Advanced Design of Steel Structures
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Lecture - 24
Plastic design -1

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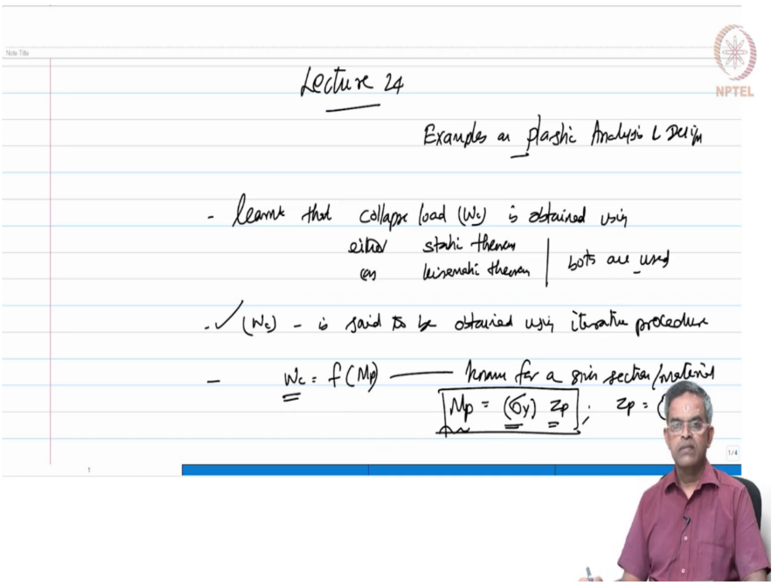

Welcome to the lecture 24. In this we are going to learn more examples on Plastic Analysis and Design. So, in the last lecture, we learnt two theorems, static and kinematic theorems which are lower and upper bound theorems respectively, which are useful in computing the collapse load. We already saw, that collapse load Wc is obtained using either static theorem or kinematic theorem. Mostly, but we have also seen as example where both theorems are used.

So, please do not mistake yourself in learning that these theorems are substitute each other. No, in fact they support each other. So, do not think that there is a substitution of this. I can only learn only one procedure. I can get rid of the other and I will still be able to get Wc. That is the first thing we learned.

Second thing we also learnt that Wc is said to be obtained using iterative procedure. But we have not seen an example so far, how this will be done from an iterative procedure. So, today we will see some examples where this will be learnt.

We have also understood that collapse load is a function of Mp and Mp is known for a given section and material, because Mp is actually yield stress multiplied by Zp, where Zp a shape factor of Zy. And for standard sections given in steel handbook especially with steel, Zy is a known value.

Of course, we have also given you the MATLAB codes. We explain you the procedure for finding out the shape factor for different standard rolled steel sections. So, I believe that Mp can be easily computed for a known material for a known cross section which is more or less identified as a geometric property.

So, now, the collapse load or the load at which this geometry is intended to collapse can be obtained by these two theorems namely static and kinematic theorems. So, having said this, let us do few more examples to learn more in detail about the plastic analysis, and also an example of the plastic design.

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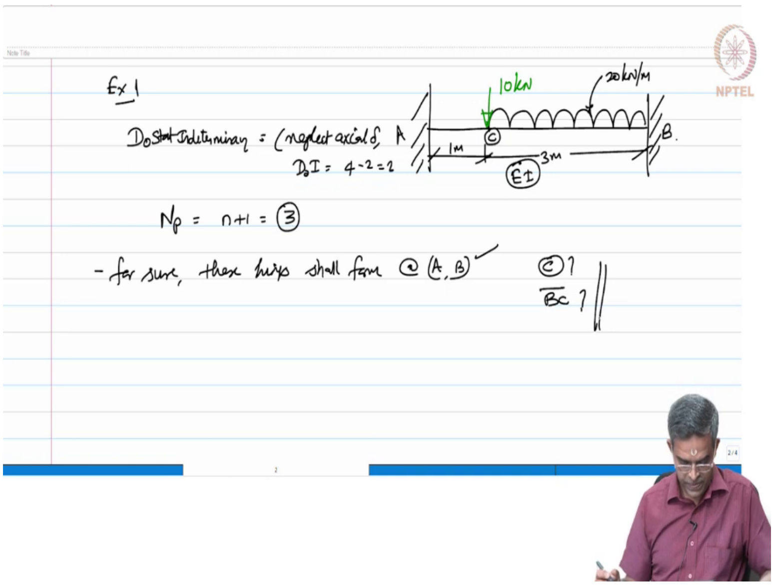

