Advanced Design of Steel Structures
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Lecture - 25
Plastic design -2

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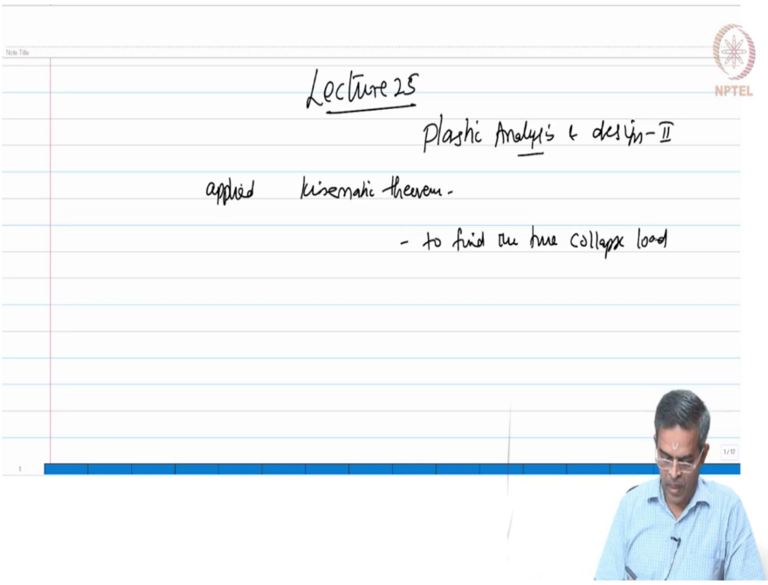

Friends, welcome to the 25th lecture on Advanced Steel Design where we will continue to discuss some more examples on plastic analysis and of course, application of analysis in design perspective I will put this II because it will be a continuation of the last lecture what we discussed. In the last lecture we are trying to apply kinematic theorem for a frame example to find the true collapse load.

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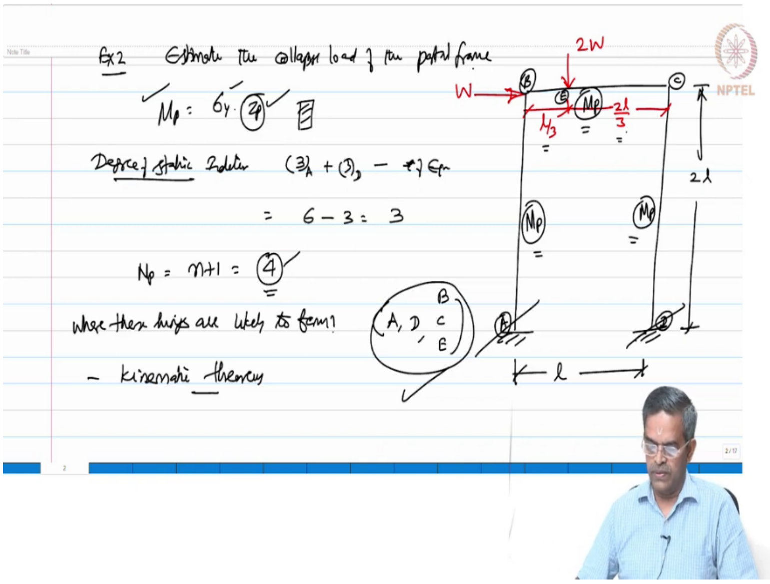

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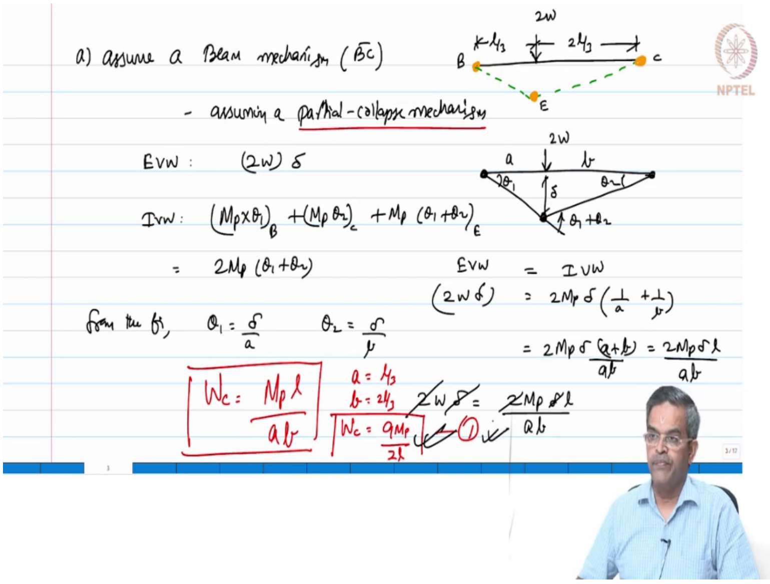

This was the problem where a frame is subjected to a lateral load of W and a vertical load of 2W at an eccentric portion of this. So, we computed from the beam mechanism that may collapse load is 9Mp by $2l$.

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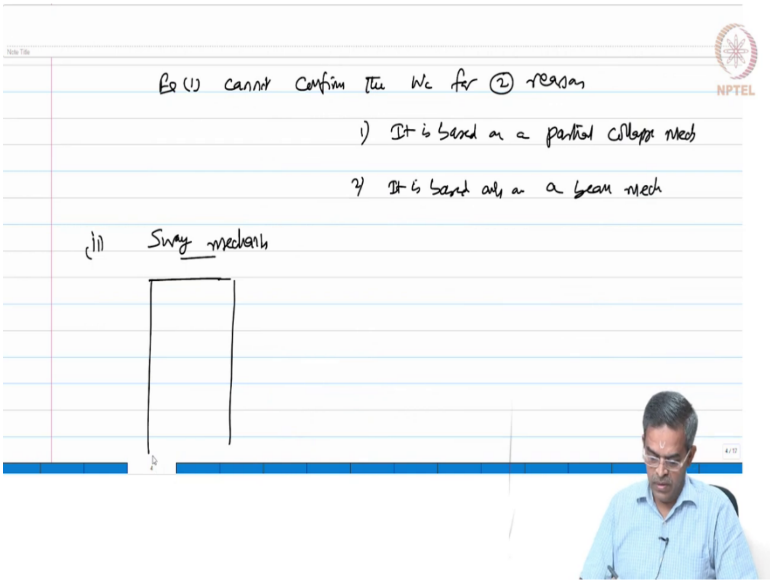

