**Advanced Design of Steel Structures**
**Dr. Srinivasan Chandrasekaran**
**Department of Ocean Engineering**
**Indian Institute of Technology, Madras**

**Lecture - 22**
**Plastic analysis -2**

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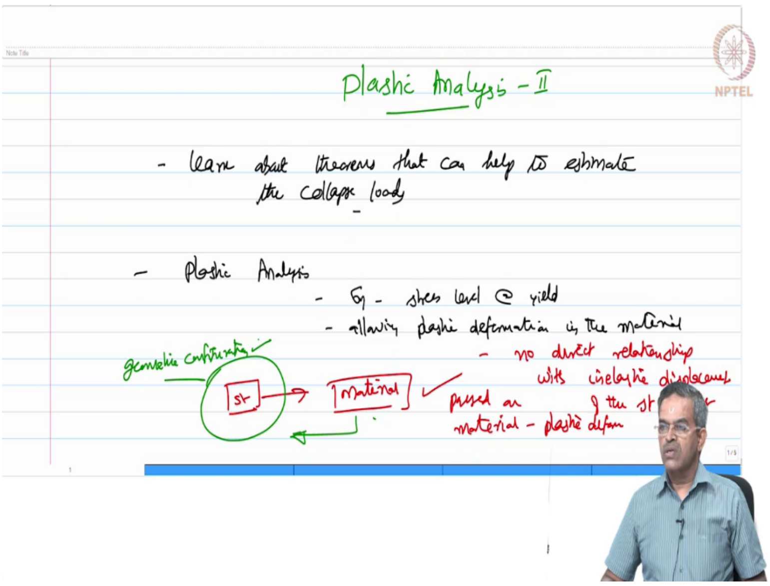


Friends, we will have this lecture on Plastic analysis-2 where we are going to talk about the theorems which are useful in estimating the collapse load. So, friends we are going to learn about various theorems that can help to estimate the collapse loads. So, you should realize an important statement when we talk about plastic analysis, you must realize that we are talking about the stress level at σ σy and allowing plastic deformation in the material.

This has no direct relationship with inelastic displacement of the structural member. However, large inelastic displacements of the structure can invoke the requirement of plastic deformation. So, from the structure to the material there is a request passed on that the material should undergo or should be capable of undergoing plastic deformation.

But from the material there is no request transferred to the system. that is important. So, therefore, we keep on repeatedly saying plastic design or the design of large displacement structures like form dominant does not depend on the material strength alone. they also depend on the geometric configuration.

So, the geometric configuration enables a system to resist the loads to a significant extent of course, you cannot have a good geometric configuration and the material can be made of zero strength like paper we cannot have a structural member like that. So, we are not oaring that extreme condition we say the material strength is important, but the design is not material strength centric alone.

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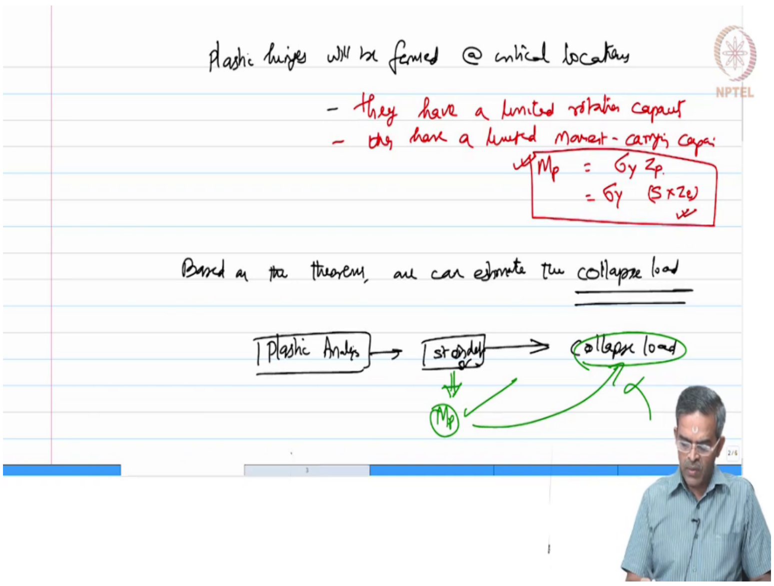


So, we also learnt in the last lecture that plastic hinges will be formed at critical locations we have discussed what all these possible locations where plastic hinges can form. So, at critical locations these hinges will be formed, but they have a limited rotation capacity and they have a limited moment carrying capacity. What does it mean?

The moment carrying capacity of the plastic hinges is referred as M p which we can recollect now confidently the σy *Zp where we can also say that is σ y in shape factor into Z e where we already know for a given standard cross sections the elastic section modulus and the shape factor can be easily computed.

So, M p is not the function of load, but it is a geometric centric property. So, plastic hinges will be formed whose rotation will be limited and based on the theorems which we are now going to discuss, one can estimate the collapse load. So, please be noting that we are referring this load from the plastic analysis.