So, we will in this lecture, we say example 1, let us say we have a fixed beam subjected to a load as shown in the figure is an UDL of 20 kN/m. There is also a point load of 10 kN and the geometric details are given like this.

So, let us name this end as A and B, and this point as C. Let us quickly ask a question before we solve this problem. Let us ask what the degree of static indeterminacy of this problem is. If we neglect axial deformation, then degree of indeterminacy is 4 minus 2 which is 2. So, how many plastic hinges do I need? The number of plastic hinges required to convert this beam into a mechanism is n + 1 which will be 3.

Now, the question comes where these hinges will fall. For sure these hinges shall form at support A and B, there is no doubt because they are fixed ends. But we have a doubt whether this will form at C being a concentrated load or anywhere on the span BC. So, let us try to examine this.

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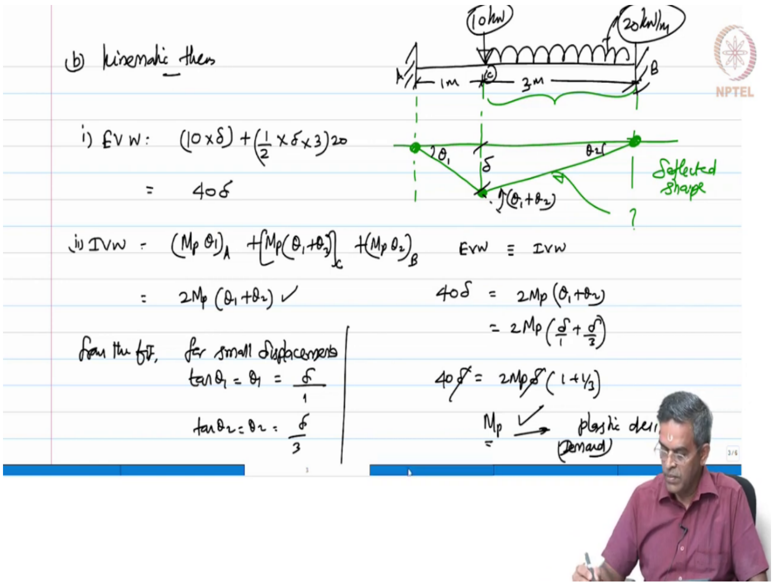

So, we know that the beam is subjected to two set of loads. One is a concentrated load of 10 kilo newton, then an UDL of 20 kN/m for a span of 1 meter and, 3 meters, respectively.

Let us use kinematic theorem. So, for kinematic theorem I must draw the plastic deformation of this beam. Let us do that. So, we will assume a hinge here for sure, we will assume a hinge here for sure, I will also have a hinge here for sure. You may wonder that for a uniform distributor load span, I am drawing a straight line.

Friends, this is not the bending moment diagram. This is the deflected shape of the beam. So, now, let us mark these rotations. Let us call θ1. If you call this as θ2 obviously, this will be θ1+θ2 by simple mathematics. And let me call this vertical displacement as δ. Then, let us write down the external virtual work.

There are two loads here, so the point load multiplied by this displacement. When there is a UDL, you have to find the area. So, half into base into height into intensity of the load, so which will give me 40 δ.

