

**Advanced Design of Steel Structures**  
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**Lecture - 18**  
**Plastic design -2**

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Lecture 18 plastic design - II

- plastic analysis permits

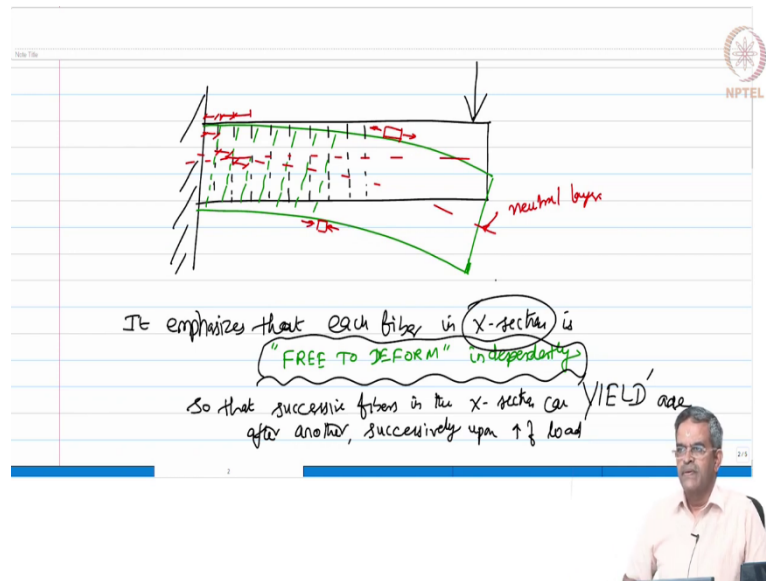
- no shear strain
- no warping

\* assumption plane sections remain plane and Lx to the longitudinal axis of the member, even after bending

Friends, welcome to the 18th lecture at Advanced Steel Design which is on continuation lecture to plastic design. We call this Plastic design 2, because in the last lecture we started with the plastic analysis design. So, friends the plastic analysis has certain basic conditions permits no shear strain and no warping.

Why? Because, there is a very strong assumption we deal with this which controls this. The very strong assumption is plane sections remain plane and normal to the longitudinal axis of the member even after bending. So, very interesting and very vital assumption we make in plastic analysis which can be expressed diagrammatically like you see here.

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Let us say I have a beam which is supported one end we just free the other end. It is a cantilever beam. Let us assume that the beam is subjected to a tip load, some magnitude. As we all know the beam will deflect. Let us say a reflected position of the beam right. So, let us say the beam initially had different segments equally cut along its length, different segments.

Now, each segment will now also get transformed, is it not in the new position; am I right? So, we clearly say that the segments in the old and new are same, they do not get disturbed. So, there is no warping which is happening. So, only the horizontal axis is get twisted that is all. So, the extreme fibers in the top will be under tension and the extreme fabric at the bottom will be under compression, I think we all know that. And, somewhere there is a layer which is the neutral layer which neither has tensile nor compression fiber.

So, that is a very interesting assumption that the plastic analysis does not permit any shear strain or warping on the cross sections. It emphasizes that each fiber in the cross section is free to deform. There is no restraint on them independently. So, that successive fibers in the cross section can yield one after another successively upon increase of load. As you keep on increasing the load, successive sections keep on yielding independently; each fiber is been given this freedom to deform independently.

That is a great advantage, that is a great boon given to every section, every fiber in the cross section, in the cross section. Every fiber in the cross section is permitted to do anything they want freely, they are independent. They can deform independently; they have a full freedom.

And, this freedom enables successive fiber in the cross section to keep on yielding one after another when you gradually increase the load; that is an assumption we make and that is how it is implemented. Having said this let us go slightly one step ahead and discuss more in detail about this.

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According to plastic design philosophy,  
 load capacity of the member is geometry dependent.  
 It is not load dependent.  
 $M_p$  of a plastic section WILL BE INDEPENDENT OF APPLIED LOAD.