So, plastic analysis applied on to a structure statically indeterminate structure will give you estimate of collapse load. Please understand very carefully the structure which is subjected to plastic analysis the moment capacity is already known, it is not dependent on load. So, from the moment capacity you calculate the load. So, it is actually a reverse engineering problem.

In convectional analysis load is given to you estimate the moment then design the section and then check the section for exceedance of its bending stress shear stress etcetera that is the conventional design procedure you know, but here the design procedure is entirely challenging. You have the moment capacity of the section you want to find the collapse load estimate.

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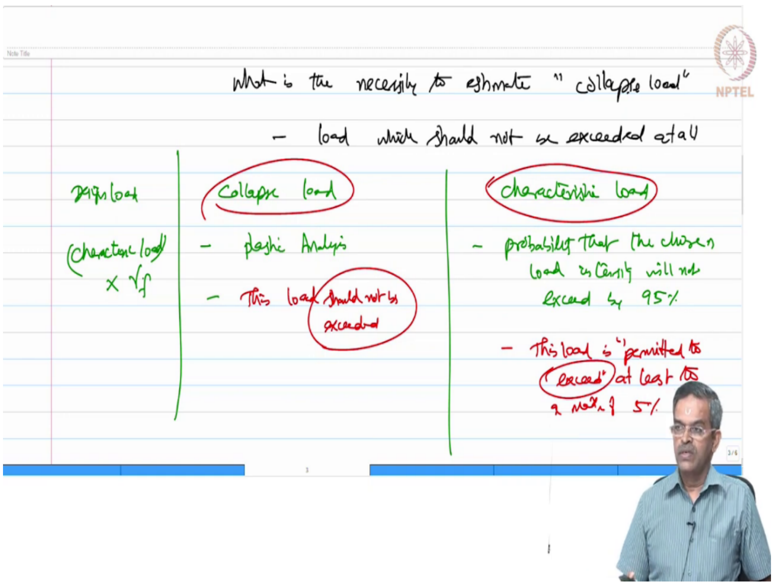


So, one can ask me a question then what is the use, what is the necessity to estimate a something called collapse load? This is a load which should not be exceeded at all. You may ask you a question what is the difference between a collapse load and the characteristic load? Let us say and the design load a collapse load is that which you get from the plastic analysis and this load should not be exceeded in the service life of the structure.

Whereas, characteristic load is the probability that the chosen load intensity will not exceed by 95 percent, for example, Indian codes, but there is a probability that this load is permitted to exceed at least to a maximum of 5 percent. So, there is a possibility that this load can be exceeded, but this load cannot be exceeded. So, there is a difference between this.

Then you ask me a question what is the design load? Design load is the characteristic load multiplied by partial safety factor for the load, you get a design load. We already saw this in the discussions earlier. So, we have a distinct values for understanding that what is a collapse load. Having understood this let us see what are the theorems which will help us to do plastic analysis of structures.

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See we have got three theorems, they are static theorem, kinematic theorem and uniqueness theorem. So, there are three theorems which are useful in estimating the collapse load and we do plastic analysis of structures we will see one by one now.

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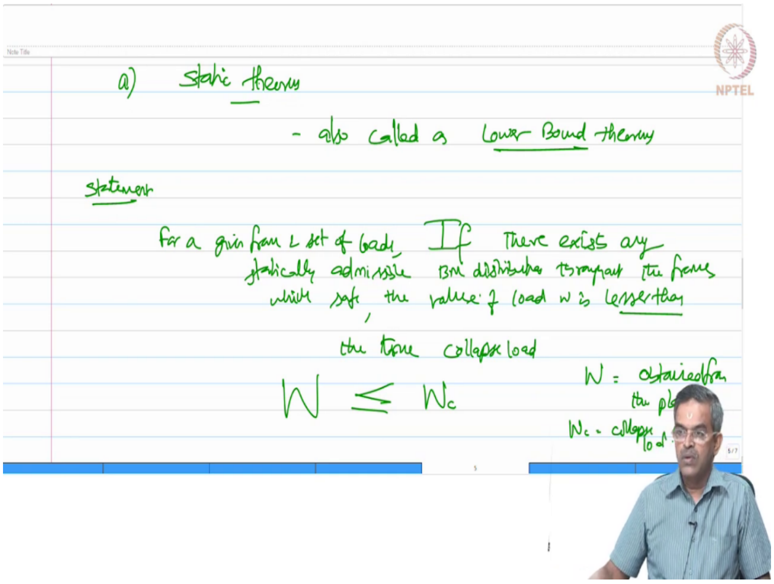


Let us talk about static theorem. Static theorem is also called as lower bound theorem. the statement of the theorem is as follows. For a given frame and loading if there exists any statistically admissible bending moment distribution throughout the frame which is safe, Then value of load W is lesser than the true collapse load.