So, second one is internal virtual work. We have assumed plastic hinges at 3 locations as marked in green color. So, this is going to be Mp of θ_1 which is at A, plus Mp of θ_1 plus θ_2 at C, plus Mp of θ_2 at B. So, which gives me it is 2 Mp ($\theta_1+\theta_2$).

So, from the figure, I can always relate θ and δ. So, from the figure, we can say for small displacements $\tan\theta_1$ is θ_1 which is $\delta/1$ and $\tan\theta_2$ is θ_2 which is $\delta/3$.

Let me substitute these θ_1 and θ_2 as a function of δ in this equation. So, we substituting and equate, external virtual work to internal virtual work, there is principle of virtual work. I should say it is $40\delta = 2\,Mp\,(\theta_1 + \theta_2) = 2\,Mp\,(\delta/1 + \delta/3)$.

So, now $40\delta = 2\,Mp\,\delta\,(/ + 1/3)$. I can find Mp now. So, this matches with the assumptions what we made in the number of plastic hinges formation and I am able to get my Mp, where I consider this load will be the subjected load on the system. So, obtaining Mp for the analysis is plastic design. So, now, I am estimating the demand.

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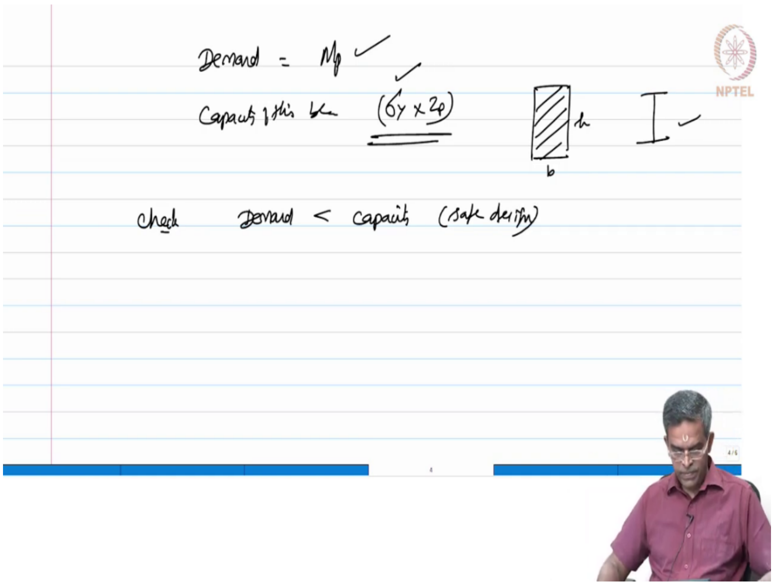

The demand is Mp. The capacity of this beam is σ_y into Z p. If we know the cross section, I can find this, if I know the material strength. I can check, demand should be always lesser than the capacity for safe design.

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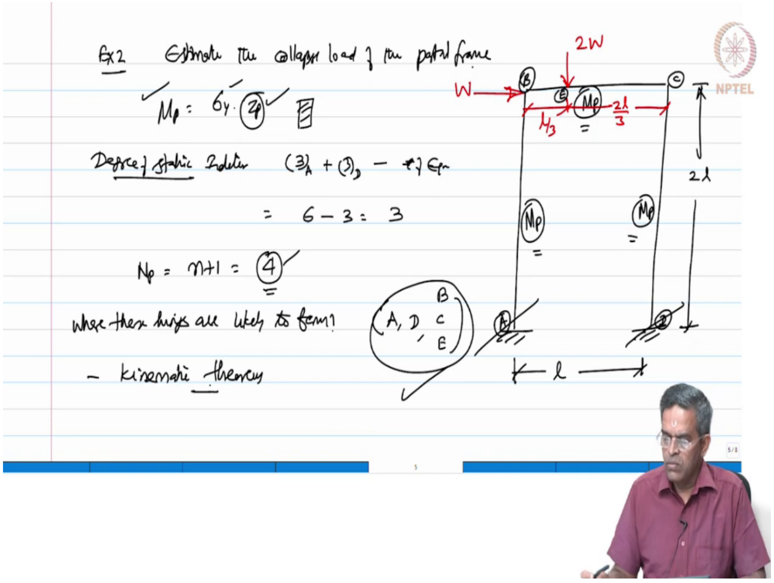

