Advanced Design of Steel Structures
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Lecture - 19
Plastic design - 3

Friends, welcome to the 19th lecture on the course Advanced Steel Design. Here we are going to continue with the Plastic design. I will call Plastic design 3, 3rd lecture. In the last lecture we learnt the comparison between the plastic design philosophy with elastic design and ultimate load design. We have learnt that in plastic design the material strength, which is in terms of ductility or plastic deformation is utilized which is one of the basic requirements in a form dominant structural system.

Apart from utilizing the reserve capacity of the geometry, in terms of structural indeterminacy, the material reserves strength in terms of ductility is also being used in plastic design effectively. So, we said that in plastic design we have got a new concept called equal area axis where the total compressive force in a cross section, meets exactly in magnitude with the total tensile force acting in the cross section.

So, each and every layer in the cross section is having freedom to reach the σ_y, because we have assumed that each and every fiber is independent to elongate or contract. So, that the stress value can be reached till σ_y, once the stretch σ_y is reached then the next section of the fiber is increase the stress of σ_y and so. So, the whole cross section gets plasticized.

So, let us now derive the plastic moment capacity of any cross section. To start with we will assume a rectangular cross section then we will go to arbitrary sections. So, to start with we will do with the rectangular cross section.

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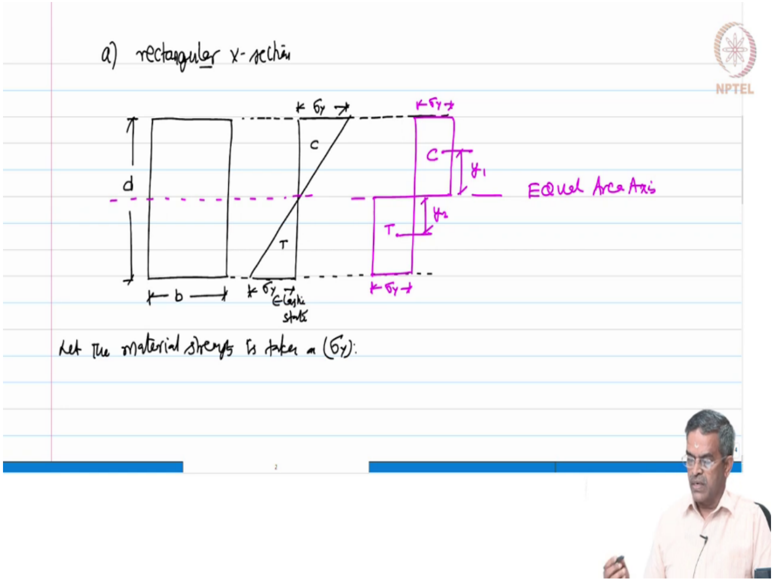

So, let me draw a rectangular cross section; a solid cross section of dimensions b and d. So, let us draw the stress diagram at elastic state and plastic state so, in the elastic state, because only the extreme fiber is yielded. As usual this is my compressive force and tensile force. This is my σ_y. In the fully plastic state we already know that the total tensile force will be equal total compressive force and every fiber will be σ_y. So, we call this centroid as $y_1$and this centroid as y_2. So, now, this axis is termed as equal area axis.

Because above this axis you see the total compressive force is equal to the total tensile force. Let the material strength is taken as σ_y. So, we can easily find the section modulus I will just rub this and write it here, because I need some space.

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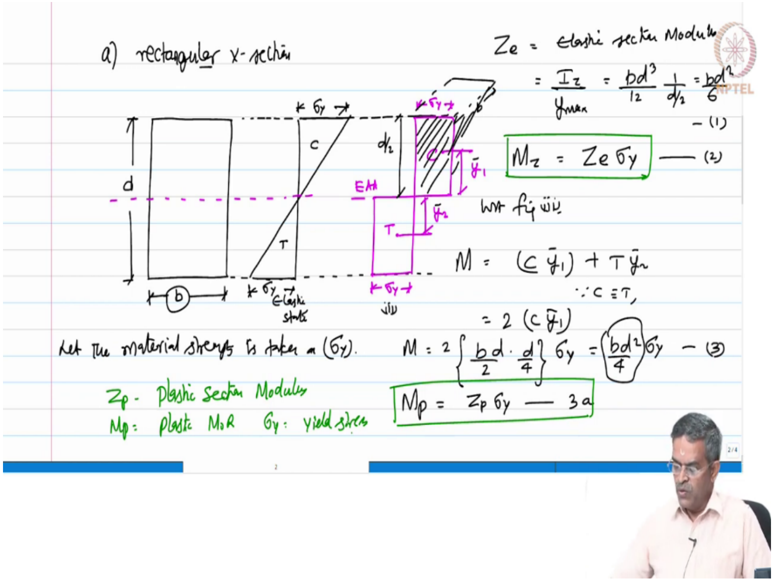

Let me write it here, I put this equal area axis. So, we can quickly find Z_e, which is the elastic section modulus which can be given by the moment of inertia about the Z axis by the distance of extreme fiber.