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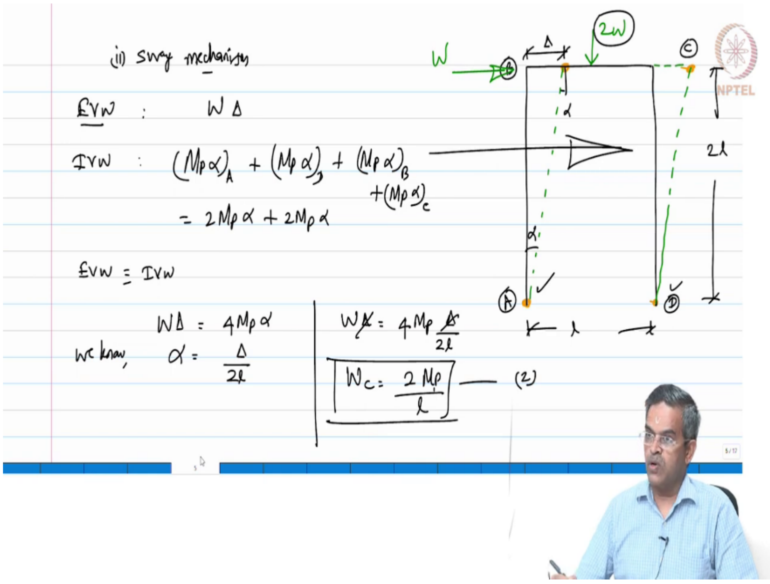

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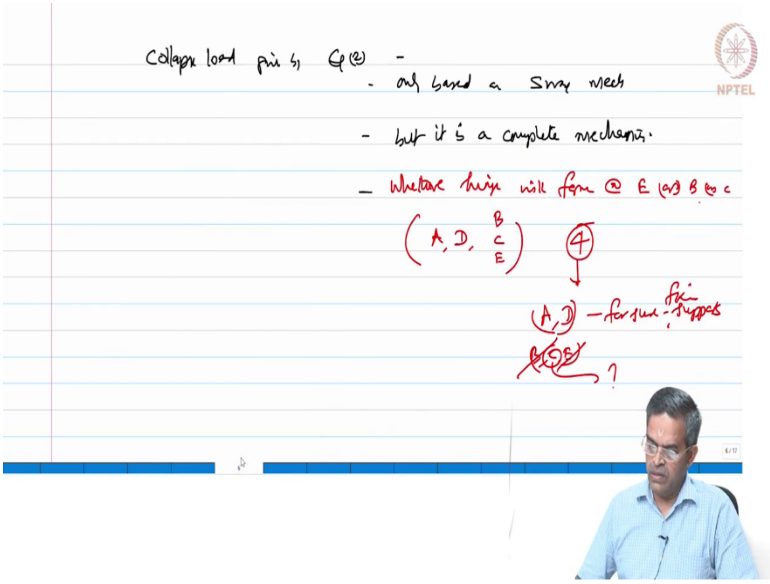

We also did the sway mechanism where we said the collapse load is 2Mp/*l*. We further went down to understand there are 5 possible locations where the hinges can form, but we need only 4 for a complete mechanism. So, we wanted to be sure using iteration procedure where this hinge will form. For A and D they are sure because they are fixed supports. So, now the hinge can form either B C or E. So, we have a doubt. So, we are going for an iteration to find out the true collapse load.

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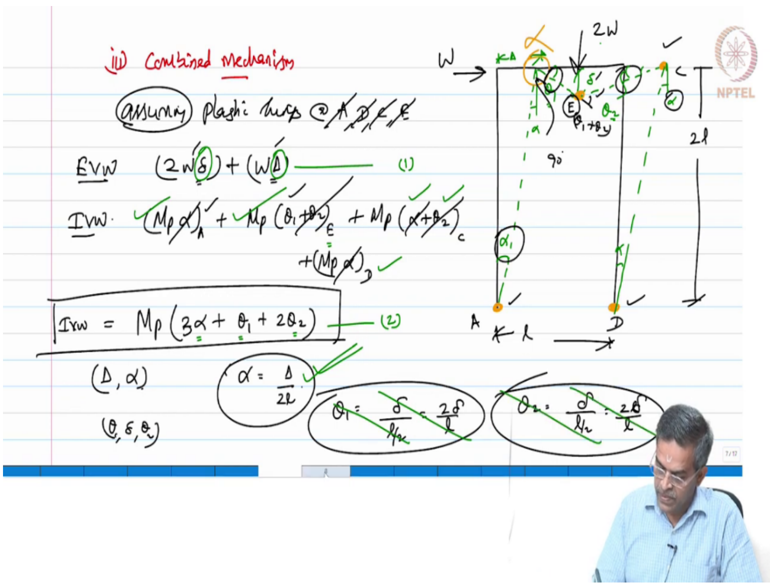

We started doing this and we said that the hinges will form at A, E, C and D it is an assumption, the hinges will form at A, D, C it is an assumption we are assuming this. The external virtual work is force*displacement. So, 2 W*δ and W*Δ the internal virtual work is Mp of α which is at A. So, that we can check it Mp*α which is at A then again there is a hinge here at E.

So, Mp*$\theta 1$and $\theta 2$, this is at E, then again at C you got 2; 1 is Mp for α other is Mp for $\theta 2$ and we do know that these inclinations will not be equal because the load is eccentric it is not at the concentric position of the beam. Then the fourth 1 is Mp α at D. So, this gives me an internal virtual work let us say this is equation number 1 we will take this 2.

So, we want to equate 1 and 2 and find out the true collapse load, but there is a problem here equation 1 has got 2 variables δ and Δ and equation 2 has got different variables α $\theta 1$and $\theta 2$, this is the issue what we had. So, then we started working out from the figure α will be Δ by $2l$ this is very simple straight forward and $\theta 1$and $\theta 2$ need to be found out. So, now, $\theta 1$cannot be this way. So, let us try to correct this and go back.

