Numerical Ship and Offshore Hydrodynamics
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Lecture - 07
Seakeeping - 6

Welcome to the Lecture 7 of Numerical Ship and Offshore Hydrodynamics.

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Now, today we are going to discuss this following topic that we are going to discuss the uncoupled Heave, Pitch and Roll Motion. And also, after that we are going to discuss the coupled equation of motion where you get to know about this mode j and k etcetera. And after that we are going to discuss something about the Froude scaling if time permit, otherwise you can continue in the next lecture.

Now, before we start this discussion on heave, let us continue from our last lecture where we are going to discuss about how I get the radiation force from the experiment, right.

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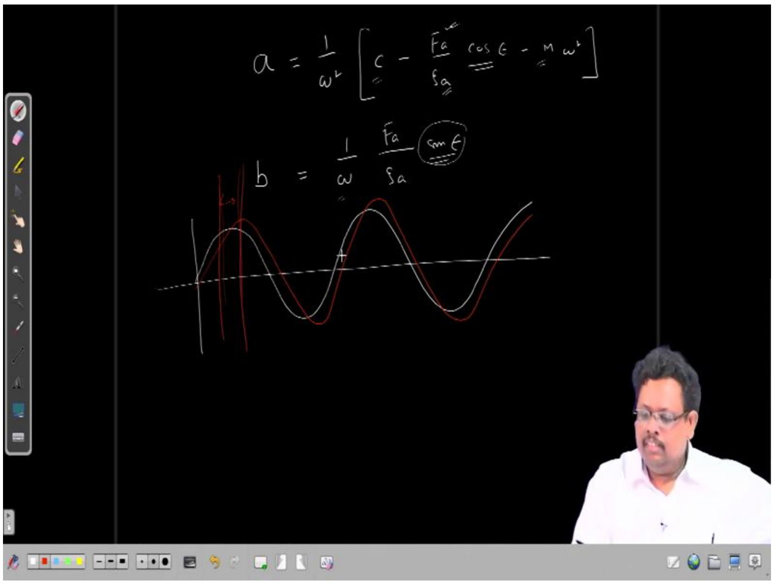

And if you remember that we are actually end up getting; end up getting a relation $a = \frac{1}{\omega^2}\left[c - \frac{F_a}{\xi_a}\cos\varepsilon - M\omega^2\right]$. And then you get the expression for damping $b = \frac{1}{\omega}\frac{F_a}{\xi_a}\sin\varepsilon$. Now here that time when in the last moment we have we did not discuss how to get the $\cos\varepsilon$ and $\sin\varepsilon$, there is a many way of getting it.

Now if you look at this expression this Fa is basically that you are getting from the signal from experiment ξ_a is your input, right M mass is you know that is the restricted c you know and omega also of course, your input. So, only thing is that you do not know the phase epsilon. Now how to get the epsilon that from the machine you know the record of the forcing function. So, this one is the let us say forcing function and again that the response also that force that you are getting from the device let us say it has this one right.

Now, if you do the simple FFT you will get the phase, otherwise you know you need to integrate in averaging the time period and then you know this how to get this phase numerically like, mostly you are going to do the FFT. Otherwise, there is another way of doing it, but nowadays you know like you know this then you need integrating a and b you know to get this you know the sin epsilon and cos epsilon, but best way to do that you have two signal do the FFT get the phase, ok.

So, this is we are going to discuss from the last class. Now today we are going to start a new topic we need to discuss the three uncoupled mode.

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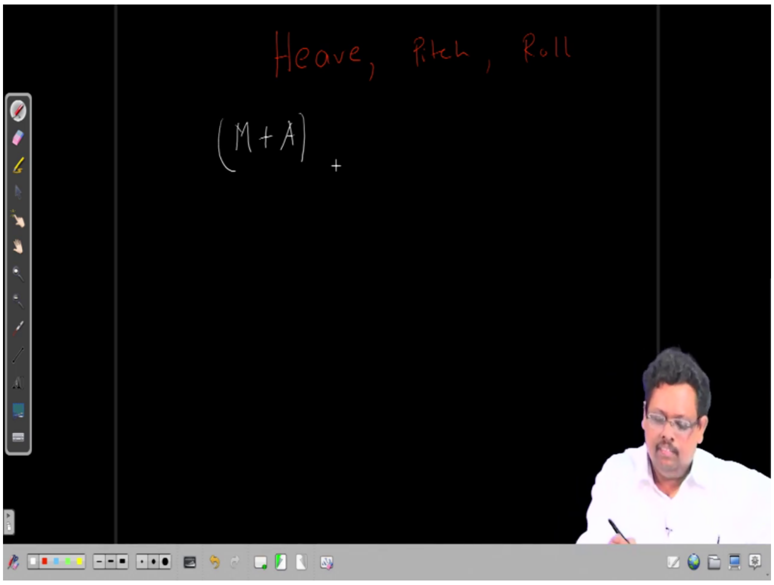

