

Advanced Design of Steel Structures
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Lecture - 26
Structural Stability

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NPTEL

Lecture 26

Stability - 1

- Compliant structural systems
 - Innovative in geometry
 - counteract the loads not by material strength
 - major contribution comes from the form
- Under any circumstances, the geometric form should be STABLE

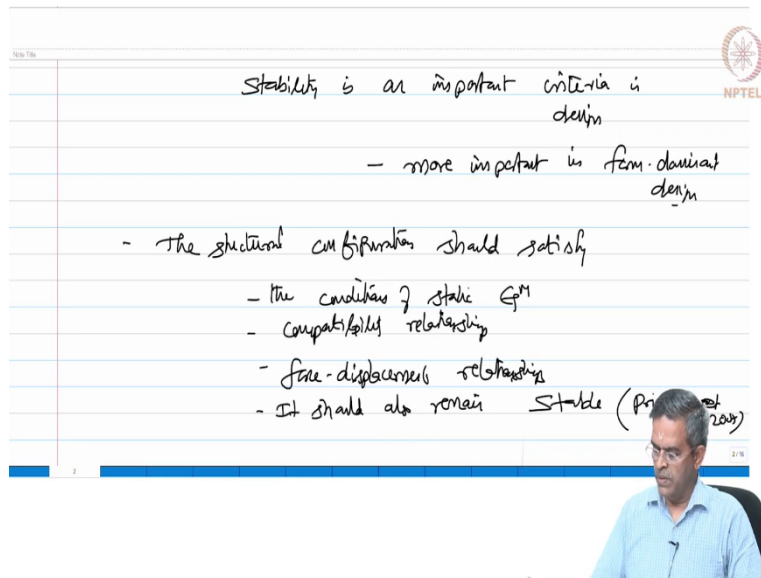
So, friends welcome to the 26th Lecture which will be on Stability of Structural systems. It is a new module stability lecture 1 we are talking about stability of systems. Stability of a system in general and to steel structures in particular is extremely important because most of the complaint offshore structures encounter the loads based upon the geometric form and not by strength.

So, now when we talk about compliant systems which are innovative in geometry they have a special characteristic they counteract the loads not by material strength alone, a major a major contribution comes from the geometry. So, form dominant, but we have got to impose a very important condition that under any circumstance the geometric form should be stable.

So, the stability of the system cannot be challenged you can use a form dominant design, you can try an innovative geometric form, you can derive the benefit of indeterminacy and arrangement of members to support the system and resist the load all are fine. Provided one

condition that the geometric form what you design should remain stable. So, the stability becomes a very important criteria.

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The slide contains handwritten text on a lined background. At the top right, there is a small circular logo with the text 'NPTEL' below it. The main text reads: 'Stability is an important criteria in design'. Below this, there are two bullet points: '- more important in form-dominant design' and '- The structural configuration should satisfy'. Under the second bullet point, there are four sub-bullets: '- the conditions of static Eqⁿ', '- compatibility relationship', '- force-displacement relationship', and '- It should also remain Stable (Presley et al 2007)'. In the bottom right corner of the slide, there is a small number '2/8'. A man in a light blue shirt is visible in the bottom right corner of the frame, looking at the slide.



So, we say here that stability is an important criteria in design, this is much more important in form dominant design let us put it this way, because there are challenges. We also know that the structural form is subjected to variety of challenges in given loads and boundary conditions. The structural configuration or the geometric form should satisfy the conditions of static equilibrium, compatibility relationship and force displacement relationships. It should also remain stable were nicely said by Presley et al in 2007 now these are conditions which are to be met.

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An unstable condition is referred to as
a FAILURE STATE

- Stability is affected significantly under compressive forces - axial compression

Geometric stability is more important than the material strength
↳ challenges the functional requirements of any structural member

Therefore, an unstable condition is referred to as a failed state. So, unstable condition is a failure therefore, stability under conditions are failure. Furthermore, stability is affected significantly under compressive forces to be very precise under axial compressive forces. Therefore, the geometric form or the geometric stability is more important than the material strength, you can use any material of a higher order in terms of its strength.



If the geometry is not able to sustain a stable configuration equilibrium condition the geometry cannot be used to resist the applied loads. Therefore, geometric stability challenges the functional requirements of any member.

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Stability refers to a stable state of E^* .

