Advanced Design of Steel Structures
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Lecture - 20
Shape factor examples

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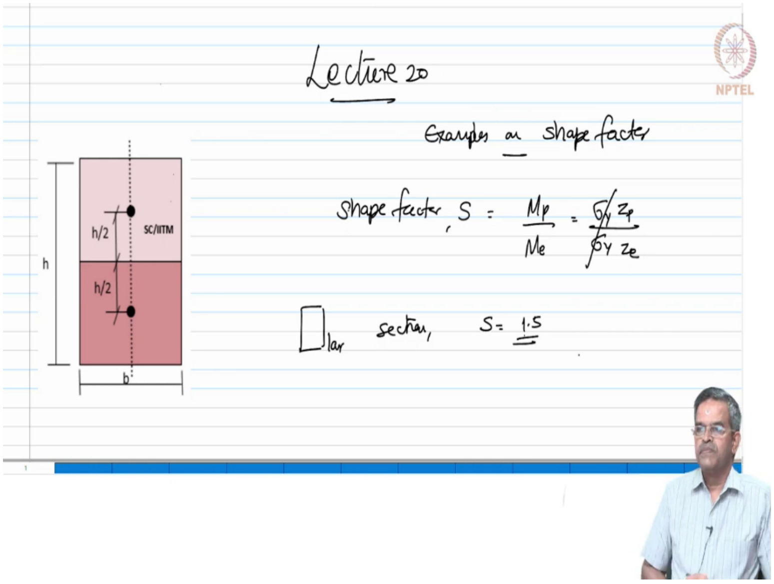

Welcome to the 20th lecture on the course Advanced Steel Design. In this lecture we are going to do more examples on Shape Factor. In the last lecture we already said that shape factor is expressed as $S = \frac{M_p}{M_e} = \frac{\sigma_y Z_p}{\sigma_y Z_e}$ and therefore, it is actually the ratio of section moduli of plastic to elastic.

So, on this example we applied and worked out a rectangular section and we found for a rectangular section we got the shape factor as S=1.5, we derived that.

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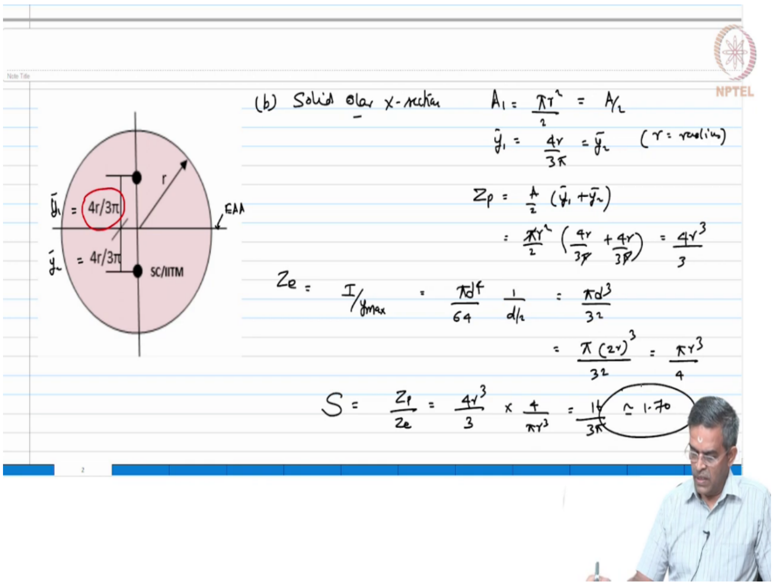

Now, we will derive further more for different cross sections. Let us take a circular cross section as you see on the screen the radius is r. So, let us say this is classical example b. A solid circular cross section radius is r and we know the center of this particular part will be $\overline{y}_1 = \overline{y}_2 = \frac{4r}{3\pi}$ as you see here. The standard expression which we know.

So, let us say the upper half is $A_1$and lower half is A_2. Let us say $A_1 = \frac{\pi r^2}{2} = \frac{A}{2}$. Because you know this is actually equal area axis, correct. And if I say this as $\overline{y}_1 = \overline{y}_2 = \frac{4r}{3\pi}$ where r is the radius of the solid cross section.

So, we already have an expression for Z_p section modulus plastic which is $Z_p = \frac{A}{2}(\overline{y}_1 + \overline{y}_2)$. Let us substitute that here A is $\frac{\pi r^2}{2}$. That is $Z_p = \frac{\pi r^2}{2}\left(\frac{4r}{3\pi} + \frac{4r}{3\pi}\right) = \frac{4r^3}{3}$.

Let us find out the elastic section modulus for this which is I by y max. So, which will be $Z_e = \frac{I}{y_{max}} = \frac{\pi d^4}{64}.\frac{1}{\frac{d}{2}} = \frac{\pi d^3}{32}$. Which will be $\frac{\pi(2r)^3}{32} = \frac{\pi r^3}{4}$.