It is heave, then pitch and roll. I discussed pitch first after before the roll, ok. Now in heave the uncoupled equation of motion is

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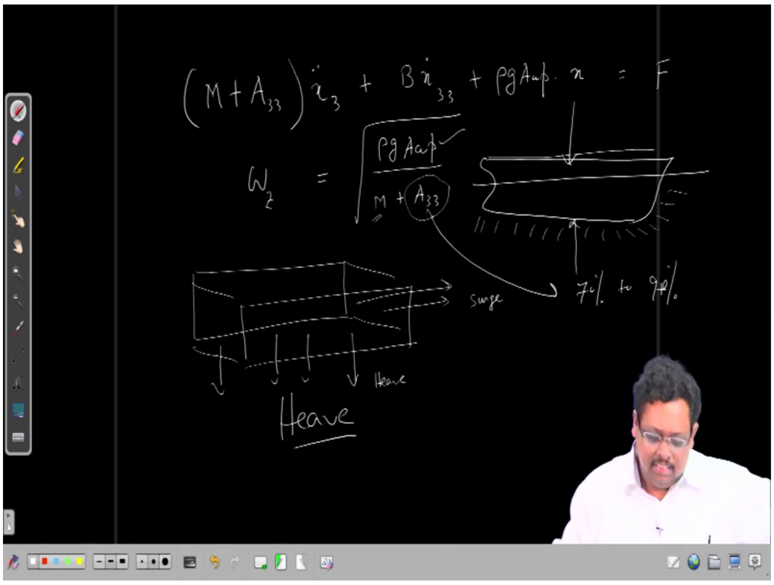

$(\mathrm{M}+\mathrm{A}_{33})\ddot{x}_3+B\dot{x}_3+\rho g A_{wp}x=F$. So, now here you know the natural frequency for the heave that is $\omega_z=\sqrt{\dfrac{\rho g A_{wp}}{\mathrm{M}+\mathrm{A}_{33}}}$ normally called because it is in about the heave see about the z direction. Now here definitely in this numerical ship hydrodynamic class we are going to discuss how we are getting this heave added mass A_{33}.

Now, suppose; however, in first shot if you ask somebody like how to estimate the natural frequency for heave, right. Now, A_{wp} is the geometry property of course, you know that rho is the density g is g you know the you can approximate 10 or 9.81. Whatever M you know the mass the only thing it is coming from the hydrodynamic side is basically the term A_{33}.

Now, ideally speaking for engineers like to get a realistic estimation how to go about it? Now if you know that that how we get the radiation force this is as follows like, we did for the experiment of the same thing we have this sorry we have this vessel, and then we oscillate this vessel in calm water and then that corresponding pressure if you integrate you will get the radiation force.

Now, if you see that essentially you can think of this ship basically pushing the water surrounded by this body. And because of that you can assume that pressure get created. So, therefore, now you can think of let us say a rectangular barge, right and suppose I am pushing it downwards; I am pushing it downwards, ok and then I push forward. So, can you tell me that you know from your basic idea like if you push this body downwards, right.

Like if I have this mobile with me if I push it downwards, I will push more water or if I push this way, I push more water. I know the answer is you know the answer the answer is basically if you push downwards, they are definitely you are going to push more volume of the water, right. Now having said that if I that it means that added mass in heave direction definitely more than the added mass in surge direction.

Like this is how I mean we could approximate like. Because we are in heave mode, we are pushing more water compared to the surge. So, therefore, in A_{33} normally we approximate some 70 percent to 90 percent of the mass of the body, ok. So, this is how actually you can approximate the added mass of the body, fine. Now let us take another thing very interesting thing.