Define "As the ability of any structural system to remain (or continue to remain) in its stable geometric form, which can perform the intended function even if the geometric form is disturbed by external forces"

STABILITY

Therefore stability is always referred as stable state of equilibrium; How do you define stability? Stability is defined as the ability to remain stable. stability the ability of any structural system to remain or continue to remain in its stable geometric form which can perform the intended function even if the geometric position is disturbed by external forces. So, we can say it is ability to remain stable-stability.

By this definition friends complaint offshore structures should remain stable because they perform their functions of disbursing the loads only by counteracting the loads with the novelty of the geometry. Say we have studied about an example of a complaint platform which is tension like platform; a tension like platform in it is disturbed position tries to reach recentering.

So, as long as the change of the geometric position does not affect its loads dispersing capacity we can say the structure is instable equilibrium. Therefore, structures do not need to remain in their original geometry to classify them as stable that is a very important statement.

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Structures do not require to stay in their original geometric form to classify them as stable

- They can remain stable even under the displaced position

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Friends, structures do not require to stay in their original geometric form to classify them as stable. They can remain stable even under the displaced position that is very important. So, form dominant structures can remain to stay in the displaced position and as long as they are able to disperse the loads and perform intended function they are considered to be stable.

So, please do not get confused that stability refers to an static equilibrium position. No, it can be a deformed position of geometry I am saying of geometry can be a deformed position. As long as under the deformed position if the structure is able to perform intended functions successfully we can call that geometric form or that structure as a stable system. So, stability is always referred to performance index.

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Stability refers to performance
functional characteristics

- i) It disburse the loads, effiently without failure

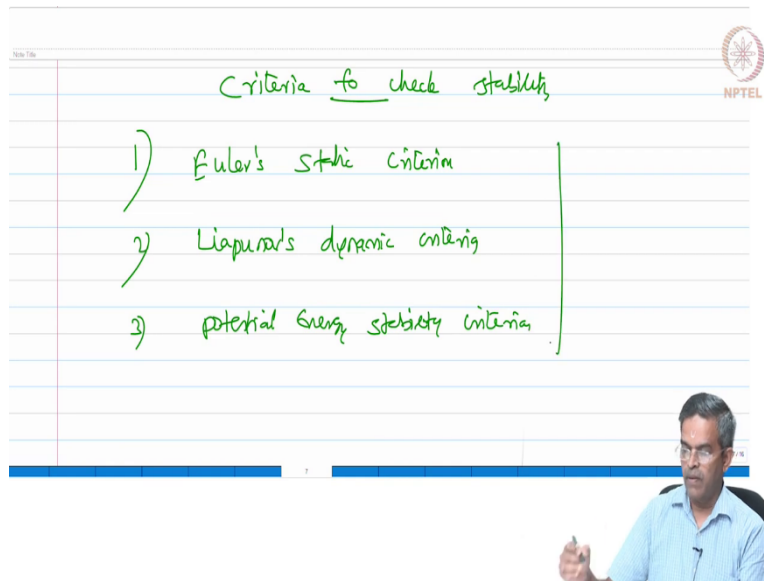
Stability refers to performance or functional characteristics. You may ask me a question, what are the functions of any given structural system? The foremost function is the structural system should disperse the loads efficiently without failure is it not, that is the intended function first. Then comes the question of it is going to stay in equilibrium, it is going to cover the roof, it is going to cover the walls, what are may be the functional properties come later.

The foremost requirement of structural system is to disperse the loads acting on the structure efficiently without failure. Having said this there are three criteria of checking stability.

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Criteria to check stability

- 1) Euler's static criteria
- 2) Liapunov's dynamic criteria
- 3) potential energy stability criteria



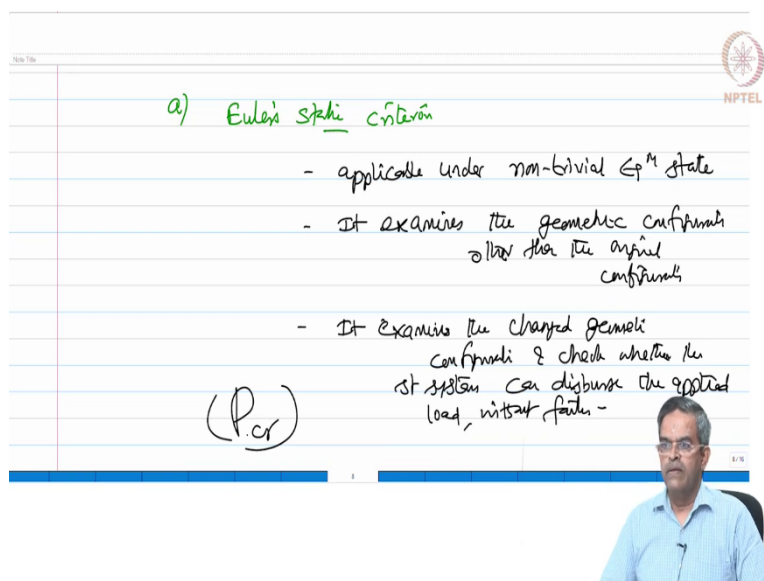
(1) Euler's static criteria, (2) Liapunov's dynamic criteria and the last one is (3) the potential energy stability criteria. Let us see one by one very quickly what are these?

