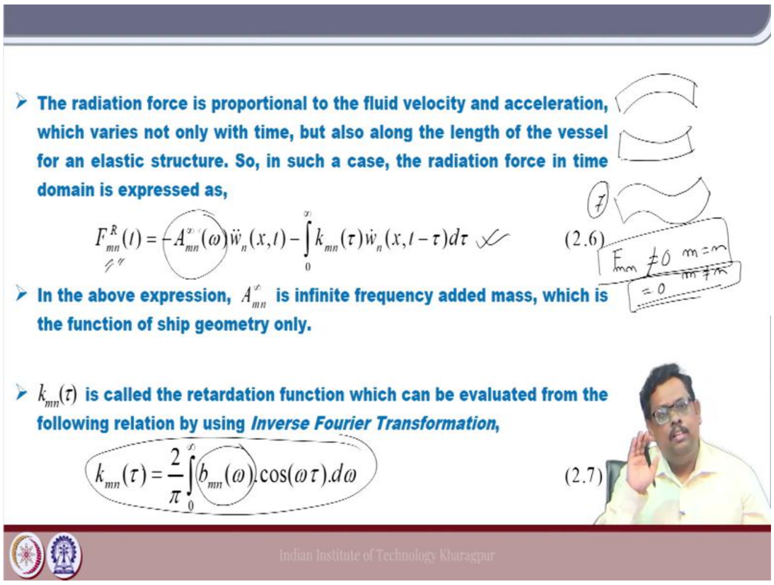

Here it is more complicated, that I am using some flexible mode let us say my 6th or the first flexible mode it is basically the 7th mode in the in general. If you consider first six as a rigid body mode the seven is the first flexible mode. Now if you oscillate, so in 7th mode how you oscillate the body? If you remember that my that picture you actually oscillate the body like this way.

Now, here when you oscillate the body like this way, then what would be the contribution like when this body is oscillating like this way? You see that difficulty it is, like here I even it is feeling (Refer Time: 19:24) even we have to have complicacy to feel that, if I oscillate the body in heave mode how I get the force at the pitch mode even it is a rigid body it is very difficult to realize.

Now, in case of a flexibility it is further, far more critical to realize that when a body is oscillated in this way then what is the contribution when you oscillate in like this mode I mean, what is the contribution in the force in that other mode. So, therefore, it to make it little bit simpler, what normally we assume that we here in case of hydroelasticity you know we use we do not you know consider the cross-coupling thing.

So, that means, we have the non-zero component when $mn \neq 0$ then $m = n$. And when $m \neq n$ then $mn = 0$. So, it is actually a simplification and you know if you read the journal in fact, for the plate this is the study that sometimes that this cross coupling added mass for the flexible body also have a big role.

However, in our case is a simplistic way not simplistic little bit simpler way, we assume that there is not much effect when actually $m \neq n$. So, when $m = n$ we have only taken the contribution. So, here though I write it is in general the expression, this is the general expression.

So however, in practice we only apply this when m and n both the index are same ok. Anyways so now, here this is again the infinite frequency added mass and this is how actually we can get the value for the memory part or the k_{mn} ok. Now here we do not have much time to discuss how we could get the you know the this added mass and frequency domain added mass and damping value for the flexible structure.

But at least for the barge there are some results available in literature in the reference actually, when we discuss the reference that time, we discuss this the paper where actually we can at least get the added mass damping coefficient the frequency domain added mass damping coefficient ok. But anyway, the idea is same, that in order to get the time domain this $k_{mn}(\tau)$ we you need to know the frequency domain added mass and frequency domain damping data for the flexible structure also.

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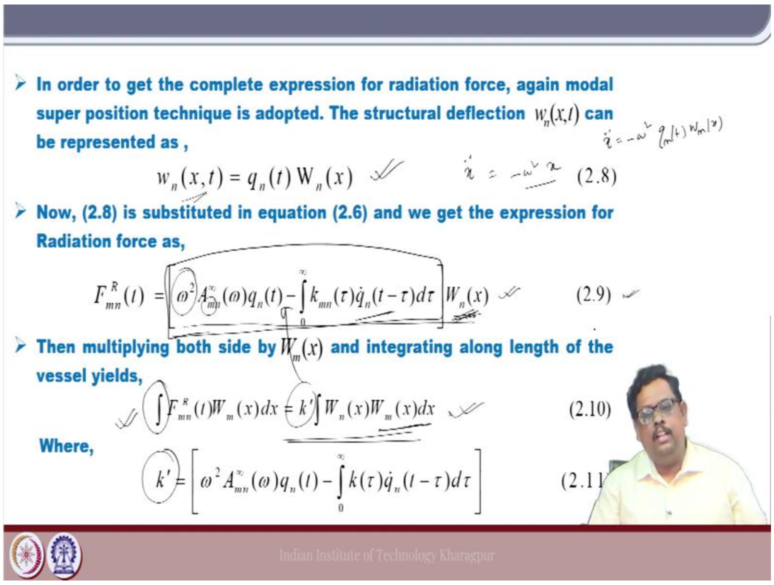