What does it mean is, from the analysis you will be getting a load W which will always lesser than the true collapse load. So, W is a load obtained from the analysis from the plastic analysis and W c is the true collapse load.

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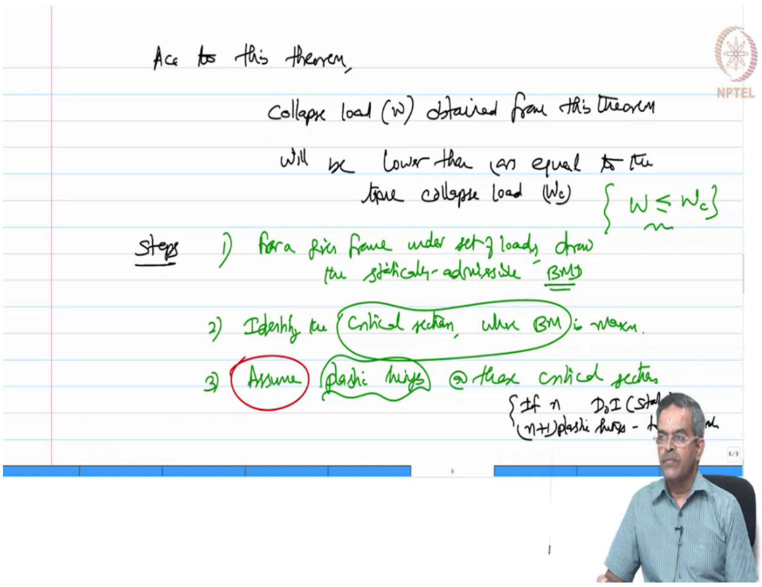

So, we could say that according to this theorem collapse load W obtain from this theorem will be lower than or equal to the true collapse load W c. So, what are the steps involved in estimating W? So, we should put a condition here W is always lesser than or equal to W c since it is always lesser than or equal to this theorem is called lower bound theorem, what are the steps involved in this?

First for a given frame under set of loads, draw the statically admissible bending moment diagram. So, once you draw the bending moment diagram identify the critical sections where the bending moment is maximum that is very simple because looking at the magnitude you can identify them. 3. Assume plastic hinges at these critical sections. Please note carefully "assume", there is no guarantee that the plastic hinges will form only at these sections in this theorem.

Assume plastic hinges at these critical locations. So, we also remember that if n is a degree of indeterminacy static. So, one need n plus 1 plastic hinges to become a mechanism.

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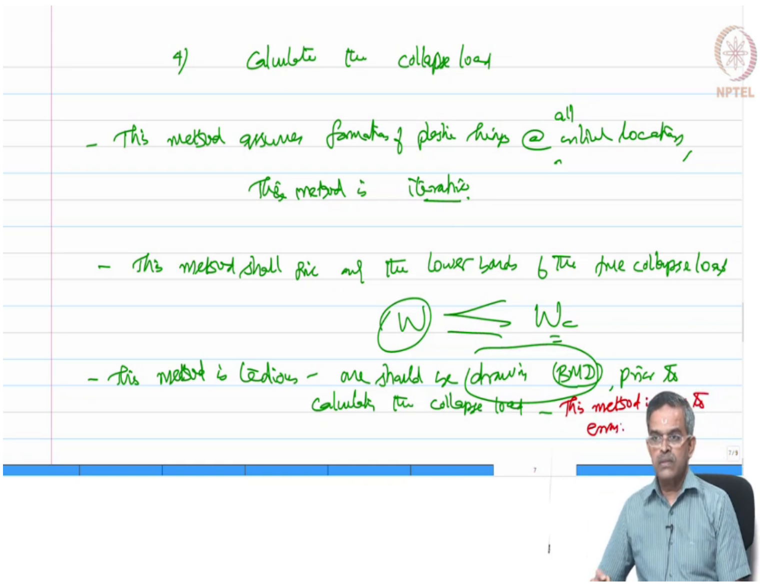


Then once you do this in the last step calculate the collapse load. Since we say this method assumes formation of plastic hinges at all critical locations this method is iterative. Further, this method shall give only the lower bounds of the true collapse load, W will be either equal to or lesser than W c what we get from the static theorem is W.

So, out of all the Ws of time you can choose the maximum and say that is means W c. So, that at any given point of time the W obtained from the iterative mechanism will not exceed W c. So, there are some comments about this method ,this method is tedious, is complex. I will tell you why because one should be drawing the bending moment diagram prior to calculating the collapse load correct.

So, this method has a prerequisite. You need to draw a bending moment diagram for the given problem now you may ask me a question why do you need to draw a bending moment diagram? Go back here. You have to draw the bending moment diagram identify the sections has got the maximum bending moment and those sections are the locations of a plastic hinges.

So, just to know where the hinges will form you have to draw the bending moment diagram they therefore, this method is prone to errors. So, that is the limitation here.