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a) Euler's static criteria

- applicable under non-trivial \mathbb{E}^n state
- It examines the geometric configuration other than the original configuration
- It examines the changed geometric configuration & check whether the st system can disburse the applied load, without failure -

(P.cr)



Let us talk about Euler's static criteria. Euler's static criteria is applicable under non trivial equilibrium state. It evaluates the stability of a structural system by examining the optimum geometric configuration of the system other than the original configuration. So, it examines the geometric configuration other than the original configuration. So, what is the new system it is examining?

It is examining the geometric configuration or it examines the changed geometric configuration and check whether the structural system can disburse the load, the applied load without failure. The applied load in this case is called P_{cr} it is called as a critical load we will talk about that later.

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The slide contains handwritten notes on a lined background. At the top right, there is a small circular logo with the text 'NPTEL' below it. The notes are as follows:

- While examining the factors that are important:
 - i) Boundary condition
 - ii) Initially, perfect, straight geometry
- Euler's criteria evaluates whether the structure is capable to carry load, P

Below the main text, there is a red handwritten note: "instead of remaining in a state where it is unable to carry any load at all (this $P > P_{cr}$)".

In the bottom right corner of the slide, there is a small inset video frame showing a man in a light blue shirt speaking.

While examining this, what are the factors which are important; (1) boundary conditions, (2) initially perfect, straight geometry. Remember; when no load is applied when the structure is not enabling itself to disperse the load by its form dominant characteristic structure has to remain initially straight and perfectly elastic is it not, that is the basic design requirement we have. So, initially the system should be perfectly straight geometry and if it is not so that will affect the stability criteria as per Euler's condition.

In addition to that the boundary conditions applied on the system will also be examined under Euler's criteria. So, therefore, friends Euler's criterion evaluates the structure evaluates whether the structure is capable of carrying load (P), remember this P can be even less than or equal to. Let us put it this way capable of carrying a load ($P > P_{cr}$) let us put it like this.

So, let us say we are looking at the failure condition instead of remaining in a state where it is unable to carry any load at all. Friends, it is very interesting to see that the load carrying capacity of the member is reduced from P to P_{cr} .

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Hence, the load capacity of the member is reduced from P to P_{cr} ($P > P_{cr}$)

still, it is an important fact that the member should at least carry P_{cr} even @ the disturbed geometric condition which is quite weak from the original form.

The slide features a white background with blue horizontal lines. The text is handwritten in black ink. A small NPTEL logo is visible in the top right corner. In the bottom right corner, there is a small video feed of a man in a light blue shirt.

Therefore, the load capacity of the member is reduced from P to P_{cr} . So, the condition is P exceeds P_{cr} that is the original capacity, but because of the change in geometry because of the incapability of the member we arrive at a load which is P_{cr} and this condition holds good. So, there is a reduction in load capacity which is estimated which is computed using Euler's criteria. But still, it is an important fact that the member should at least carry P_{cr} even at the disturbed geometric condition which is quite weak from the original form.

So, please understand friends, the load capacity at the changed or disturbed geometric form is not 0. The load capacity at the changed disturbed geometric form is P_{cr} and we believe that P_{cr} is much lower than P the original capacity. But still the load capacity is not 0, it is having some critical load carrying capacity that is what is estimated using Euler's criteria.

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This value of the load (P_{cr}), which is reduced magnitude in comparison to original load capacity (P) - is termed as

Euler's - CRITICAL BUCKLING load

So, this value of the load P_{cr} which is reduced magnitude in comparison to the original capacity of load is termed as critical load or buckling to be very precise is called as Euler's buckling load.