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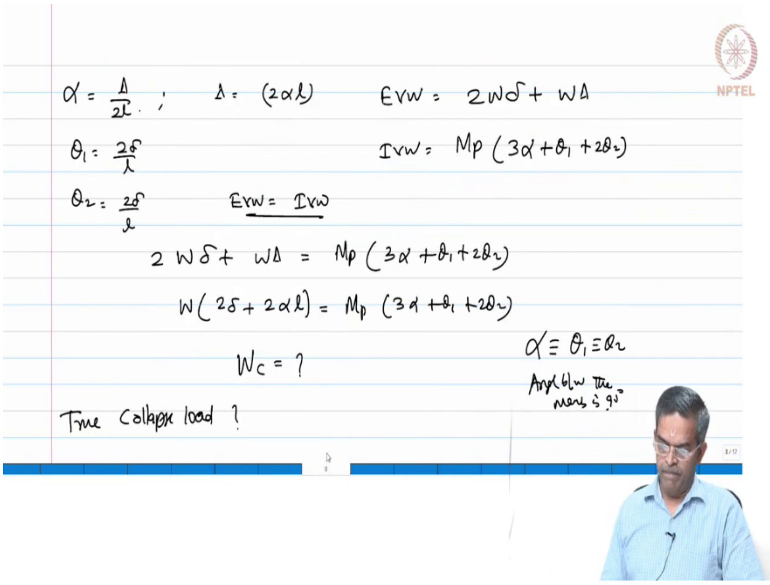

So, we will simply say that the correction is going to be there for this case and let us do this.

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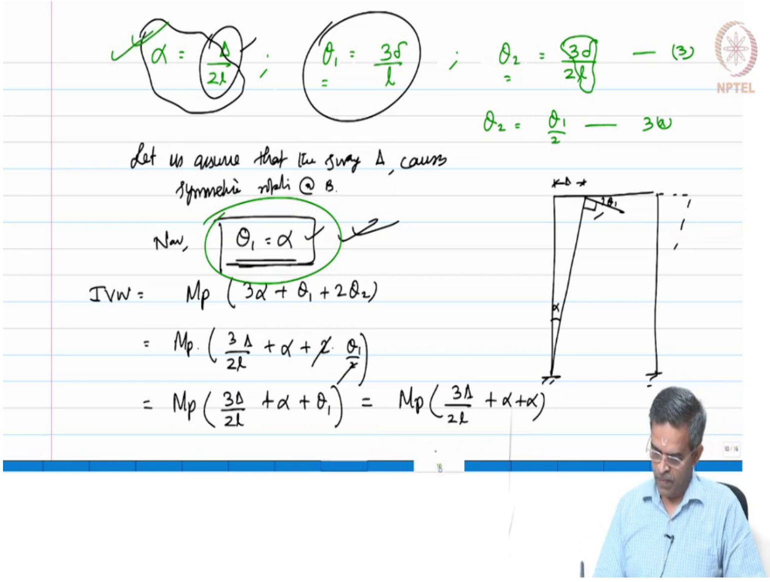

So, now α is Δ by $2l$, $\theta 1$is $3\delta/l$ and $\theta 2$ with $3\delta/2l$ from the geometry. So, let us call this equation number 3. So, from this equation we can also say trying to look at $\theta 1$and $\theta 2$ we can straight away say $\theta 2$ is $\theta 1$by 2 is it not? Because 3 δ/l is $\theta 1$. So, $\theta 2$is $\theta 1/2$. Now, for the frame to sway towards right and maintain this angle as 90 if this is α and this is $\theta 1$. The frame has to sway towards the right.

So, let us assume that the sway Δ causes symmetric rotation at B what does it mean this is Δ. So, we say now $\theta 1$is α. So, we maintain its angle 90 degree. So, this is true. Now, we already know the internal virtual work from equation 2, let us copy that equation. So, internal virtual work is Mp times of 3 α + $\theta 1 + 2\ \theta 2$, let us see this equation.

So, which will be Mp times of α is here. So, 3 times of Δ by $2l$ plus $\theta 1$is already α. So, can I say this is α plus let us say $\theta 2$ is $\theta 1/2$ which will become Mp times of 3 Δ by $2l$ plus α plus $\theta 1$, which I can now say is Mp times of 3 Δ by $2l$ plus α plus α, because we have this equation here.

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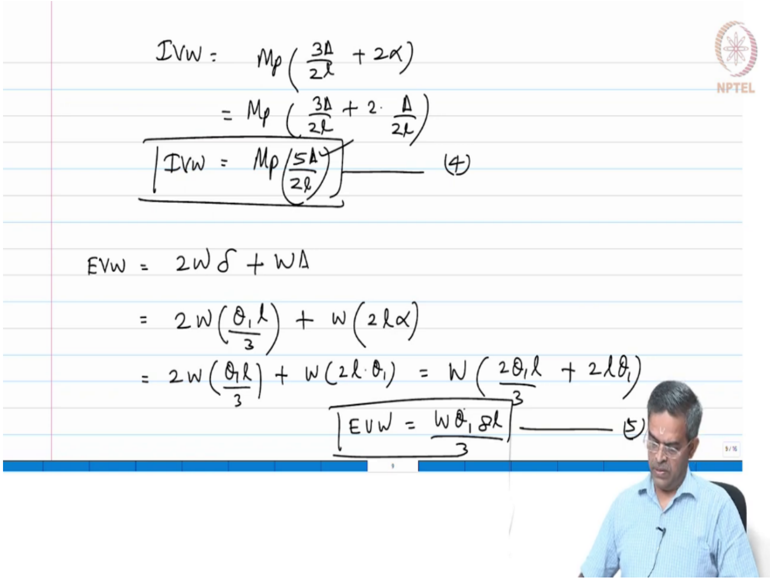

So, now I can say internal virtual work is $Mp*(3\Delta/2l) + 2\ \alpha$ which is $Mp * (3\Delta/2l + 2\Delta/2l)$ which is $Mp *5\Delta/2l$, equation number 4. I have modified the internal virtual work in a simplified form. Look at the external virtual work there are two variables, but we know they are connected to each other.

So, let us say external virtual work which was $2\ W*\delta$ plus $W*\Delta$ which can be said as, $2W(\theta 1l/3)$. Plus $W*2l\alpha$, which will now become $2W\theta_1 l/3 + W(2l\theta_1)$ because we know α and $\theta 1$ are same.

So, can you simplify this as $W*2\theta_1 l/3 + 2l\theta_1$ which will become $W\theta_1*8l/3$. So, I simplified this equation further in this form $W\theta_1*8l/3$. So, now, you see I have internal virtual work in terms of Δ, but here I have this in terms of θ 1.

