Advanced Design of Steel Structures
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Lecture - 21
Plastic analysis -1

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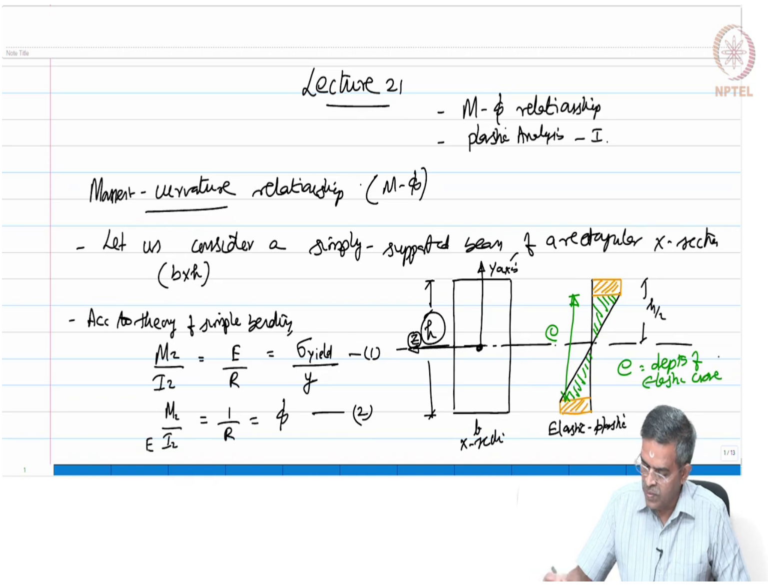

Welcome to the 21st lecture on the course advanced steel design, in which we are going to talk about the moment curvature relationship and Plastic analysis I. We are going to learn these two things in this lecture now. Now, let us talk about the moment curvature relationship, which is M-φ curve. Let us consider a simply supported beam of a rectangular cross section b cross d.

Say this is b and this is d, now let us keep it as h. So, let us draw the stress distribution diagram of an elastoplastic section. Let us say this is partly plasticized on the extreme tension compression fibers, then the remaining is elastic. So, let us say this is the plasticized part and the remaining is elastic part; it is an elastoplastic section and this is the cross section.

And of course, we know this is h/2 and this is my y axis, this is my z axis and this is my x axis, . We know according to theory of simple bending, $M_z/I_z = E/R = \boldsymbol{\sigma}_{yield}/y$, that is equation number 1. Now, if you derive this further from this; we can say , $M_z/EI_z = 1/R = \phi$, which I call as the curvature and this is my z axis .

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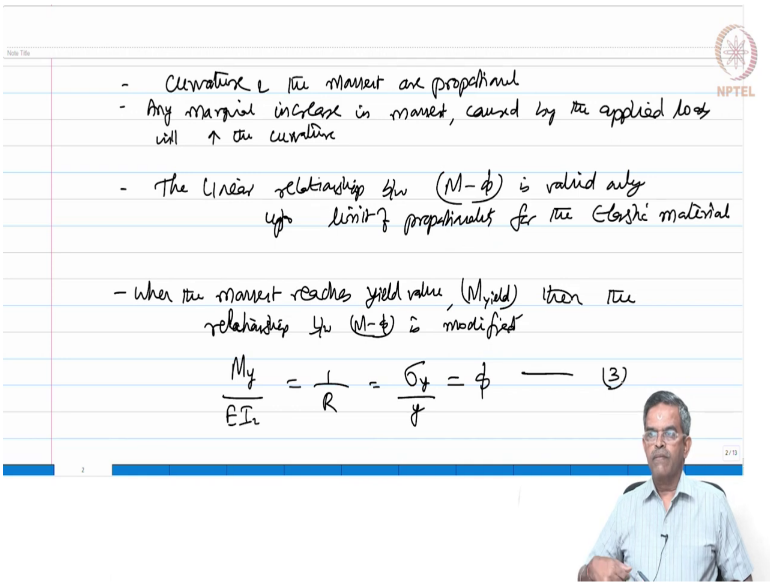

Now, one can see from equation 2 that, the curvature and the moment are proportional. Any marginal increase in moment which is caused by the loads will increase the curvature. But there is a big issue here, the linear relationship between moment and the curvature is valid only up to limit of proportionality for the elastic material. When the moment reaches yield value, that is the moment reaches moment at yield value, which I called as M yield; then the relationship between M-ϕ is modified.

So, I should say $M_y/EI_z = 1/R = \boldsymbol{\sigma}_{yield}/y = \phi$, which is the curvature, call this equation number 3. Now, look at the cross section here; the overall depth of the section is h.

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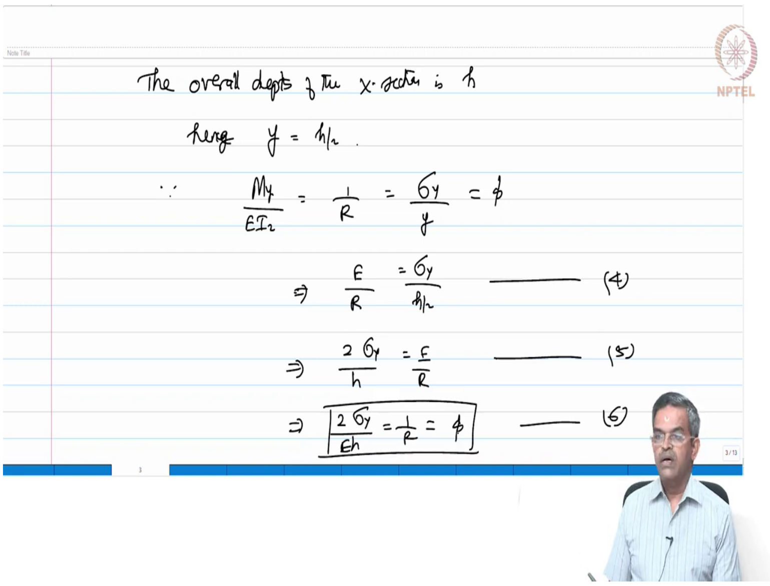

So, the overall depth of the beam is h. So, hence y=h/2, that is the distance of extreme fibre; therefore, $\boldsymbol{\sigma}_{yield}$ /(h/2) which should be equal to, let us let me write the equation again here, let us say M_y/EI_z =1/R= $\boldsymbol{\sigma}_{yield}$/y =$\phi$. So, let us write , $\boldsymbol{\sigma}_{yield}$ /(h/2)=E/R or 2$\boldsymbol{\sigma}_{yield}$ /(h)=E/R, 2$\boldsymbol{\sigma}_{yield}$ /Eh=1/R is the curvature. Let us call this as equation 4, equation 5 and equation 6.