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P_{cr} - dependent on

- 1) Boundary conditions
- 2) cross-sectional properties of the member
- 3) slenderness ratio

Instability occurs when two or more adjacent eqm positions correspond to different mode shapes

So, it can be easily seen that P_{cr} is dependent on boundary conditions cross sectional properties of the member and slenderness ratio. I believe all of us know how to estimate a slenderness ratio for a given structural member. Furthermore, instability occurs when two or more adjacent equilibrium positions correspond to different mode shapes. That is why friends

you note; in well-designed structural systems there exist distinctly different well separated mode shapes.

On the other hand the frequencies of the system will not be closer they will be well separated. So, the mode shapes do not overlap very closely with each other because if they have very closely related condition there can be an overlap and that can cause instability. So, when you talk about stability of offshore structures to be more interesting because we have form dominant design well practice in offshore structures therefore, we have to learn about this in detail.

We should at least know about them the advanced steel design is not meant only for offshore structural applications, form dominance in existing for many years for example, arches, domes, cable stay bridges these are all statically indeterminate systems they are all form dominant.

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When stability of offshore compliant platforms is assessed, it disagrees with Euler's criterion

why? (Boundary conditions) of the member do not permit examination of Euler's criterion

- In such cases, things recentering

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But when we assess stability of offshore compliant platforms it disagrees with Euler's criterion. So, Euler's criterion cannot be applied to a compliant platform, maybe why? Let us ask this question why? Why because, the boundary conditions of the member do not permit examination of Euler criterion. So, the problem it is the boundary conditions, because we are talking about floating systems, we are talking about floating structures anchored to the seabed using pre tensioned axial tethers.

So, such boundary conditions cannot be implemented in Euler's criterion, because they completely float under hydrodynamic stability what we call as compliancy. So, in that case stability is largely dependent on the pre-axial tension in their tethers. Then how do you ensure stability in such cases when Euler's criteria is not applicable? We ensure stability in such cases through recentering.

Friends, please recollect we discussed this property of recentering, recentering is an ability of the structural system to return back or regain its original geometric position inherently in the presence of load. So, that is exactly the condition what we discuss in stabilities, stability analysis is the capability of the structural system to assess its condition of disbursing loads and performing the intended function at the displaced geometric configuration.

It is well understood, well established, it is very well clear that the load capacity of disturbed geometric form will be far lesser than the original load capacity there is no question about that. The disturbed geometric form can always carry only a lesser load than its original geometric form which is initially straight perfectly elastic etcetera. But what we want to emphasize here is if that load capacity at the change geometric form is P_{cr} we call that as a stable condition that geometric capacity or the load capacity at the intended geometric form is not 0, it has some capacity.

We all agree that their capacity is much lesser than the original load carrying capacity of the structural system in its original geometric shape and size and position agreed. But the new geometric shape size and position is having some load capacity which is not 0 it is that condition which is called as the stable condition.

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Liapunov's condition of stability

- It examines the stability of structures under dynamic excitations
- when P is fct, it is necessary to check its eq/stability conditi during the excitati process

Let us talk about Lyapunov stability; Lyapunov's condition examines the stability of the structural system under dynamic excitations. If a member is subjected to continuously varying disturbing force, it is necessary to examine whether equilibrium under the dynamic force is satisfied. So, when P is function of time, it is necessary to check its equilibrium or stability condition during the excitation process, as seen in the literature stability can be influenced by two ways.

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stability of systems under dynamic loads
can be influenced by 2 facts

- 1) amplitude (varying)
- 2) period of excitation

load-carrying capacity (strength-dependent) ←

Unconditional response
@ near-resonance state

Liapunov's assessment is more towards dynamic response behavior

Stability of systems under dynamic loads can be influenced by two factors one is the amplitude I should say varying amplitude, second is the period of excitation. These two criteria can influence the stability assessment of a structural system under dynamic conditions. So, the amplitude can influence the load capacity directly. So, load capacity is strength dependent parameter we all know that whereas, the period of excitation can influence an unconditional response at near resonance state.

Hence, Lyapunov's assessment of stability is focused more towards dynamic response behavior of the system.