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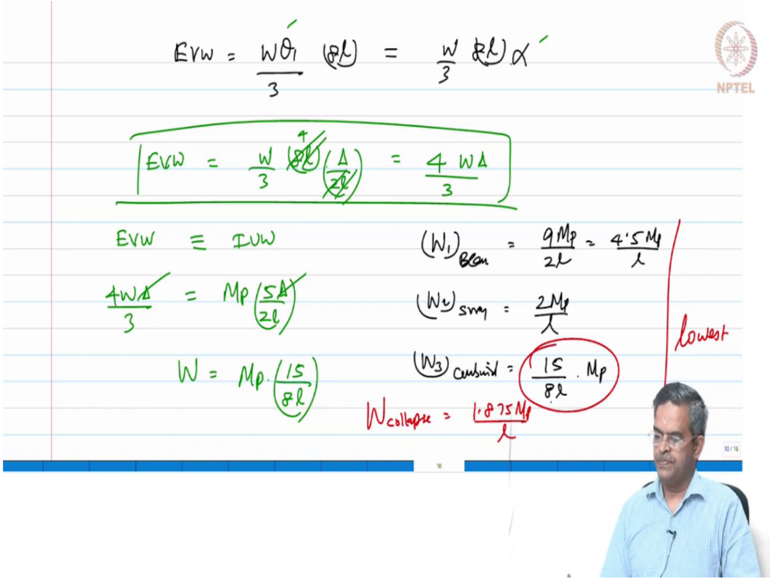

Let us convert this which now become Wθ_1/3 (8l) which can be said as W*8lα /3 you can see here θ1is α. Now α and Δ are connected, by this equation. So, let us do that which will now become (W/3) *8l(Δ/2l).

So, l goes away and this becomes 4. So, that becomes 4WΔ/3, that is my external virtual work. So, now, you see external virtual work is in Δ internal virtual work out in Δ let us compare this. So, external virtual work should be equal to internal virtual work. So, 4WΔ/3 should be equal to Mp*5Δ/2l. So, now Δ goes away, I can state to be find W, as Mp*15/8l.

So, now, we have different values. We have W1 which is from the beam mechanism which was equal 9Mp/2l which is 4.5 Mp/l. We have W2 from the sway mechanism which is here which is 2Mp/l. We have now W3 is a combined mechanism which says it is 15Mp/8l. So, we must choose the least of all. So, now, the true collapse load will be 1.875Mp/l which is given by here.

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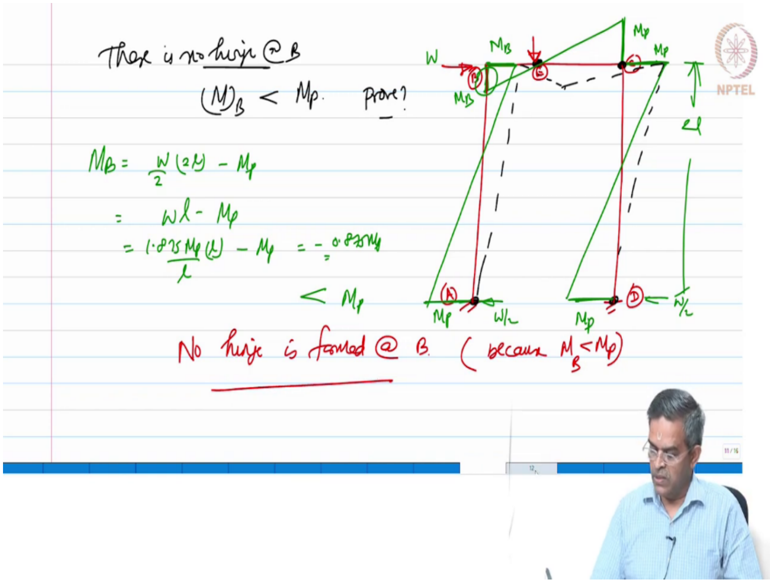

So, I have a frame loaded by lateral and vertical these are the points A, B, C, D and E this was the mechanism what we had which gave us the true loads. So, we have hinges at here, at here, here and here is it not. So, now we say there is no hinge at B. So, M at B is lesser than Mp.

So, we have to prove this, because we do not have a hinge at B. So, let us draw the bending moment diagram on the tension side. So, this is Mp and again this is M_B this is M_B and this is of course, Mp and Mp and Mp I am drawing B M D on the tension side. So, I am just joining this.

So, I want to find M_B. So, if you look at this M_B will be actually equal to (W/2)*2/- Mp. Now if I say this W is my Wc, let us say W/- Mp which is 1.875 Mp by /*/minus Mp. So, you get minus 0.875 Mp, negative is due to a sagging moment and it is lesser than Mp.

So, no hinge is formed at B, hinge will only form when M is equal to Mp.

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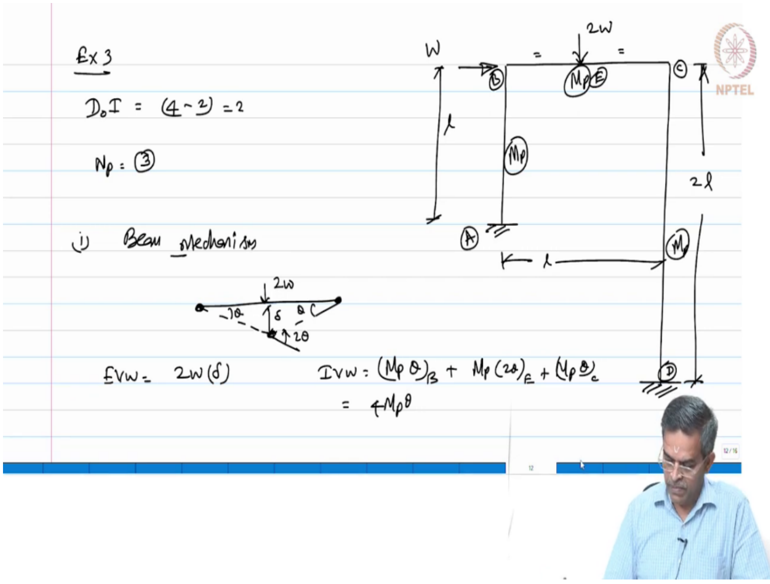

Let us say, we have an un symmetric frame both ends fixed, there is a later load W and there is a central concentrated load 2W, which is at the center and the capacities of columns and beams are same. So, let us say this height of the column is l and the span of the beam is also l and the height of the other column is twice of this. So, the question is found out the true collapse load. So, let us quickly see the degree of static indeterminacy for this problem neglecting axial deformation will be 4 - 2 which will be 2 because we neglected the axial deformation.

So, this is A, this is B, this is C, this is D and this is E. So, we need number of plastic hinges as 3 to make it as a mechanism. So, we will start with the beam mechanism. So, we assume hinge at B, E and C. So, it is a beam mechanism. So, there is a hinge here, here and here. So, this is 2W and this is θ and θ and this is 2θ and of course, this displacement is δ.

So, the external virtual work in this case is 2 W*δ, the internal virtual work in this case will be Mp*θ this is at B plus Mp*2 θ this is at E plus Mp*θ this at C which makes it as 4 Mp θ.