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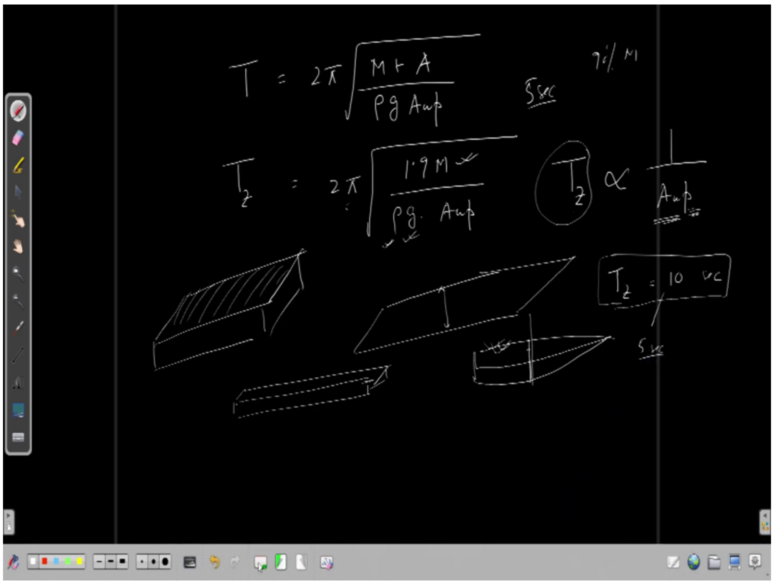

Now, let us let me write the time period of course, I mean it is the $T = 2\pi\sqrt{\dfrac{M+A}{\rho g A_{wp}}}$. Now what I am trying to say is like let us, now we assume that A let us say some you know 90 percent of the mass, ok. So, then I can redefine this now we can call T z the time period in heave which is $T_z = 2\pi\sqrt{\dfrac{1.9M}{\rho g A_{wp}}}$.

Now here you can see that mass is constant ρ you know this water of water the sea water g is constant, π is constant. So, I can very easily see $T_z \propto \dfrac{1}{A_{wp}}$; that means, in ship now if I look at the from the top. So, it is the top part of the ship or the barge, ok. Now if you increase the area of the barge, right.

Then basically your natural time period basically decreasing, right. Now if you increase sorry if you decrease the barge this natural time period will increase, and if you increase the water plane area your natural time period will be decrease. Now you see there is a; there is a totally different from the hydrostatic thing like, I will tell you the fun here you increase this then natural time period decrease.

Now, let us say in normal situation suppose you have this barge little bit slender little bit slender. And then you can what you say that if it is slender then your you can that natural time period T_z becomes some let us say 10 second or something like this let us say like this then, but you are not happy because if your ship becomes slender, right.

Then you are fearing that it might not be hydrostatically stable because the restoring moment may be less, because what you did to make it slender you compromise with the beam of the ship. So, it means that your beam of the ship is smaller, right. So, in that case you are you know having some kind of scarcity like your the ship may be capsized with this beam; however, you are getting very nice natural time period of 10 or 15 seconds something like this, right.

Now, if you want to compromise with this. So, and you want to put more importance to the slave stability of the ship and if you increase this now what are we needing more. So, definitely it comes down to the now 5 second, something like this. Now what is happening now what is the difference between these two when it is 10 second then actually if you are on board, you can have little bit easier way of going in inside the ship. Because ship naturally moving very slowly with his natural frequency 10 second or 15 second something like this.

However, you are thinking your ship is not stable stat hydrostatically, and now what is happening it is now at 5 second you your ship may be now more stable, but your natural period become increase. So, therefore, you might feel uncomfortable while you are boarding on the ship.

Now see in one sense your if you compromising on the comfort of the person so your ship might be stable more. And if you are thinking about the comfort of the passenger maybe you have to; you have to think about the stability, how stabilities and you have to look out the.

So, that is why design become very crucial or I mean you know very interesting in this such situation. Because it is so counter intuitive like I am looking for the comfort of the passenger I am compromising on the stability of the ship and the other time I try to make the ship will be very very stable, increasing the beam and then because you have limitation of the length, of course.

Like you cannot make it 300, 400 a long to increase the water plane area something like you cannot. And that does not help also, because your this momenta is not change. So, anyways. So, therefore, these are something very interesting thing like well there especially for the engineers like who was going to design a vessel inland vessel or something like this, and some passengers going there some road or zero packs if that time they have to think very you know logical way how to go with this, ok. Now coming back coming to the pitch.