So, for this cross section we can easily find out it is going to be $\frac{bd^3}{12}$, divided by $\frac{d}{2}$ which becomes $\frac{bd^2}{6}$; let us call this equation number 1. If you want to find the elastic moment capacity this is given by a simple expression this section modulus multiplied by the corresponding stress we know that. Let us look at the plastic stress distribution. Let us look at this is let us look at this figure 3.

So, now with reference to figure 3, let us find out the moment $M = (C\overline{y}_1) + (T\overline{y}_2)$, since C is equal to T we can say this is $M = 2(C\overline{y}_1)$. Can I say that? So, what is the compressive force, which is going to be 2 times of so, you know here the breadth of section is b is it not see here. So, b and d by 2, because this is d by 2 am I right?

So, I am looking at this area now, b into d by 2 into. The centroid y bar will be d by 4, I think that is simple geometry and I multiply this with stress, because this has got to be force into this stress. So, which gives me $M = \frac{bd^2}{4}\sigma_y$. So, if I say this equation number 3. So, I replace this M as M_p and I will term this as Z_p to σ_y, this is only to keep the similarity of this equation with that of this. $M = M_p = Z_p \sigma_y$.

Instead of keeping some bd etcetera, I want to keep it like this. So, Z_p is called plastic section modulus. M_p is called plastic moment of resistance and of course σ_y is the yield stress. You can easily see there is a perfect compatibility between the equations 2 and 3a, both of them indicate in the same manner and one refers to elastic section modulus, other refers to plastic section modulus.

Therefore the corresponding moments are elastic moment of resistance and plastic moment of resistance is it not. So, it is.

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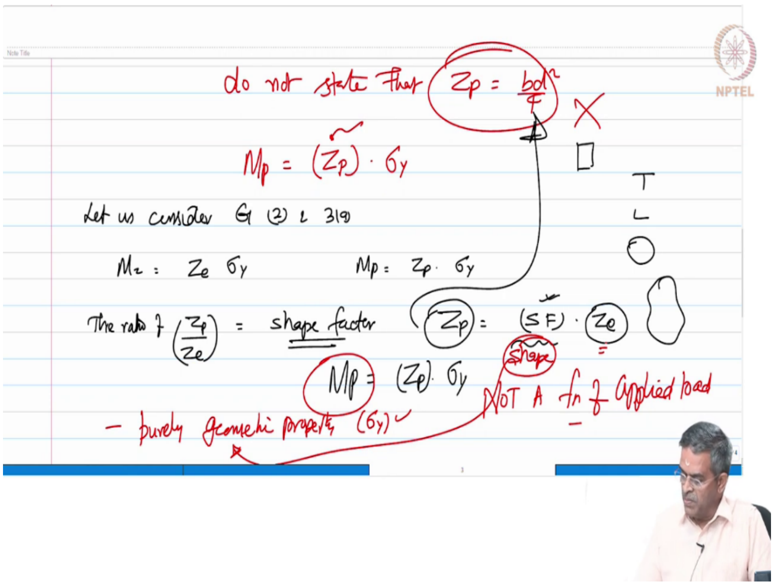

But we do not want to say please understand, we do not want to say do not state that $Z_p = \frac{bd^2}{4}$. We will not do this. We will not do this; we will not remember Z_p in this manner. Well, simply say plastic moment of resistance is plastic section modulus multiplied by σ_y.

We will come to this argument slightly later, how do you get this. Now friends, we can easily find out the moment capacity. Let us take these 2 equations 2 and 3a. Let us consider equations 2 and 3. So, equations 2 say it is $M_e = Z_e\ \sigma_y$ and this is $M_p = Z_p\ \sigma_y$. The ratio of Z_p to Z_e, the ratio of Z_p to Z_e is called shape factor. On the other hand, I would say Z_p is shape factor multiplied by Z_e. That is why I said let us not remember $Z_p = \frac{bd^2}{4}$ of rectangular section.

So, our job is to now find out the shape factor for various cross sections. So, for a given cross section which is T L circle or any arbitrary, if I am able to find out the shape factor. I already have the second moment of area divided by the y_{max}, I get Z_e I can easily find the moment capacity. And also please note friends the moment capacity of plastic section is plastic section modulus multiplied by σ_y.