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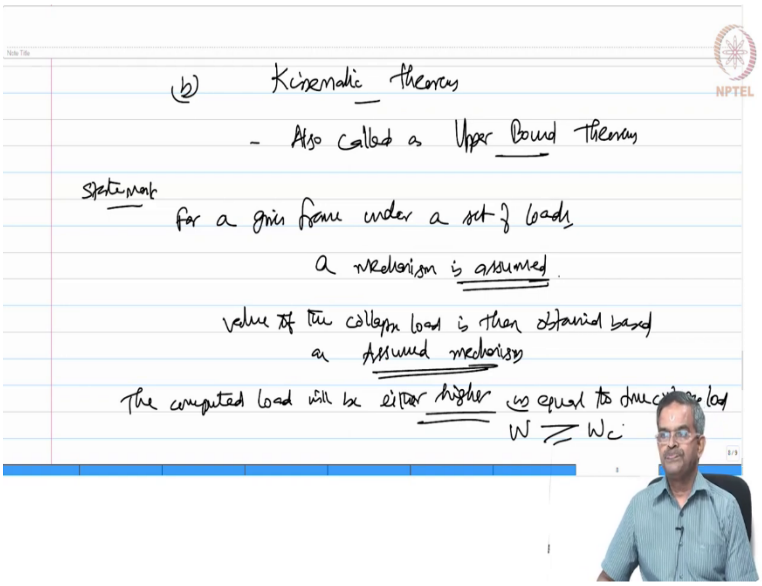


Now let us talk about the next theorem. Next theorem is named as kinematic theorem. Kinematic theorem is also called as upper bound theorem the statement of this theorem is as follows. The statement says for a given frame and loading under set of loads assume a mechanism a mechanism is assumed, value of the collapse load is then obtain based on the assumed mechanism . The computed load will be either higher or equal to the true collapse load.

So, we can say W will be either greater than or equal to W c since it is greater this is called as upper bound theorem.

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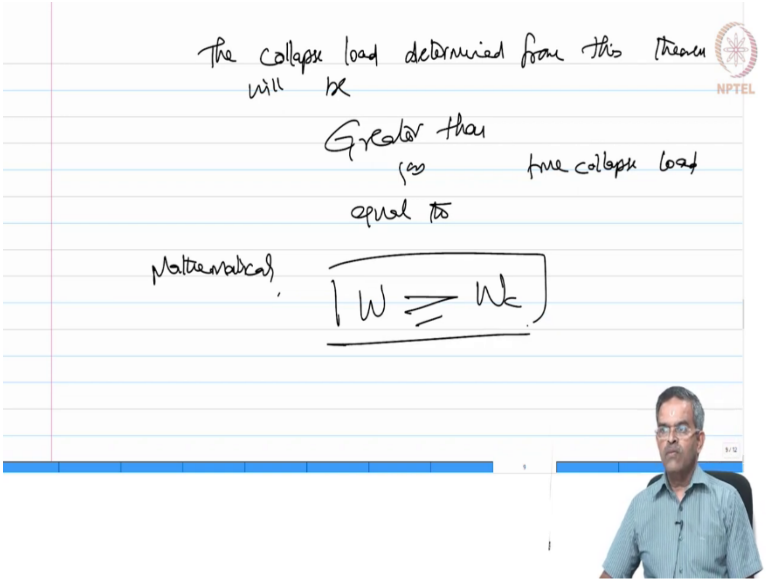


So, therefore, friends the collapse load determined from this theorem will be greater than or equal to the true collapse load. Mathematically, W will be either equal to or greater than W c. Let us see what are the steps involved in computing W from this method.

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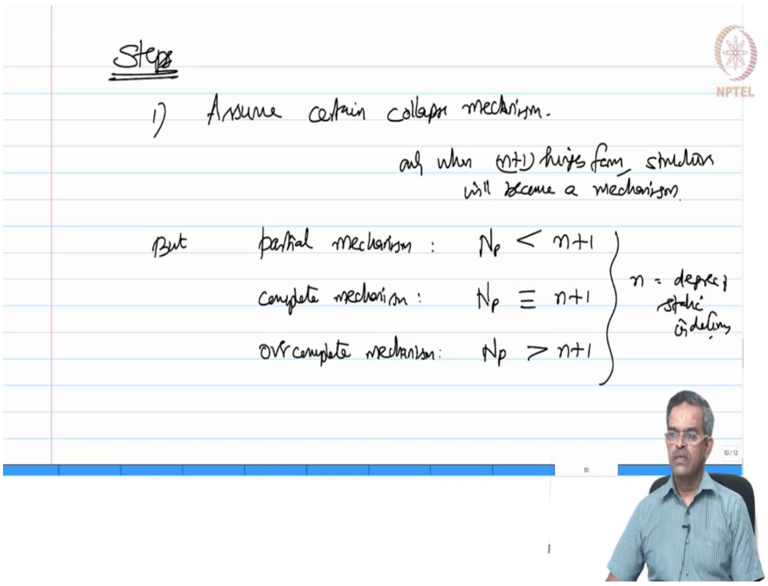

Step number 1. Assume certain collapse mechanism. We already know mechanism will form in a given structure only when n plus 1 hinges form the structure will become a mechanism. But there is something called partial mechanism. This is the condition where the number of plastic hinges formed is lesser than n plus 1 there is something called complete mechanism.