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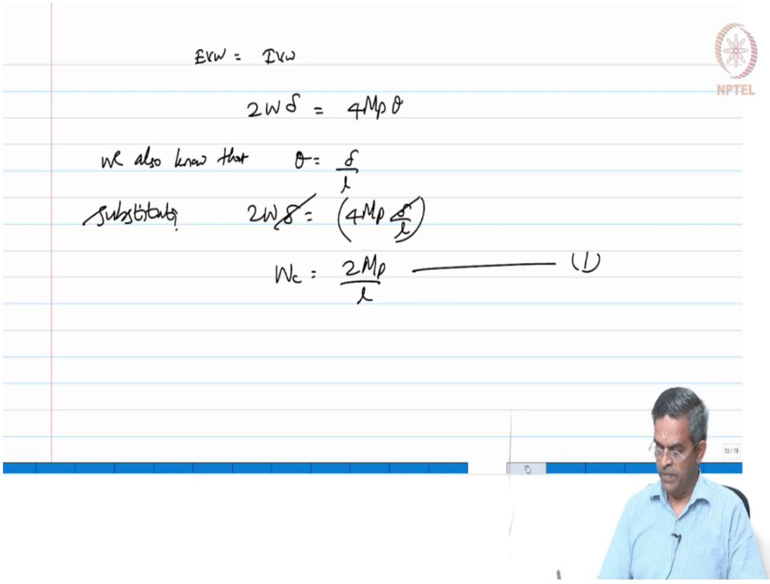

So, equating external virtual work to internal virtual work 2Wδ should be equal to 4 Mpθ and we also know from the figure that θ is δ/ℓ. The span of the beam is 2ℓ. So, δ/ℓ is θ. So, substituting 2 W*δ should be 4 Mp*δ/ℓ. So, Wc in this case is going to be 2 Mp/ℓ the first iteration.

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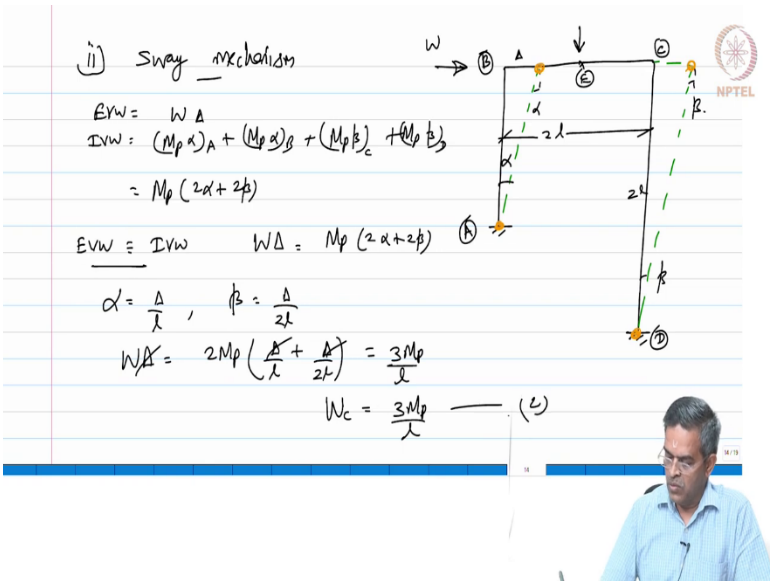

Let us go for the next one which is going to be a sway mechanism. So, the frame is going to sway to the right this point is A, B, C, D and somewhere it is E there is a load here and now there is a lateral load here W. If I call this as Δ and this as α this is also going to be α and this is β this is going to be β and we know this is 2ℓ this is also 2ℓ.

So, let us assume hinges at A, B, C and D. So, now, the external virtual work is going to be equal to W*Δ the internal virtual work is going to be equal to Mp*α this is at A, plus Mp*again α at B, plus Mp*β this is at C, plus Mp*β at D. So, which makes Mp*(2α+2β). So, we know external virtual work should be equal to internal virtual work by principle of virtual work.

So, WΔ should be equal to Mp*(2α+2β). Now we know from the figure α is Δ/l and β is $\Delta/2l$ let us substitute that here. So, W*Δ is going to be 2Mp($\Delta/l + \Delta/2l$) , so which will become 3 Mp $/l$. So, now Δ goes away W c is 3 Mp $/l$.

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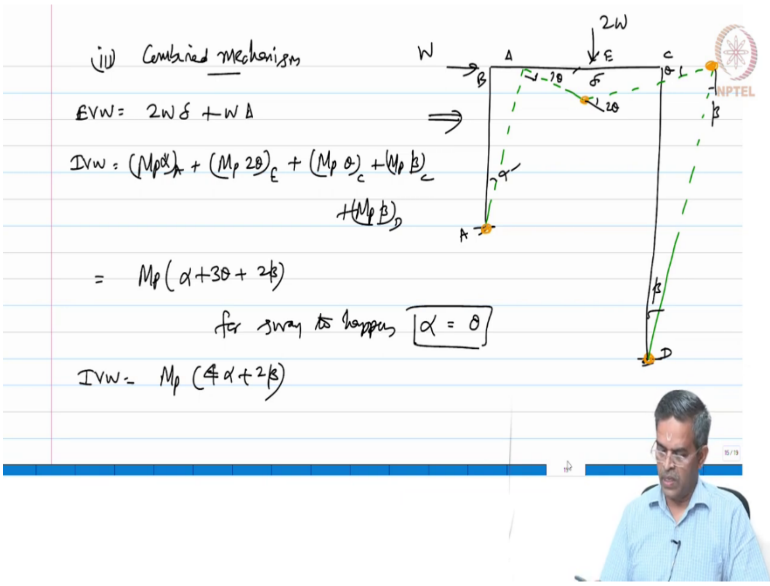

In the first iteration we got 2 Mp $/l$ now, we have 3 Mp $/l$ let us also do a combined mechanism; Let us say this is my combined mechanism I am going to assume hinges at this place and this place here and here. Let us say this is α and this is θ, this is 2 θ because this is also θ and this is β and this is β.

So, the loads are this is W and here there is a vertical load of 2W. So, the external virtual was going to be 2 W*δ + W*Δ. internal virtual work is going to be, Mp*α this is at A, plus Mp*2 θ that is at E, plus Mp*θ that is at C, plus Mp*β that is at C, plus Mp*β that is at D.

So, one can say Mp(α+3θ+2β). Now, keeping this angle as 90, for sway to happen α should be θ. So, therefore, internal virtual work will become Mp(4α+2β) and we already know the value of α and β, let us substitute and get the true collapse load.