It is not a function of applied load, am I right? Usually $M = \frac{wl^2}{12}$ etcetera whatever maybe. It is not a function of load it is purely a geometric property of course, σ_y is present. So, that is why we always wanted to introduce a factor called shape factor. So, shape is related to something of geometry. That is a reason why we want to introduce a term called shape factor.

So, for different cross sections, if I am able to find the shape factor derive them or is it available in steel tables or any standard handbooks I can easily find the moment capacity, because we all know in structural steel handbooks. The moment of inertia or second moment of area of the cross section and depth of the section; therefore, Z_e section modulus is available in the steel table is it not.

Please look at structural engineering handbook IS-SP 6 1. If you look at for different cross sections of i c l angles, you will find Z_e. That is elastic section modulus this is available readily. If by any chance if the table also gives you a shape factor of that section, you also have Z_p with you and if you have Z_p you can have M_p with you. So, it is just a minute to calculate the moment capacity of the section for plastic design.

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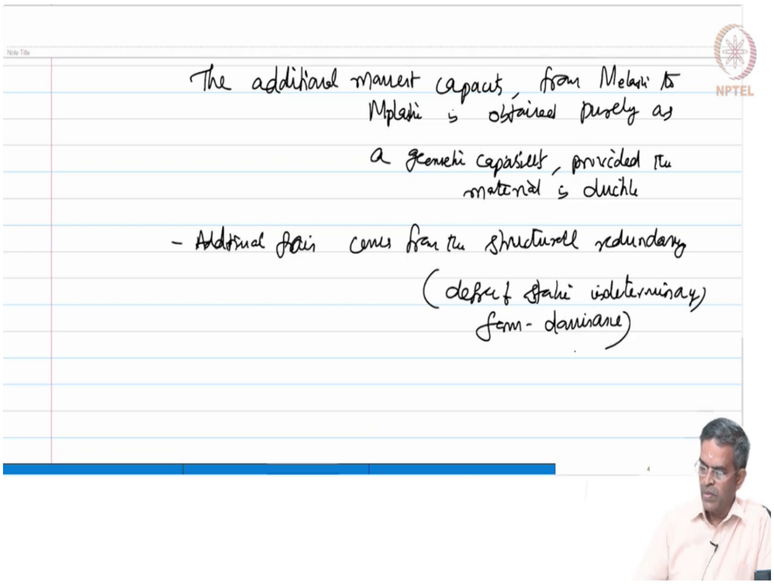

So, let us try to understand a very important concept here. That the additional moment capacity, from M elastic to M plastic is obtained purely as a geometric capability, provided the material is ductile. So, the additional gain comes from the structural redundancy, which is degree of static indeterminacy is one part of it form dominance is other part of it.

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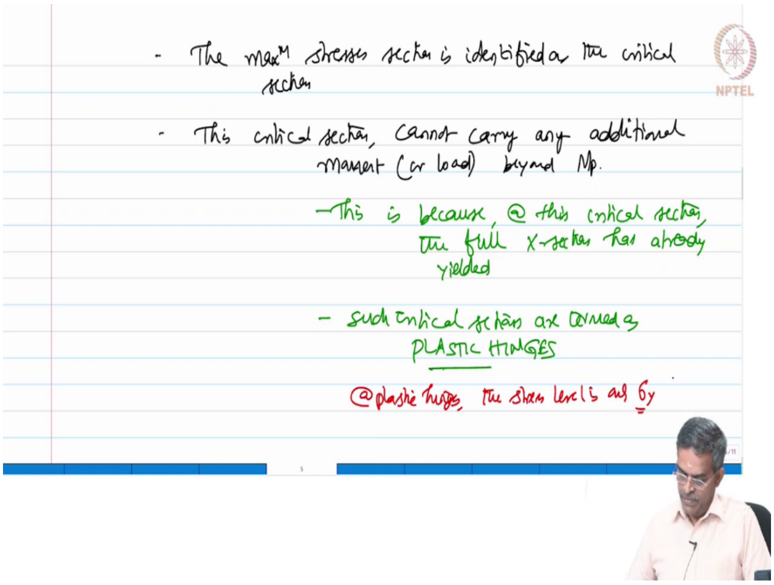

Now, friends let us also make a comment that; the maximum stressed section is identified as the critical section. So, what do you mean by a critical section? This critical section cannot carry any additional moment or let us say load beyond M_p. This is because at this critical section the full cross section has already yielded. So, such sections are termed as plastic hinges and the corresponding section is fully plasticized.

But please note at plastic hinges, the stress level is only σ_y, but still the term plastic is being used, because the deformation at the section is in a plastic stage. Having said this let us try to derive a shape factor.