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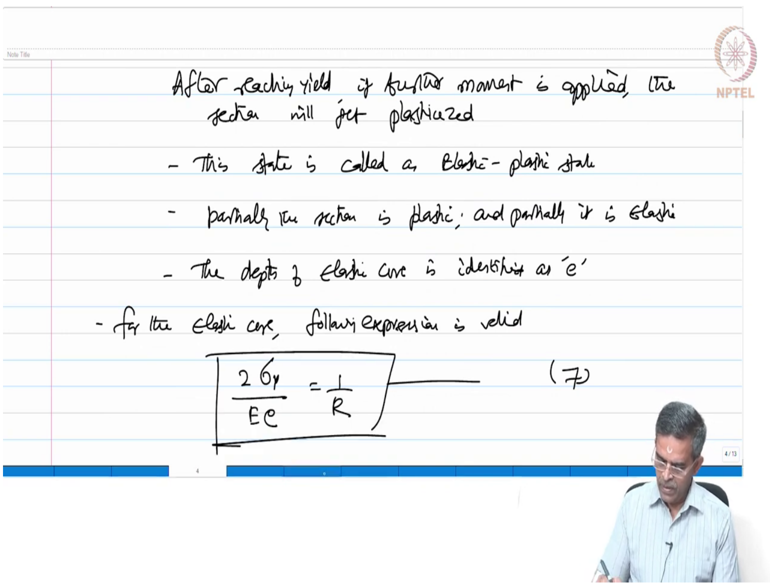

Now, after reaching yield, if further moment is applied; the section will get plasticized. So, this state is called as elastic-plastic state.

So, at this stage partially the section is plastic and partially it is elastic; therefore, the depth of elastic core is identified as e. Let us look at this figure; we call this as depth of elastic core, so e is depth of elastic core. So, for elastic sections or for the elastic core following expression is valid; because there is a linear relationship existing, therefore I can say $2\sigma y / Ee = 1/R$.

We already said $2\sigma_y / Eh = 1/R$; I am replacing this h with e for an elastic core right, equation number 7.

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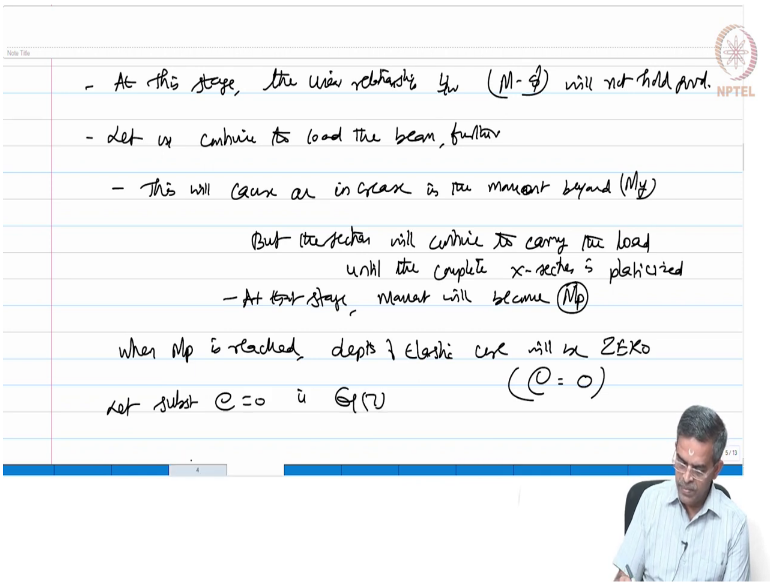

So, at this stage the linear relationship between moment and curvature will not hold good. So, as stated in plastic design, let us keep on continuing to load the beam. This will now increase the moment, beyond M y. But the section will continue to carry the load, why?

It is because the section is not fully plastic, still some part of the section is elastic. So, the section will keep on continuing to carry the load until the complete section or the complete cross section is plasticized, at that stage moment will become Mp. So, when Mp is reached, what would be the depth of elastic core? Elastic core will be zero; because this elastic section will go away, therefore I can say e will become 0.

So, let us substitute, let us substitute e as 0 in equation 7. So, if you put e as 0 in equation 7, what happens?