It is a case where the number of plastic hinges found will be exactly equal to n plus 1 there is also something called over collapse mechanism or over complete mechanism where the number of plastic hinges are more than n plus 1 where n is the degree of static indeterminacy of the frame.

So, you can assume either a partial mechanism a complete mechanism or an over complete mechanism depending upon the number of hinges what you assume and we all know that hinges cannot be assumed at any location of your choice, hinges can only form on certain critical locations which I already discussed in the last lecture. So, we know for a given frame what are those critical geometric sections, where hinge plastic hinges can form likely? So, choose those points, assume plastic hinges there and assume therefore, a collapse mechanism.

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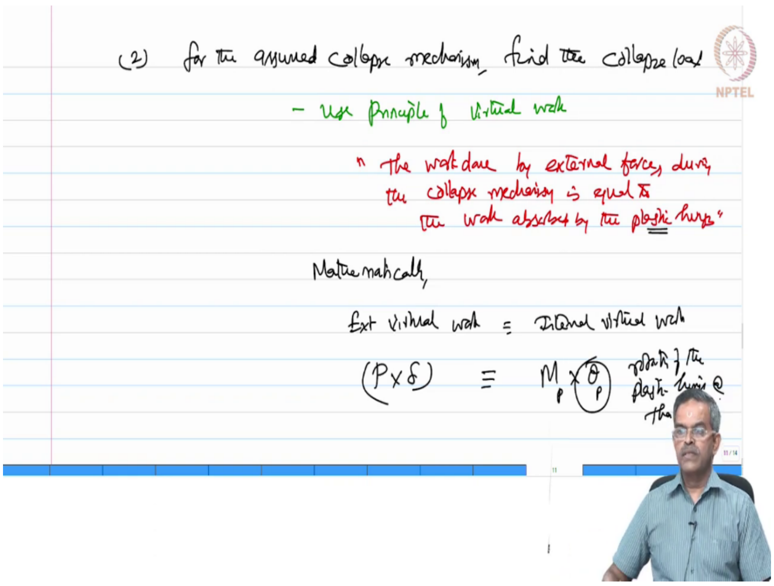


Now, step number 2. For the assumed collapse mechanism find the collapse load to find the collapse load use principle of virtual work. So, one may ask me a question what is the principle of virtual work which we all know, but still we will write this principle states that the work done by external forces during the collapse mechanism is equal to the work absorbed by the plastic hinges.

So, mathematically external virtual work is equal to internal virtual work. External virtual work is actually done by load multiplied by displacement, internal virtual work will be moment multiplied by theta. Since we are talking about plastic analysis the work done will be by the plastic hinges. So, I should say this M p this is theta p where M p is the plastic moment capacity of the section and theta p is the rotation of that plastic hinge at that location.

It is not plastic rotation it is rotation of the plastic hinge at that location

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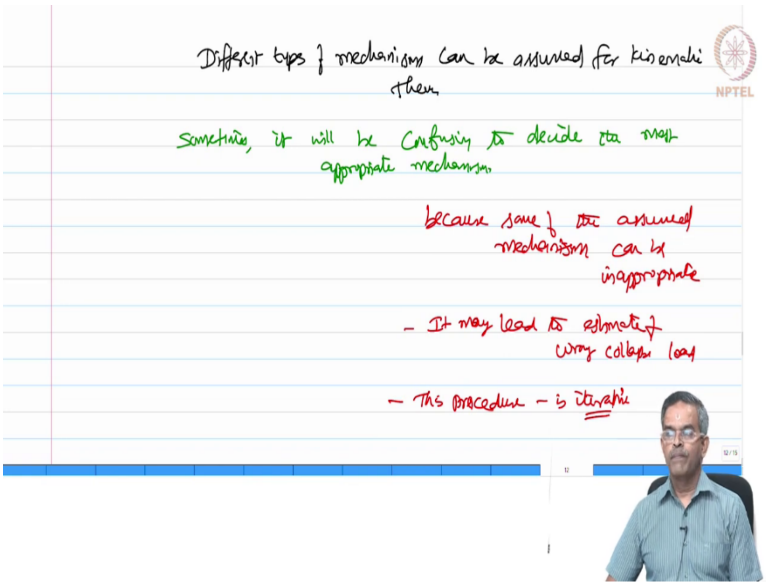


So, now, friends the question comes different types of mechanisms can be assumed for kinematic theorem. Sometimes, it will be very difficult for you to choose which the appropriate mechanism is. Sometimes it will be confusing to decide the most appropriate mechanism, because some of them can be inappropriate also because some of the assumed mechanisms can be inappropriate  that may lead to a confusion or it may lead to estimate of wrong collapse load.