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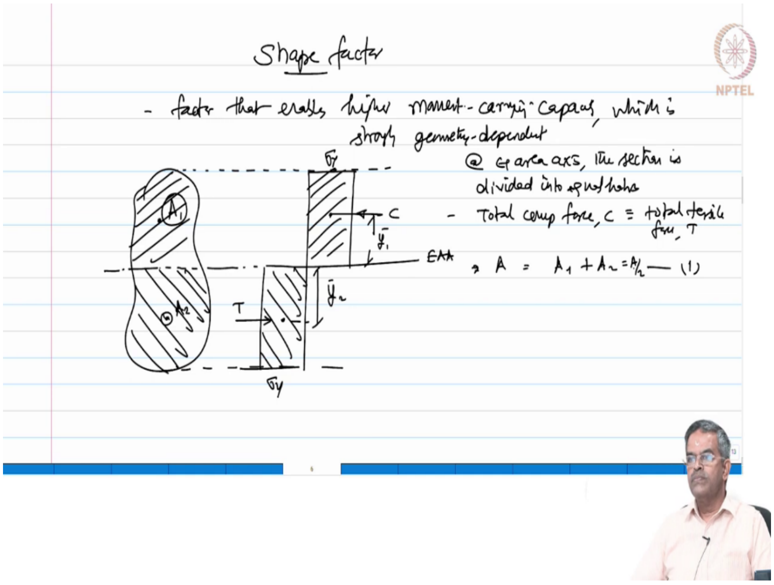

Let us quickly see what is the shape factor? Shape factor is a factor that enables higher moment carrying capacity, which is strongly geometry dependent. That is why it is called shape factor.

Let us derive a shape factor for an arbitrary section. Let us derive a shape factor for an arbitrary section, then we will apply this logic to all sections. We will take an arbitrary section let us say extreme fiber we have a field of plastic stress distribution and we also mark the equal area axis. This is C compressive force and this is tensile force. Let us say the corresponding area here is A_1.

And the corresponding area here is A_2. So, now, at equal area axis, the section is divided into equal halves, is it not? So, that is the total compressive force is equal to the total tensile force, T. So, I can now say the total area A = $A_1 + A_2$. Can you say this as A by 2?

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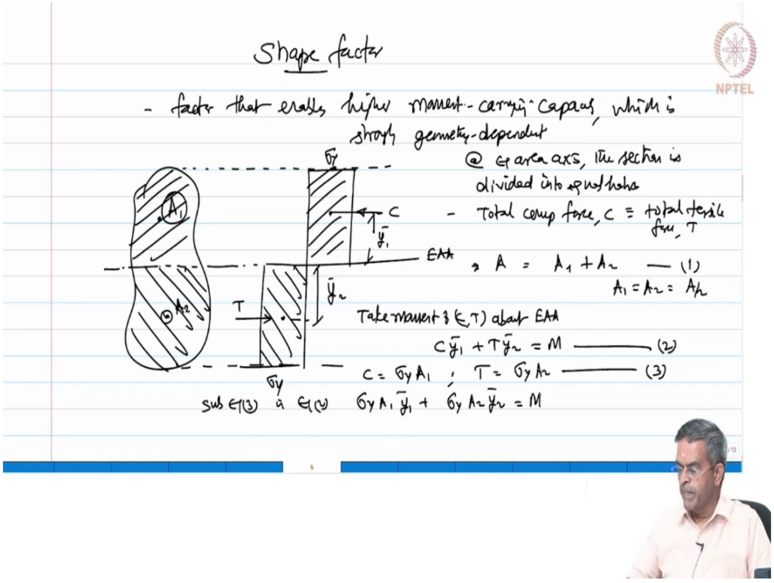

That is we can also say $A_1 = A_2 = \frac{A}{2}$. Now, let us take moment about of C and T, about equal area axis. See what happens. $C = \sigma_y A_1; T = \sigma_y A_2; \qquad \sigma_y A_1 \overline{y}_1 + \sigma_y A_2 \overline{y}_2 = M;$

We should say C into y bar 1 plus T into y bar 2 should be the total moment. We also know C is σ_y into $\frac{A}{A_1}$ and T is σ_y into A_2. Let us substitute them let us substitute equation 3 in equation 2. So, σ_y A_1 y bar 1 + σ_y A_2 y bar 2 is M. Let me carry it in the next page.

