Numerical Ship and Offshore Hydrodynamics
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Lecture - 09
Hydrodynamics - 1

Hello, welcome to the Numerical Ship and Offshore Hydrodynamic course.

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Today is the Lecture 9 and today we are going to discuss on the Basic Hydrodynamics ok.

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And these are the keyword that you are going to get this in YouTube.

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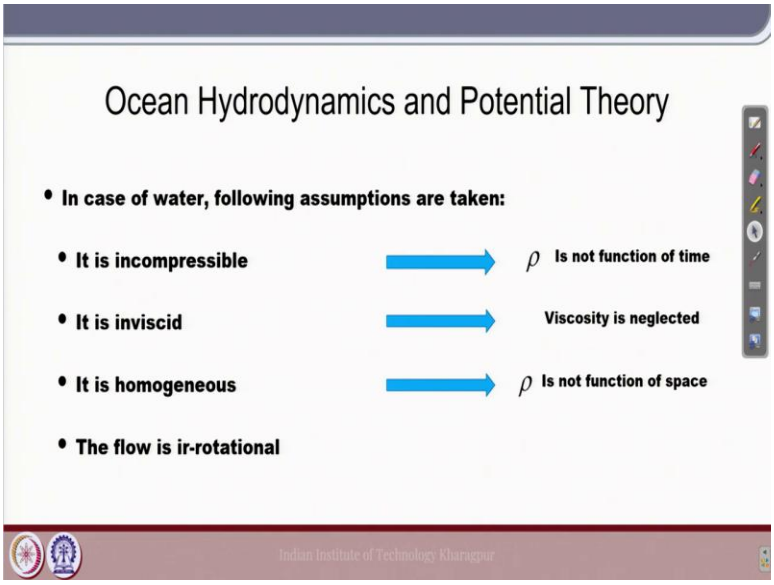

Now, we are mainly discuss the potential theory. Now, what is potential theory? That definitely we are going to discuss now. Now, in case of water, that we can take the following assumptions. So, the first assumption is that it is incompressible, second one is this it is inviscid, third one is it is homogeneous and then, fourth one is it is ir-rotational.

Now, what is the meaning of the incompressibility or inviscid or when you say it is homogeneous or it is ir-rotational, let us try to understand. Now, when I said it is incompressible it means that the density is not the function of time. Now, it is not the case for gases right, because in case of a gas it is adiabatic and it is compressible, it is the density is not constant over the time also, but in case of a water we assume that it is incompressible. So, ρ does not change with respect to time ok.

Now, when you call it is inviscid it means that I ignore the viscosity and it is quite natural like to ignore the viscosity for water because it understand is it is very low right. And most of the thing like because this course mainly focuses on the normal forces like radiation, diffraction, Froude-Krylov or so, that is why we really do not deal with tangential forces ok, like drag forces because wave resistance part is not considered over here.

It is not the case always that we are not considered the viscosity of course, we consider in case of sea keeping also when you calculate the roll motion. So, that time we do consider viscosity and sometimes like when you consider very very small tube type structure and we do for the long wave approximation and we can use some Morison equation that time also we are considered this viscosity.

However, in our ships wave's interaction and where if we try to understand that where objective is to get the normal forces which is radiation force, diffraction force, etcetera at that time we can consider that viscosity is you know negligible ok. Next, we call this is homogeneous. Now, when you call it is homogeneous it means that ρ is not the function of space also. Now, what is the meaning that ρ is not a function of the space?

Now let us take a cup of tea and you add some sugar to it and you do not stir it not firmly like you just do it and then you try to take the sip. So, then what you get is that the first time when you take a sip, you cannot find, it is not much sweet right. So, as you take more and more you can find the sweetness is more and more it means that it is not homogeneous that is that the sweetness is not homogeneous, it is a function of space also, in the top level the sweetness is low, in down level sweetness is high.

So, it is not homogeneous. Now, if you stir it firmly and quite for quite some time then you can find that throughout the tea in all time we are getting the same level of sweetness. So, then we call this a homogeneous. Now, in context of our situation we are talking about the ocean, now in ocean what is happening that you can say that salinity is not constant right. So, of course, I agree salinity is not constant and it is varying over the depth.

However, mostly you are dealing with some in in case of the deep water approximation also it will be hardly 80 to 100 meter that is what actually you are going to take for this all this numerical evaluation and all and we consider that is a deep water case. Even if you take 150 meter depth still the rho is does not change much the side does not change much in this region.