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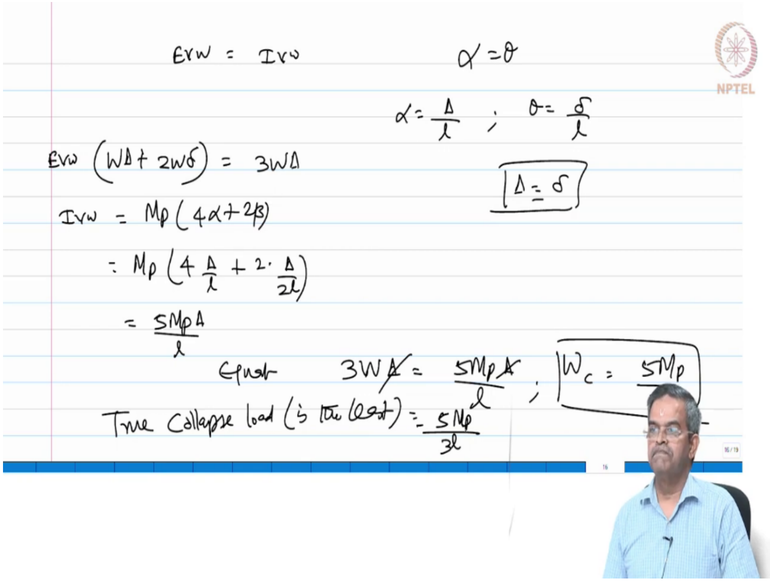

So, now I can say external virtual work should be equal to internal virtual work. Now see here α is equal to θ. So, where α is Δ $/l$ whereas, θ is also δ $/l$. So, I can say these two are identical. So, the external virtual work is WΔ + 2Wδ which can be written as 3 W Δ that is external virtual work. internal virtual work is going to be Mp*(4α+2β) and α is δ $/l$ and this is δ $/2l$.

So, Mp *(4Δ $/l$ + 2Δ $/2l$) Which makes this as 5 Mp Δ/l. So, equating 3 W Δ is 5 Mp Δ/l. So, I get Wc as 5 Mp/3l. So, now, this is the third one I have which is about 1.67. So, now, I have this is 3, this is 2 and this is 1.67 So, the true collapse load is the least of all which is 5 Mp/3l and for a given cross section we already know Mp, I can find Wc.

So, friends these are some examples where we have obtained very easily the collapse load for different mechanisms combine them then we got a test of how to get the iterative mechanism about this and get the collapse load.

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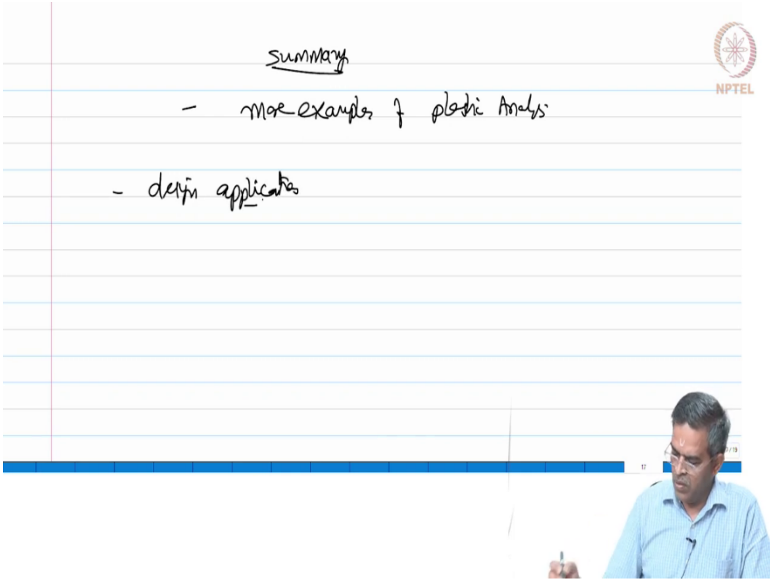

So, as a summary in this lecture we learnt more examples of plastic analysis, we will also do one design application very quickly now.

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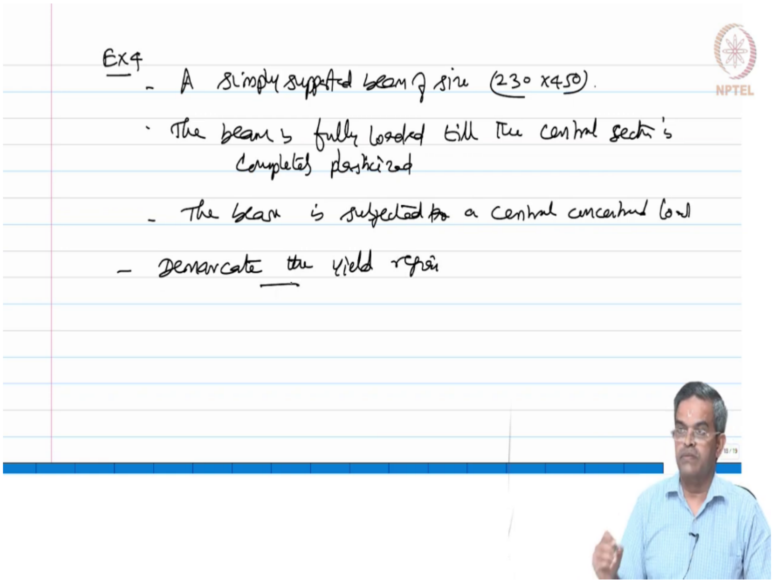

Let us do a design application, say I have a simply supported beam of size 230 by 450 in cross section. The beam is fully loaded till the central section is completely plasticized the beam is subjected to a central concentrated load. What is want is demarcate the yield region.

