Advanced Design of Steel Structures
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Lecture - 23
Plastic analysis -3

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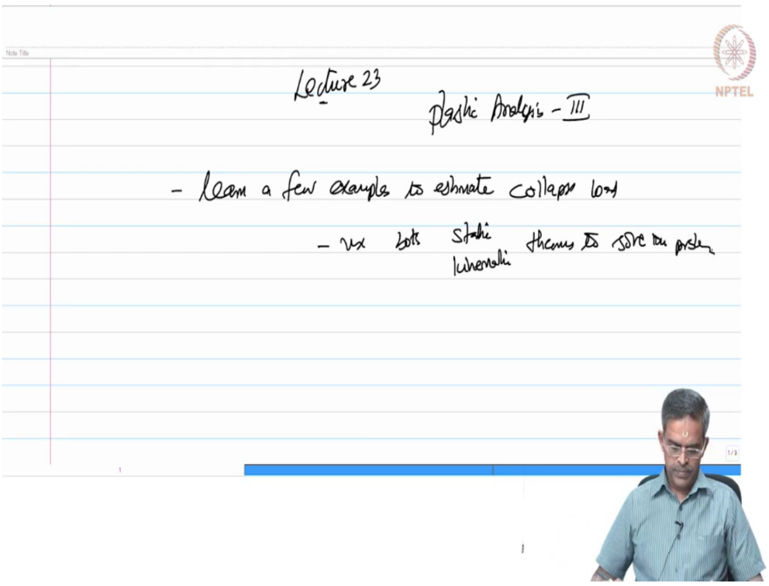

Friends, welcome to lecture 23, which is Plastic analysis III where we will learn few examples to estimate collapse load. We will use both the static theorem and the kinematic theorem to solve the problems. We will start with simple examples and see how we can understand this theorem applications.

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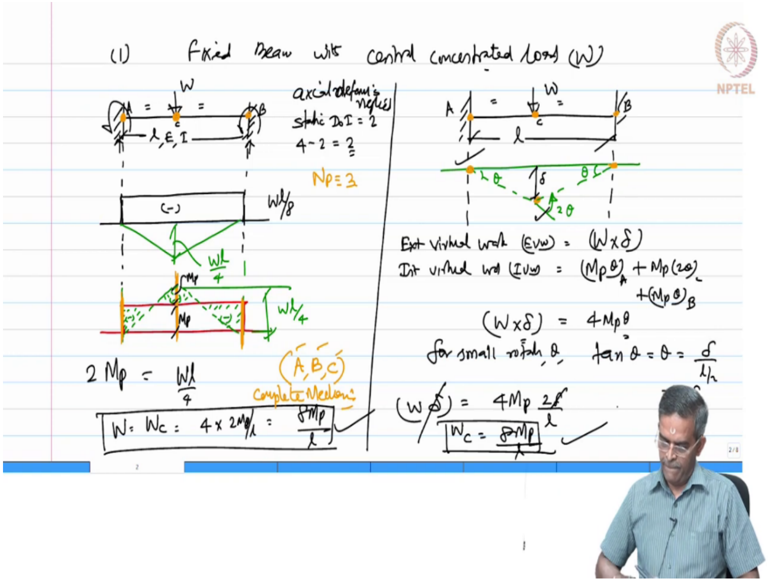

So, example 1, we consider a fixed beam with central concentrated load, we call this load as W. So, I have a beam which is fixed at both the ends with the central concentrated load W, let me name the points as A B and the midpoint as C and we say this span is equal and this is l E comma I A .

So, as per the static theorem we have first to draw the admissible bending moment diagram, for a fixed beam we know if this is my base line. So, there is a fixed moment which is equal to Wl/8 negative bending moment, then there is a positive bending moment which is the midspan is equal to Wl/4.

I superimpose these on the same side. So, I draw the negative bending moment diagram here and the positive bending moment diagram I just swap it like this. So, this is negative and this is positive again negative, we already know that this value is Wl/4.

So, now the critical sections are at three different places. The critical sections can be here, can be here, and can be here. So, what is the static degree of indeterminacy for this problem let us ask a question what will be the static degree of indeterminacy for this problem.

The static degree of indeterminacy for this problem is 2 by neglecting axial deformation that is 4 are the unknowns moment, reaction, moment, reaction. There are two equations of equilibrium Σ fy = 0 and Σm=0. So, the degree of indeterminacy becomes 2 by neglecting axial deformation.

So, I need 3 hinges to get a mechanism, let us say these are my 3 places where the hinges can form. So, the governing equation now going to be 2 Mp because this is also Mp this is also Mp hinges is formed here. So, 2 Mp should be equal to Wl/4. Therefore, Wc which I said W will be 4 into 2 Mp by l which is 8 Mp by l. So, that is my collapse load.

Ultimately we assume that, hinges are formed at three places hinges are formed at A B and C. making it as a complete mechanism, is it not? I need 3 hinges I got 3 hinges. On the other hand let us try to solve this problem using kinematic theorem. So, I have a fixed beam subjected to central concentrated load W the locations are A B and C.

Let me now assume a mechanism, so the assumed mechanism is the beam mechanism. So, I have one rotation here another rotation here and another rotation here by symmetry you will see this is 2 θ, is it not? So, I need 3 hinges let us assume the hinges here, here and here which are marked in this figure also.

So now, let us see what is the external virtual work? External virtual work is load into displacement let us call this as Δ. What is the internal virtual work? Internal virtual work will be the work done by the plastic hinges and its rotation which will be Mp into θ which is at A plus Mp into 2 θ which is at C plus Mp into θ is at B which gives me 4 Mp θ.