So, as a classical definition shape factor is $S = \frac{Z_p}{Z_e} = \frac{4r^3}{3}.\frac{4}{\pi r^3} = \frac{16}{3\pi}$. which can be 1.70. So, that is my shape factor for a solid circular bar.

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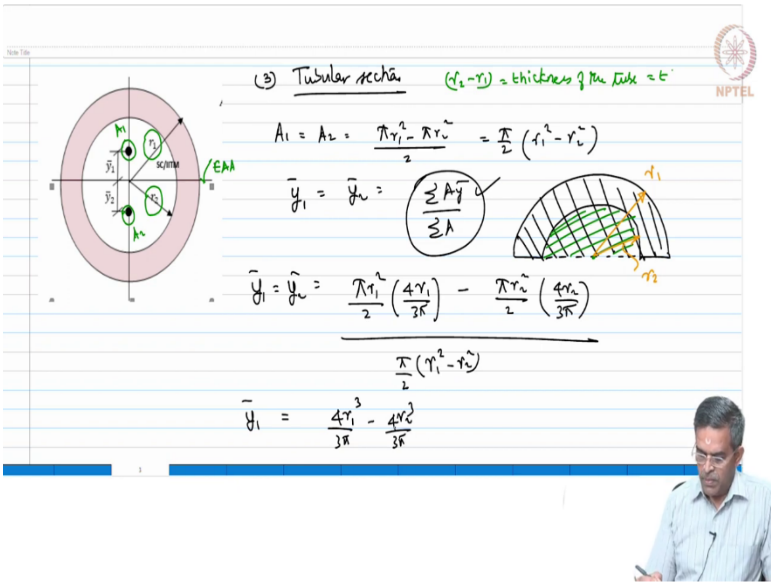

Having said this let us do one more example, which will do it now for an annular ring tubular section. Let us say a tubular section you know the outer radius is r_1 and the inner is r_2 so; obviously, $(r_2 - r_1)$ will be the thickness of the tube, which is t= $(r_2 - r_1)$.

Let the upper centroid be at y bar 1 from the equal area axis and the bottom centroid be y2 and we call this as A_1 and this as A_2. So, from the figure you can very well easily calculate $A_1 = A_2 = \frac{\pi r_1^2 - \pi r_2^2}{2} = \frac{\pi}{2}\left(r_1^2 - r_2^2\right)$. We will also know $\overline{y_1} = \overline{y_2} = \frac{\Sigma A\overline{y}}{\Sigma A}$.

Let us do this way $\frac{\Sigma A\overline{y}}{\Sigma A}$ we will employ this equation. So, let us try to find out. So, I am taking a semicircle. So, this has got 2 areas. So, this one is separate and this one is separate we know this is at the radius r_1 and this is at the radius r_2. So, with the help of this we will employ this equation and we will calculate this $\overline{y_1}$ which is as same as $\overline{y_2}$ will be $\overline{y_1} = \overline{y_2} = \frac{\Sigma A\overline{y}}{\Sigma A}$.

Let us say $\overline{y_1} = \overline{y_2} = \frac{\frac{\pi r_1^2}{2}\left(\frac{4r_1}{3\pi}\right) - \frac{\pi r_2^2}{2}\left(\frac{4r_2}{3\pi}\right)}{\frac{\pi}{2}(r_1^2 - r_2^2)}$. am I right. Which we simplify will be $\overline{y_1} = \frac{4r_1^3}{3\pi} - \frac{4r_2^3}{3\pi}$.

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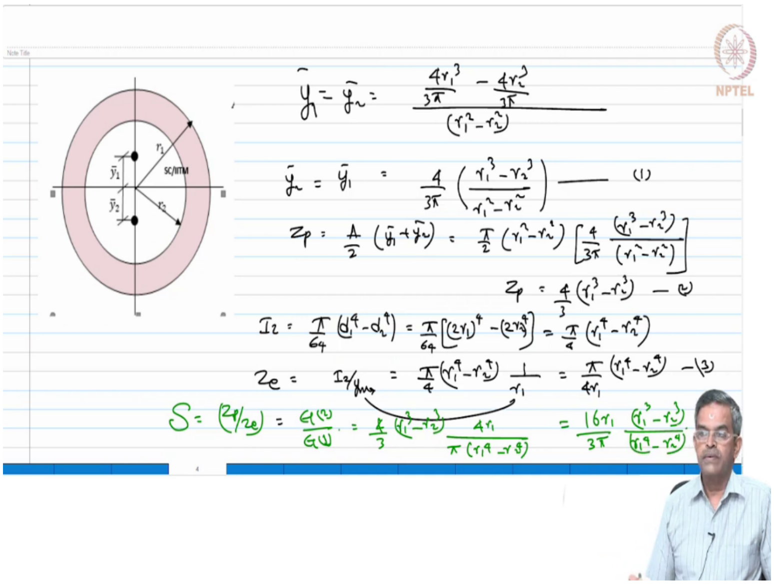

Let me write it this way. $\bar{y}_1 = \bar{y}_2 = \frac{\frac{4r_1^3}{3\pi} - \frac{4r_2^3}{3\pi}}{(r_1^2 - r_2^2)}$. Let us take it this way. Which will be said as $\bar{y}_1 = \bar{y}_2 = \frac{4}{3\pi}\left(\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right)$.