Simply supported beam:  $M = \frac{wl^2}{8}$

Fixed beam:  $M = \frac{wl^2}{12}$ ,  $M = \frac{wl^2}{24}$   
 $M = f(w)$

So, friends according to plastic design philosophy, the capacity of the member; let us say very clearly load capacity of the member is geometry dependent. It is not load dependent. I will later on prove that the moment capacity of a plastic section is independent of load. I will prove that the moment capacity of a plastic section will be independent of load frames is very interesting you know.

For example: if you really wanted to find a moment capacity of any section, let us say simply supported beam you want to find moment, you will say it is  $\frac{wl^2}{8}$  where moment is a function of load. You want to find a fixed beam any section, you say  $\frac{wl^2}{12}$  or  $\frac{wl^2}{24}$ . So, the moment is again a function of  $w$  is it not, but I will prove in the derivation later in this class that the moment capacity of a plastic section will not depend on. It is purely a geometry dependent parameter.

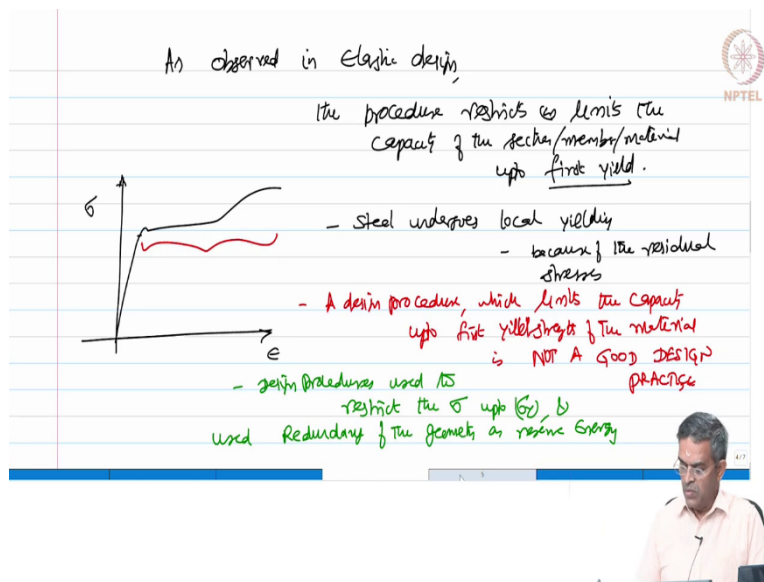
So, can I say that we are approaching towards form dominance? So, plastic design is in existence for the past 25-30 years in analysis and design friends. So, as I said in the beginning form dominance is not a recent philosophy. It is well established, well practiced in

engineering application, in structural engineering problems both in classrooms codal provisions and in fact, industrial practice very well versed in the application. So, here is a standing example where the moment capacity of the section is not your load dependent criteria.

It is a geometric property, it depends purely on the cross-section property of the system and of course, the material. It is purely not geometric dependent; it has got a component coming from; the strength has to come from the material friends. You cannot ignore the material strength anyway. For example, you have a shell, you have an arch which are all form dominant; can it constructed with the paper to carry the load? You cannot say sir material is going to be paper which is having no strength, but can the form alone this is the strength.

The answer is big no, we all know that engineering is not possible. So, material strength is not ignored, but that is not the only criteria that is the emphasis in form dominance. Geometric dominance also plays a vital role in sharing, distributing, using or dispersing the load very economically, that is the point we are discussing here.

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So, friends as seen in elastic design, the capacity you will say the procedure restricts or limits the capacity of the section or the member or the material whatever you want to say up to first yield is it not. So, we know look at the stress strain curve of classical structural steel which we plotted n number of times after the first yield steel undergoes local yielding. This happens because of the residual stresses. Now, we all know where these residual stresses come from.