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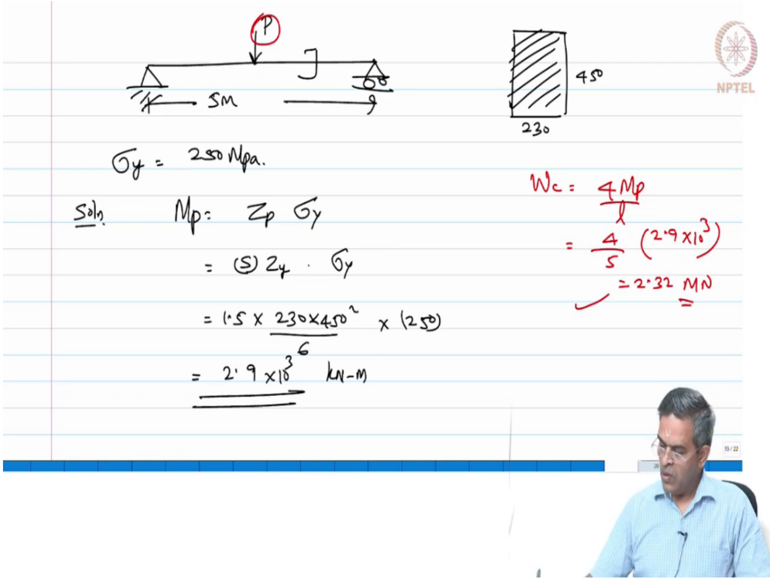

So, the beam is like this the simply supported beam. Central load for a span of 5 meters the cross section is a rectangle of size 230 by 450. Take yield value as 250 MPa. we know Mp is going to be equal to Z p*$\boldsymbol{\sigma}$y which will be shape factor*Zy*$\boldsymbol{\sigma}$y which is (1.5*230*450^{2} /6)*250 which becomes 2.9*10^{3} kNm for this problem, by the way Wc the collapse load will be 4 Mp / l.

So, if I know Mp, I can easily find the collapse load and so, on which will be 2.32 MN that is the load at which the beam will collapse. Let us also find out what is the yield moment of this.

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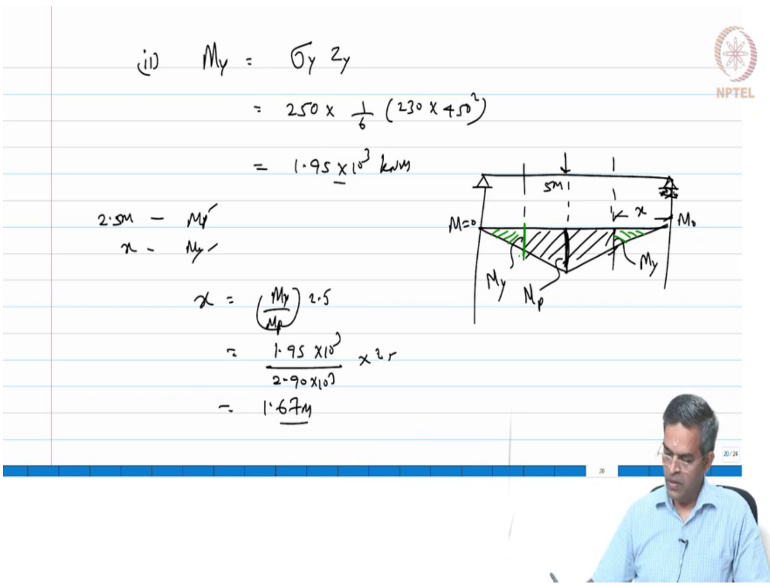

So, step number ii let us try to find out the yield moment which will be **σ**y *z y which will be 250*1/6 of 230*450^2 which will be 1.95*10^3 kNm. Let us draw the demarcated region. This is my beam this is my bending moment diagram this is my deflected profile. Here the moment is 0, at this point it is Mp, at some section this value is My from a similar triangle we can find this.

The span of the beam is 5 meters. So, we can say for 2.5 meter this is Mp, therefore, for x it is My, I know Mp and My, I can find x which will be My by Mp of 2.5, M y is 1.95 and Mp is 2.9*2.5. I get x as 1.67 meters this is 1.67 meters. So, only this portion is trying to get yielded remaining this portion is still elastic, because up to this point it is My.

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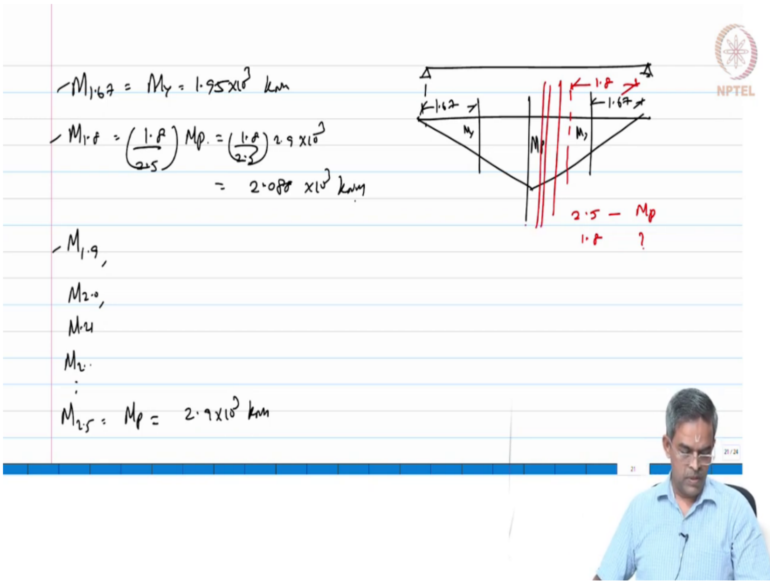

So, now let us say this is my beam in a deflected profile is the bending moment diagram as well this, this is Mp here, this is My here and this distance is 1.67 is also 1.67 by symmetry. So, I can say M at 1.67 is My which is 1.95*10^{3}kNm. Can I find M at 1.8 from the similar triangle principle? Which will be (1.8/2.5)Mp.

I can find this 1.8 /2.5*2.9x10^3. So, I can know this value as 2.088 x 10^3 kilo newton meter. So, can I this way find M at 1.9, M at 2.0, M at 2.1, 2.2 and so, on at M at 2.5 will be equal to Mp, which will be 2.9x10^3 kilonewton and meter. So, I have moment at different sections I have moment at different sections I have computed them.

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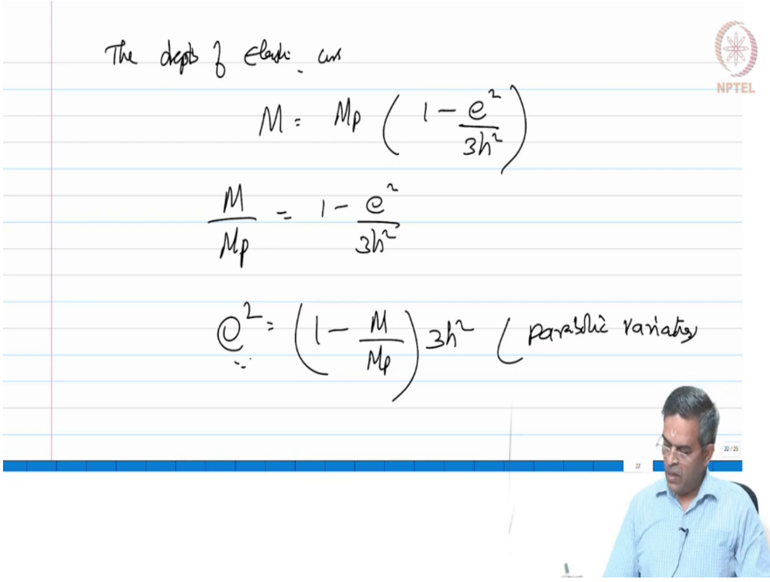

So, let us also find the depth of elastic core. We know the equation M is equal to Mp $*(1-e^2/3h^2)$. So, I can say M /Mp is $*(1-e^2/3h^2)$ by this logic e^2 will become $(1- M/Mp)*3h^2$. So, it will have a parabolic variation as e^2.