We will retain this equation we will call this equation number 1 which is $\bar{y}_1$, which is also equal to $\bar{y}_2$ instantaneously. So, we can find quickly the $Z_p = \frac{A}{2}(\bar{y}_1 + \bar{y}_2)$, which will be $Z_p = \frac{A}{2}\left(\bar{y}_1 + \bar{y}_2\right) = \frac{\pi}{2}\left(r_1^2 - r_2^2\right)\left[\frac{4}{3\pi}\frac{(r_1^3 - r_2^3)}{\left(r_1^2 - r_2^2\right)}\right]$. Which gets simplified to $Z_p = \frac{4}{3}(r_1^3 - r_2^3)$. So, that is my Z_p.

Let us now quickly find out $I_Z = \frac{\pi}{64}\left(d_1^4 - d_2^4\right) = \frac{\pi}{64}\left[\left(2r_1^4 - 2r_2^4\right)\right]$. So, I can now find Z equivalent as $Z_e = \frac{I_Z}{y_{max}} = \frac{\pi}{4r_1}(r_1^4 - r_2^4)$.

So, I can now quickly find the shape factor as $S = \frac{Z_p}{Z_e}$. Let us say equation this we call as equation number 3. So, I should say now this is equal to equation 2 by equation 3. So, which you simplify which will be $S = \frac{4}{3}\left(r_1^3 - r_2^3\right) \cdot \frac{4r_1}{\pi(r_1^4 - r_2^4)}$. Which will become $S = \frac{16r_1}{3\pi}\frac{(r_1^3 - r_2^3)}{(r_1^4 - r_2^4)}$.

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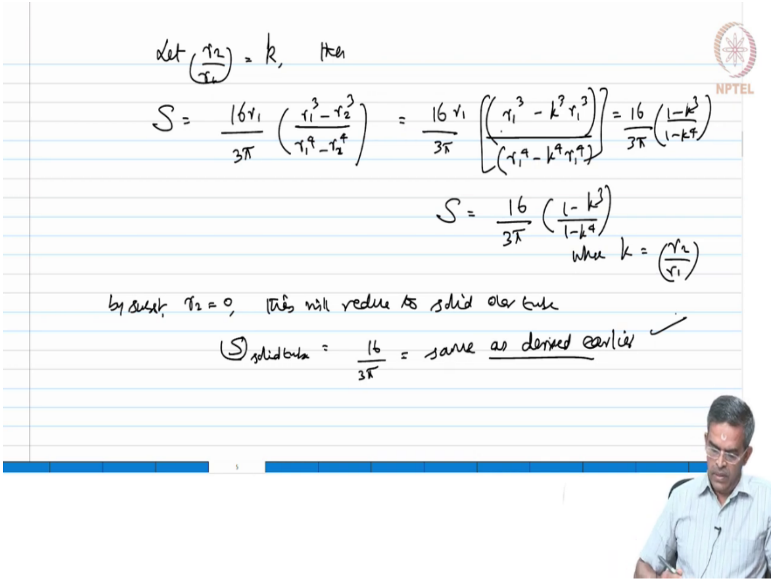

Now, let $\frac{r_2}{r_1} = k$. Then shape factor S was actually equal to $S = \frac{16 r_1}{3\pi} \frac{\left(r_1^3 - r_2^3\right)}{\left(r_1^4 - r_2^4\right)} = \frac{16}{3\pi}\left[\frac{\left(r_1^3 - k^3 r_1^3\right)}{\left(r_1^4 - k^4 r_1^4\right)}\right]$. Which will now become $S = \frac{16}{3\pi}\left(\frac{1-k^3}{1-k^4}\right)$.

Where k is a ratio of the radii. Yes, interestingly friends by substituting $r_2 = 0$. This will reduce to solid circular tube, is it not? So, therefore, the shape factor s for a solid tube is $\frac{16}{3\pi}$ which we already have. So, we landed up in the same equation same as derived earlier. So, we have a very interesting example of the solid section and an annular section.