They come from fabrication process, they come from preheating of the member. There are many reasons, we discussed them in the previous lectures. Therefore, friends it is not a good design practice to limit or to restrain the material capacity at the first yield is it not. So, I should say here a design procedure which limits the capacity up to first yield strength of the material is not a good design practice.

Why? There is a reason for this. Beyond the first yield it has got a lot of ductility, lot of energy, lot of reserve energy present in the material. So, what the design procedures used to do? The design procedures used to restrict the strength up to yield and used the redundancy of the geometry as reserve energy.

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Static degree of Indeterminacy is a redundant strength in the structural system

- purely developed from the geometry form
- It is capable of offering great internal resistance capacity
- Also a type of form-dominance
- undergo large displacement

Tip  
Cable stayed bridge  
load

You know static indeterminacy is a redundant strength in the present in the structural system. This is purely developed from the geometry of the system. So, it is capable of offering great internal resistance capacity. This is also a form, it is also a type of form dominance. But, we are not talking about this, we went one step beyond this. We allow the structure to undergo large displacement and by the virtue of large displacements, they interact or counteract the loads. So, this is the form dominance what we expect which is happening in special kind of structures like TLP, cable state bridges etcetera.

Offshore structures are most prominently formed dominant which undergo large displacement and the relative displacement or relative force helps the structure to counteract the platform. There is a big beautiful idea conceptually implemented in the design procedures of offshore

structures. So, I think it is very important for us to learn this. Coming back to that we understood that the static degree of indeterminacy is capable of offering a good internal resistance capacity.

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The slide contains handwritten text in black ink on a white background with horizontal lines. The text is as follows:  
- material reserve, which is present beyond yield  
in the form of ductility  
can also add additional resistance  
But in elastic design this was neglected  
(material remain under-used)  
In plastic design, apart from using the geometry, material reserve

The NPTEL logo is visible in the top right corner of the slide. In the bottom right corner, there is a small video inset showing a man with glasses and a light-colored shirt speaking into a microphone.

Having said this the material reserve which is present beyond yield in the form of ductility can also add additional resistance is not, its capable. So, but in elastic design this was neglected. So, the material remain under use, its not used properly. So, now, this idea is violated is. So, in plastic design apart from using the geometry, material reserve capacity beyond first yield is also utilized.

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capacity beyond first yield is also utilized.

- first yield, in many  $\sigma$ - $\epsilon$  curve is a falsified indication of yielding
- This occurs primarily due to presence of residual stresses
- first yield cannot be allowed to govern the design
- Steel, as a material has significant reserve energy beyond first yield. That should be utilized

The slide includes an NPTEL logo in the top right corner and a small inset image of a man in a light-colored shirt at the bottom right.

And, let me also put a mark here. First yield in many  $\sigma - \epsilon$  curve is a falsify indication of yielding. This occurs primarily due to presence of residual stresses. So, therefore, first yield cannot be controlled or cannot be allowed to govern the design, that is the first argument we have. Secondly, steel as a material has significant reserve energy beyond first yield and that should be utilized. So, that is the focus of plastic design. The focus of plastic design is that.

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Considering the standard  $\sigma$ - $\epsilon$  curve of structural steel

one can understand that when material fails under load, following observations are also important

- failure could be also due to instability
- " " fatigue
- " " excessive deformation