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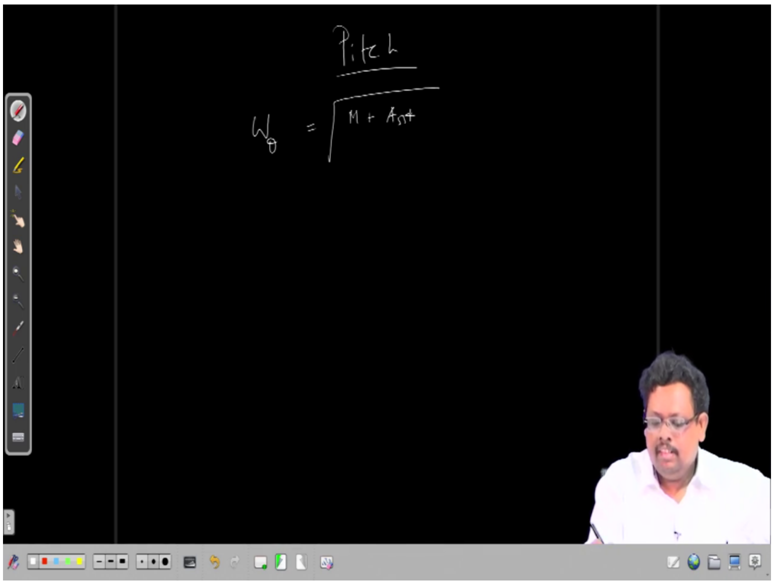

Now, I am just now the equation of motion is same now I am just talking about that natural frequency.

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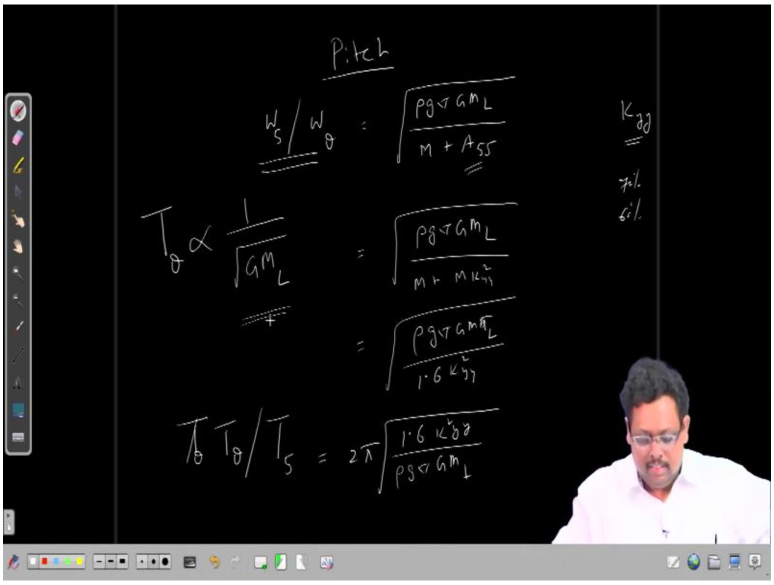

Now you can call this $\omega_5 or \omega_\theta = \sqrt{\frac{\rho g \nabla GM_L}{M + A_{55}}}$ Now, here also; now here also that how I approximate A_{55}. So, now, we can have the radius of gyration k_{yy}. So, then we can think of this we can again rewrite $\omega_5 or \omega_\theta = \sqrt{\frac{\rho g \nabla GM_L}{M + MK_{yy}^2}}$. Now here also as you know that again I am trying to get some percentage of M, right. So, here also let us take let us again pitch also decrease of course decrease, it gives more water compared to the roll of course. So, we can take some again some 70 percent or 60 percent of the M.

So, then we can approximate this as $\omega_5 or \omega_\theta = \sqrt{\frac{\rho g \nabla GM_L}{1.6 k_{yy}^2}}$. So, that may be the natural time period, sorry frequency. Now on the natural time period $T_\theta or T_5 = 2\pi \sqrt{\frac{1.6 k_{yy}^2}{\rho g \nabla GM_L}}$.

Now here also you can see my T z, sorry T theta is varies with 1 upon square root of G M L same thing. But now this G M L is so high you really do not have to compromise much on this, because it does not affect the design as such, right. But in case of a roll definitely it is going to affect now let us see what happened for the roll.