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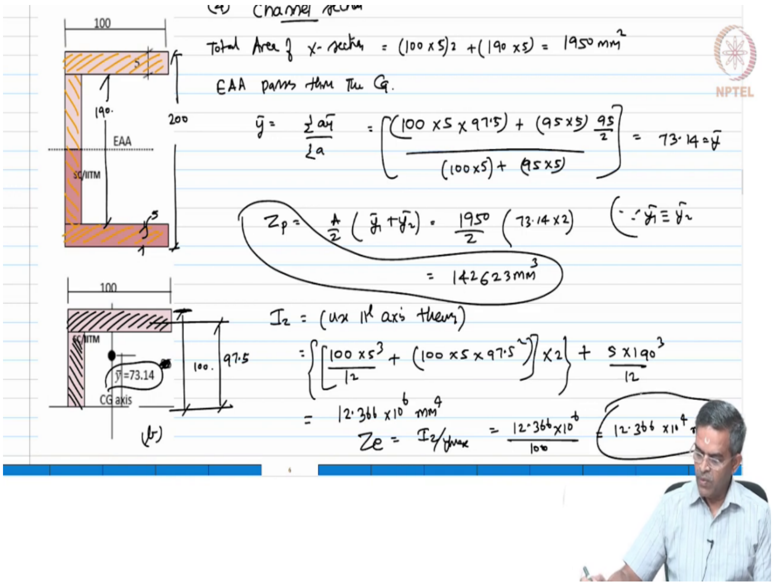

Let us do one more example as you see in the figure, which is a channel section. Because these are common sections used in steel design. So, you are trying to work out the shape factors for all of them. So, the design becomes easy and can handle it comfortably. So, look at the section available on the screen. Now let us say this dimension the overall size is 200 mm, and the thickness is 5 millimeters, thickness is 5 millimeters through and through, right.

Let us try to find out the total area of cross section. You can see this is going to be I have divided like this, this is my first piece. This is my second piece and my third piece. So, which is going to be now 100 into 5, there are 2 such pieces plus 200 minus 10. So, 190 into 5. So, the total area becomes 1950. Since the section is symmetrical equal area axis passes through the CG. $\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \left[\frac{(100 \times 5 \times 97.5)\frac{95}{2}}{(100 \times 5)+(95 \times 5)}\right] = 73.14 = \bar{y}$

One can easily find out $\bar{y}$ which can be $\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a}$. We will try to find out look at this figure b here. So, that is going to be 100 into 5 into 97.5. That is the CG of this piece, right. 100 into 5 97.5 because this is 100 from here till here it is 100. So, now, this will not be. So, from the CG to here this will be 97.5 plus then the second piece is this which is 95 into 5, that is the area of this piece and CG will be 95 by 2.

Divided by total area 100 into 5 plus 95 into 5. If you work it out you get this value as 73.14 which is y bar which is indicated here. Now, I can work out $Z_p = \frac{A}{2}\left(\bar{y}_1 + \bar{y}_2\right)$ is already there. So, I can simply say it is 1950 by 2 73.14 into 2. Because $\overline{y}_1$ and $\overline{y}_2$ are identical. So, if you do that, I will get this value as $142623 mm^3$.

$Z_p = \frac{A}{2}\left(\overline{y}_1 + \overline{y}_2\right) = \frac{1950}{2}(73.14 \times 2) = 142623 mm^3$.

Let us try to find out the I_z value of this channel section. We can use parallel axis theorem. So, let us do that 100 into 5 cube by 12 plus 100 into 5 into 97.5 square that is my first piece. I have 2 such pieces plus these are for the flanges. Now for the web going to be 5 into 190 cube by 12, you know if this is 200 this will be 190. Because the thickness is 5 in both cases. So, if it make the total of I_z this becomes $12.366 \times 10^6 mm^4$.

$$I_z = \left\{\left[\frac{100 \times 5^3}{12} + \left(100 \times 5 \times 97.5^2\right)\right] \times 2\right\} + \frac{5 \times 190^3}{12} = 12.366 \times 10^6 mm^4.$$

I can find Z elastic which is $Z_e = \frac{I_z}{y_{max}}$ which will be $Z_e = \frac{12.366 \times 10^6}{100}$. Now this is the extreme fiber from the neutral axis which will be $12.366 \times 10^4 mm^3$. So, I have Z_p, I have Z_e can I quickly find the shape factor.