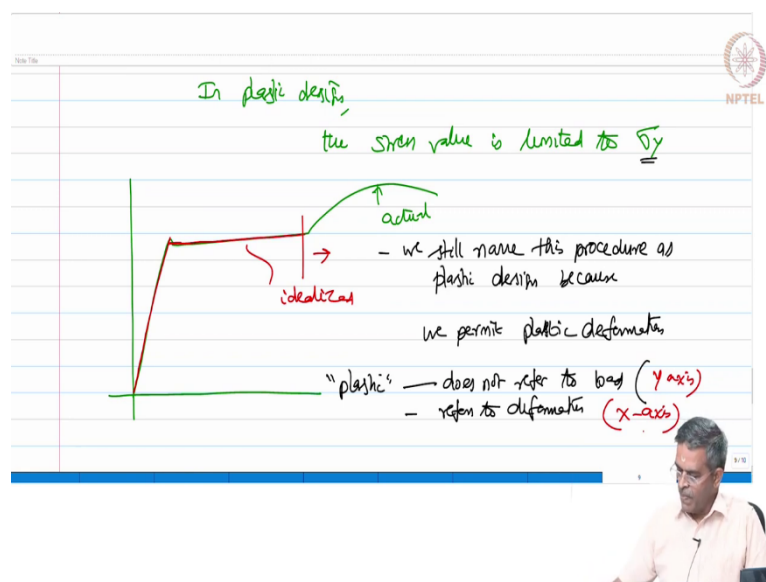
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So, now let us extend this argument further. Considering the standard  $\sigma - \epsilon$  curve of structural steel, let us plot it here; we all know that. Let us consider the standard  $\sigma - \epsilon$  curve

of structural steel. One can understand that when material fails under load following observations are also important which are the failure could be also due to instability, can also due to fatigue or it can be due to excessive deformation.

So, if any of these types of above factors does not initiate the failure then the member can carry more load beyond the elastic limit, is it not. Therefore, friends in plastic design the gain is the increase in load capacity. The loss is permanent which we call as plastic deformation. So, there is a gain, there is a loss. It is also interesting note that plastic design misnamed even though the stress is up to  $\sigma_y$ , please note that.

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In plastic design the stress value is limited to  $\sigma_y$ . If you ask me a specific question what would be the  $\sigma_y$ , upper or lower yield? Because, many steel many material does not show both the yield points. So, we are talking about an idealized curve where. So, we are not exceeding beyond this. There is an idealized curve, the red one is an idealized curve and the green one is the actual.

So, the stress value is limited only to  $\sigma_y$ , then why we call this a plastic design? Because, we still call we still name this procedure as plastic design because we permit plastic deformation. So, the term plastic does not refer to load, it refers to deformation. To be very specific it does not refer Y axis, it refers X axis. So, in plastic design the yield of the cross section is assumed to be idealistic as we are going to see now.



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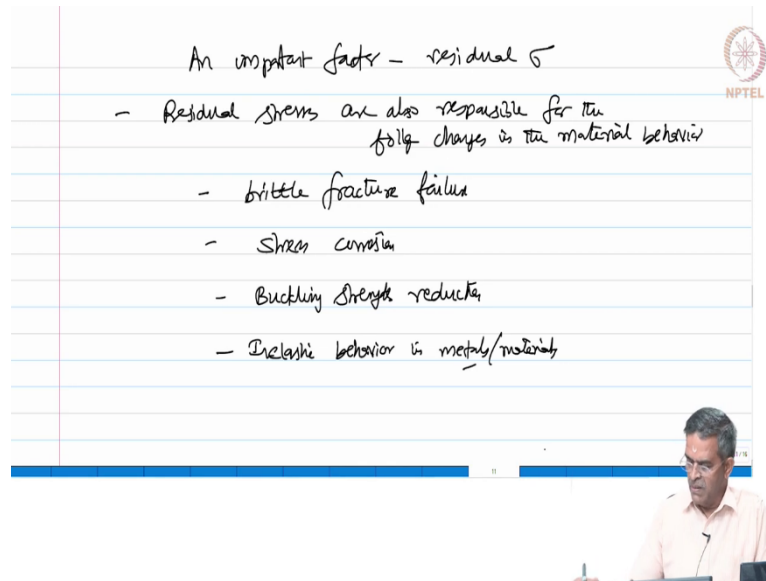
The slide contains handwritten text on a lined background. At the top right is the NPTEL logo. The main text reads: 'Plastic behavior of structures', 'Ultimate limit state design deals with the collapse modes which will convert the structure into a mechanism', '- The formation of mechanism enables excessive yielding, which finally leads to collapse.', '- Yield strength of a material should not be a deciding factor to assess the failure of the structure', 'Failure cannot be assessed purely based on the strength', and '- we should also look into deformation characteristics'. A small inset image of a man speaking is visible in the bottom right corner of the slide area.