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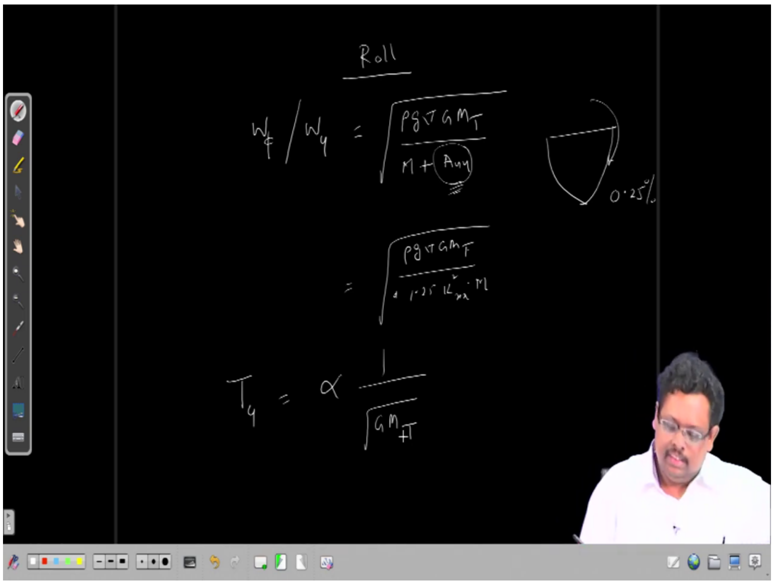
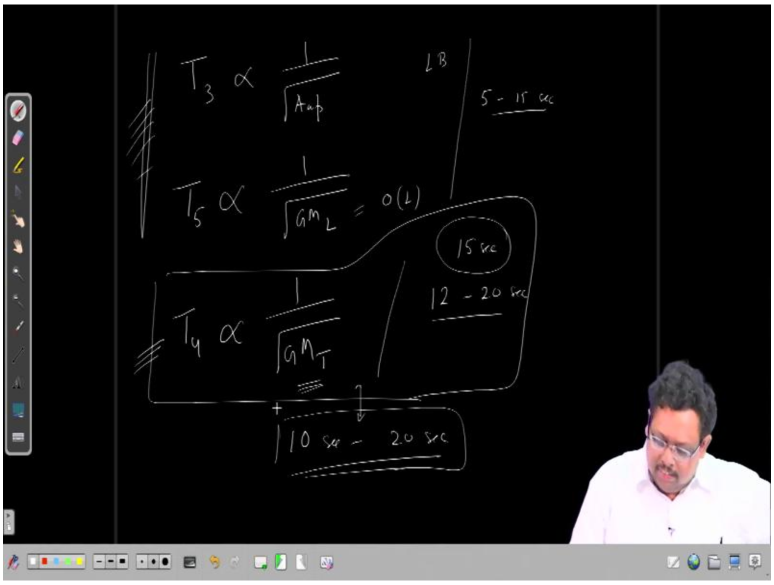


Now in case of a roll that your natural time period that you can say $W_{\varphi} or W_4 = \sqrt{\dfrac{\rho g \nabla GM_T}{M + A_{44}}}$. Now, it is M_{44} like and also it is A_{44}. Now, here again the same thing if you again although do all these things and now you really actually this is much much lesser compared to the pitch because here you can see only you have this thing, right.

And, now if it is rolls like it really does not replace much water as such, right. So, therefore, this could be taken as let us say maybe some 2.5 percent of the whole the mass of the body, ok. So, now again you can again approximate this as now here it is just in a is a very crude approximation, ok. This we are going definitely going to get from our that hydrodynamics that we are going to discuss in from the next week, ok.

But here it is just very quickly I try to estimate what is the value, right. So, then again if you do this everything then we can just substitute everything here, ok I miss the it is let us say $1.25 k_{xx}^2 M$. Something like this, ok. So, also you can find out that time period $T_4 \propto \dfrac{1}{\sqrt{GM_T}}$.(Refer Slide Time: 18:21)



So, then you understand from these three for the heave mode the $T_3 \propto \frac{1}{\sqrt{A_{wp}}}$, then your $T_5 \propto \frac{1}{\sqrt{GM_L}}$ and then $T_4 \propto \frac{1}{\sqrt{GM_T}}$. Now, what I am trying to say is as follows now G M L is much much higher is the order of length, right. And then you can see the water plane area also you can come you can let us take a rectangular barge it is also L into B, right something like this.