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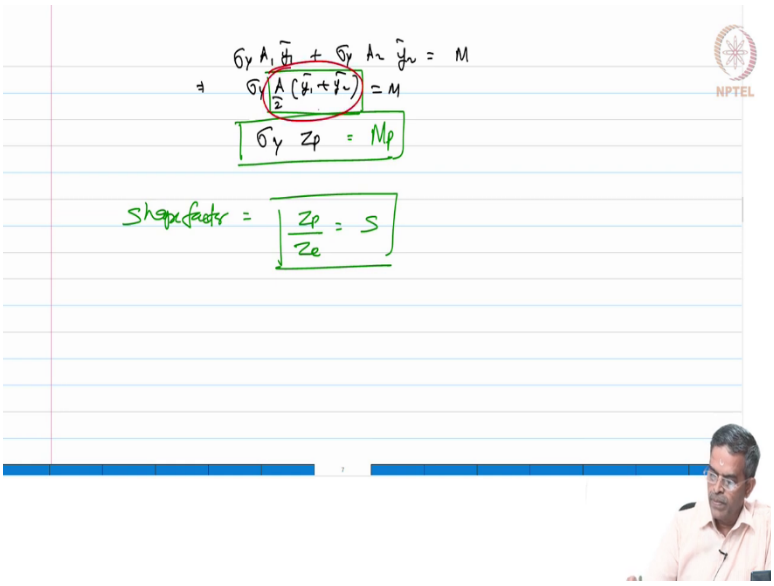

$$\sigma_y A_1 \overline{y}_1 + \sigma_y A_2 \overline{y}_2 = M;$$

$M_p = \sigma_y Z_p$;So, can I say this as σ_y into Z_p, if I use this term as Z_p then this becomes my M_p am I right. Now, interestingly friends the shape factor is Z_p by Z_e, Z_p by Z_e is the shape factor. So, now, we can try to find out the shape factor for different cross sections. Before we do that let us try to find out what is an elastic core?

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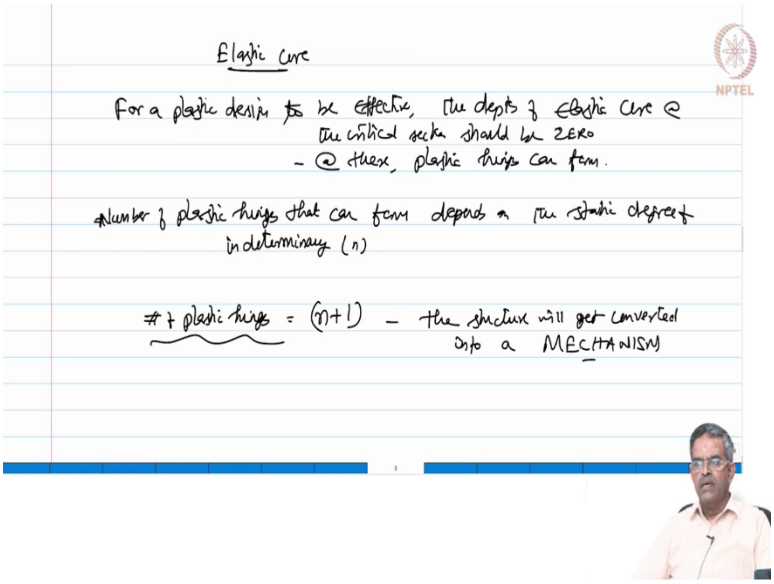

So, let us try to understand what is an elastic core and what its depth? So, for a plastic design to be effective, the depth of elastic core at the critical section should be zero. Is it not? Because at that section it is fully plasticized so, elastic core should be zero. Then only at these, sections plastic hinges can form. Now, the number of plastic hinges, that can form depends on the static degree of in determinacy. Let us call this as n. So, the number of plastic hinges in a given structure should be equal to number of degree of static indeterminacy plus 1.

When so many hinges have been formed the structure will get or will get converted into a mechanism. So, more the degree of static indeterminacy more the possibility of plastic hinges, because we need n plus 1 hinges. So, it becomes a mechanism. It will also enhance the moment carrying capacity of the structure apart from enhancing it is attained the shape factor. In simple terms the moment carrying capacity is achieved by the plastic design in two ways.

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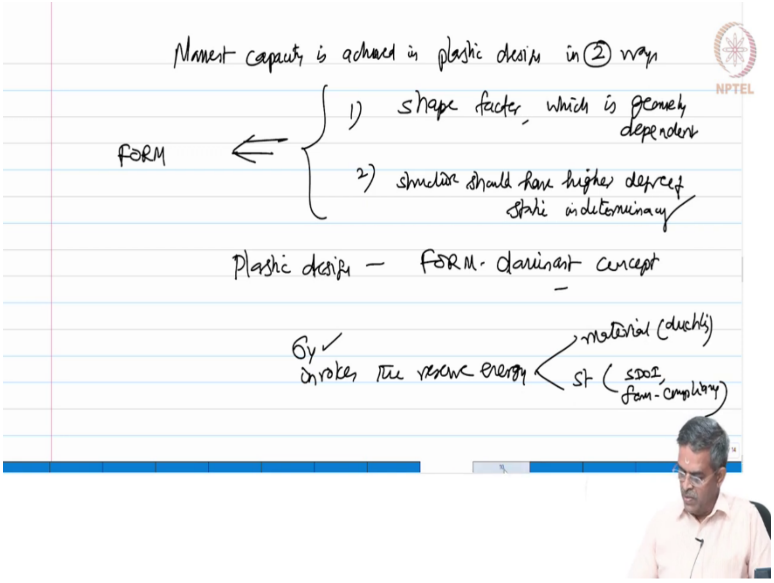