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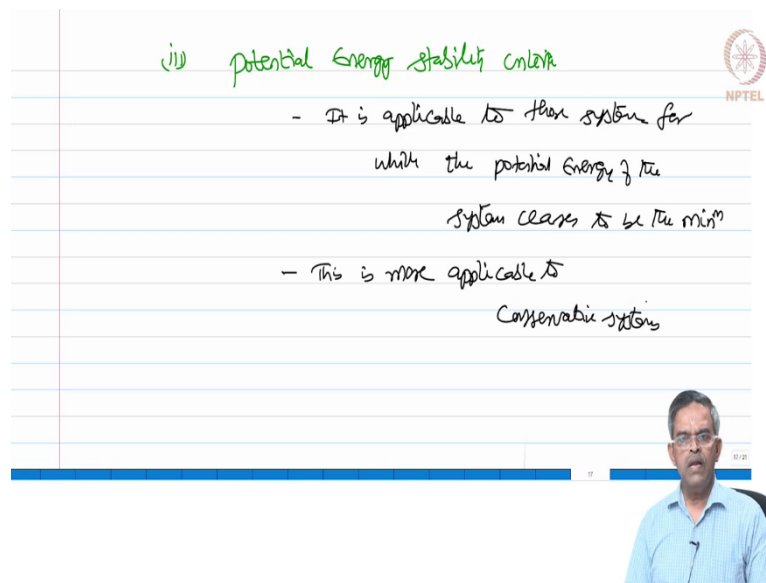
The slide contains a handwritten comparison of Euler and Lyapunov stability criteria. The text is written on a grid background. On the left, under the heading 'Euler', it says '- purely to assess load capacity'. On the right, under the heading 'Lyapunov', it lists three points: '- stability assessment under dynamic loads', '- focused more towards the response behavior', and '- greater challenge is the geometric form @ any instant of time (displaced configuration)'. The NPTEL logo is visible in the top right corner of the slide. A small inset image of a man in a light blue shirt is visible in the bottom right corner of the slide area.

So, let us quickly compare Euler and Lyapunov, Lyapunov system is stability assessment under dynamic loads. So, it is focused more towards the response behavior whereas, Euler's condition is purely to assess the load capacity. So, that is the quick comparison here. Therefore, friends Lyapunov's condition is more significant for structures designed to perform intended function under varying or time varying external forces.

The most challenging part of stability of such structures is a geometric form assumed at any instant of time. So, the greater challenge here geometric form at any instant of time that is we can say displaced configuration. Structural systems like ships, offshore compliant platforms, floating production platforms, fall under this category of stability check.

The stability calculations of ships for example, focus on estimating the center of gravity, buoyancy and meta center vessels and their interaction. In case of structures which are form dominantly designed like TLPs they are permitted to undergo large displacements. So, one should ensure that under such large displaced condition the geometric design ensures proper recentering and therefore, the load carrying capacity is not towards close to 0, that is very important condition.

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The image shows a slide with handwritten notes in green ink. The title is 'Potential Energy stability criteria'. There are two bullet points: '- It is applicable to those system for which the potential energy of the system ceases to be the min^m' and '- This is more applicable to conservative systems'. The NPTEL logo is visible in the top right corner. A small video inset of a man in a blue shirt is in the bottom right corner.

(iii) Potential Energy stability criteria

- It is applicable to those system for which the potential energy of the system ceases to be the min^m
- This is more applicable to conservative systems

The third condition which potential energy stability criteria is applicable to structural systems for which the systems potential energy ceases to be minimum, this is more applicable to conservative systems.

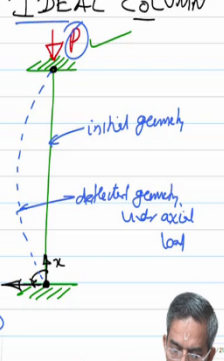
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II Euler's critical load

- Based on application to an IDEAL COLUMN

- Conditions

- 1) column is pinned @ both the ends
- 2) Uniformly slender
- 3) assumed to be laterally restrained in position @ both the supports
- 4) negligible weight \Rightarrow perfectly elastic



The diagram illustrates an ideal column under axial load. A vertical column is shown with a load P at the top. The initial geometry is a straight line, and the deflected geometry under axial load is shown as a dashed curve. A coordinate system with x and y axes is shown at the bottom right. The NIPTEL logo is in the top right corner.

So, friends having said and discussed different stability criteria as from the literature, let us pay more attention towards estimating Euler's critical load. Euler's critical load is based on application to an ideal column. So, here we are talking about the compression member which is axially loaded and the column is ideal let us say how an ideal column looks like. Let us say this is my positive x and anticlockwise 90 is positive y anticlockwise 90 is positive y , the deflector profile is shown here. This is the initial position of the column or initial geometry this is deflected geometry under load.