So, therefore, we consider here the ρ is not the function of space also and that is why I call it is homogeneous. So, this is at the property of the fluid right. Now, again let me read out the first property it is incompressible. So, we are not considering the ρ is changing with the time, second one is the I consider this inviscidness; that means, that that water is not I mean we are ignoring the viscosity term of course, in the Navier-Stoke equation and thirdly we assume it is a homogeneous.

So, ρ is not a function of space also now this is about the fluid, but what about the flow we consider that flow is ir-rotational right. Now, what is the meaning of when you say that flow is ir-rotational?

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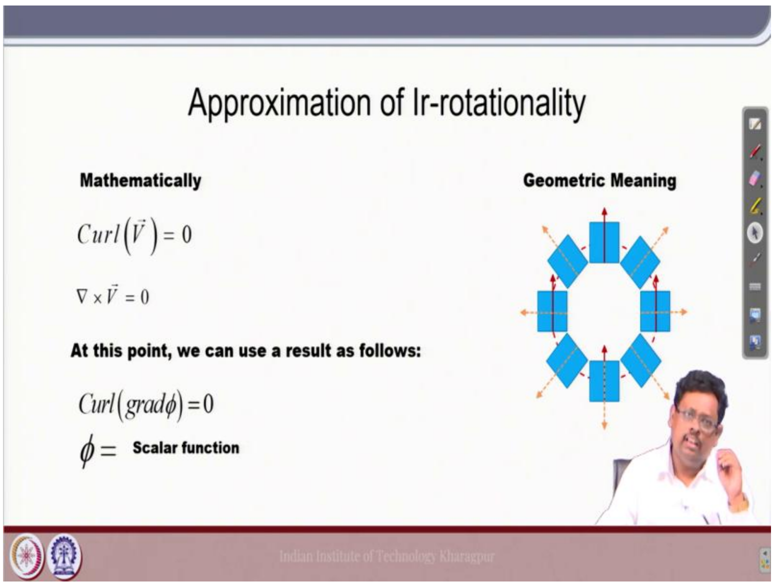

Now, let us try to understand from the geometric point of view, now here you can see let us say these are the fluid particle it is you know the path is in the red dotted line it is moving in this path. Now, how it moves, now let us what we do is let us put a flag and try to see the flag when it actually moves right.

So, firstly, I said when you call ir-rotational it does not mean that the fluid particle moving in a straight line not necessary so, that is why I take this circular path. Now, if we look at the fluid particle now if it is coming from this to this point you can see that flag does not change.

So, it does not have any rotational velocity about its own axis right. Now in case of a rotational flow if I try to do the same then what we get is as follows, now you can see that as long this particle is moving from here to here and then from here to here it has a rotational velocity right. So, therefore, you can see that as it moves along this path it also rotate about its own axis so, in that case we can call this is a rotational flow.

However, mathematically speaking if we consider it is a ir-rotational flow so then what we do is, we can say that $Curl(\vec{V})$, which is $\vec{V}$ is the velocity $\vec{V}=0$ or $\nabla\times\vec{V}=0$. Now, this is a very important you know the mathematical concept that we are going to use and for the potential flow theory of course, we consider this is a ir-rotational flow and we use this property and also use a property like this is the typical is from the vector calculus we know that, that $Curl(\text{grad}\,\phi)=0$ right?; that means, $\nabla\times(\text{grad}\,\phi)=0$ ok.

Now, look at this, now if it is a ir-rotational flow then $Curl(\vec{V})=0$ right and also I know that in case of a it is from vector calculus you know that curl of curl cross I mean curl of grad of something equal to 0; that means, del cross gradient of something should be equal to 0. Now suppose take a scalar potential ϕ.

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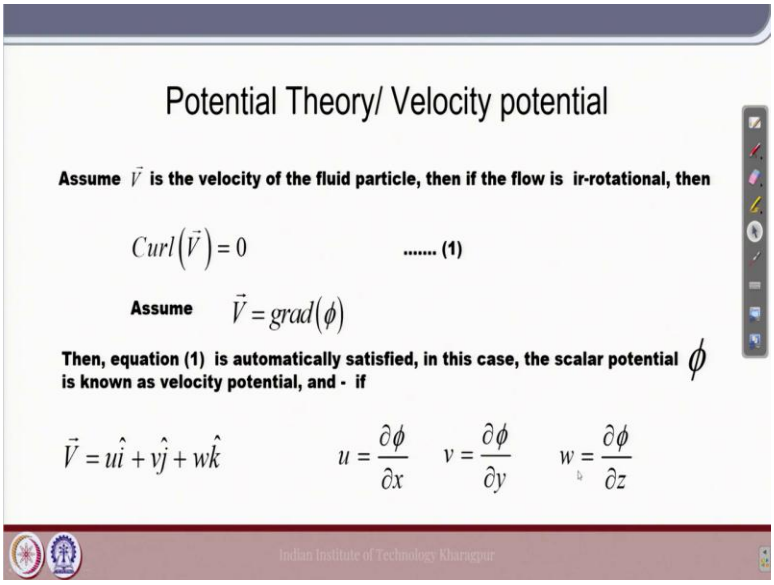