Let us do one more example. Let us say we have a portal frame. We say estimate the collapse load of the portal frame. Let us draw the portal frame. Let us say the length or the span of the beam is ℓ, and height of the column is 2ℓ, and the column has a capacity Mp, whereas the beam was also a capacity Mp.

I am marking Mp because I know Mp, because Mp is actually σy into Zp. Zp depends on the cross section. So, I know Zp, I know the material strength, I know Mp therefore, I can mark these values. Let us say that the beam and column has the same strength.

Now, this beam or this frame is subjected to two loads, one is lateral load W, other is a vertical load which is 2W which is eccentric. This distance of the load is $\ell/3$ and of course, this distance is $2\ell/3$. Let us name these joints or nodes, this as A, this as B, this as C, this as D, and we call this point as E. Let us try to find out the degree of static indeterminacy for this frame.

So, there are two fixed supports. So, there will be 3 unknowns. So, the degree of indeterminacy will be 3 reactions at A, plus 3 reactions at D, minus number of equations of static equilibrium, will say it is 6 minus 3, so 3. So, the number of plastic hinges required to form or to convert this portal frame into a mechanism will be n +1 which is 4. So, we need 4 plastic hinges.

Let us ask a question where these hinges will form. For sure they will form it A and D, because they are fixed ends. And there is a possibility it can further form at B, C and E. So, there are 5 possible locations, I need only 4. So, plus recollect that there are different mechanisms available for analysis like beam mechanism, column mechanism, or sway mechanism, combined mechanism, partial collapse mechanism, complete collapse mechanism, over collapse mechanism and so on.

We learnt all of them in the previous lectures, please turn back and learn and understand. Now friends, this problem is slightly tricky because we have got 5 possible locations whereas, I need only 4. So, we will use kinematic theorem to solve this problem. The kinematic theorem stays I must assume a collapse mechanism first.

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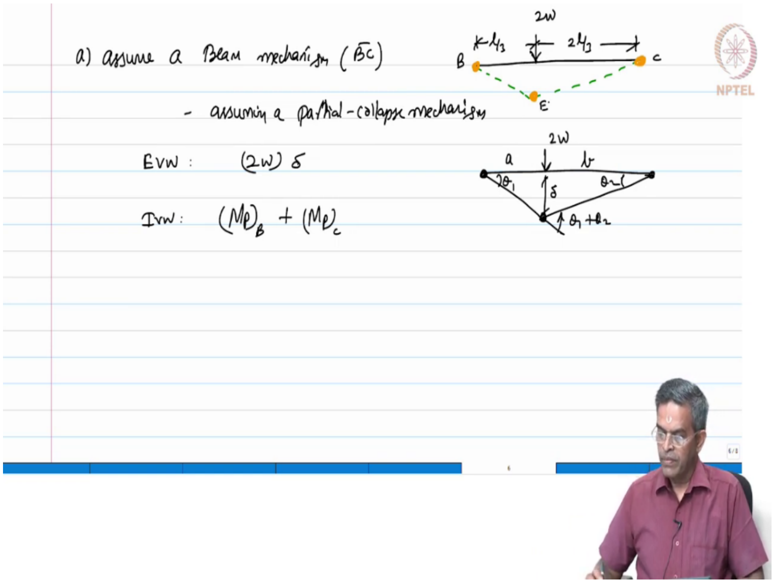

Let us assume a beam mechanism which is going to be on the segment A, E, C or on the segment BC, on the beam BC. Let us draw that beam separately. So, there is a load 2W here, this is $\ell/3$, this is $2\ell/3$. This is my deflected shape of the end B. So, let us assume that I have 3 hinges formed here, so it is a partial collapse mechanism. Why a partial collapse mechanism? So, for a complete collapse mechanism I need 4 hinges, I assume only 3.