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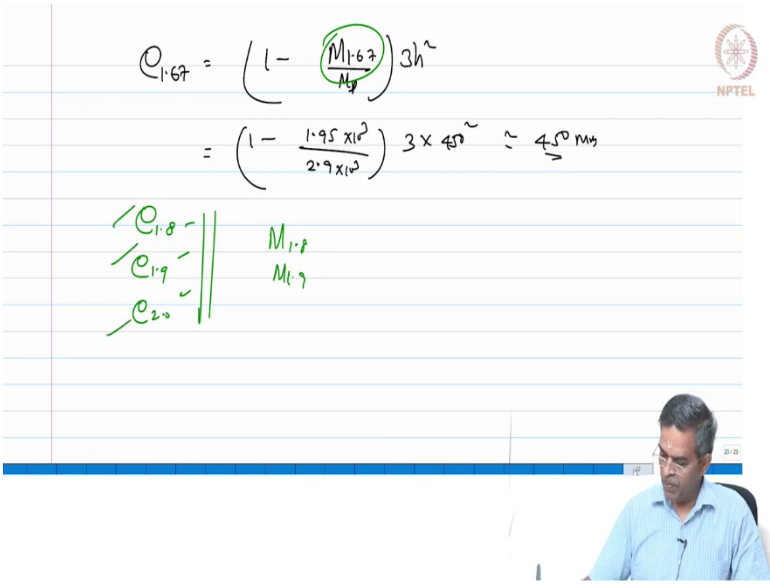

So, can you now find what is the depth of elastic core at 1.67? How do you do that? It is 1 minus M at 1.67 by Mp*3 h^2, h is the depth of the section. So, it will be 1 minus M at 1.67 is $1.95*10^3$ you can see here 1.95 $x10^3$ by Mp is $2.9x10^3$ 3 h^2 which is 450^2. So, I can find very easily this will be 450mm which is completely elastic at 1.67, see here at this section it is fully elastic is it not?

That is what we are getting similarly can we find e at 1.8, e at 1.9, e at 2, how can you do that? Because at all these sections I have moment at 1.8 moment at 1.9 I have here seen here 1.8 1.92 I have all the moments you can substitute here and I can find the depth of elastic section here.

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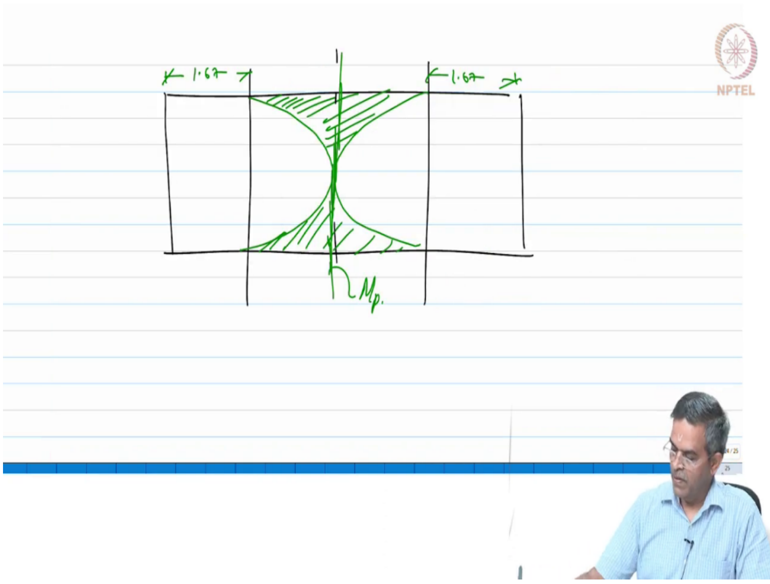

So, if you try to plot that the plot will look like this if this is my depth of the section if this is the center and this is 1.67. So, it is 1.67 from here and it is 1.67 from here and you see the hinge formation will be like this. So, this is how it is plasticized and at the mid span it is fully plastic it is Mp.

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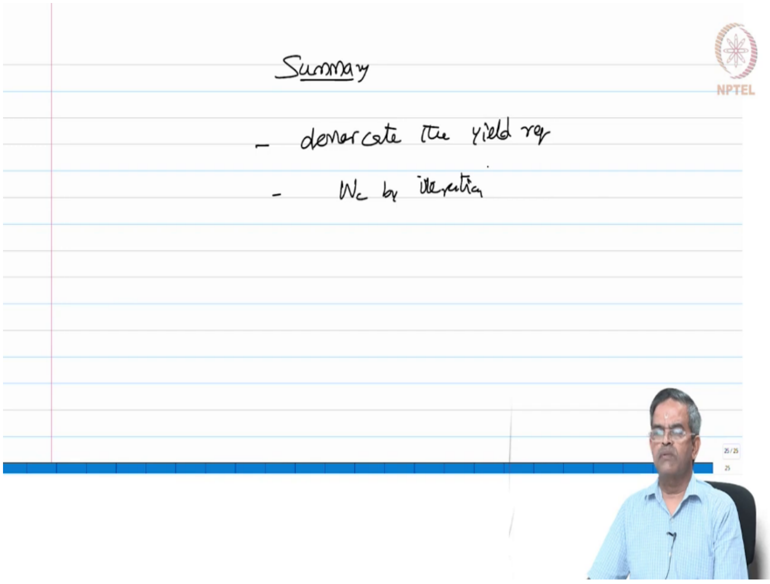

So, friends this is how we have demarcated the yield region. So, we have also learnt how to demarcate the yield region and we learnt how to find out Wc by iteration. So, friends, in this lecture we learnt about the analysis and design in this module we learnt about plastic analysis and design in detail now we will move on to the second module the third module which is on stability of members.

Thank you very much have a good day.