Since you are assuming the mechanism and estimating collapse load this procedure is iterative. So, friends, both the theorems demand an iterative scheme because in one case you plot the bending moment diagram and keep on assuming the location of plastic hinges and derive the collapse load in the other case you assume the mechanism itself and estimate the collapse load. So, if you make a mistake in either of the procedure which is prerequisite to estimate the collapse load then the estimate of collapse load can become wrong that is number one there is a danger there.

Number 2. In both cases some of them may be inappropriate also of assumption of hinges therefore, the system requires iteration. Having said this let us try to see what the possible independent mechanisms which are available are.

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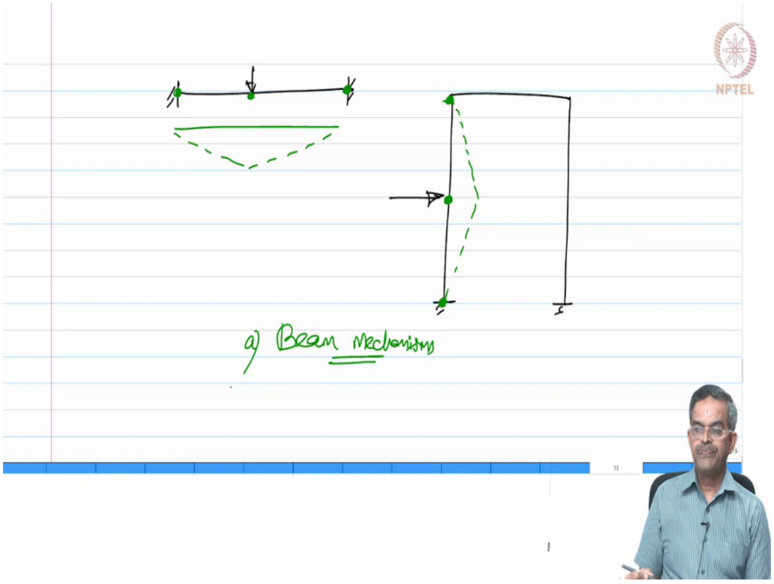


So, if I have a beam subjected to a load then I can always assume plastic hinge the fixed support plastic hinge here and plastic hinge here. So, the deflected profile will become like this. If you have a frame then I can assume and subjected to a lateral load like this I can also assume a collapse mechanism like this  where hinges are formed here that is not here and here or here . So, these two are called beam mechanisms.

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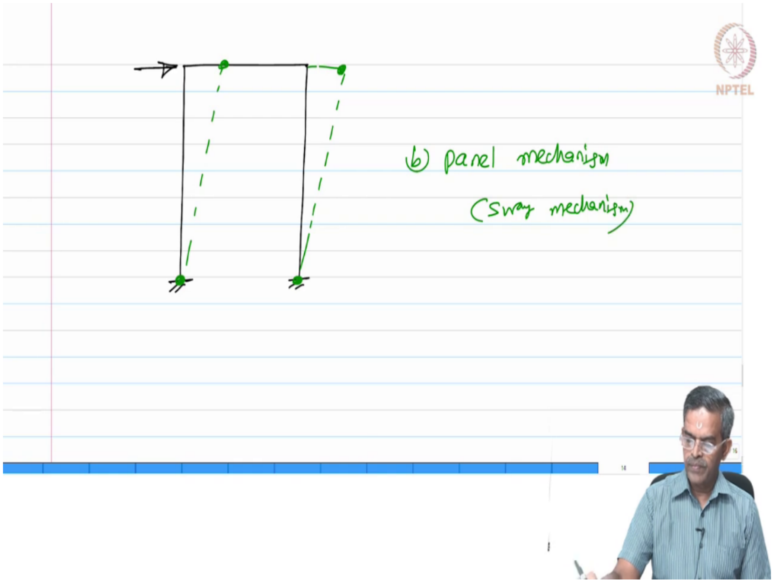


Suppose, if you have a frame subjected to a lateral load like this the frame can sway and there can be hinges formed here, here, here and here. this is called a panel mechanism, some literature advises this as sway mechanism.