Let me redraw this figure with this load 2W. I call this as A and this as B. So, let us say I will have hinges here. I can call this angle is θ1, this angle as θ2, and this displacement as δ and this angle will be θ1 plus θ2.

Let us write down the external virtual work which will be $2W * \delta$. The internal virtual work will be the work done by the plastic hinges. So, I have one plastic hinge formed at B, I have one plastic hinge at C.

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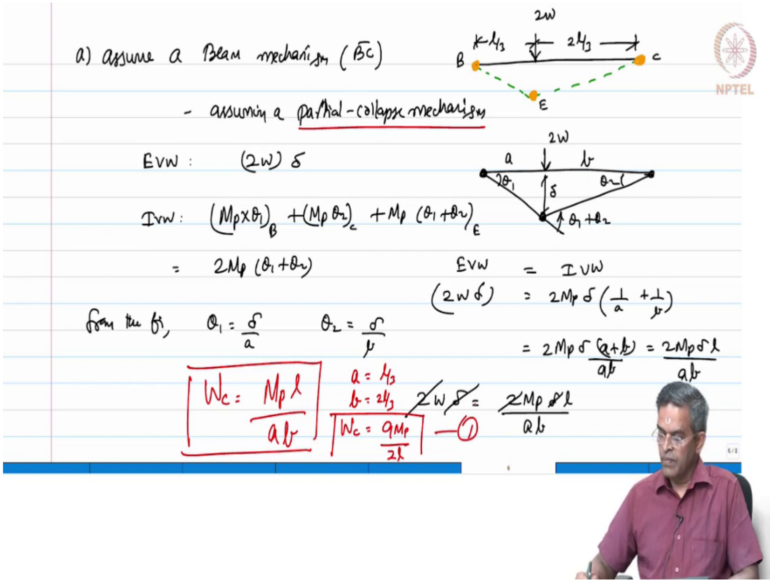

So, this multiplied with, ; let me $M_p * \theta_1$ this is at B, plus $M_p * \theta_2$ this is at C, plus $M_p * \theta_1$ plus θ_2 this is at E, so which becomes $2M_p * \theta_1$ plus θ. So, from the figure we also know θ_1 is δ/a and θ_2 is δ/b. So, let us equate external virtual work to internal virtual work, which is $2W\delta$ should be equal to $2M_p$ times of δ $(1/a + 1/b)$, which will be $2 M_p\delta(a+b)/ab$. Can I say $a + b$ as ℓ from the figure? So, now, $2W\delta$ is $2 M_p \delta \ell / a b$. Therefore, I can say now the collapse load W is $M_p \ell / ab$.

But I cannot guarantee is the final answer of the collapse load, because this has been obtained based on the partial collapse mechanism. I will looking only at the beam mechanism, right. There are other mechanisms possible.

Let us extend this further let us say a is $\ell/3$ and b is $2\ell/3$, I can quickly get W_c as $9 M_p/2\ell$. That is my answer. I call this equation number 1.