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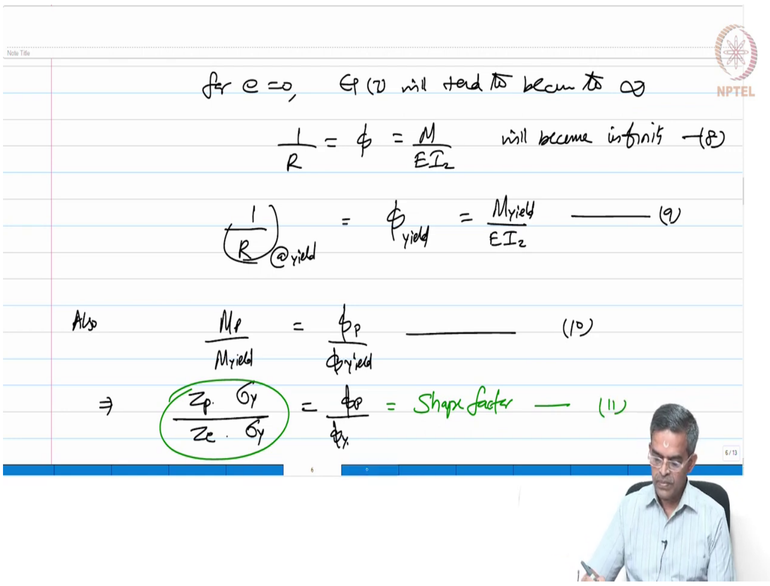

So, for e become 0, equation 7 will tend to become infinity; that is 1 by R which is a curvature, which is M by E I z will become infinity, right. Therefore, we can say 1 by R; let us write this equation as 8, 1 by R at yield will be equal to curvature at yield will be equal to M, at yield by E I z.

Also, M p by M yield is phi p by phi yield; we already have this relationship with us, moment and curvature are directly proportional till the elastic limit. So, we can use this relationship now. So, therefore, friends M p we know it is Z p into **σ** y and M y is Z e into **σ** y, which will be phi p by phi yield. Now, friends we all know that this relationship is actually shape factor, equation number 11.

So, having said this, let us rewrite this equation of elastic core; we already know M equals $Mp(1 – e^2/3h^2)$.

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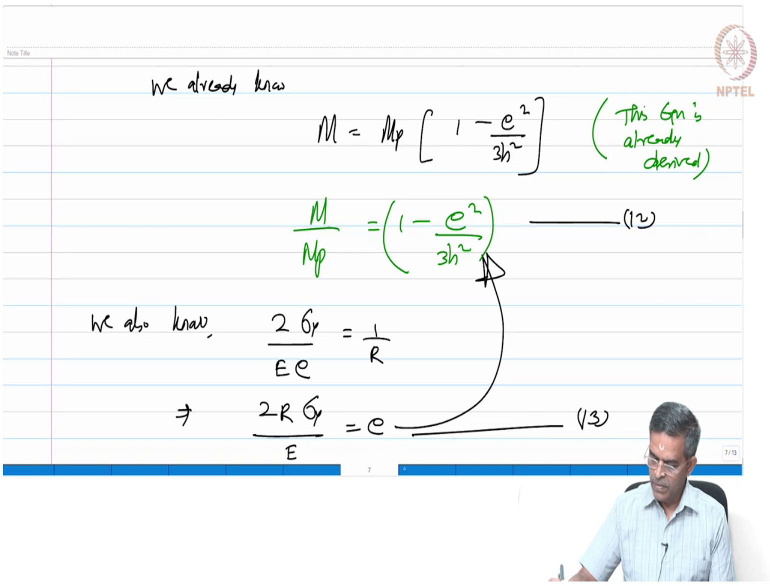

We also know twice of **σ** y by E e is 1 by R; this equation we already have, see here, is it not.

Which tells me 2 R into **σ** y by E is the depth of elastic core. Let us substitute this value here. So, I call this as equation number 12.

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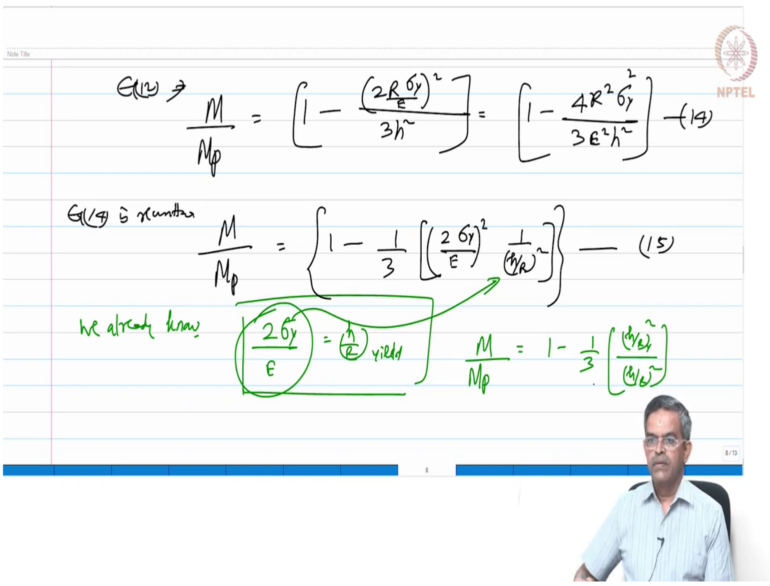

So, equation 12 now becomes M by M p = 1 minus; e will be 2 r **σ** y by capital E. So, 2 R **σ** y by capital E the whole square by 3 h square, which becomes 1 minus 4 R square **σ**y square by 3 E square h square, equation number 13 and 14.

So, therefore, M by M p is 1 minus 1 by 3 of, I am rewriting equation 14 slightly in a different form, 2 **σ** y by E the whole square 1 by h by R the whole square. 2 **σ** y by E the whole square into 1 by h by r the whole square.

So, let us see this equation of curvature. We already know 2 **σ** yield by E is h by R at yield, we already know this equation. So, let me substitute h by R yield from here to here. So, can I write this as now M by M p will become 1 minus 1 by 3 of h by R yield square pi h by R the whole square, can I say this?