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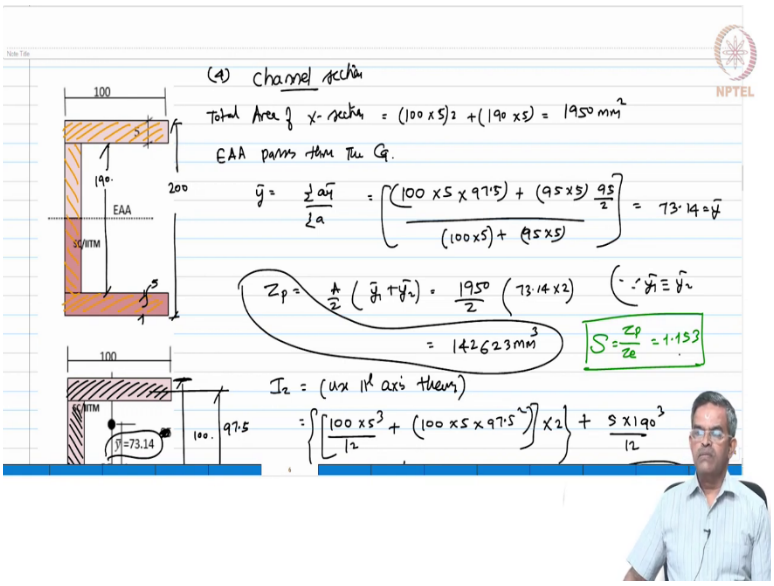

So, the shape factor will be now equal to $S = \frac{Z_p}{Z_e}$. which will be you can work it out S=1.153, friends. So, very simple example which is easily understandable.

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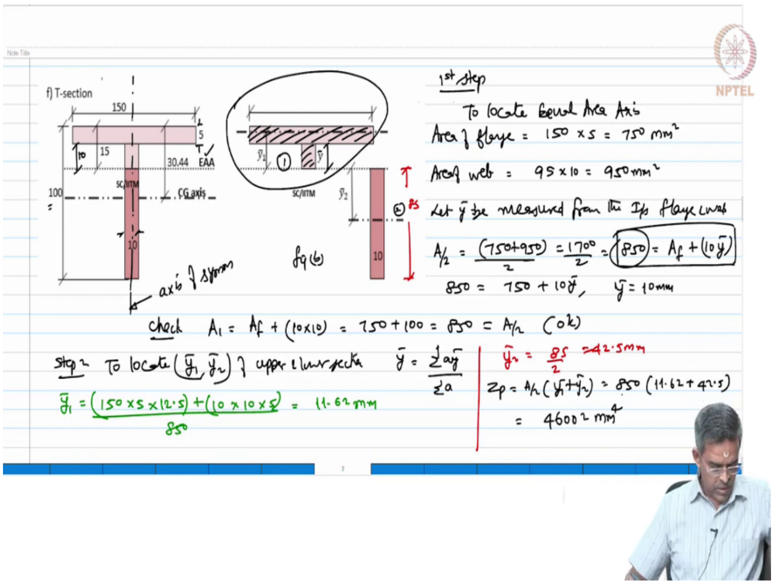

Let us do one more section the T-section the dimensional T are given, 150 and the overall thick depth is 100 mm, thickness of the flange and the web are different in this case. Flange is 5 mm whereas; web is thicker 10 mm. Now in this case the first step is to locate the equal area axis.

In the previous example this problem was not there because there was an axis of symmetry. In this case of course, you have vertical axis of symmetry there is no doubt, you have vertical axis of symmetry. But I want to find what would be this value. So, we would like to locate this, right. So, the first job is to find out the equilibrium axis. So, let us divide this into 2 parts.

Let us say this is my first piece and this is my second piece. Considering these 2 areas has got to be equal because an equal area axis no. So, let us find out area of the flange which is 150 into 5. Let us find out the area of the web which is 95 into 10 which is 950. So, area of the web is more than the flange. So, therefore, the equal area axis has to lie at a distance $\overline{y}$ down into the web.

So, we have to find out this distance $\overline{y}$ now, which is measured from the. Let us mark $\overline{y}$ like this not here let us mark $\overline{y}$ here. Let us mark y bar which is going to be from the flange web junction let us say this is my $\overline{y}$. Let us do this. So, let y bar be measured from the intersection of flange and web.

So, I can now say $\frac{A}{2} = \frac{(750+950)}{2} = \frac{1700}{2} = 850$. And that should be equal to area of the flange plus 10 times of $\overline{y}$ am I right? Look at this figure. $850 = A_f + (10\overline{y})$.

So, I have this the governing equation I have area of the flange I know which is $850 = A_f + (10\overline{y})$ which will be telling me that $\overline{y}$ is 10 mm. So, at 10 mm from here I locate the equal area axis. So, I have located the equal area axis. One can also check A_1 going to be area of the flange plus 10 into 10 because I am working out this area. Which is 750 plus 100 which is 850 which exactly equal to A by 2 as you see here.

$$A_1 = A_f + (10 + 10) = 750 + 100 = 850 = \frac{A}{2}.$$

Now I want to locate the CG of both the sections. Now step number 2 will be to locate the CG's y bar 1 and y bar 2 of upper and lower sections. So, look at the figure b. So, 2 sections are drawn separately let us try to find out and apply this equation. So, $\overline{y} = \frac{\Sigma a\overline{y}}{\Sigma a}$ .let us apply this and try to find out for $\overline{y_1}$ first. So, let us say $\overline{y_1} = \frac{(150\times5\times12.5)+(10\times10\times5)}{850} = 11.62mm.$

So, it will be lying in the flange, as marked in the figure. Let us do $\overline{y}_2$. $\overline{y}_2$will be you know this dimension the total is 95 you lost 10 there. So, its 85. $\overline{y}_2 = \frac{85}{2} = 42.5$. So, can I quickly find Z_p? $where\ Z_p = \frac{A}{2}\left(\overline{y}_1 + \overline{y}_2\right) = 850(11.62 + 42.5) = 46002mm^4$. We got Z p.