If I do this then if I assume that $\vec{V} = (\text{grad}\,\phi)$ ok. So, therefore, since $Curl(\vec{V}) = 0$. So, we can assume that $\vec{V}$ should be some gradient of ϕ whether ϕ is some kind of scalar function right, it is very natural it is coming. Now, if I assume this then you know this equation 1 is automatically satisfied right. So, in this case we called this ϕ is basically a scalar potential right and then, the velocity I can define this velocity of the gradient of the scalar function. Now, three component of velocity is u, v, w that everybody knows it right.

Now if it is so, then what is u? So, u basically the i-th component $\frac{\partial \phi}{\partial x}$, v equal to the j-th component $\frac{\partial \phi}{\partial y}$ and w be the k-th component $\frac{\partial \phi}{\partial z}$, okay?. So, this is the- you can see that these three basically my three component along the x y and z direction right. So, that is how we can define the velocity.

And this ϕ is now is known as velocity potential ok and this is basically the backbone of the potential theory. So, when you called about the potential theory we are talking about a function a scalar function phi such that $\vec{V}$ can be represent as gradient of the scalar function and therefore, $u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$ and $w = \frac{\partial \phi}{\partial z}$.

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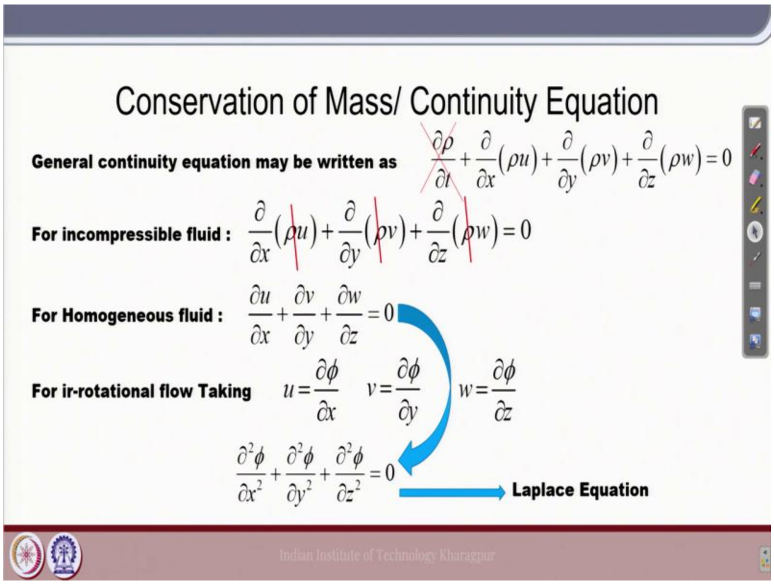

Having said that now let us try how I can rewrite my conservation of mass equation. Now, you know this is the general equation $\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$, this is the standard equation continuity equation. Now, in this course we are really not going to discuss how this equation comes, we start from this equation ok.

So, here I assume that it is the ρ is not function of time it is incompressible. So, therefore, I strike out this term right. So, if I assume it is incompressible fluid. So, then it turns out to be this equation. So, I just ignore the first term. Now, I use the, this homogeneity that ρ is not the function of space also.

So, in that case you know that, that ρ will come out and see $\rho \neq 0$. So, I can strike out the ρ also and then we can find out that our equation is $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$, till this point this is the equation for continuity if we do not consider the that u is ir-rotational that mean now I need to use the concept of the ir-rotational fluid.

Now, if you consider the ir-rotational flow. So, therefore, we can write this $u=\frac{\partial \phi}{\partial x}$, $v=\frac{\partial \phi}{\partial y}$ and $w=\frac{\partial \phi}{\partial z}$ right?. It is now actually we are just almost at the you know striking point where actually we need to substitute del u, $\frac{\partial \phi}{\partial x}$ here, $\frac{\partial \phi}{\partial y}$ here and $\frac{\partial \phi}{\partial z}$ here, the once you do that then we will get what? We get the Laplace equation right.