So, let us equate external virtual work to internal virtual work. So, there are two unknowns here Δ and θ, but in the figure they are connected. One can say very well if the span is l, you can say for small rotations tan θ which is θ which is actually Δ by l by 2 which is 2 Del by l.

So, substituting back here W into δ will be 4 Mp into 2 δ by l. So, δ goes away I get Wc as 8 Mp by l which is same as this. So, friends both the theorems will ultimately converge to give you the same answer if the problem is simple. So, either methods or both procedures give you the same answer in this example. Now let us take another problem.

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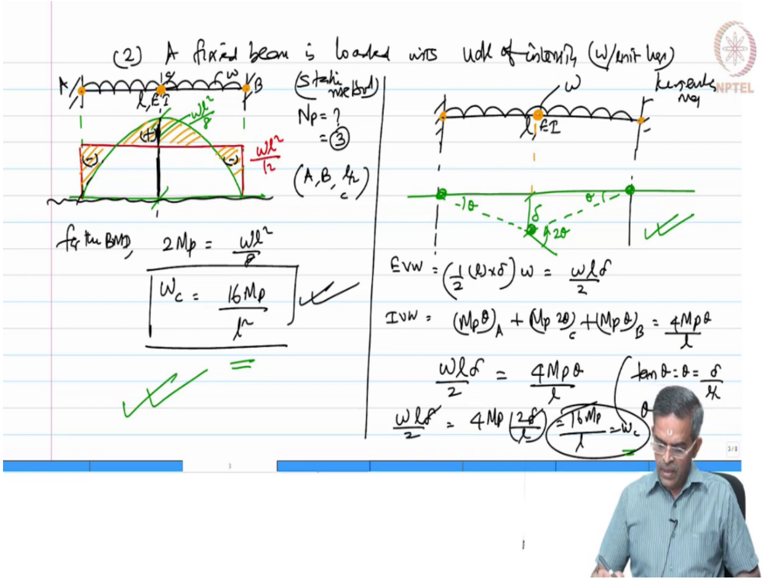

Let us say a fixed beam is loaded with uniform distributed load of intensity W per unit length, obtain the collapse load. So, let us do that. Let us say I have a fixed beam subjected uniform distributed load over a span of l E I let us mark this as A and B. There is a critical section at the center which will be see because we all know for a beam under uniform distributed load the maximum moment can also occur at the mid span of the beam.

So, what we should do in static theorem, we must first draw the static admissible bending moment diagram, the static method and this is a kinematic method. Let us draw that, so I am drawing statically admissible bending moment diagram. So, let me draw the fixed end moment diagram separately we all know this intensity is $Wl^2/12$.

Then let me superimpose the bending moment diagram of this which will be equal to $Wl^2/8$. So, now, the net bending moment diagram is what I have here right out of which this is positive this is negative. So, this neutral line has shifted to this line.

So, now we have drawn a statically admissible bending moment diagram, let me then do for the statically admissible bending moment diagram. Let us first ask a question how many Mp is required for this beam? it is a fixed beam, it has got static degree of indeterminacy as 2. So, we need three hinges to make it as a mechanism.

So, let us assume these hinges are at A, B and mid span which is C. So now, I have a hinge here, I have a hinge here, I have hinge. Now I say there is a hinge here, there is a hinge here also, is it not? So, I should say, 2 Mp= $Wl^2/8$. So, which will give me Wc = 16 Mp/l^2, which is my collapse load. Let us try to do this problem using kinematic theorem a fixed beam, under uniform distributed load for a span of l and E I .

Let me try to draw the mechanism. Let us say the hinges are allowed to form at 3 locations one at the mid span and therefore, this becomes my assumed mechanism. So, one hinge here, one hinge here and one hinge here. Now let us take this deflection as δ let us say this rotation is θ.

Now let us see what the external virtual work is. So, the external virtual work is done by the load on this area. So, it is nothing, but if it is udl it is half into base into height into W which will be $Wl\delta/2$. Internal virtual work will be equal to $M_p\theta$ which will be at A, plus Mp into 2 θ which will be at C which is the mid span of the member plus Mp θ that B which becomes 4 $M_p\theta/l$.

So, by principle of virtual work I must equate external to internal virtual work. So, $Wl\delta/2$ should be 4 $M_p\theta/l$. So, from the figure we know that tan θ is $\theta = \delta/(1/2)$. So, θ becomes $2\delta/l$ let us substitute that. So, $W*l*\delta/2$ is $4M_p(2\delta/l)$, so δ goes away. So, that becomes $16M_p/l^2$ which is Wc, which is same as this.

So, I get the same collapse load by both the methods. if the problem is simple where I can draw the statically admissible bending moment diagram easily, I can assume a perfect collapse mechanism easily and I can get the same answer by both the methods. And you will also notice that there was no iteration involved. You may be wondering when will the iteration come, when you have got more than one collapse load obtained from the analysis, then the iteration will start.