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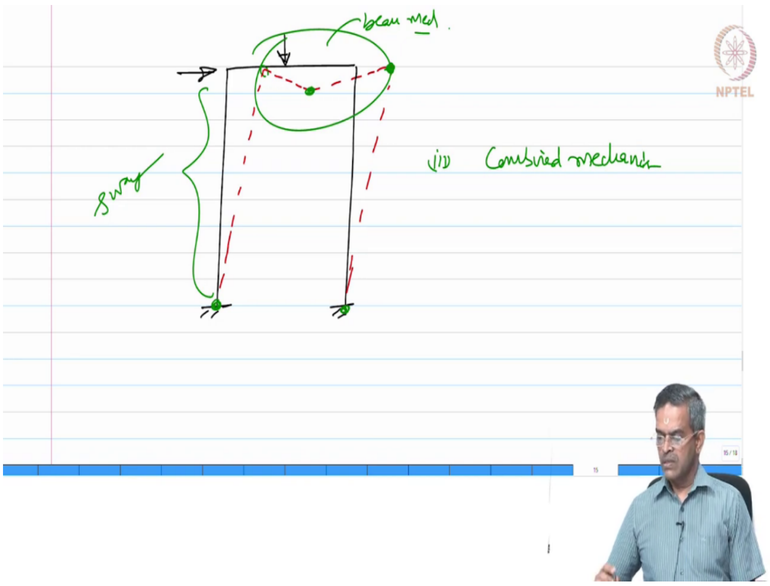


Suppose, if you have a frame which is having a combination of loads like it has got a lateral load and the gravity load then the deformation of the structure could be as marked now where the hinges can form here, here, here either here or here. So, this is called a combined mechanism. why it’s called as a combined mechanism? You can see there is a sway mechanism here and there is a beam mechanism. So, it’s called as a combined mechanism.

So, these are all different collapse mechanisms other than partially collapse mechanism over collapse and perfectly collapse mechanisms and so on. So, kinematic theorem demands to assume a mechanism first then estimate the collapse load.

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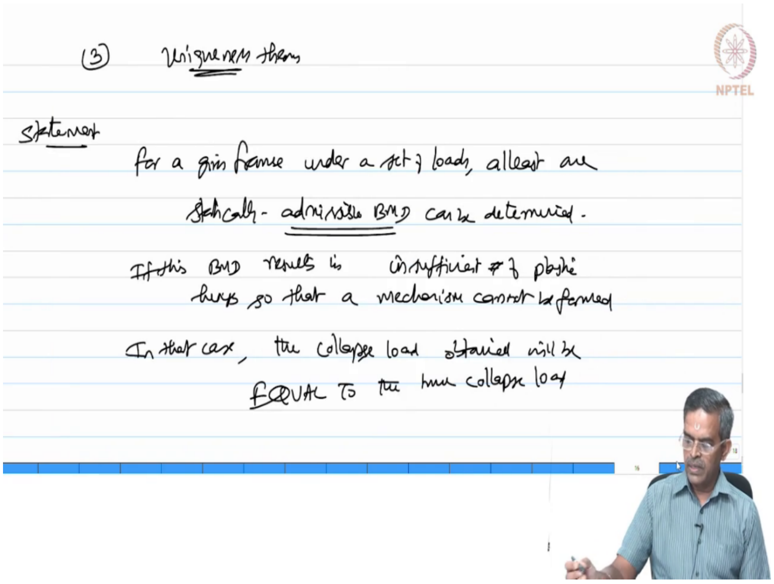


Whereas, there is a third theorem called uniqueness theorem. let us see what does this theorem does. The statement of the theorem says for a given frame under set of loads, at least one statically admissible bending moment distribution can be determined. If this bending moment distribution results in insufficient number of plastic hinges, so that a mechanism cannot be formed.

Let us ask a question how many numbers of plastic hinges are required to form a mechanism? If n is the degree of indeterminacy static degree of indeterminacy, we at least need n plus 1 hinges to convert the frame into a mechanism. But there is a possibility in the given bending moment distribution you may not get n plus 1 critical sections where you can assume plastic hinges.

So, there can be a possibility that the bending moment distribution results in insufficient number of critical sections where plastic hinges can be assumed. Therefore, it cannot become a mechanism in that case, the collapse load obtained will be equal to the true collapse load, mathematically W will be equal to W c.

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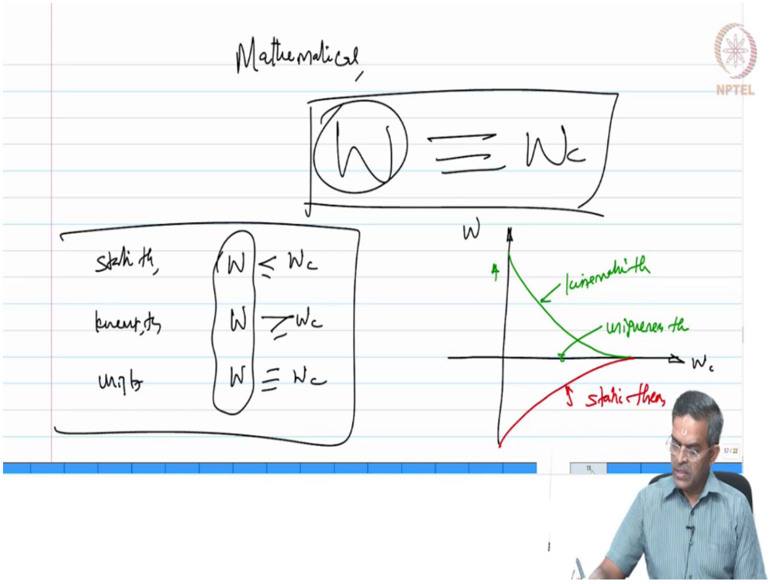


So, in static theorem or lower bound theorem we have W less than or equal to W c in kinematic theorem or called as upper bound theorem W is greater than or equal to W c in uniqueness theorem, we say W is equal to W c. We can also plot this graphically as shown here. let us say in the x axis I try to plot W c in the y axis I am trying to give W is what we estimate from the theorem W is what we estimate from the theorem W c is the true collapse load.