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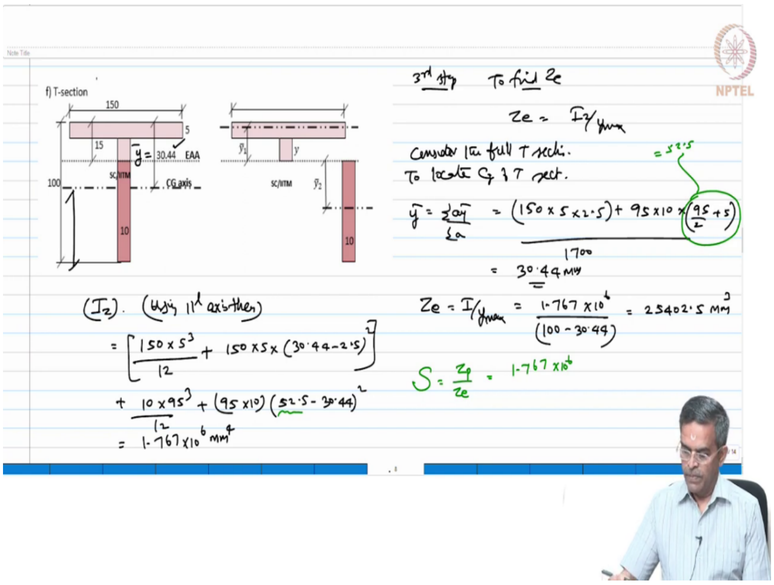

Let me find Z_e we know $Z_e = \frac{I}{y_{max}}$. Let us now consider the full T-section. Now to locate the CG of the T-section. So, let us say the CG is taken as y bar in this case. So, $\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a} = \frac{(150\times5\times2.5)+95\times10\times(\frac{95}{2}+5)}{1700} = 30.44mm$. So, if you work out this $\bar{y}$ will become 30.4 millimeters which is indicated here.

So, now, let us find out the moment of inertia of the section about the z axis, which will be using parallel axis theorem. Which can be said as $\frac{bd^3}{12}$ for the first piece $I_Z = [\frac{150\times5^3}{12} + 150\times5 + (30.44 - 2.5)^2$, that is for the first piece. Do it for the second piece. $[\frac{10\times95^3}{12} + 95\times10 + (52.5 - 30.44)^2$.How do you get this 52.5? Is exactly this. So, can you find out this? Which will be $I_Z = 1.767\times10^6 mm^4$. So, now, I can find $Z_e = \frac{I}{y_{max}} = \frac{1.767\times10^6}{(100-30.44)} = 25402.5mm^3$. Because this dimension is 30.44 and this value will be 100 minus 30.44. So, I substitute that here I get 25402.5, this should be y_{max} remember that friends this is y_{max}.

So, now I have say d with me. So, can I find the shape factor shape factor $S=\frac{Z_p}{Z_e}$.