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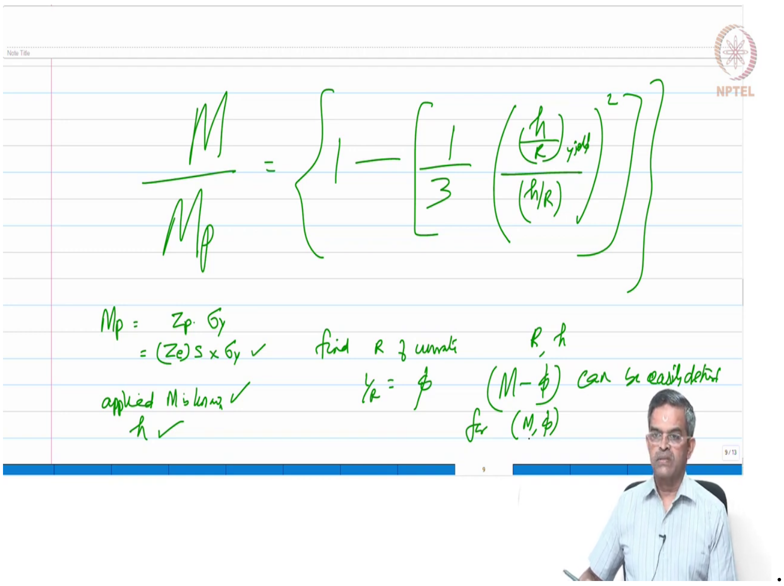

Which I can say now as M by M p is 1 minus 1 by 3 of h by R-at yield divided by h by R in general of the whole square.

So, what does it mean? If you have an elastic plastic section, whose radius of curvature and depth of section are known or if I know the M p of the section which is Z p into **σ** y, which we already know that Z p is Z elastic into shape factor into **σ** y. In a given tables steel table Z e for different sections available and shape factor is also known. So, M p is known; applied moment M is known, because you know the load, depth of the section is known, you can find the radius of curvature and 1 by R will give you the curvature.

So, moment curvature relationship can be easily defined and derived for various pairs of M-ϕ; you can easily know this here, you can plot this. That is a very interesting understanding we learnt from the moment curvature relationship using plastic design concept.

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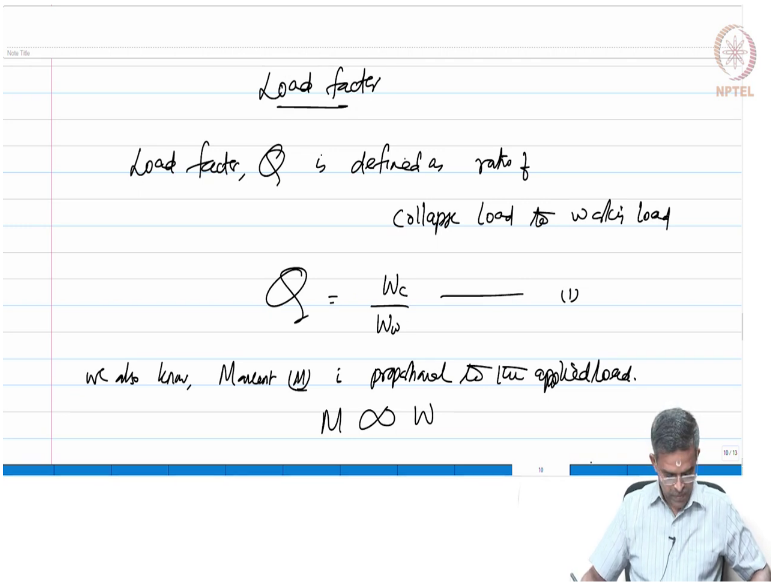

Let us go to another important segment of plastic design, which we call as a load factor. load factor is generally defined as; let us call this as Q is defined as the ratio of collapse load to working load. Let us call this Q as collapse load to working load. We also know that applied moment M is proportional. Let us say moment M is proportional to the applied load, hence moment is proportional to W.

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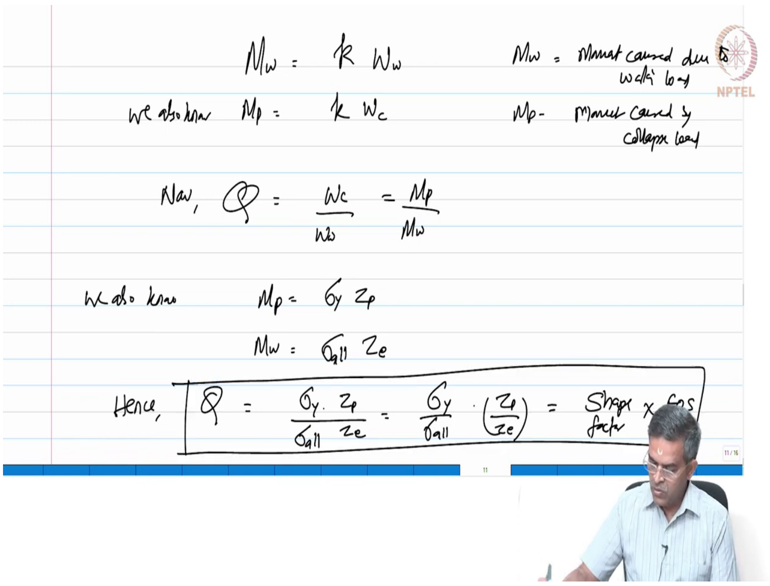

So, therefore, Mw is sometimes of Ww. Let us equate it to some proportionality constant; we also know Mp is k times of collapse load. So, where Mw is the moment caused due to working load and Mp is the moment caused by collapse load. So, in plastic design, we always estimate the load as a collapse load because it is M p. Now, Q is W c by Ww; by this logic, this can be simply M p / M w.

We also know M p is yield stress of Z p and M working is **σ** allowable of Z e; we are talking about working loads. Hence, Q now will become **σ** y of Z p by **σ** allowable of Ze, which can be **σ**y by **σ**allowable into Zp by Ze, which is actually equal to the shape factor multiplied by factor of safety.