So, the kinematic theorem gives you upper more than W c or equal W c, the static theorem gives you a lower bound and this itself is a uniqueness theorem . So, that is what graphically we can explain.

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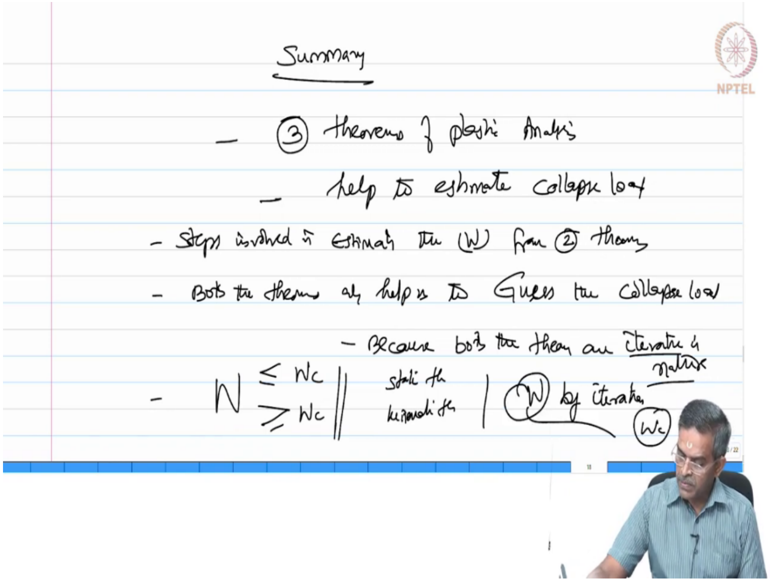


So, now, let us quickly see what we have learnt from this particular lecture. This lecture helped us to learn three theorems of plastic analysis. It also told that plastic analysis will help to estimate collapse load. We also learnt different steps involved in estimating the collapse load from two distinctly different theorems. Both the theorems only help us to approximately guess the collapse load because both the theorems or the solution from both the theorems are iterative in nature.

So, the moment I say iterative there is a possibility that there can be some error in computation. We also learnt that worse by worse the collapse load estimated will be either lesser than or equal to true collapse load will be greater than or equal to true collapse load uniqueness theorem is an hypothetical theorem it’s not applicable. So, we do plastic analysis either by static theorem or by kinematic theorem and estimate W by iteration. So, that our objective is to then conclude what will be the W c.

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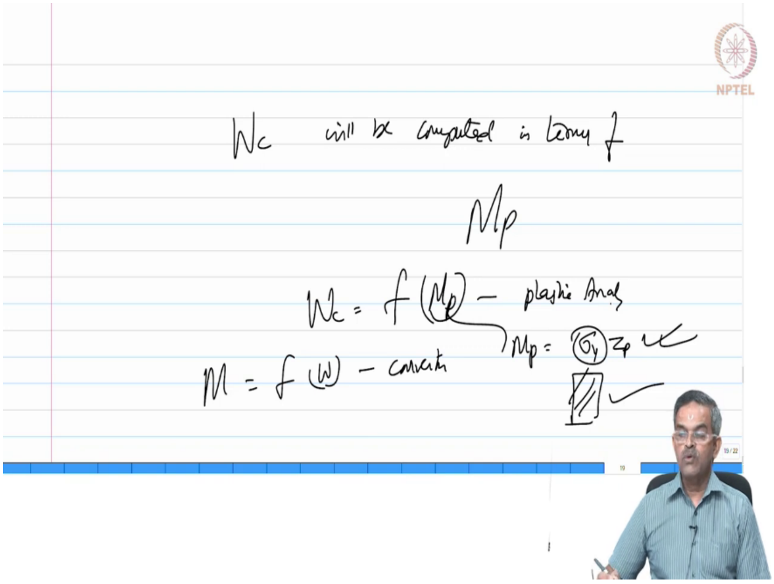

In terms of W c will be computed in terms of M p, very interesting in conventional analysis M is a function of load know this is conventional. In plastic analysis W c is a function of M p, because M p is known to me which is known independent of load it's a section property it of course, depends on material it's the section property . So, that is the beauty what we have learnt so, far in this lecture.

Thank you very much have a good day.