1. By having a higher shape factor, which is geometry dependent. 2. The structure should have higher degree of static indeterminacy. Friends, you will recollect that both of these are related to the structural form is it not. Therefore, one can say plastic design is a form dominant concept. It utilizes strength till σ_y of the material and it invokes the reserve energy from the material in terms of ductility from the structure in terms of static degree of indeterminacy and form compliancy. That is very interesting.

So, let us try to derive the elastic core and check, what is the condition for a plastic design. So, to do an elastic core let us take again a rectangular cross section.

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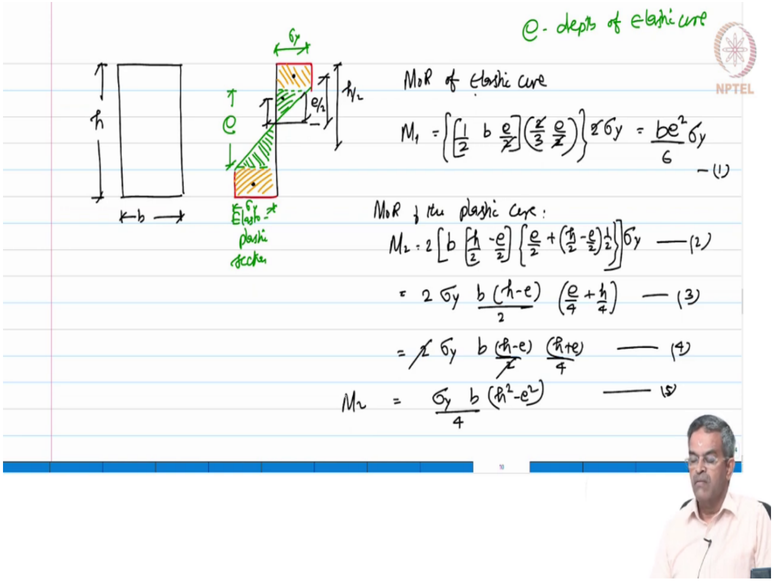

The cross section has the dimensions b and depth as d. Let us h, let us design a hybrid section where the section is partially plastic and the remaining is elastic. Let us have an elastoplastic section. So, let us have an elastoplastic section.

We call this elastic part as e, where e is called depth of elastic core. Of course, we know that the stress at the extreme fiber remains σ_y. And we also know from this figure that this dimension is h by 2 and this dimension of core is e by 2 am I right. Let us consider this cross section and the stress distribution diagram, which is elastoplastic as shown in the figure right.

First let us find out the depth of elastic core. So, this is the elastic part and this is my plastic part. Let us find out the moment of resistance of elastic part or elastic core at any cross section. How do you get that? Let us say that is M_1, which will be area of a triangle. So, half base height is e by 2 and the cg of that is going to be 2 3 e by 2 am I right it is a triangle know? I am taking moment about this point.

2 3 e by 2 and I have two such things 1 on the top and 1 on the bottom. And there is only the force. So, I will multiply this with the stress to get my moment right. So, half base e by 2, 2 3 e by 2 and twice of that. So, let us cut the common values. So, can I get this as be square by 6, σ_y be square by 6, σ_y equation number 1. Now, let us work out the moment of resistance of the plastic section or plastic core.

I call this as M_2 which is the red one right. So, let us find out that. So, let us say it is a rectangle. So, b into h by 2 minus e by 2 right and the cg of this from here will be equal to e by 2 plus h by 2 minus e by 2 of half am I right, e by 2 plus h by 2 minus e by 2 half of that correct. So, that is going to be the distance and I multiply this with the stress and there are two such pieces one on the top and one on the bottom.

Let us simplify this. So, 2 h by 2 minus e by 2, e by 2 plus h by 2 minus e by 2 half of that correct fine. So, let us simplify this please simplify and see what happens. So, it becomes 2 σ_y b into h minus e by 2 of e by 4 plus h by 4, which will further become twice σ_y b h minus e by 2 h plus e by 4 am I right, which can be further 2 σ_y b or let us remove this 2.

Because this goes away, a plus b into a minus b we can say a square minus b square by 4. So, that becomes maybe equation 1, equation 2, equation 3, equation 4, equation 5. So, that becomes my M_2. So, now, friends the total moment carrying capacity of this elastoplastic section will be sum of these.