So, this is the continuity equation for our potential theory, now it is very well known equation right and so, in our case our continuity equation basically the Laplace equation. Now, you know that I mean somebody is from maths they know very well the about the Laplace equation and also here who is dealing with the potential theory, yeah for them also this is a very well-known function right.

So, here we get my governing equation or you can say that equation of mass or continuity equation is basically my Laplace equation. Now, you have once you have the Laplace equation there are so many things you can do actually so many things the numerical solution you can think of also right. We will discuss that like once you know this this governing equation is Laplace equations and then how to solve this problem ok analytically or numerically. So, let us.

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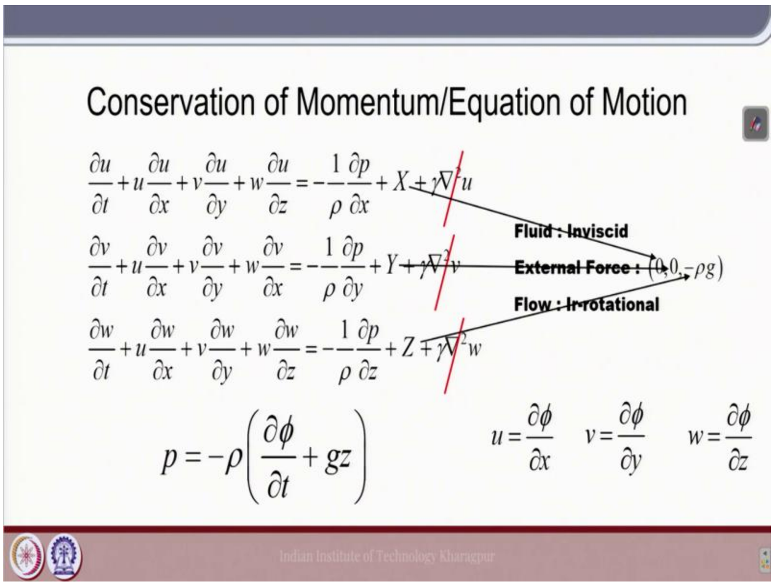

Before we go into this let us try to get what would be the equation of motion, now this is a Navier-Stoke equation right. So, here again we are not going to discuss how it comes ok and so, is. So, now, let us try to impose one by one the conditions, now first if I impose is a inviscid so, then definitely we are going to strike out this right.

And, now if you consider this external force is basically under the gravity now this is the; this is the force minus ρg now if you divided this by ρ. So, it is simply becomes $-g$ of course, right? and if you assume this flow is ir-rotational so definitely you can see that $u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$ and $w = \frac{\partial \phi}{\partial z}$. Now, if you need to do little bit of mathematics to impose everything into this equation ok and then you need to get the final term is nothing but the $p = -\rho(\frac{\partial \phi}{\partial t} + \text{gz})$. Now, how to do this? So, let us do this here. Now, here what we have done that. So, we are here it is.

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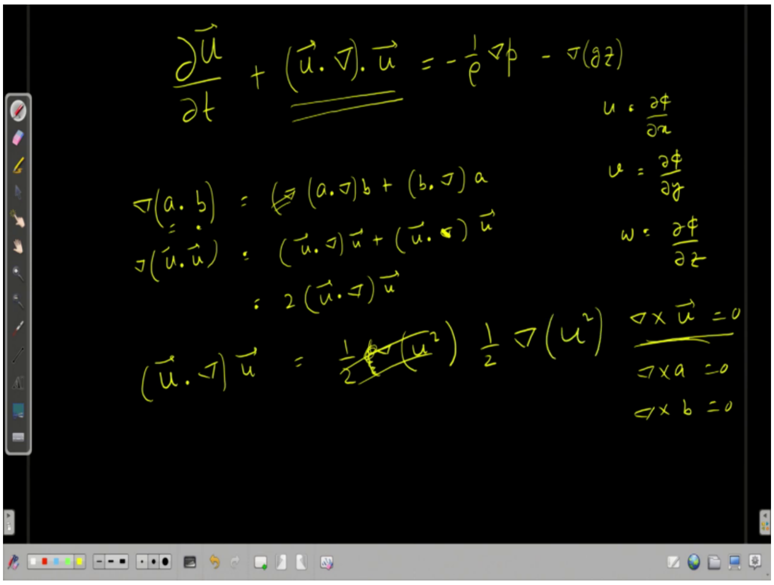

So, let me write in bigger way. It is $\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla).\vec{u} = -\frac{1}{\rho}\nabla p$ and minus as I said you can simply write $\nabla(gz)$ ok. So, now, we need to substitute that $u = \frac{\partial \phi}{\partial x}$, I mean these are the component remember this is the vector u and this is the component both are not same say $v = \frac{\partial \phi}{\partial y}$ and $w = \frac{\partial \phi}{\partial z}$ right that you need to do.