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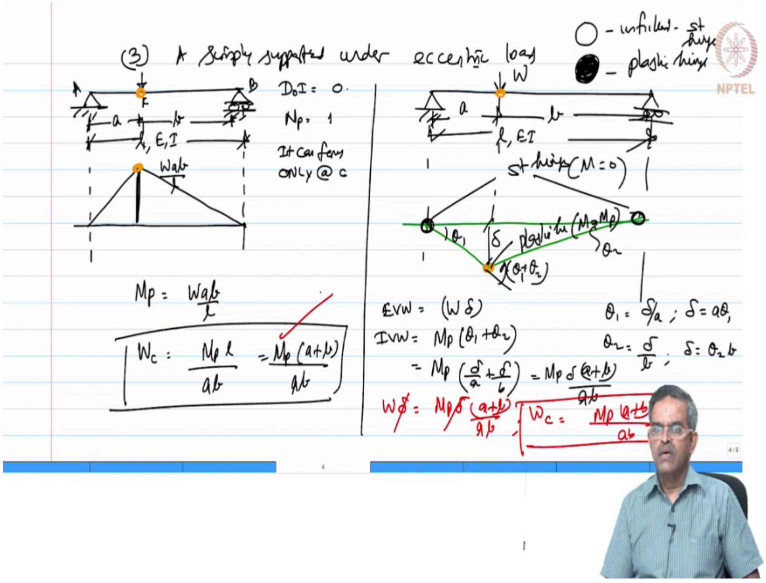

Let us do another interesting problem, where I have a simply supported beam subjected to eccentric load. So, let us say I have a simply supported beam one end hinged, other on roller, I have a load which is eccentric. So, this is A and this distance is B, of course, the span of the member is l and the member has got EI property. Let me draw the statically admissible bending moment diagram, let us first ask what is the degree of indeterminacy for this beam it is 0.

So, how many plastic hinges I need? I need only 1 plastic hinge, where it can form? It can form only at C. It cannot form at A and B, because A and B are already hinged connections. So, let us draw the bending moment diagram.

So, the hinge can form only at one point, that is here and we all know that value will be Wab/l, So, this is the point where the hinge is formed therefore, I can say Mp = Wab/l therefore, Wc is a collapsed load will be Mp*l/ab which will be Mp(a+b)/ab. So, that is collapse load, no iteration, and straight forward solution.

Let us do this using kinematic theorem. Let us draw the admissible collapse mechanism here is going to be beam mechanism. So, hinges can form only at one location that is here. Now you may wonder how I am drawing the hinges very interesting friends.

If I draw a circle which is unfilled, this is a structural hinge. If we form a circle with filled this is called plastic hinge. So, these two are structural hinges where the moment is equal to 0, this is a plastic hinge where the moment is equal to Mp. So, now since it is unsymmetric the load this will be θ_1 and this angle will be θ_2.

And this rotation will be $\theta_1 + \theta_2$ by simple geometry. Now we also know that θ_1 let us call this value as δ, so δ/a. So, that is δ =a*θ1. θ2 is δ/b which means δ is also equal to θ2*b. So, let us say the external virtual work done for this problem is W*δ, the internal virtual work for this problem is Mp (θ1+θ2), which will be equal to Mp*δ/a plus δ/b where we say it is Mp δ (a+b)/ab.

So, we should equate this Wδ = Mp δ (a+b)/ab, so δ goes away. So, Wc is Mp(a+b)/ab which is same as we have here. So, we are able to estimate quickly the collapse loads using both the theorems without iteration, because in both the assumed mechanisms or the bending moment diagram the critical sections are easily identifiable and we are able to mark the location of plastic hinges conveniently without any ambiguity.

The confusion will come only when it is either a partial collapse mechanism or over collapse mechanism.

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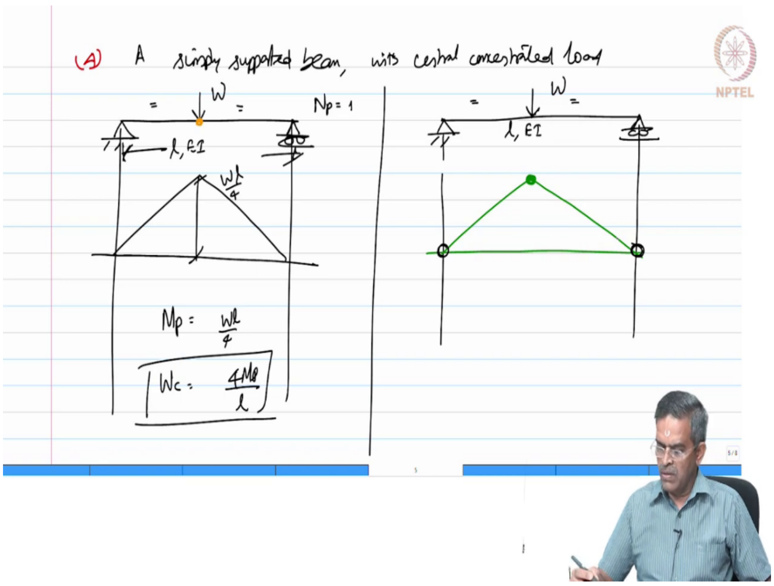

Let us quickly find out another example, where a simply supported beam with central concentrated load. So, I have a simply supported beam with a central concentrated load W over a span of l comma E I. So, we know the number of plastic hinges required for this problem is 1 because the degree of indeterminacy is 0 and that will be forming the hinge at this point.

Let us draw the statically admissible bending moment diagram which will be this and this value is Wl/4. So, let us quickly find out what will be Mp? Mp will be Wl/4 therefore, $W_c=4$ Mp/l. Let us do the same problem using kinematic theorem, let us draw a mechanism. We know that the hinge will form here and here at these two locations, there will be structural hinges or let us draw it with the other way let us do it with the other way.