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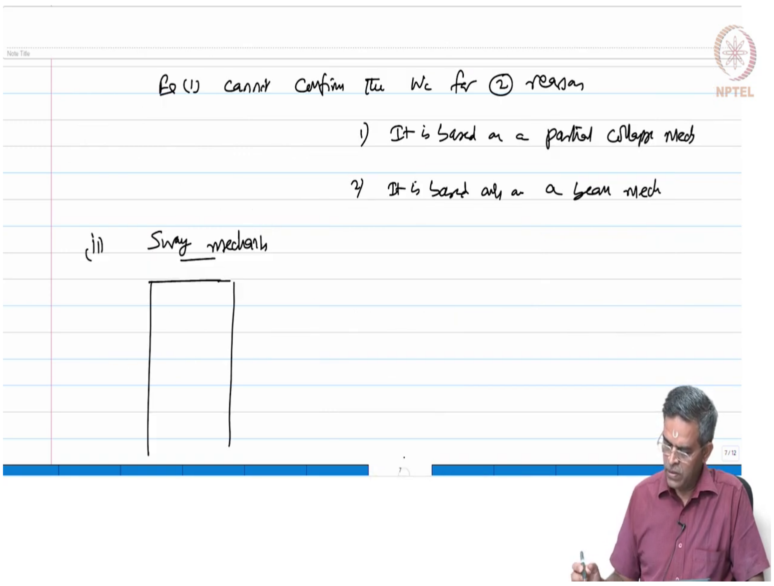

Equation 1 cannot confirm the collapse load for two reasons. One, it is based on a partial collapse mechanism. Two, it is based only on a beam mechanism. There are other mechanisms possible. Let us look into the second option which is a sway mechanism.

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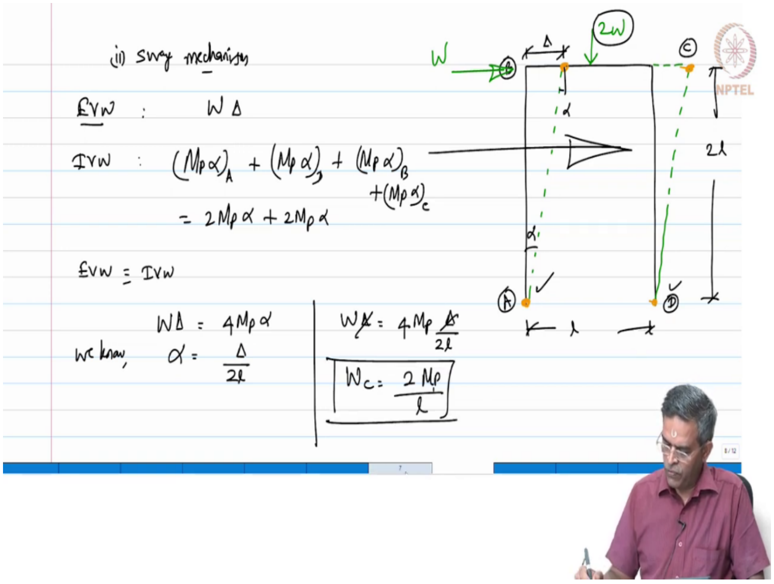

Let us draw the deflected profile of the frame in the sway mode. So, I assume the plastic hinges are at A, B, C and D. That is I think I can mark them here. Let me call this angle as α which is also α, let me call the displacement as delta. Let us write down the geometry this is ℓ and this is 2ℓ. And this end is A, this is B, this is C and this is D.

There is a load applied which is W here; and of course, there is a load applied here which is 2W. Let us write down the external virtual work, which is going to be only W into delta because this will not do any work as it has not undergone any displacement.

And now let us estimate the internal virtual work. So, one work done is here it is rotated, other work done is here is rotated, the angle between the normal's should be same, but you know there is only a stick to sway here. The frame has just to swayed. Therefore, I can now say the internal work virtual work is Mp*α at A plus Mp into α at D. So, can I say this as 2 Mp α?

So, external virtual work equated to internal virtual work W into Δ should be 2 Mp α, but α and Δ are related. You can write down the relationship from the figure. We know α for small angles is Δ by 2ℓ. So, substituting this in the above equation we get W into Δ is 2 Mp times of Δ by 2ℓ.

So, therefore, Wc is; there will be work done at sections B and C which is again Mp α plus Mpα at C. So, let us say this is also added to this. So, let me put this as 4 Mp. So, this becomes 4 MpΔ /2ℓ, so its 2Mp/ℓ. So, friends, one value gives me collapse load as 9Mp /2ℓ which is about 4.5Mp /ℓ, the other one gives me it is 2 Mp/ℓ. So, now, second equation.