Let us see, how I can incorporate this idea in this semi analytic method ok. Now here I know this is my definition. So, so in previous equation this here I replace in this equation this expression and once I replace this expression here, I know this is my value right, because here this mn as I said that it is only possible when $m = n$ though I mention this mn whatever.

So, anyways this if we multiply whole by this $W_n(x)$. So, finally, you will get this expression right because here this all this space terms will be coming out of the integration sign, now just I will show you here this if you do this if you substitute here. So, it is double derivative right. So, if you do over here this $-\omega^2$ will come comes out and then because this is not depending on the t, so it will remain as it is and again if it is a vector because it is a vector.

So, $\dot{q}_n$ will be there and then $W_n(x)$ will be coming out ok. Now here, again I am using this $\ddot{x} = -\omega^2 \times x$. So, that is why I can write that $\ddot{x} = -\omega^2$ and this x actually $W_n(x)$ it is nothing but my $q_n(t) \times W_n(x)$. So, this $W_n(x)$ will comes out and this part is entirely depending on the t right.

So, now as you know that initially, I multiply everything with the $W_m(x)$ which is the arbitrary mode shape. So, when you multiply this then finally, you have this expression

2.10 because here if you multiply the $W_m(x)$. So, here I just have a $W_m(x)$ and then you we need to integrate from 0 to l. So, therefore, I integrate it from 0 to l and therefore, I have this expression right.

And this is very trivial just you just see it here substitute here and you can get this expression right. Now this is k' nothing but this expression which is the independent of the $W_m(x)$. So, I just to make it you know little bit eye soothing I just I mean I just rewrite the whole expression, I simply I replace by the k'; k' is nothing but the term which is inside the inside here inside the square bracket in 2.9 ok, fine.

So, this formulation is not that it is just you have to sit and replace the thing you will get this expression 2.10 and 2.11.

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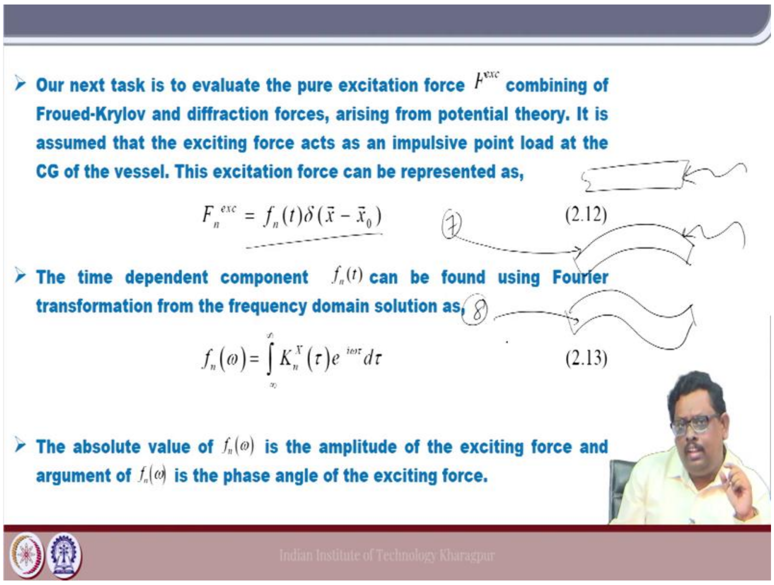

Now, this idea is actually little bit complicated when you calculate the same thing for the exciting force. Now if we remember, what is the expression for the exciting force or what is the physical phenomena that we are going to replicate for the exciting force. Now, you see that when you do the exciting force you actually take the shift as it is and then you hit a wave and then we try to figure out what is the force right.

Now the question is in case of a rigid body I understand it, now in case of a flexible body then wave will hit in which type of I mean or which position of the ship. For example, you know suppose I can I approximate the ship is like this and then I allow the wave is hit into this object are you getting my point?

Now, this thing is I understand that that body does not move right; however, the waves is coming and hitting to the object. Now the question is that that body does not move, but it is a flexible body then which mode I consider? Like can I do this for all modes? Like in case of when I try to figure out, let us say try to find out my exciting force and then for the mode let us take 7.