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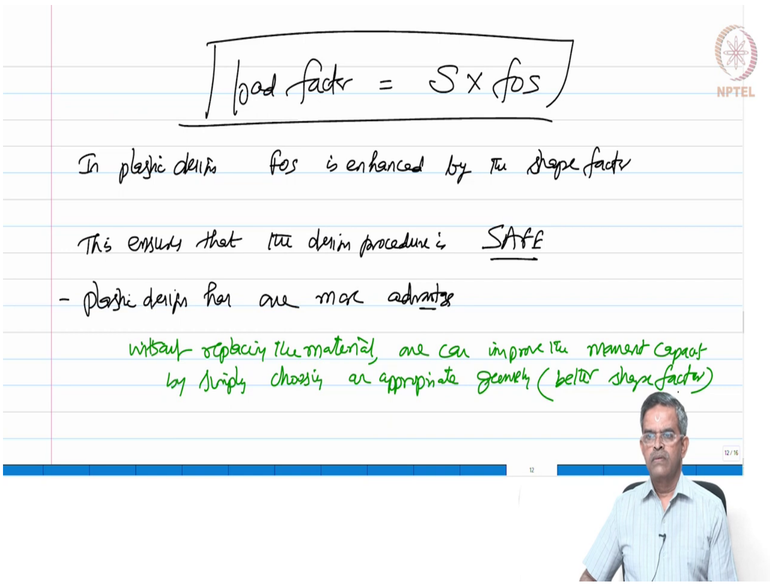

So, we have a very interesting relationship, load factor is a product of shape factor into factor of safety. So, the load factor which is used in the plastic design is enhanced by the shape factor. So, in plastic design, factor of safety is enhanced by the shape factor. So, this ensures that the design procedure is safe. So, plastic design claims one more advantage. Without replacing the material, one can improve the moment capacity by simply choosing an appropriate geometry.

So, I will put it as not the word geometry, I will put it as a better shape factor; can I use this? So, now, we have done the shape factors for different rolled sections like open sections of L, T, channels, etcetera.

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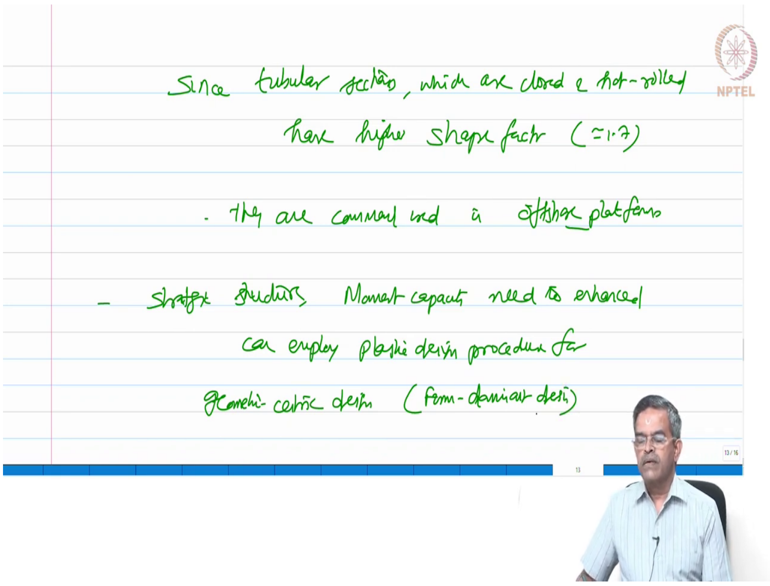

So, since tubular sections which are closed and hot rolled have higher shape factor, if you recollect; It was approximately equal to 1.7, higher shape factor they are commonly used in, for example offshore platforms.

So, offshore structures are mostly tubular members; this is because of the reason that they have a very high shape factor. So, any strategic structure where the moment capacity needs to be enhanced, can employ plastic design procedure for geometric centric design; that is for a form dominant design, one can use plastic design and one can use cross sections of higher shape factor.

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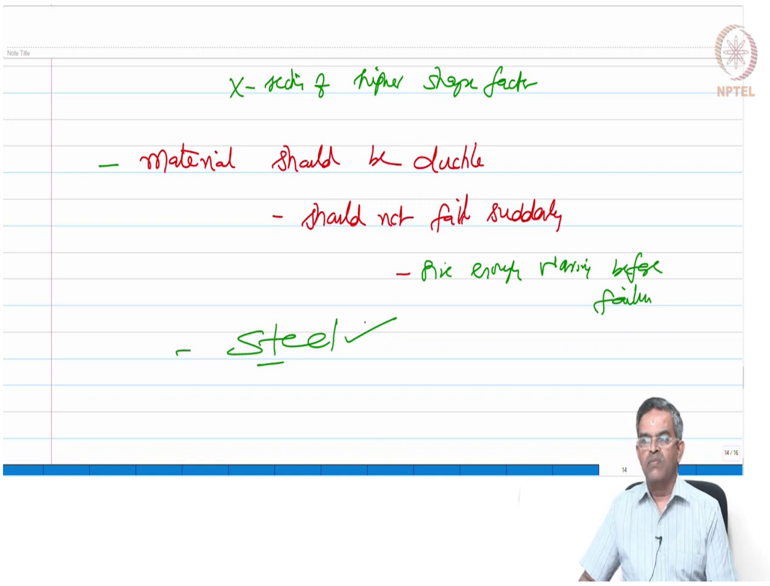

But there is only one catch here, the material what you use should be ductile, should not fail suddenly; it should give enough warning before failing. So, if you think about a material immediately in mind, you will find the steel possess this advantage; therefore, steel is the most successful and most prominent material recommended by engineers for plastic design of sections.

That is the reason, otherwise there is no very fancy attraction about steel as a construction engineering material. Steel almost possesses all type of advantages in the geometric side, in the material side, in the fabrication, in the recycling, quick return on investment and of course in maintenance; but the only issue what steel has is an environmental influence on the material, which is strength degradation because of corrosion.