So, these two is actually you know these are fall in more or less same range, ok. However, you can see that GM_T is much much lesser compared to all these values. So, therefore, comparison to the heave natural time period which is normally 5 to 15 second the roll natural period actually much higher, it is something around let us say 12 to 20 second something like this or on an average let us say 15 second.

So, idea is it is much higher than this now why this is severe, because in day to day life that you are travelling in ocean and all that most common waves are having this time period. They mean they are mostly from 10 second to 20 second. So, this is the range of the you know the time period of the ocean waves. Now, so you can see that it is very much possible that roll natural time period falls within the range of this so called regular waves.

So, therefore, in ocean we have to very careful about the capsizing of the vessel especially we have to take care about the role and also actually because of this we have huge discomfort to the passenger also. Because, sometimes if we start oscillating in some resonating mode some waves hitting here is close to the natural time period of the roll, then the passenger having lot of uncomfortable situation. Like some people vomit also you can call the sea sickness, right ok.

So, this is more or less that is what I am going to discuss today about the heave pitch and roll. And, now let us start something on the coupled equation of motion.

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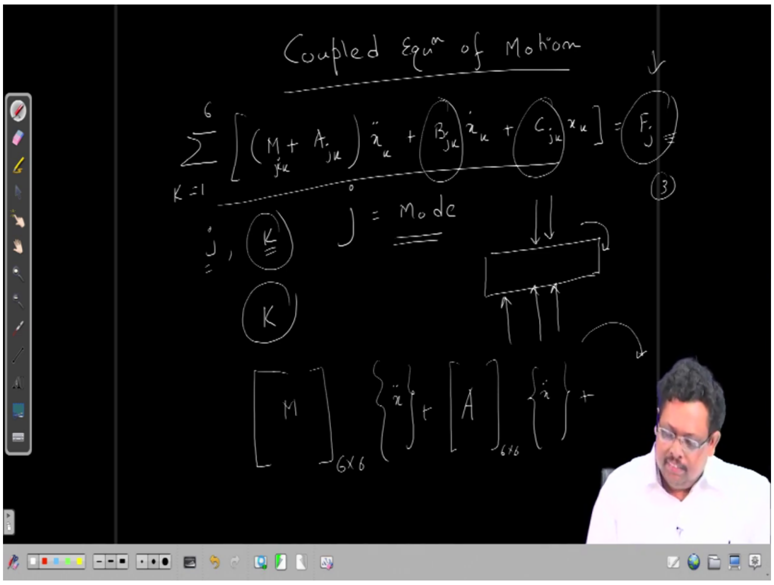

Now, again I am going to say that like here this one more concept is the encounter frequency and we are not discussing here, right. Now when we discuss about the forward speed problem that time again, we come back and we discuss something on the encounter frequency.

Here we assume that vessel does not move forward it is floating, ok. Now in coupled equation motion if you remember that we are using here $\sum_{k=1}^{6}\left[\left(M_{jk}+A_{jk}\right)\ddot{x}_k+B_{jk}\dot{x}_k+c_{jk}x_k\right]=F_j$. Now, here what is j, what is k? Now j is basically the mode j refers the mode at which the ship got excited, ok.

Now, that is why it is a right hand side, fine. And then K basically that because of this happened, then what effect in the other mode also. Now let us give an example now suppose you have a box and then you start oscillating in this mode 3, ok. Now tell me one thing even if you oscillate in mode 3, right still you can get some moment in direction of pitch is it possible right; that means, if I am oscillating in heave mode like again taking it if I oscillate in heave mode.

So, there is a possibility definitely I can get the force in this direction also, right. Now, therefore, now if it does not happen then definitely then definitely there is no coupling, but; however, in most of the situation that many many modes are coupled to each other; that means, for example, heave and pitch definitely coupled to each other. So, if I do this exercise definitely, I can get some force like this.

As similarly if I oscillate the body in this direction definitely, I can get some force along the heave direction and. So, this is what we are talking about I am always looking for the force along jth direction in the left hand side also, because this is my exciting force, right. So, we have to match the dimensions. So, therefore, if it is a heave force. So, then the left hand side everything should be heave.