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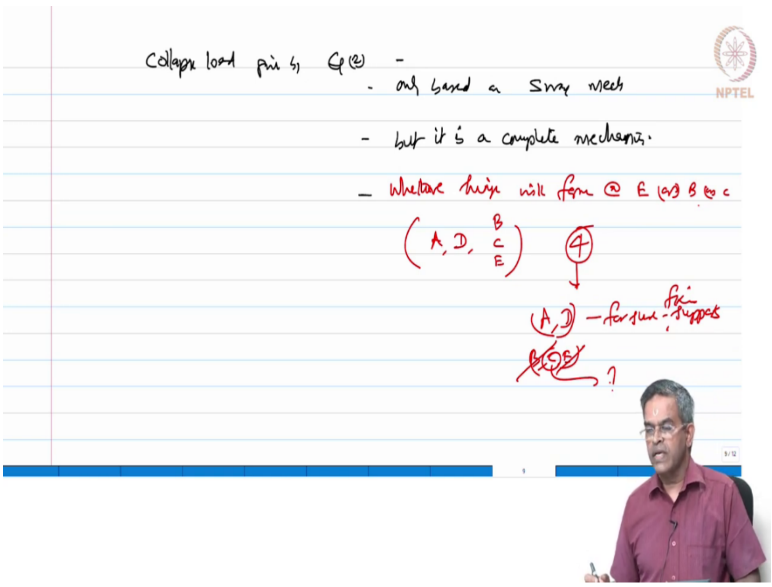

The collapse load given by second equation is again only based on sway mechanism. But it is a complete mechanism because there are 4 hinges, but still there is a question whether the hinge will form at E or B or C. See the possible locations are A, D, B, C, and E.

We have 5 locations, but we need only 4 out of which A and D are sure because they are fixed supports. But out of B, C and E, we have assumed that B and E, but we left out C because there is a question. So, this is also may not be the correct answer.

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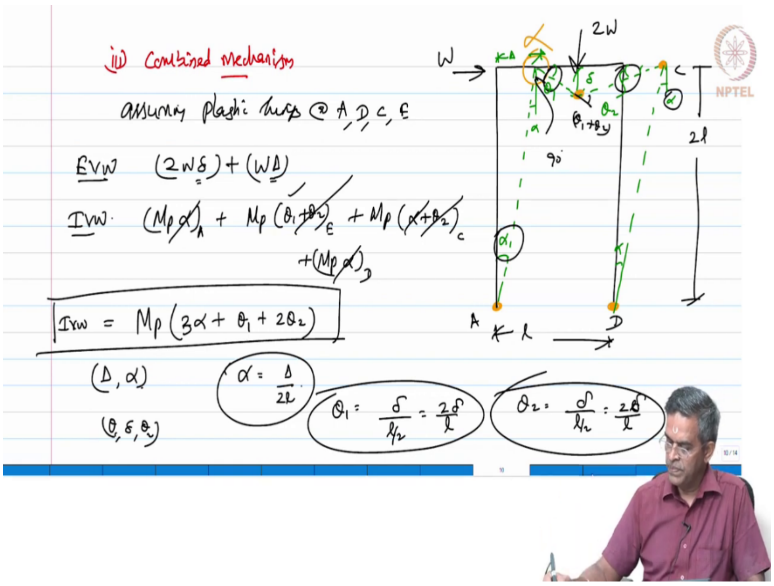

Let us do one more attempt, one more iteration to find out using a combined mechanism. Let us draw the frame and let us draw the combined mechanism including the sway and the beam. Let us assume the hinges of course, at A and D for sure. Let us assume the hinge here and here. We will not assume hinge here.

So, we are assuming hinges at A, D, C and E. So, there is a point load of 2W here, there is a lateral load W. So, we call this value as α which is also α here, also α, is also α, is also α. And we call this as Δ and we call this as δ. And we call this angle as θ1 and this angle as θ2, let us not put this α.