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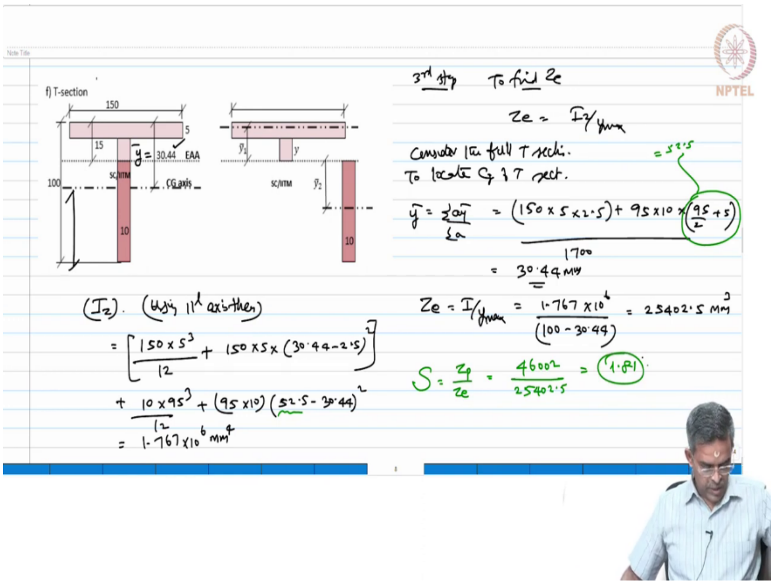
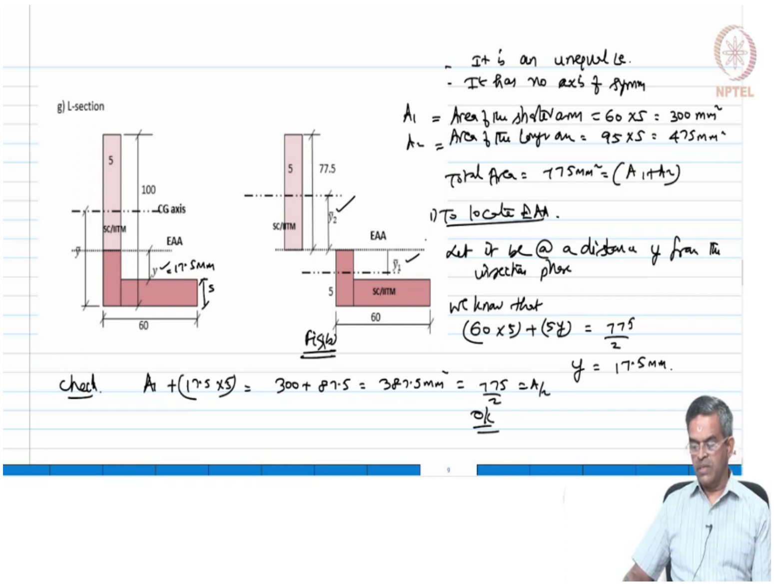


$S=\frac{Z_p}{Z_e} = \frac{46002}{25402.5} = 1.81$(Refer Slide Time: 36:11)



We will do one more example for an L-section. Let us say then L-section the dimensions are given on the screen the overall thickness is same which is 5 millimeters. So, let us first locate the centroidal axis of this.

Is an unequal angle. So, it has no axis of symmetry. So, we need to find out the equal area axis and the CG both. So, let us find out the area of the shorter arm. The area of the shorter arm is 60×5 the area of the longer arm is 95×5, the total area is

$A = A_1 + A_2 = 300 + 475 = 775mm^2$. So, now, let us locate the equal area axis first. Let the equal area axis be at a distance y from the intersection phase as shown in the figure, y.

So, we know that 60 into 5 plus 5 into y. Should be equal to 775 by 2, by this logic y will become 17.5 millimeters. So, y will now become 17.5 millimeters. We can check also. So, A_1 plus 17.5 into 5 should be equal to 300 we will call this as A_1 and this as A_2, total area is A_1 plus A_2. $(60\times5) + (5y) = \frac{775}{2}$.*where y=17.5mm.*

So, A_1 plus 17.5 into 5 which is 87.5 which is 387.5 mm square is actually equal to 775 by 2. So, its, is it not? So, we have located now the equal area axis now, our job is to locate the centroid, centroid of upper and lower areas. So, let us look at figure b. So, I want to find the centroid of the upper part centroid of lower part then I should find out the Z_p, right. $A_1 + (17.5\times5) = 300 + 87.5 = 387.5mm^2 = \frac{775}{2} = \frac{A}{2}$.

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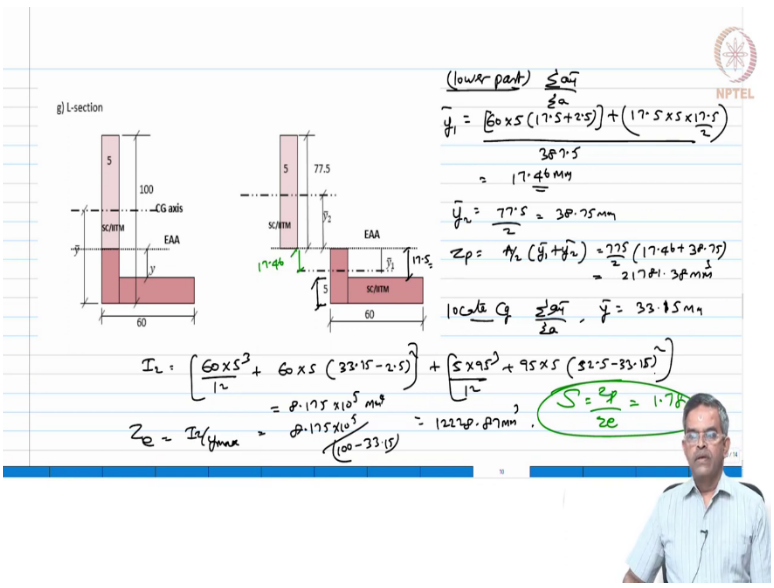

Let us take the upper part. Let us find out the lower part, let us say let us take the lower part. So, we will apply this equation $\bar{y} = \frac{\Sigma a\bar{y}}{\Sigma a}$.So, we get $\bar{y}_1 = \frac{[60\times5(17.5+2.5)]+(17.5\times5\times\frac{17.5}{2})}{387.5} = 17.46mm.$