But; however, I can get the force in the direction of heave; however, I oscillate the body in may be the direction of pitch there must be some component along the heave, right. Even I do something sway still I can get some component along the heave, what is I am assuming if I oscillate in roll I can expect some component along the heave, right. So, that is what is all about my k mode, ok.

Now, now see if you write in this as a matrix form. So, definitely you can see that it should be something like this you can have this M is a 6 cross 6 matrix, right with respect to this x double dot, and also you can have the matrix A also 6 cross 6 matrix, right and with this again x double dot, right and then you can have all this B also the 6 cross 6 matrix this also 6 cross 6 matrix and then of course, is for the energy, right. Now, thing is that not all are coupled right for example, like if we consider the coupling.

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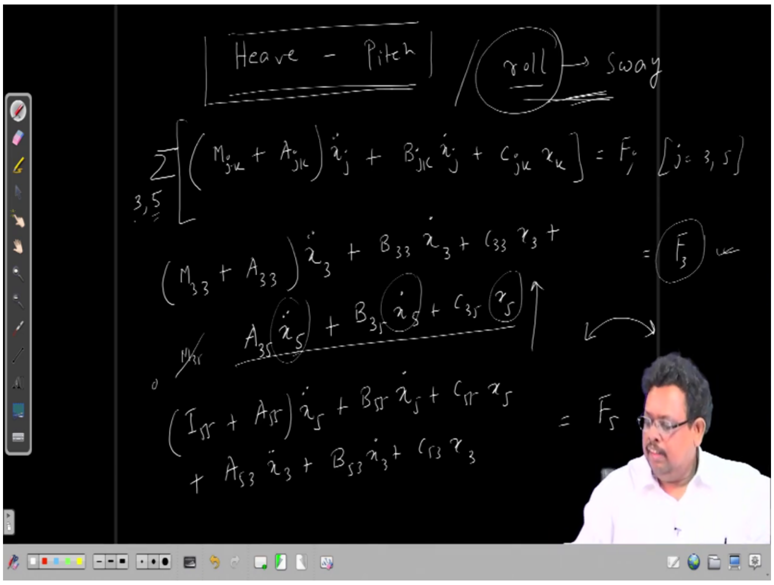

Normally that is this heave is coupled with pitch that is this is the most we do that and roll we can consider that uncoupled. But, however, roll is coupled with sway mostly, right. So, now, if you write the equation of motion normally most of the softwares what they do is they write the coupled heave and pitch equation of motion and they write the roll single degree of freedom equation emotion somebody, also coupled the roll with sway especially for the strip theory and, ok.

So, let me write now that coupled heave pitch equation, ok. Now in that case in the right hand side it should be only 3 and 5, right. And then here I am now just here I am just $\sum_{k=1}^{6}\left[\left(M_{jk}+A_{jk}\right)\ddot{x}_k+B_{jk}\dot{x}_k+c_{jk}x_k\right]=F_j\left[j=3,5\right]$. Now let us take j equal to 3. So, right hand side throughout it is F_3.

Now, let us write the left hand side now it should be $(M_{33}+A_{33})\ddot{x}_3+B_{33}\dot{x}_3+C_{35}x_3$. Now, plus I am talking about the 5. So, A so here you can see that j should be 3, but now I run k for 5, but M. So, there, but M_{35} is 0. So, I do not take this. So, I have $A_{35}\ddot{x}_5+B_{35}\dot{x}_5+C_{35}x_5$.

Now, you see that what are the second component, second component is this like if I oscillate the body in the modes fifth, fifth mode that in the in direction of the pitch then, what is the component of the force along heave direction? Now you see that this equation all is it is not possible if I solve it solely, I have to take the another equation, right. Because to compute this term that x double dot 5 x dot 5 and x 5 I need to have another equation which is $\left(I_{55}+A_{55}\right)\ddot{x}_5+B_{55}\dot{x}_5+C_{55}x_5+A_{53}\ddot{x}_3+B_{53}\dot{x}_3+C_{53}x_3=F_5$. Now, you can extend this for your all 6 degrees of equation of motion, right. And you can get that many term is 0 many term is non-zero again we are going to discuss these things again we discussed later on, when we dealing with the hydro dynamics getting all the coefficient and going to solve this equation definitely I will going to discuss this. But right now let us stop here today.

Thank you very much.