Before that try to find out what is this thing. Now, remember since this $\vec{u}$, I know it is as you know that I assume it is a ir-rotational flow. So, definitely you have to understand that $\nabla \times \vec{u} = 0$. Now, if you use this and then we are going to use one results from our vector calculus which is that $\nabla(a.b)$, now assume that that $\nabla\times a=0$ and also $\nabla\times b=0$ if it is so, then it is $(a.\nabla)b+(b.\nabla)a$.

Now, in case of a and b if you assume both are u so then, I can write $\nabla(\vec{u}.\vec{u})=(\vec{u}.\nabla)\vec{u}+(\vec{u}.\nabla)\vec{u}$ and then you can have $2(\vec{u}.\nabla)\vec{u}$. So, we understand that $(\vec{u}.\nabla).\vec{u}$ is nothing, but $\frac{1}{2}\nabla(u^2)$ right ok. So, let me write here as a $\frac{1}{2}\nabla(u^2)$, okay?. So, now, once I know this results now I can apply $u=\frac{\partial\phi}{\partial x}$, $v=\frac{\partial\phi}{\partial y}$ and $w=\frac{\partial\phi}{\partial z}$ and then the same thing I can get is.

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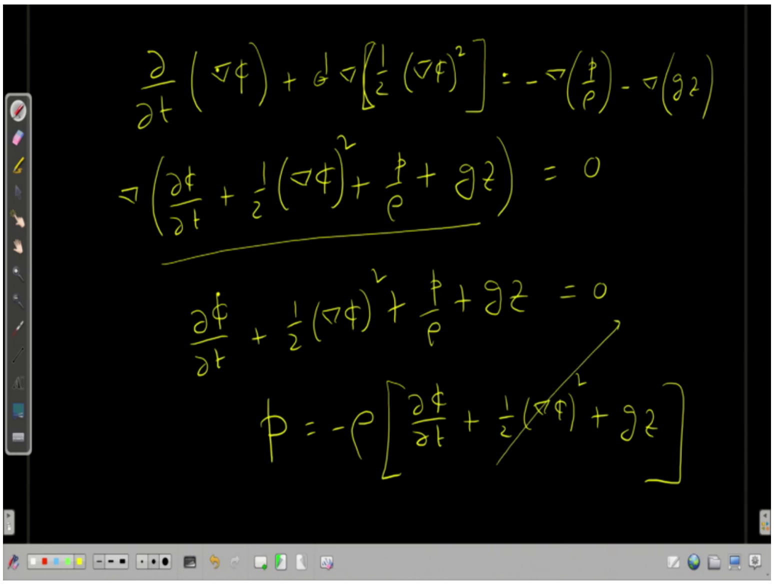

It is $\frac{\partial}{\partial t}(\nabla\phi)$ plus or we can say the $\nabla\frac{1}{2}(\nabla\phi)^2=-\nabla\left(\frac{p}{\rho}\right)-\nabla(gz)$. Now you take everything this side and then you can write the $\nabla\left(\frac{\partial\phi}{\partial t}+\frac{1}{2}(\nabla\phi)^2+\frac{p}{\rho}+gz\right)=0$. Now, since you can say this cannot be 0. So, this term equal to 0.

So, you get finally, that $\frac{\partial\phi}{\partial t}+\frac{1}{2}(\nabla\phi)^2+\frac{p}{\rho}+gz=0$ and then finally, you can get $p=-\rho\left[\frac{\partial\phi}{\partial t}+\frac{1}{2}(\nabla\phi)^2+gz\right]$. So, now, if you would if you ignore this this quadratic term so, you can get this one right ok. So, now, so if you now you can see from here that if you ignore this quadratic term, so this is your pressure equation.

Now, here in this context I would like to tell you that this is basically the major difference between the conventional fluid dynamics to this particular course or maybe the when we do talk about the hydrodynamics or numerical ship hydrodynamics, thing is that this is known also that Bernoulli equation of course you know, thing is that in case of all other fluid flow that we are considering in mechanical in all other discipline, aerospace all other discipline normally we keep the second order term and we ignore this transient term which is $\frac{\partial \phi}{\partial t}$ right.