So, if that is addressed by suitable alternate suggestions on the material of steel, like functionally graded materials; for example, which we studied in detail in the previous lectures, then the construction practices use of steel or FGM on strategic structures can go very sound and that can give you a very good performance as well as economic design, that is the idea what you want to emphasize in this.

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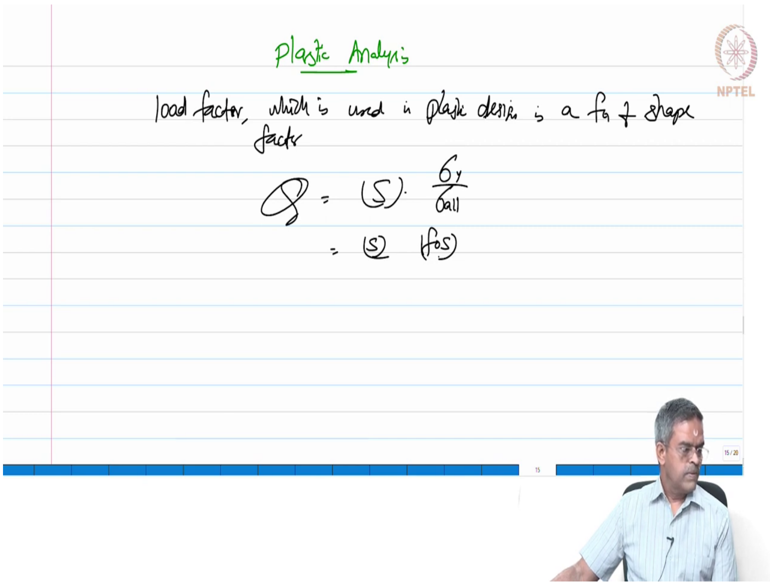

So, now let us take it forward; how do I do a plastic analysis? Let us do that load factor which is used in plastic design is a function of shape factor, right. So, the Q of S is shape factor **σ** y by **σ** allowable; if I say this is my factor of safety, so I can say that in plastic design the load factor is enhanced shape factor times the factor of safety. So, it is a very safe design. In plastic analysis as we go further, we have a concept called mechanism.

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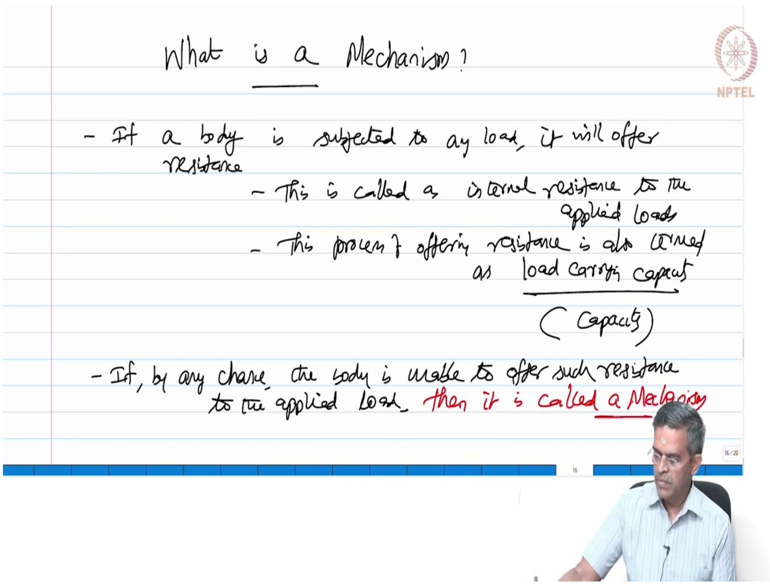

Let us ask a question what is a mechanism? If a body is subjected to any load, it will offer resistance; we call this as internal resistance to the applied loads, is it not. This process of offering resistance is otherwise termed as load carrying capacity; in general, this is the capacity. Now, if by any chance the body is unable to offer resistance; there may be many reasons for this, then it is called a mechanism. So, mechanism is a simple definition of no resistance to load.

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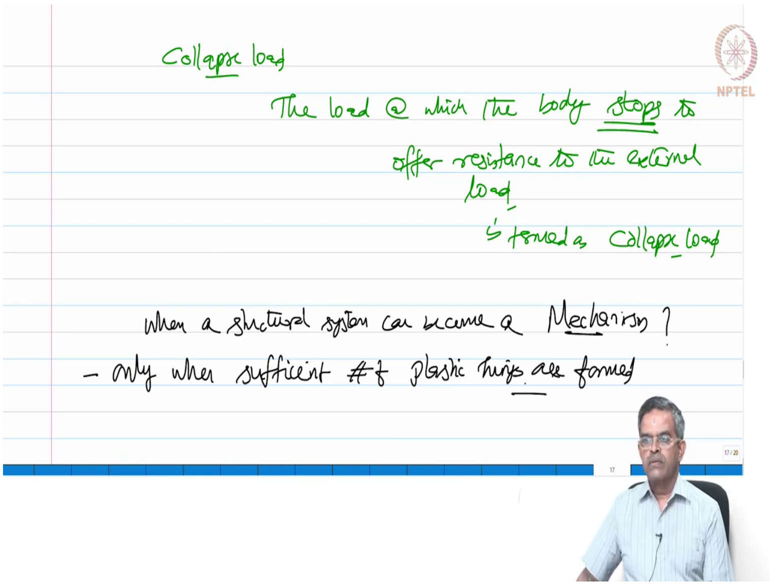

Then what is the collapse load? The load at which the body stops to offer resistance to the external load is termed as collapse load. So, the structural system will become a mechanism under certain conditions. So, the question comes, when a structural system can become a mechanism? A structural system can become a mechanism only when sufficient number of plastic hinges are formed.