Let us write down the external virtual work. The external virtual work will be 2W * Δ that is by the vertical load, 2 W * δ + W * Δ that is the horizontal load. Let us write down the internal virtual work, we have 4 hinges, So, we should say Mp * α which is at A, plus M p Say, this is θ_1 plus θ_2, θ_1 plus θ_2 is going to occur at E, plus Mp into α plus θ_2 plus this is at C, plus Mp into α at D. So, which can be said as Mp times of 3 α plus θ_1 plus 2 θ_2, the internal virtual work.

Now, I can find the relationship between Δ and α. So, α is $\Delta/2\ell$. So, I can replace α in terms of Δ.

Let us also find out relationship between θ and δ and θ_2. That is θ_1 is δ by ℓ by 2 which is 2 δ by ℓ and θ_2 is also δ by ℓ by 2 which is 2 δ by ℓ. So, we will substitute this. So, we have θ_1, we have θ_2. and we have α and δ. So, we have equations in terms of δ and Δ. So, we now we will replace this in δ and Δ. So, let us do that. We know α is Δ by 2 ℓ.

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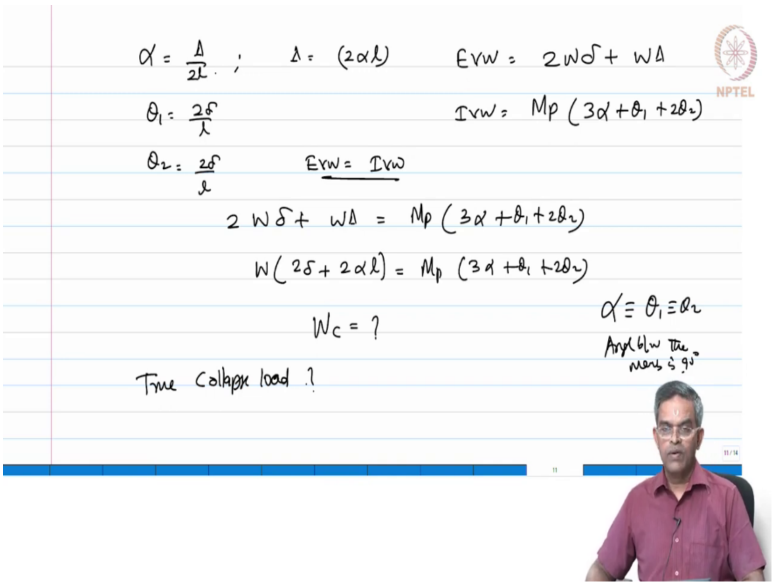

So, I should say Δ is 2 α ℓ. And we have θ_1 as 2 δ by ℓ and θ_2 also as 2 δ by ℓ. So, let us write down external virtual work which is equal to 2 W δ plus W Δ. Internal virtual work which was this one, Mp times of 3 α plus θ_1 plus 2 θ_2. Mp times of 3 α plus θ_1 plus 2 θ_2. Let us substitute them back.

We will equate external virtual work to internal virtual work. So, 2 W δ plus W Δ should be Mp times of 3 α plus θ_1 plus 2 θ_2. So, let us express everything. So, let us say W times of 2 δ plus 2 α ℓ which will be equal to Mp times of 3 α plus θ_1 plus 2 θ_2.

So, in this figure if you look at friends, I can say α and θ2. and α and θ1 because the angle between the normal has got to be same. So, using that condition I can say α should be equivalent to θ1 to θ2. because angle between the normal's is same. That is in the displaced portion we consider this angle as still 90 degrees.

We simplify this and try to find Wc. We will give you the answer in the next lecture, what would be this value. Please work on this, and check up and tell me what is the Wc from the combined mechanism, and therefore, what is the true collapse load of this.

Thank you very much. Have a good day. Bye.