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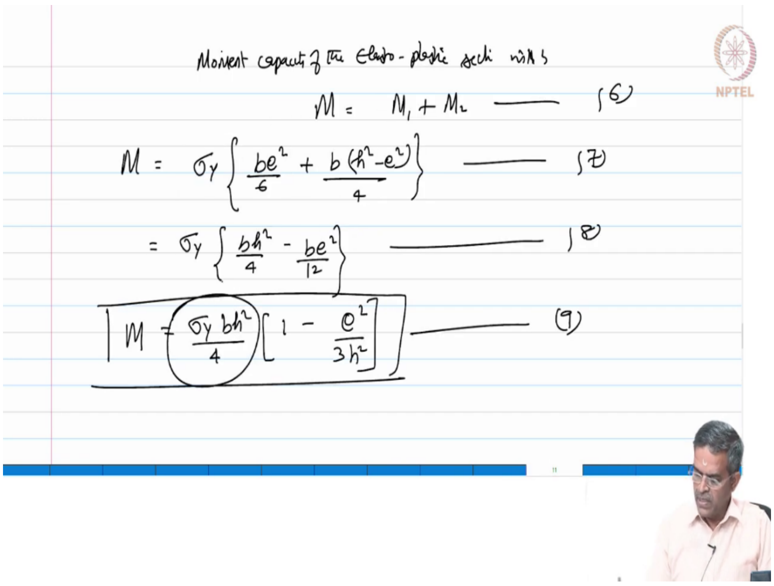

So, the moment capacity of the elastoplastic section will be M=$M_1 + M_2$. So, let us sum these two what happens let us see. So, M is going to be σ_y is anyway common in both cases the first one was $\frac{be^2}{6}$, the second one was $\frac{b(h^2-e^2)}{4}$, am I right, which on simplification will become $\sigma_y\{\frac{bh^2}{4} - \frac{be^2}{12}\}$, on simplification $\frac{\sigma_y bh^2}{4}\left[1 - \frac{e^2}{3h^2}\right]$.

Please check this let us call equation number this was 5. So, I call this as 6, this is 7 these as 8, this as 9. So, that is my total moment capacity of the elastoplastic section, which is $\frac{\sigma_y bh^2}{4}\left[1 - \frac{e^2}{3h^2}\right]$.

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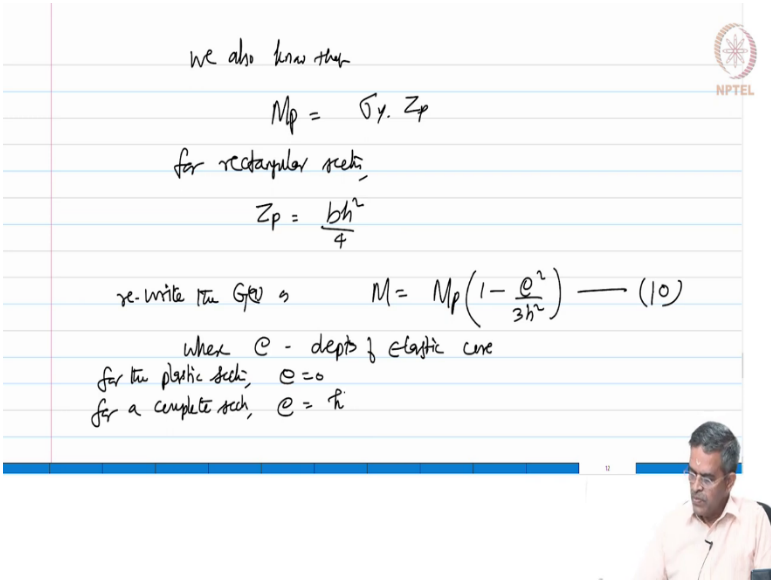

We also know, we also know that M_p is σ_y into Z_p for rectangular section, Z_p is bh square 4. We already said that. So, we can now write.

We can rewrite the equation 9 as M equals this part I am replacing as $M_p(1 - \frac{e^2}{3h^2})$. That is the equation number 10; where e is the depth of elastic core in the section. So, for the section for a plastic section, e will be 0 for a complete elastic section, e will be equal to what? Depth of the cross section is it not ok. So, one can easily find out this. So, we will also try to expand this.