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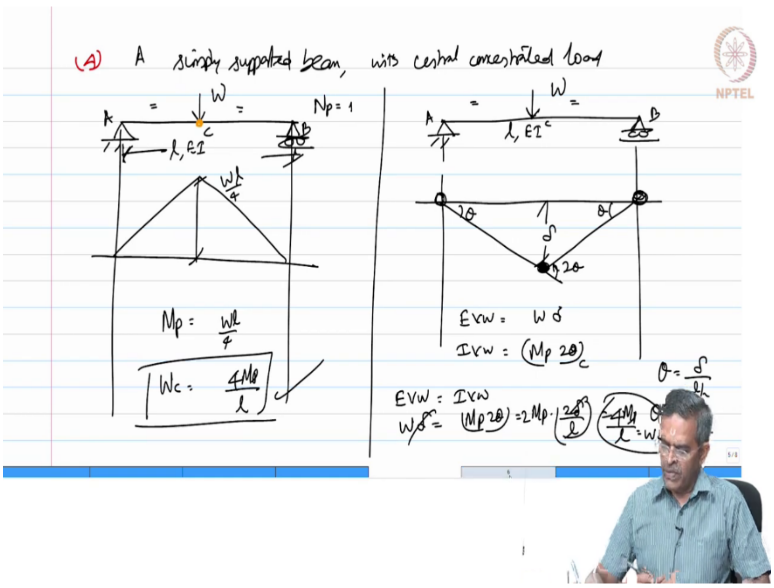

Here there will be structural hinges and here there will be plastic hinge. So, this is θ this is 2 θ this is θ I call this as δ and the load is W. So, the external virtual work will be W into δ the internal virtual work will be Mp into 2 θ which is at C, so this is A B and C. We also know θ is $\delta/(l/2)$, so θ is 2 δ/l. So, equating external virtual work to internal virtual work $W\delta$ is $Mp(2\theta)$ which will be $Mp^* \, 2\delta/l$, so δ goes away. So, that is 4Mp/l which is exactly same as this, so that is Wc.

So, these are simple problems where we are able to find out the collapse load straight away without any iterations. Now let us quickly compare with this understanding the plastic and elastic analysis.

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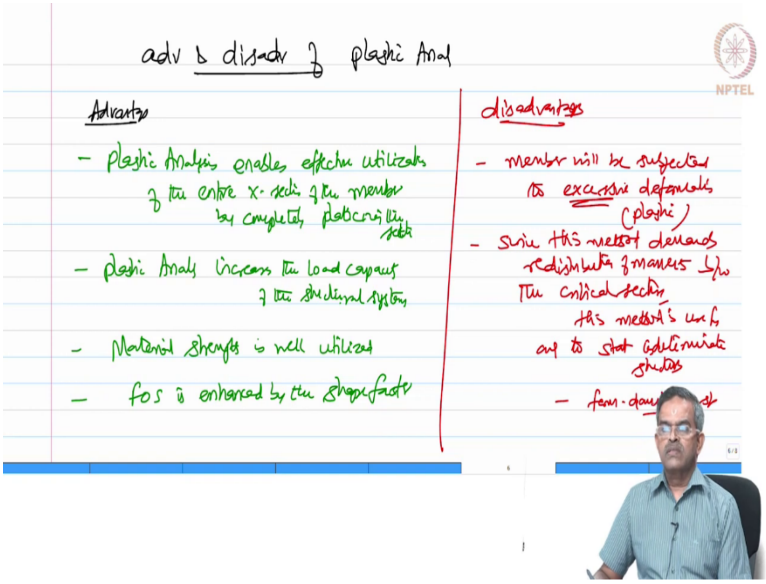

Let us say what are the advantages and disadvantages of plastic analysis.

1. The plastic analysis enables effective utilization of the entire cross section of the member by completely plasticizing the section.
2. Plastic analysis increases the load capacity of the system.
3. Material strength is well utilized.
4. Factor of safety is enhanced by the shape factor.

What are the demerits of plastic analysis? The disadvantages could be the member will be subjected to excessive deformation. In fact, it is subjected to plastic deformation. since this method demands redistribution of moments between the critical sections, This method is useful only to statically indeterminate structures.

Of course it is applicable to form dominant design structures as well. The system undergoes excessive deformation.

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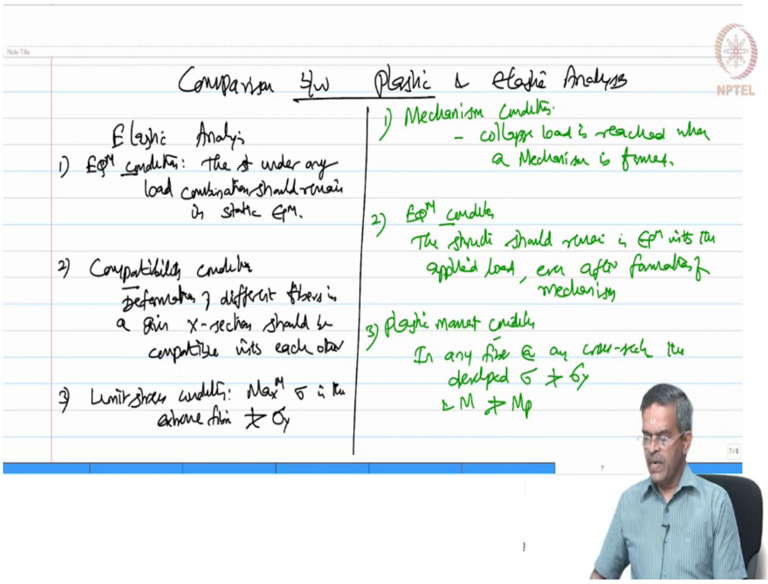

Let us quickly compare the plastic and elastic analysis. Let us say an elastic analysis what do we do is, we first satisfy something called equilibrium condition. It states that the structure under any load combination should remain in static equilibrium.