So, let us try to take a beam. Before we get into the derivation of this, let us try to have a serious note furthermore on plastic behavior of structures. We all know friends that ultimate limit state deals with the collapse modes which will convert the structure into a mechanism. Now, the formation of mechanism enables excessive yielding which finally, leads to collapse. We all know friends that steel is a very common construction material which has got enough ductility. Looking at the  $\sigma - \epsilon$  curve of steel, it is evident that the material has got good capacity beyond the first yield.

Therefore, any design procedure which limits the strength till the first yield like an elastic design is not a good design procedure. So, the material strength beyond yield which is having enough reserve value is not utilized. So, there should be a design procedure which utilizes this excessive reserve strength beyond yield value, is it not. The load carrying capacity will therefore, can be increased because of the strength.

But however, this does not mean that the structural capacity should be extended beyond a value that results in a catastrophic failure. Therefore,  $\sigma_{yield}$  of a material should not be a designing factor to assess the failure of the structure; because failure cannot be assessed purely based on the strength. We should also look into deformation characteristics.

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An important factor - residual  $\sigma$

- Residual stresses are also responsible for the following changes in the material behavior
  - brittle fracture failure
  - stress corrosion
  - Buckling strength reduction
  - Inelastic behavior in metals/materials

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So, an important factor which will govern the behavior of material is as we know is a residual stress. So, presence of this will have a false yielding point, because of imperfections in the material which is developed during fabrication. So, let us recollect that residual stresses are also responsible for the following changes in the material behavior. What are those changes? Brittle fracture failure, stress corrosion, buckling strength reduction. We already said that, but we are reemphasizing for us to understand more.

Last one is inelastic behavior in metals or let us say materials we are talking about steel therefore, does not wrong in saying metals. But you know friends residual stresses development cannot be avoided. It is an inherent part of manufacturing a fabrication process, because when you start building the connections there is always a high probability that will land up in development of residual stresses. So, this will now start prompting a long  $\sigma_{yield}$  which will govern the design.

So, elastic design of structures controls the design till this point. Therefore, they are not considered to be a good method of design. Therefore, friends designer shifted the design procedure from elastic to ultimate load design. The most advantageous factor of steel as we all know is ductility which is a sign to use the reserve strength more conveniently and confidently.

So, the ductility of the material enables a thorough redistribution of moments from highest strength region to the next highest stress region. Thus, the load carrying capacity of the member is kept on increasing beyond the  $\sigma_{yield}$  which is enabled in the other methods of design like plastic design which is completely different and deviated from elastic design.

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plastic design is an effective design procedure