You look at the Bernoulli equation ask anybody it says that $\frac{p}{\rho}+\frac{1}{2}q^2+gZ=0$. So, we are ignoring the term the linear term which is time dependent term and we are keeping that q^2 term, you see that in all Bernoulli equation we are keeping it.

But here what we are doing is, we are ignoring that quadratic term under the linearity, what is the linearity, we are going to discuss later on, but here this is my pressure equation where we are keeping the transient term which is $\frac{\partial \phi}{\partial t}$ and we are ignoring the second order that $(\nabla \phi)^2$ right.

So, here there is a big difference, here we are really do not give much value to the quadratic term; however, our the linear transient term is much more valuable it is unlike the other discipline. So, it is you know when you see what is the actually what is the difference what is the catch. So, these are the first time you can see there is a difference in the pressure equation itself right ok.

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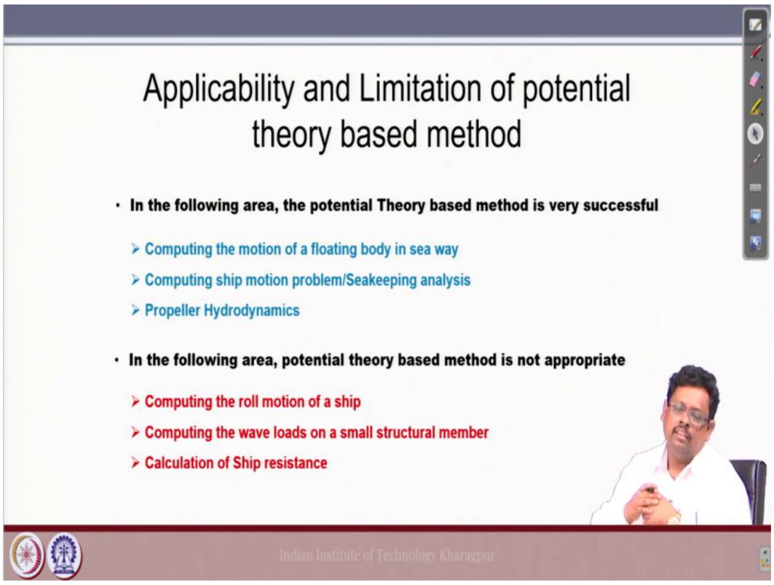

Now, let us move on, now this is the all areas where actually this potential based method are very very successful. So, let us see, the first one is that we are going to discuss here the computation the motion in the floating body in sea way, which is the sea keeping problem this is this is very much successful. Remember that we say that it is for the floating body it is we are not considering the forward speed without the forward speed we are considering the floating body problem right. So, it is very successful.

Second one is the computing a ship motion problem; that means, the sea keeping analysis. Now, considering the ship motion or forward speed it is not that successful compared to the floating body problem now nowadays everywhere any offshore industry, they are having some kind of solver based on the potential theory ok. But we do not see that popularity when you consider the forward speed motion ok as such, of course, the steep theory is very popular, but not the 3 dimensional methods as such. And of course, the in propeller hydrodynamics also this potential theory is very important.

Now, let us discuss some areas where this potential theory is not that appropriate or not that popular. So, one is the computation of the roll motion of a ship of course, because it is viscous dominant as I said that in beginning that when you consider the roll. So, here the viscosity is very important ok. So, this is one area where actually we have to consider the viscosity and then what we normally do is with respect to the potential theory we add some viscous term to deal with this; however, it is better to use direct the Navier-Stoke equation.

Second one is the computation of the wave loads on a small structural member, as I said that Morison equation here normally we use this. So, here what it is under the long approximations theory; that means, that the tubular structure really small compared to the wave which is very high. So, in that case actually potential theory is not that popular people try to because that viscous drag also very important in that particular situation and of course, that ship resistance problem it is directly related to the drag force.

So, you cannot ignore the effect of viscosity. So, therefore, we cannot I mean normally we do not use popular the potential theory in this in such problems ok.

So, today we are going to stop here and from the next class we are going to discuss that how we apply this boundary value problem, I mean how to generate the boundary value problem using our basic governing equation is the Laplace equation and what would be the boundary conditions and then how I obtained the pressure using the Bernoulli equation ok. I mean that is comes from the Euler equation. Okay till then.

Thank you.