The second is compatibility condition, which states that deformation of different fibers in a given cross section should be compatible with each other to deform freely. The third condition is called limit stress condition, this condition states that the maximum stress in the extreme fiber should not cross yield stress. Relatively what are the equivalent conditions in plastic analysis, the first condition equivalent to this is mechanism condition.

According to this condition ultimate load or collapse load is reached the mechanism is formed. Two, equilibrium condition this states that the structure should remain in equilibrium with the applied loads even after the formation of mechanism. Third condition is the plastic moment condition. According to this condition in any fiber at any cross section the developed stress, should not exceed sigma y and the moment cannot exceed Mp. So, quick comparison between these two method of analysis.

Having said this let us now look at examples where some tricky iteration is also involved.

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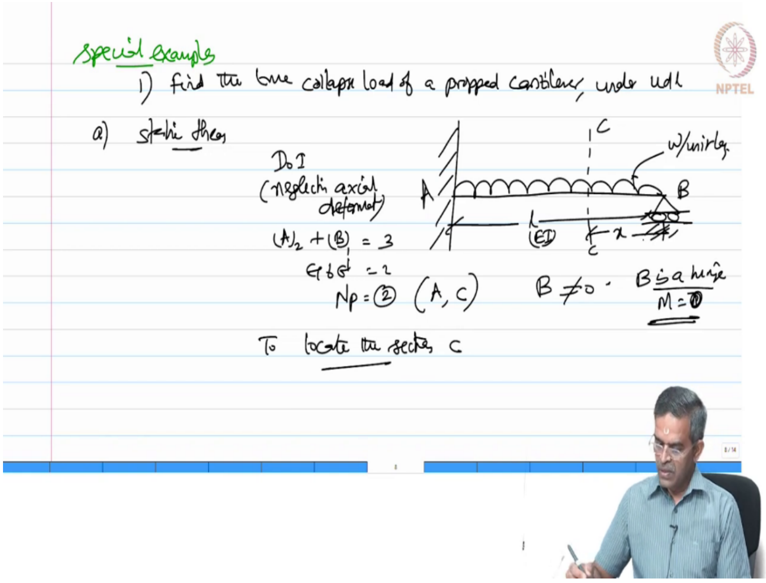

Let us do special examples, where obtaining the collapse load is not that easy slightly tricky we will take an example 1. Let us say find the true collapse load of a proper cantilever subjected to or under uniform distributed load . Let us take the example where static theorem is applied we will do a static theorem now. So, let us take a cantilever which is propped at one end subjected to uniform distributed load W per unit length. Let us say the span of the beam is l and has got EI properties.

Let us call this section as A and this is B. Now we really do not know where will be the maximum moment happening in the span A B. Let us call that section CC and we want to find this. So, it is that section where the moment is going to become maximum. Obviously, I think it is very clear for all of us that CC cannot be at the mid span because one end is propped either end is fixed. So, there will be an uneven distribution of the critical section it cannot happen at the mid span.

So, by the way let us see what is the degree of indeterminacy of this problem, neglecting axial deformation. So, let us say there are two reactions at A and there is one reaction at B is it not. So now, the total is 3 equations of static equilibrium is 2, so I need the number of plastic hinges to be 2. They can form at A and they can form at C. They cannot form at B , because B is already a hinge where the moment is 0.

So, now it is very important to locate the section C, is it not? Then only we can. So now, the problem is I cannot draw quickly a statically admissible bending moment diagram and locate the hinge because I do not know the distance at which this is going to form let me call that as x.

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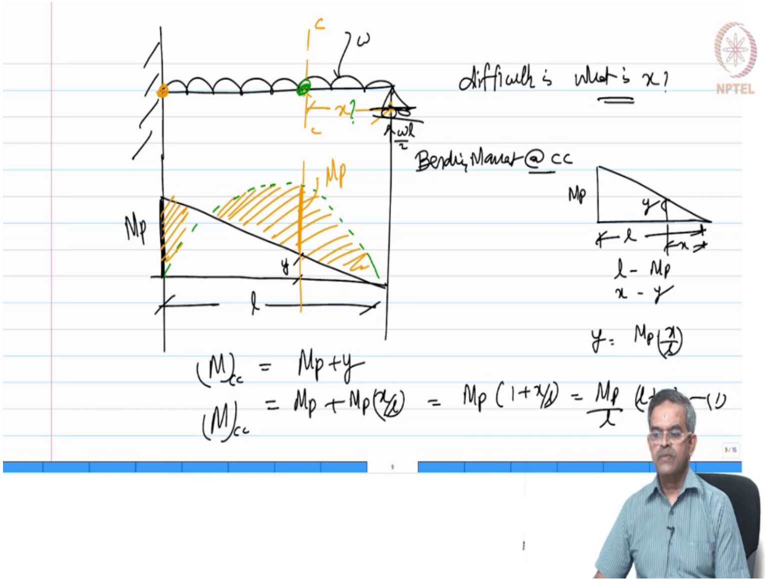

So, let me draw the bending moment diagram. So, we know this is going to be Mp this is going to be Mp. So, one hinge will anyway form here. So, let us say the other diagram is here. So, the section is going to be cc which is a distance x from here and there also the net diagram is going to be Mp, so this is the bending moment diagram.

But the difficulty is what is c or what is x, we do not know this is it not? So, the first job is to find out that. So, let us say bending moment at c or a section cc. So, this is Mp let us call this as small y this is l, so use similar triangle. This is l and this is Mp and this is y. So, I want this and of course, this is x. So, for l it is Mp for x it is y. So, y will be actually equal to Mp into x by l.

So, the bending moment at a critical section Mcc will be equal to Mp+y which is Mp+Mp(x/l) which is Mp(1+x/l) which is Mp(l+x)/l. That is my bending moment at cc. Let us also find out this bending moment from the first principles.