plastic design - ultimate load design

-  $\sigma \leq \sigma_y$

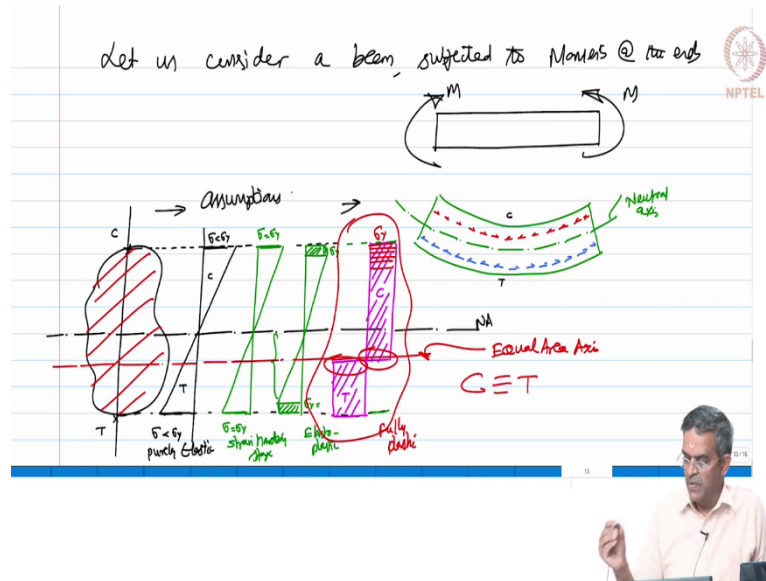
but enables plastic deformation

uses the material ability to the max<sup>m</sup> positive point in design perspective

Therefore, friends plastic design is an effective design procedure. Let us quickly compare the plastic design with ultimate load design. The ultimate load design uses the material ability to the maximum positive point in design perspective whereas, in plastic design the stress is limited to yield value, but enables plastic deformation. So, one method uses the material ability the maximum strength, other method uses the material ability to the maximum deformation.

So, simply look at this curve. One method uses it to the maximum strength, other method uses it to the maximum deformation. So, I mean we are looking for a different perspective of conservatism. So, now, I want to design the plastic moment carrying capacity which we call as a plastic design you know of sections. How do we do that?

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Let us imagine that I have a beam subjected to moments at the ends. So, I have a beam, I subject this beam to a moment at the end. The beam will bend. There will be a neutral layer which will have neither tension nor compression. Above the neutral layer things will be under compression, below the neutral layer things will be under tension. So, we call this as neutral axis. This is the bending action of the beam. Let us take an arbitrary cross section.

In this cross section, let us mark the extreme fibers. So, when the member is fully elastic, we all know that this fiber will be in compression and this fiber will be in tension. You see from here this is tension, this is compression. So, when the member is fully elastic. So, the stress value in compression and stress value in tension that is also mark the neutral axis and by here the neutral axis.

So, this becomes my here the stress is less than  $\sigma_y$ , here also the stress is less than  $\sigma_y$ . And, this is tensile and this is compressor. So, this is purely elastic case. Now, I apply more moment, I increase the moment now. There is a stage possible where the stress will become equal to  $\sigma_y$ .

I say the stress is now equal to  $\sigma_y$ . So, I call this as strain hardening ray stage. Now, as I said independent fibers can elongate or contract with the full freedom available. So, what happens is apart from this fiber extreme becoming  $\sigma_y$ , we will have additional fibers also becoming  $\sigma_y$

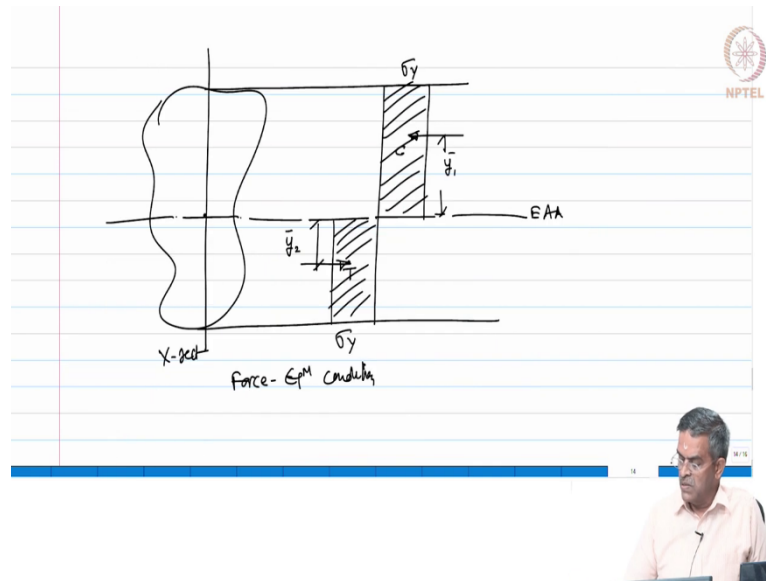
. We will have additional fibers also becoming  $\sigma_y$ . So, the stress distribution curve. So, there is a small region in tension and compression which has become  $\sigma_y$ . So, I can call this as elastoplastic condition. Why elastoplastic condition? Some part is still elastic, because the stress is less than  $\sigma_y$ . Some part is plastic, because  $\sigma = \sigma_y$ .