Let us work out $\overline{y}_2$. *where* $\overline{y}_2$ is very simple because we know the overall dimension. So, this is going to be , $\overline{y}_2 = \frac{77.5}{2} = 38.75mm$. So, can you quickly find Z_p friends, using this equation which is $Z_p = \frac{A}{2}\left(\overline{y}_1 + \overline{y}_2\right) = \frac{775}{2}(17.46 + 38.75) = 21781.38mm^3$. So, similarly 1 can locate the CG. I am not giving the answer; I mean I am not giving the procedure I think you should be able to work it out.

$\overline{y} = \frac{\Sigma a\overline{y}}{\Sigma a} =$ 33.15 mm. Once I get $\overline{y}$ I can find I_Z which will be $I_Z = \left[\frac{60\times5^3}{12} + 60\times5\times(33.15 - 2.5)^2\right] + \left[\frac{5\times95^3}{12} + 95\times5\times(32.5 - 33.15)^2\right] = 8.175\times10^5 mm^3$, $Z_e = \frac{I_Z}{y_{max}} = \frac{8.175\times10^5}{(100-33.15)} = 1228.87mm^3$. Therefore, shape factor will now become $S = \frac{Z_p}{Z_e} = 1.78$.

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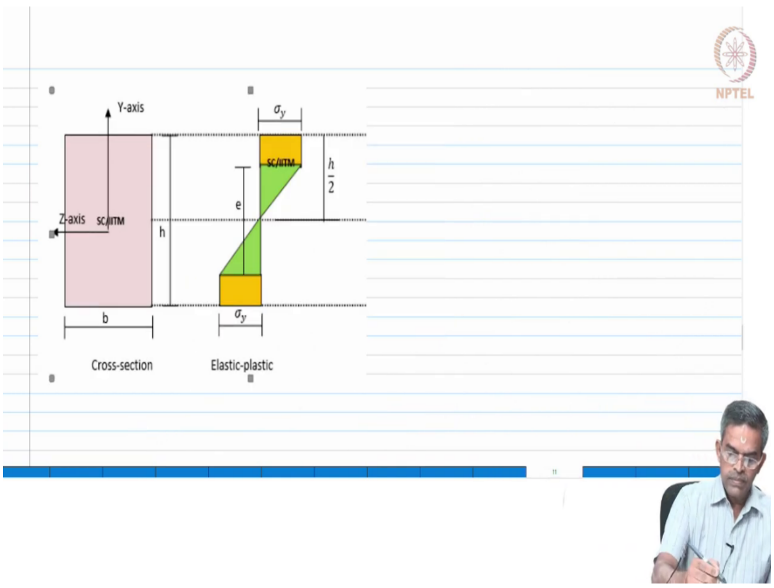

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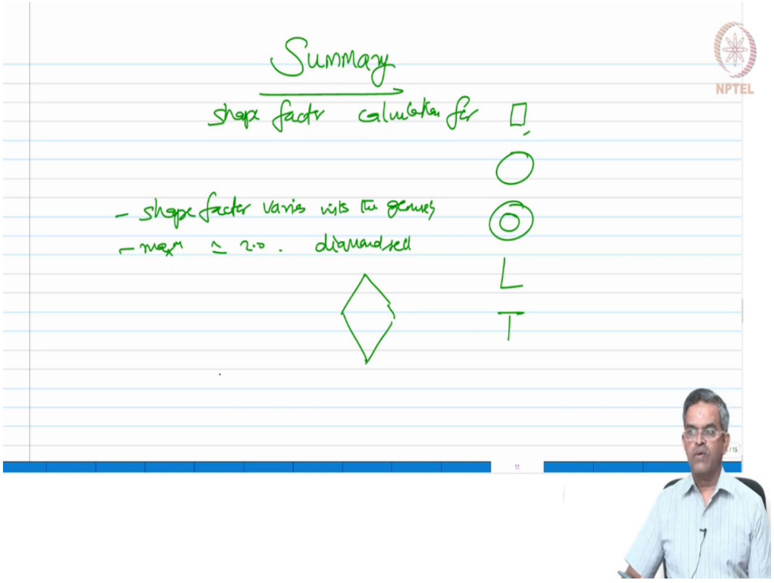

So, friends in this lecture we learnt, the shape factor calculations for rectangular section, solid cylinder, tubular member, L-section, T-section and we learnt that shape factor varies with the geometry it is not same, right. The maximum value of the shape factor is about 2.0 which is for a diamond section, you can try and find out.

So, shape factor is a geometric property and if I know shape factor, I can easily find the capacity of the section. Which is very simple to find out from the derivations what we discussed in the last lectures.

Thank you very much have a good day, bye.