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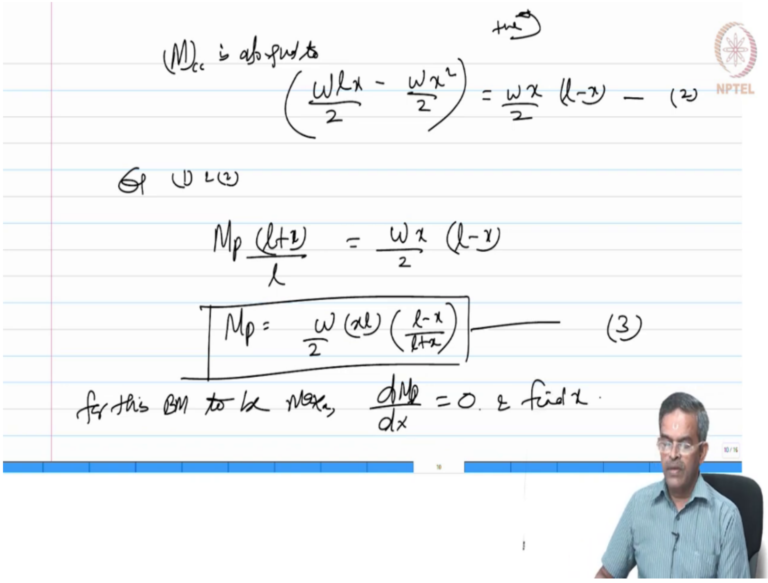

We know Mcc is also equal to $Wlx/2 – Wx^2/2$. So, this reaction is Wl by 2 and this x . So, Wl by 2 I am taking anti clock as positive. So, which will become $Wx(l-x)/2$. So now, this equation 2 we call this as equation 1, let us equate 1 and 2. So, we can easily do that.

So, $Mp(l+x)/l$ should be equal to $Wx(l-x)/2$. So, I can straight away say Mp is Wxl $(l-x)/2(l+x)$. Can we say this? Equation 3. Now for this bending moment to be maximum the first derivative of this with respect to the variable should be 0. Because I want the bending moment to be maximum here. So, differentiate this, set that to 0 and find x.

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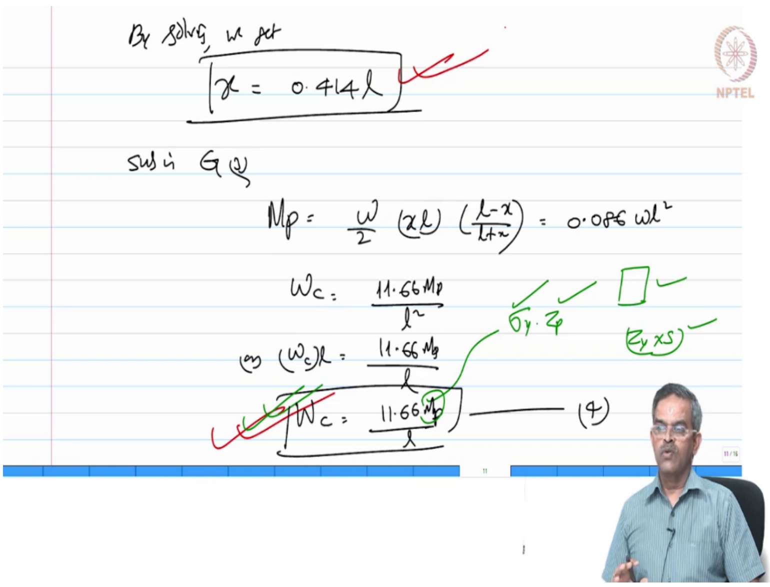

You will find x as 0.414 l. Once I know x let me find Mp. So, substitute in equation 3 we get Mp as which is Wxl (l-x)/2(l+x) which will become 0.086 Wl^2. So therefore, w_c is now going to be 11.66 Mp/l^2 or w_c* l will be 11.66 Mp/l, where this I can say as Wc= 11.66 Mp/l.

So, friends you can see the procedure of finding out collapse load if the system is complicated and you cannot draw the statically admissible bending moment directly to locate the plastic hinges then the solution is tricky. So, you know in this case Wc is given by 11.66 Mp/l. And we all know very quickly that Mp= σ_y * Zp. σ_y for the material is known, Zp for a cross section is known, because Zy* shape factor is known.

So, the right hand side equation is known. therefore, Wc can be computed, so collapse load is estimated . So, look at this example very quickly that how we used the static theorem in a roundabout manner to estimate the collapse load. In the earlier examples they were straight away because no such complication was there. I could easily draw the statically admissible bending moment diagram and mark the required number of plastic hinges, readily at those cross sections and get the collapse load directly.

But in this case I could locate the section, but I do not know where it will happen. So, I found out that and I could locate this.