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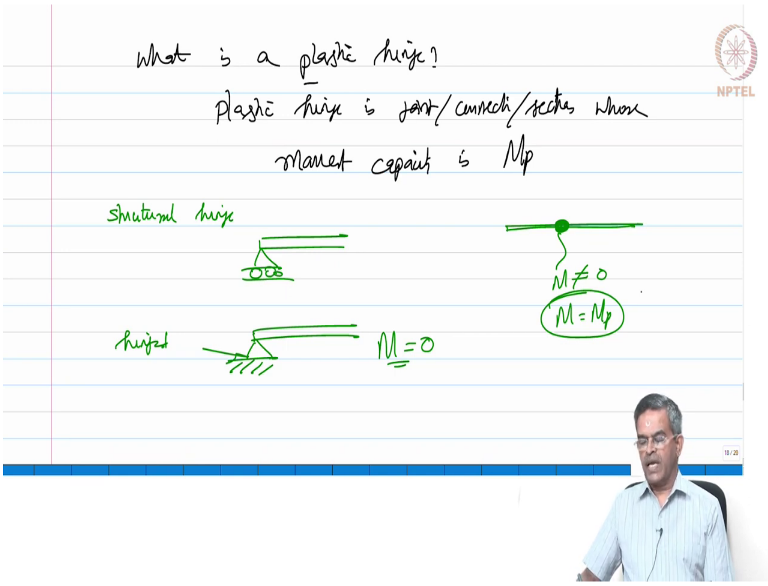

So, what is the plastic hinge then? Plastic hinge is a connection or a joint or a cross section or a section; not cross section or a section, whose moment capacity is M p.

Remember, you have a structural hinge like roller support, like hinged support, here the moment is equal to 0, whereas a plastic hinge is a section where the moment is not 0, the moment is equal to Mp. Now, the question comes if sufficient number of plastic hinges are formed in a structural system, the system can become a mechanism.

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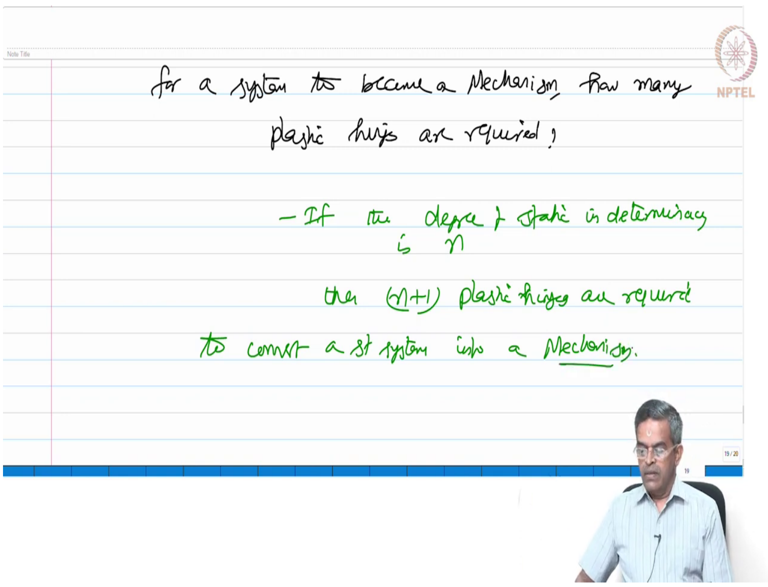

Then let us ask a question for a system to become a mechanism, we agree that sufficient number of hinges are to be formed. How many plastic hinges are required? The answer is very simple; if the degree of static in determinacy is n, then n plus 1 plastic hinge are required to convert a structural system into a mechanism. Then the question comes, where these hinges can form, where plastic hinges can form?

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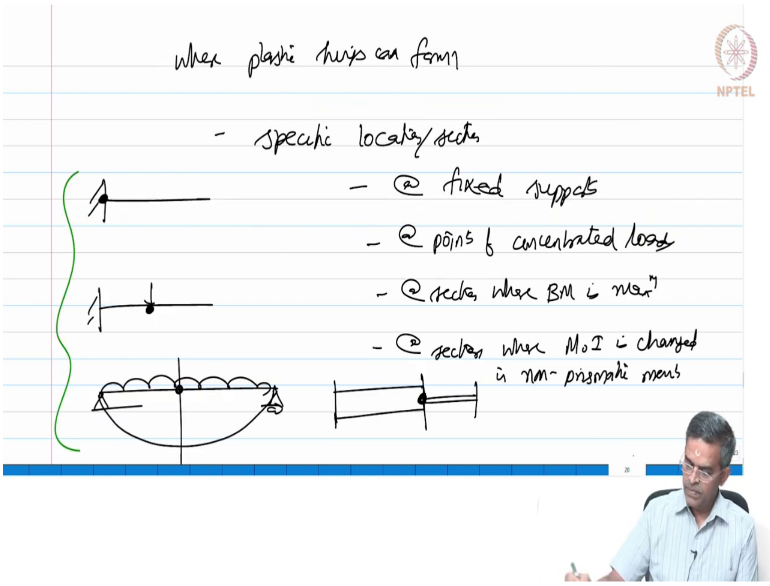

They can form at specific locations; like at fixed supports, at points of concentrated loads, at sections where bending moment is maximum, at sections where moment of inertia is changed in non-prismatic members. Let us say at fixed support, plastic hinge can form; at point of concentrated load, plastic hinge can form; at sections of maximum bending moment, plastic hinge can form; if a section is having a non-prismatic cross section, plastic hinge can form. So, these are the possible locations where plastic hinges can be formed.