So, at that time, do I consider that this is the fundamental structure and it will be stand still and then I hit the wave here and try to figure out what is the pressure distribution along the hull and then I can I and then from this pressure distribution, I can get the exciting force.

When I am when I doing it for the mode 8 finally, finally, the same thing? Like in rigid body also finally, you have you know if six degrees of freedom equation you have six ordinary differential equation for each mode. So, here also we are having the in if you take some ten modes we have ten differential equations, if you take hundred modes you have hundred differential equations.

So, then now in case when you solve for mode seven shall I take this body. When I say for the mode eight shall I take this body no this is something actually confused us. And this course is all about discussing those part mainly not the map part as such, because map part you sit and you can do that. I know that it is not that difficult the difficult is the realization. Here the realization is that we do not really do this.

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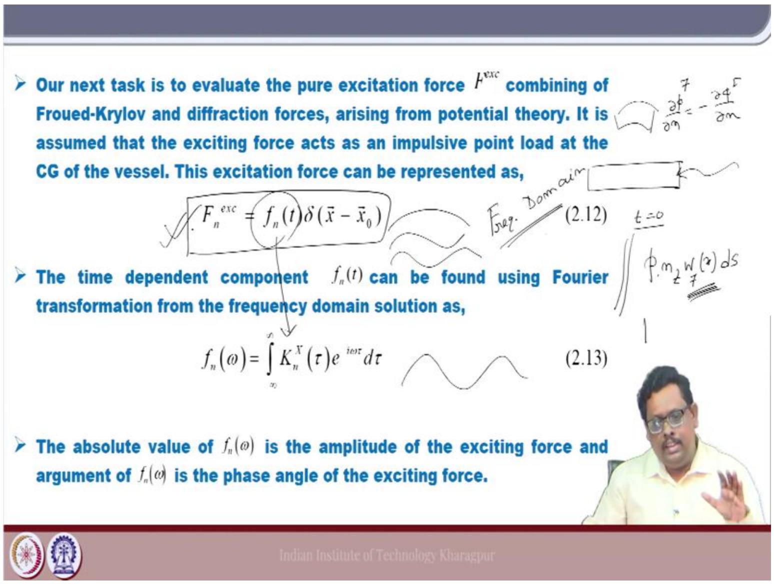

We always take we always take the body should be when it has rest at t = 0. We always take this body and then I allow the wave is hitting the body and when actually I try to figure out the force what I do is that what is the pressure field, I am getting I am integrating along the hull with the normal and that particular time it should be now.

$\iint p n_z w_7(x) ds$. Now this is the idea about the whole thing ok.

Now, when we calculate the pressure when we apply the boundary condition, the boundary condition is always you know del phi del n del phi or whatever the mode it is always diffraction mode is $\frac{\partial \phi^7}{\partial \eta} = -\frac{\partial \phi^I}{\partial \eta}$. This should be the always the boundary condition, whether rigid body or flexible body does not matter this is the boundary condition.

Now, apply this boundary condition when you get the force phi and then from that when you get the pressure P and then when you integrate the pressure P to get the force that time only, I multiply with the respective mode shape. So, this is the idea ok. Now this is how I can get the pressure in frequency domain this all actually we have to do all this exercise to get the data in frequency domain ok.

So, now when you draw this data in frequency domain see the way actually, I replicate the radiation force in time domain. So, this is the expression I can get the radiation force in the time domain, where this $f_n(t)$ I can get again through the Fourier transformation right, because in Fourier transformation, we have the all these frequency domain data right.

Now, time domain and frequency domain for the zero speed only the difference is the Fourier transformation. So, I have the frequency domain data, I do the Fourier transformation then I can get the time domain data. So, this is how actually we can get the time dependent exciting force ok. So, today let us stop at this point we are now we discuss how we get the radiation force or how I get the exciting force.

So, now, therefore, I know how I can how I get the total force over here the total for $f_x(t)$. So now, I now in the next class I write this $f_x(t)$ in the right hand side I multiply by the $W_m(x)$ and then I will show you how semi analytic how to use this you know the structural equation that that beam equation Euler Bernoulli beam equation and how we are using some semi analytic technique to find out the deflection and the velocity of the each mode.

That means we are trying to figure out how I get this $q_n(t)$ right which is actually again it is in this equation. So, we are trying to figure out the $q_n(t)$ in the next class. So, till so now just to conclude I have this expression $f_x(t)$ in the right hand right we know the mode shape in the left-hand side.

So, how I convert this into the ordinary differential equation for the time mode and then how to solve this the ODE in t in analytic method ok.

Thank you.