Let us quickly see what are the ideal conditions of the column? We say ideal column know, what are the conditions? The conditions are (1) column is pinned at both the ends, (2) the column is uniformly slender, (3) the column is assumed to be laterally restrained in position at both the supports, (4) it has got a negligible weight and (5) it is perfectly elastic. What does it mean upon removal of this load P the reflected profile of the column will reach its original geometry, it is not recentering. Please understand, I am talking about the material here.

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- Stresses developed by the axial force are assumed to be within the proportional limit of the material.

Let us consider that this column has a load capacity (P_{cr}).

If the applied force (load) is lesser than the critical load, the column will remain straight and undergo only axial compression.

The slide includes the NPTEL logo in the top right corner and a video inset of a man in a light blue shirt in the bottom right corner.

What does it mean is to be very specific, stresses developed by the axial force are assumed to be within the proportional limit of the material. So, this is purely material property it is not a recentering property. Now let us imagine let us consider that this column has a load capacity P_{cr} . Now, if the applied force or the applied load is lesser than the critical value, the column will continue to remain straight and undergo only axial compression, under this state the column is said to be in static equilibrium.

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At this condition,

$P \leq P_{cr}$, the column is said to remain in static equilibrium condition.

Stable Equilibrium condition ✓

The slide includes the NPTEL logo in the top right corner and a video inset of a man in a light blue shirt in the bottom right corner.

What is this condition? P lesser than equal to P_{cr} , the column is set to remain in static equilibrium condition this is called stable equilibrium condition. Therefore friends, under such condition if the lateral load applied at any point. Please understand, now I am talking about two loads one is axial load which is causing these ideal conditions and I say we have worked out a critical load and we have declared that the column is in stable equilibrium.

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Under such condition, if a lateral load is applied at any point, it will cause a large lateral deflection @ the mid height of the column.

- P is applied to the column
- capacity of P_{cr}

i) $P \ll P_{cr}$ - column will be stable
 ii) $P \leq P_{cr}$ - " stable (upon removal of P column will return to its original geometry)

The slide also features the NPTEL logo in the top right corner and a small video inset of a man in a blue shirt in the bottom right corner.

Under such condition, if a lateral load is applied at any point, it will cause a large lateral deflection at the mid height of the column. However, importantly the column will regain and return to the original position in terms of geometry shape and size of cross section. Upon a continuous increase of axial load P the straight form of equilibrium tends to gradually become unstable.



Let us say P is applied to the column, the column has a capacity of P_{cr} which is known imagine that P applied is far lesser than P_{cr} , the column will be stable no disturbance to the column. Let us say P is gradually reaching P_{cr} column will still be stable and no change. Remember, upon removal of load column will return to its original geometry. That is very important that is the condition we are assessing at.

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$P \equiv P_{cr}$ stable: - no worry 😊

(iv) P is gradually $\uparrow > P_{cr}$
- then the column will become unstable



Under this condition, even a lateral load of small magnitude can cause large lateral deflections - which will not disappear upon removal of load 😞



If you continue to increase the load beyond this imagine P has reached P_{cr} column is still stable no worry, but if P is gradually increased more than P_{cr} , then the column will become unstable, this is the point of worry. Now, under this condition, even a lateral load of small magnitude can cause large lateral deflections which will not disappear upon removal of load.

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This - will be a permanent deflection.



So, this will be a permanent deformation. Remember, this was not true under these conditions, under these conditions this was not true, this was true only when this P exceeds

the critical capacity. Therefore, friends it is very important for a given column member to compute this P_{cr} and we should be very clear if this critical load capacity of the column is exceeded the column will become unstable. And we already said instability or instability is a form of failure.

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Summary

- stability criteria
 - E
 - L
 - ?
- Euler's critical load

$P > P_{cr}$, column becomes unstable

instability is a form of failure

So, friends in this lecture we learnt about stability criteria; Euler's, Lyapunov's and the third one which is based on potential energy. We started learning more about Euler's critical load and we have understood that when the load capacity or the load applied load exceeds the critical capacity column becomes unstable and instability is a form of failure.

Friends, we will close the lecture here thank you very much for your patient learning and you will be keep on referring to the notes and the reference material which has been available in the website of this course for additional learning. Stability is a very difficult topic to understand I am going very gradually and slowly and I wish that all of you should pick up this completely at single pace and try to learn this thoroughly undoubtedly. And then disseminate it to your students to your fellow researchers and use it effectively in consultancy and design offices.

So, that the area which is gray in structural engineering domain which is stability studies becomes very simple and user-friendly treatment of this subject in the advanced steel design domain for structural steel designs.

Thank you very much and have a good day bye.