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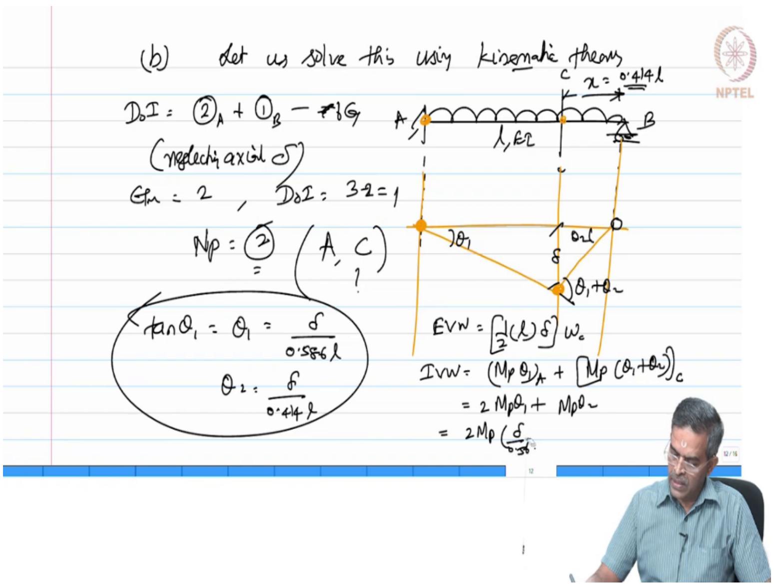

Let us try to solve the same problem using kinematic theorem.

Let us see is it easy for us to use kinematic theorem in this case. So, kinematic theorem says for a propped cantilever under uniform distributed load over a span of l E I. I have to draw the mechanism, we know the degree of static indeterminacy in this case let us say this is A this is B and this is C.

So, we know there are two unknowns at A plus, 1 unknown at B, we are neglecting the axial deformation, this will be minus number of equations. So, equation of equilibrium will be 2. So, number of plastic hinges required will be 2 because the degree of indeterminacy is which is 3 - 2 = 1.

So, any two plastic hinges where will they form? They will form at the fixed support they will form at C but C location earlier was not known using the static theorem, now I know this. So, I get hinges here and here. Now, I can draw the deflected profile and draw the hinges, say this is θ1 this is θ2 this is θ1 +θ2.

So, from the figure we know tan θ_1 is θ_1 for small deformation is δ/0.586. Then θ2 is δ/0.414 l. So, let us say the external virtual work is half into base into height into Wc, the internal virtual work is Mp into θ_1 at a plus Mp into θ1 plus θ2 at c, which will now Mpθ1 plus Mpθ2.

Let us substitute for θ1, θ2 from here.

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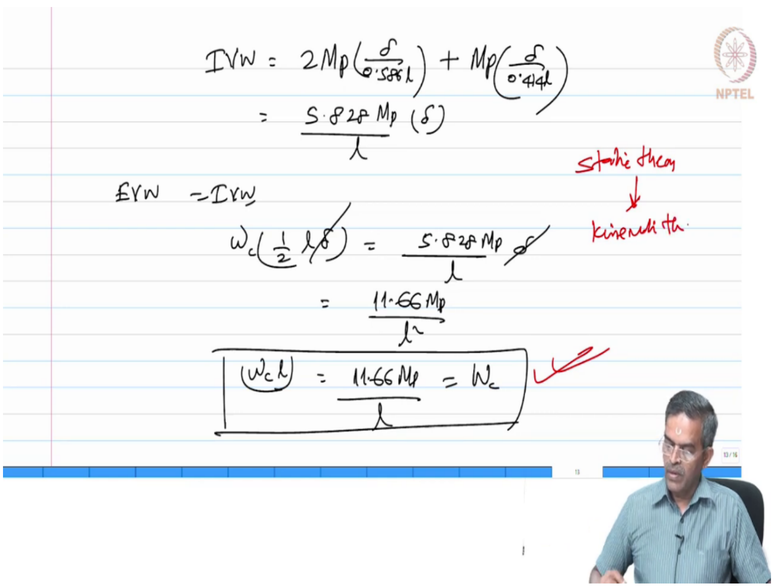

internal virtual work will be 2 Mp(δ/0.586l) plus Mp times of θ2 which is δ/0.414 l , which becomes 5.828 Mp/l of δ . So, we equating external virtual work to internal virtual work we get Wc *(l δ)/2 is 5.828 Mp δ/l, so δ goes away. So, it will become 11.66 Mp/l^2 or this is small Wc. Wc * l is 11.66 Mp / l which is capital Wc.

So, friends you will see that this value is same as we obtained here, but there is a very interesting conclusion in this problem we have used static theorem solution to check the kinematic theorem solution, How can you say this see here I have borrowed this data from the static theorem.

So, please friends note that all the time these two theorems are not independently applied some of the problems they are dependent they will be helping each other. So, please understand that it is important that I must know both the theorems, how to employ solution of finding both the theorems then only I will be able to successfully find out the collapse code there is a very standing example we have here. Let us do quickly a design problem and see how this can be helpful.

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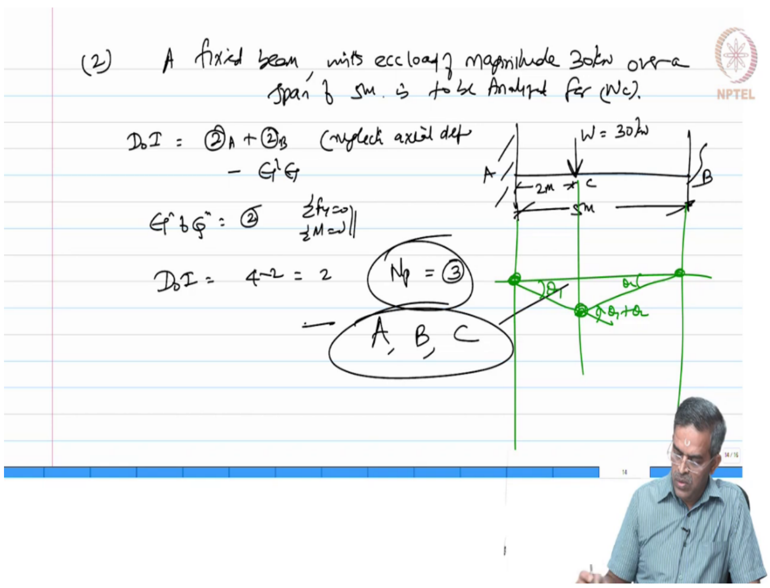

Let us say a fixed beam with eccentric load of magnitude 30 kilo newton over a span of 5 meters is to be analyzed for collapse load. So, the beam is here fixed beam there is a load which is W which is 30 kilo newton and the load is 2 meter from the end. So, this is 2 meter from the left end and this is 5 meters.