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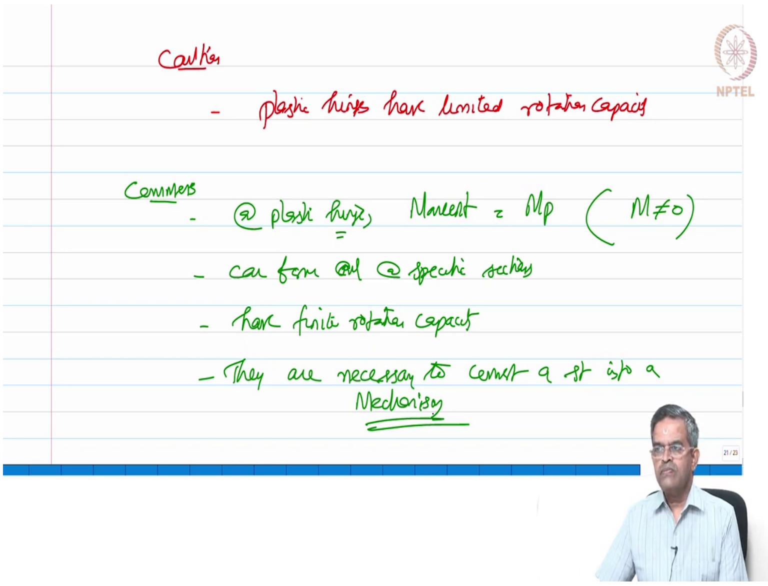

Let us add one more point to our knowledge; there is a great caution with plastic hinges, the caution is the plastic hinges have limited rotation capacity.

So, now there are some comments, observations what we can make, at plastic hinge, moment is M p; just because it is a hinge, do not say moment is equal to 0, moment is not equal to 0, it is Mp. It can form only at specific locations; where are they, we already said that. They have finite rotation capacity; they are necessary to convert a structure into a mechanism.

Mechanism is that condition of a structure which cannot offer any resistance to the load; the structure is completely out from its performance, it is collapsed, the structure cannot offer any resistance when it becomes a mechanism.

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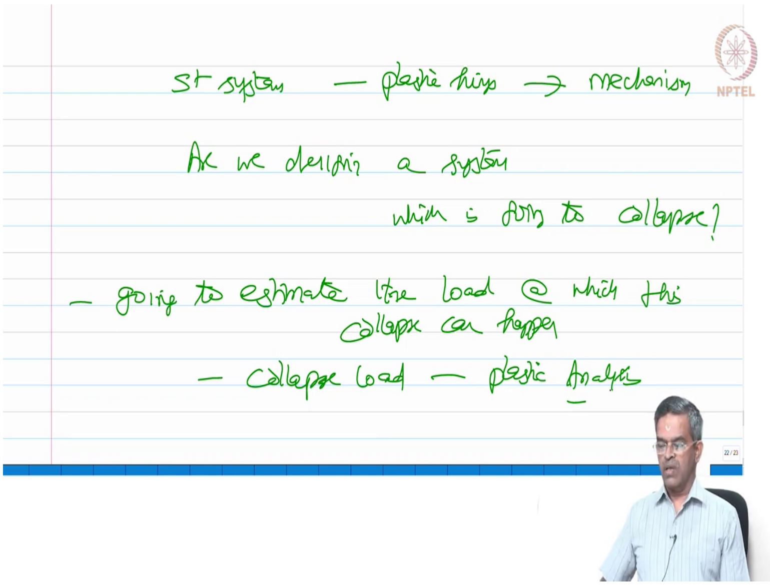

A structural system by the virtue of plastic hinges is converted into a mechanism. So, my question is, are we designing a system which is going to collapse?

The answer is not, we are designing a system going to collapse; we are going to estimate the load at which this collapse can happen, that is called as the collapse load. The process which helps us to estimate the collapse load is called plastic analysis. So, plastic analysis is a procedure which helps us to find out the collapse load on a given structure.

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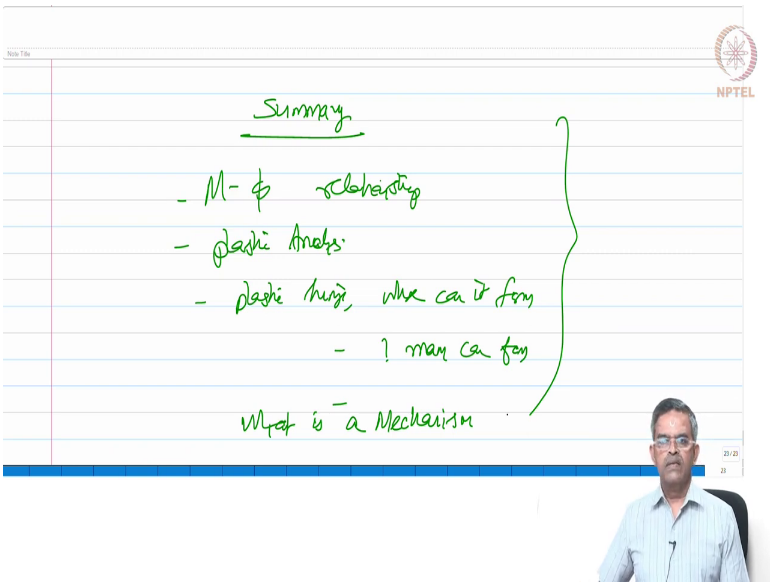

So, friends in this lecture, we have learnt the moment curvature relationship, we have learnt what is plastic analysis.

We have also understood what is a plastic hinge, where can it form, if it how many of them can form; if it forms, it converts itself into a mechanism and what is a mechanism. So, in the next lecture, we will start extending the learning process of estimating the collapse load using different theorems, which are static theorem, kinematic theorem and uniqueness theorem.

Friends, you please revise these lecture notes, try to access to my textbooks referred in the website of this particular course and keep on doing lot of exercises for shape factor and estimating for different cross sections of geometric novelty; because they are very helpful in making you to hold the answers for this.

Friends, there is one good news; for working out the shape factor, there are MATLAB equations available for different problems in my textbook. You can copy paste them and run in MATLAB window, you will get the answers of shape factor directly; you can also write simple programs of this order for different cross sections and try to practice this for your understanding.

Thank you very much, have a good day, bye.