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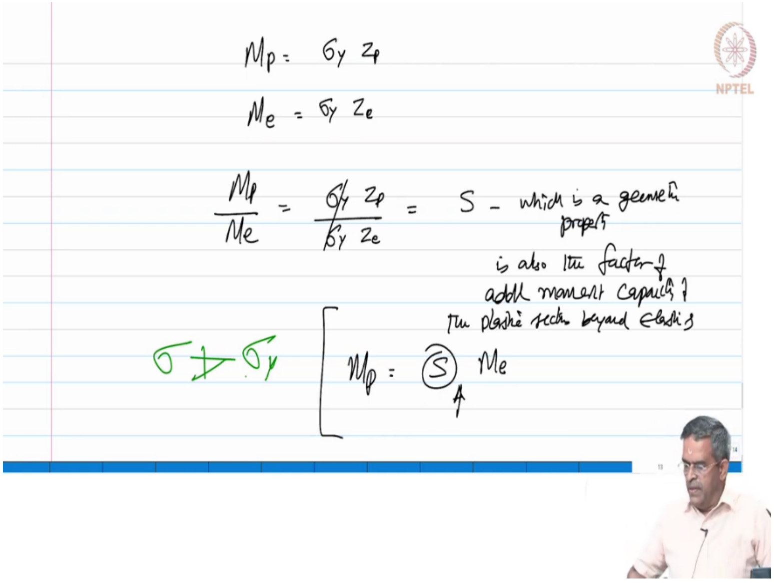

Let us say $M_p = \sigma_y . Z_p$. $M_e = \sigma_y$ Z elastic. So, let us say $\frac{M_p}{M_e} = \frac{\sigma_y Z_p}{\sigma_y Z_e}$, which gives me that shape factor. So, shape factor which is a geometric property is also the factor of additional moment capacity of the plastic section beyond elastic section correct, because M_p is shape factor of M_e. So, shape factor is an additional capacity indicator of the fully plastic section from the fully elastic section.

Please understand friends, at this stage stress is not exceeding yield, but still the term plastic is used, because the deformation is plastic not the stress. That is what it is. Having said this let us try to work out shape factor for a rectangular section.

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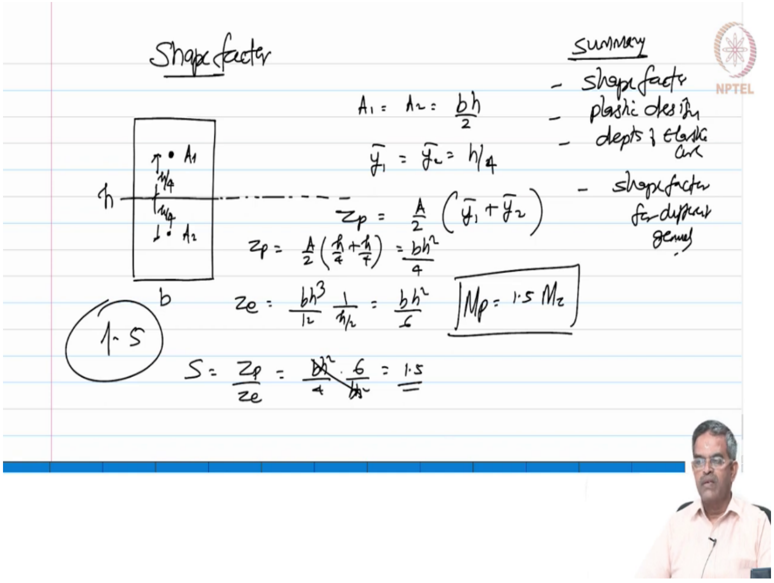

The geometric parameter let us do it for rectangular section. Let us take this h. So, this is top area bottom area. We call this as $A_1 = A_2 = \frac{bh}{2}$. This is also h by 2 is it not? So, we know $A_1 = A_2 = \frac{bh}{2}$. We also know y bar 1 is y bar 2, which is $\frac{h}{4}$. So, Z_p is actually the equation is A by 2 of y bar 1 plus, y bar 2 is it not, that is a Z_p value. We had somewhere here this. So, this Z_p is it not this is Z_p. So, that is what you are trying to say here y bar 1 plus y bar 2 correct.

Let us substitute that here which is going to be $\frac{A}{2}(\frac{h}{4} + \frac{h}{4})$, which becomes $\frac{bh^2}{4}$ and $Z_e = \frac{bh^2}{6}$. So, shape factor as we know is Z_p by Z_e which is bh square by 4, bh square which becomes 1.5. So, shape factor for rectangle is 1.5. So, the plastic capacity of rectangular section is 50% more than elastic capacity of the same section.

So, friends in this lecture we discussed the importance of shape factor, the important philosophy of plastic design, we also learned how to find out the depth of elastic core and we understood how to arrive at the shape factor for different geometry is it not. I hope you will follow these lectures and revise them regularly and you will apply for more examples look into my advised textbooks recommended textbooks for this course acquire a copy learn them there are many MATLAB programs available in this, which will help you to solve many problems as exercises and for intensive learning.

Thank you very much have a good day.