So, now the degree of static indeterminacy is let us say this is A this is B this is C. So, there are 2 unknowns at A and there are 2 unknowns at B neglecting axial deformation minus equations of equilibrium. Equations of equilibrium will be 2 that is sigma f y_0 and $\sigma_{M\,0}$.

So, therefore, degree of indeterminacy is 4 minus 2= 2. So, the number of plastic hinges required is 3 where will they form they will form at A, they will form at B, they will form at C; it is a straight forward solution. Let us draw the deflected profile hinge here hinge here and hinge here θ1, θ2, θ1 + θ2.

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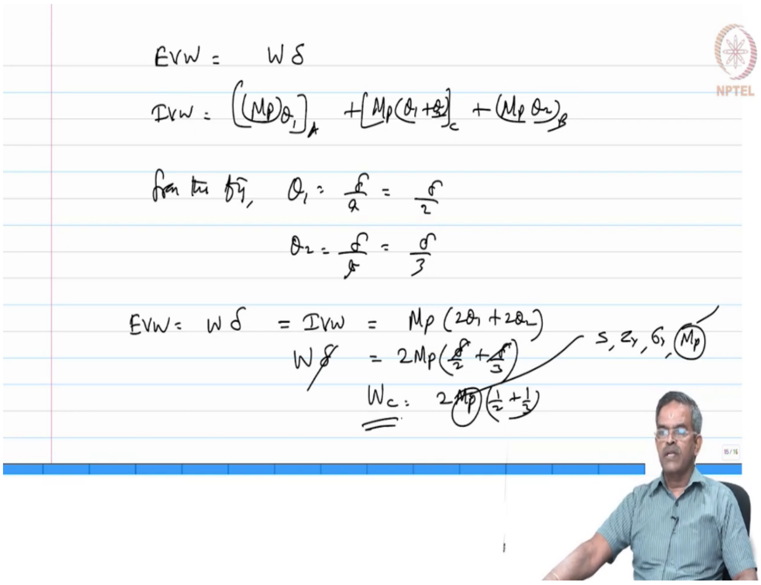

So, I could say now the external virtual work is W * δ, internal virtual work is $(Mp * \theta 1)_A$ + Mp * $(\theta 1+ \theta 2)_C$ + $(Mp * \theta 2)_B$. So, from the figure θ1, is δ/a which is δ/2 meters and θ2 is δ/b which is δ/3 meters.

So, external virtual work is W * δ which is equal to internal virtual work which is Mp times of $2\theta_1 + 2\theta_2$, is it not? So, 2 Mp*δ/2 plus δ/3 is W * δ. So, δ goes away. So, I can find the true collapse load as 2 Mp* (1/2 plus 1/3) .

For a given cross section if I know the shape factor if you know Z y I can find Mp. So, I can find the collapse load. So, plastic analysis is very simple using these theorems if they are not iterative.

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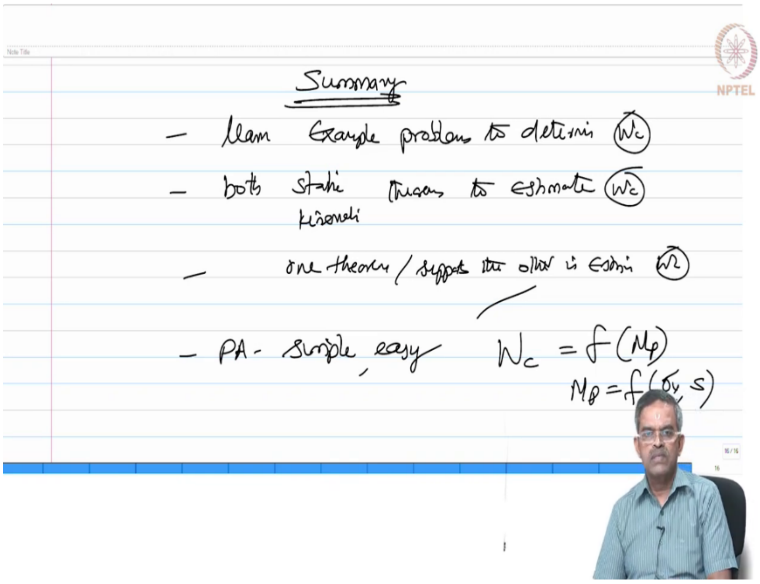

So, let us write down the summary what we learned from this lecture. This lecture helped us to learn example problems to determine the collapse load we have used both static theorem and kinematic theorem to estimate Wc. We have also seen examples where 1 theorem supports the other in estimating Wc.

So plastic analysis is simple, easy and here the load is function of Mp where Mp is a function of material and shape factor it is a geometric property of course, material is also involved that is interesting. And I am sure you will find more examples in my book and other literature papers and you will be able to enhance, your interest towards estimating plastic analysis or doing plastic analysis on collapse load estimates for varieties of problems which we will be doing furthermore in the coming lectures, but however, I wish that you should do more examples by taking these case studies or looking at examples illustrated in my textbook and my reference notes.

Thank you very much and have a good day friends. Bye.