As I said please recollect, we are restricting the stress only up to yield. We are only using the X axis capacity of the material, not the Y axis capacity to the material to be very precise mathematically. Now, if you still further increase the moment applied on the beam, a stage will come where the stress diagram looks like this. So, this is C, this is tensile, this is compressive. But, one can notice here that the interface of tension and compression force does not happen at neutral axis.

Now, it is a new axis being formed here. We call this as equal area axis which means that above this line, the compressive force is equal to tensile force. So, I can call this section as fully plastic. How can I say it is fully plastic? Each and every fiber equal to  $\sigma_y$  whereas, in this case only few fibers where equal to  $\sigma_y$ . Each and every fiber in this case equal to  $\sigma_y$  equal area axis.

So, this sketch expresses how a plastic deformation occurs in a given system. The stress variation is shown like this. Now, I extract only this figure and this figure, from here. Let me do that.

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So, I am drawing an arbitrary cross section, taking the extreme fibers, drawing the equal area axis. I mark this as equal area axis, then I only draw the compressive force and the tensile force. So, this is my total tension force, total compressive force. So, this will have a centroid. Let me say the centroid is here. This will have a centroid. Let us say the centroid is here, they are opposite.

I call this centroid as  $\bar{y}_1$  from the equal area axis. I call this centroid as  $\bar{y}_2$  from the equal area axis. And, we all know that at this stage the stress is  $\sigma_y$  in the entire cross section. So, we can call this as force equilibrium condition is the cross section is the stress diagram. When the cross section was loaded and the stress distribution is happening by stages from here to here, we had made some assumptions.

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Assumptions

- material obeys Hooke's law until the stress level reaches the first yield point
- on further straining, stress remains constant =  $\sigma_y$
- upper/lower yield points in tension & compression are same
- Material is homogeneous, isotropic in both elastic & plastic states
- Transverse plane section will remain plane & normal to longitudinal axis of the member after bending



What are those assumptions? The valid assumptions could be material obeys Hooke's law until the stress level reaches the first yield point. On further straining stress remains constant which is equal to  $\sigma_y$ . The second assumption we make is the upper and lower yield points if shown in the  $\sigma - \epsilon$  curve both in tension and compression are same. Then material is homogeneous, isotropic in both elastic and plastic states. The last assumption which is the foremost important assumption.

The transverse plane section will remain plane and normal to the longitudinal axis of the member even after bending.

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The image shows a handwritten summary on a lined paper. The text is as follows:

- Summary
- plastic design philosophy
- plastic design
  - Elastic design
  - ultimate load design
- PD - enables plastic deformation

Below the text is a small diagram showing a coordinate system with a vertical axis labeled 'Y' and a horizontal axis labeled 'X'. The 'X' axis is drawn at an angle, indicating a direction of freedom or deformation.

In the bottom right corner of the paper, there is a small inset image of a man with glasses, wearing a light-colored shirt, sitting at a desk and speaking into a microphone.

So, friends in this lecture, we have started understanding the importance of plastic design philosophy. We learnt the difference between plastic design and compare it with elastic design with ultimate load design. We learn that plastic design enables plastic deformation, that is why the name plastic has come. The freedom is in the X axis, it is not in Y axis.

So, in the next lecture, we will talk about how to work out the moment capacity of a plastic section. And, we will show that the moment capacity of the plastic section is load independent.

Thank you very much, have a good day.