Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 28
Rotation coefficients for stability functions

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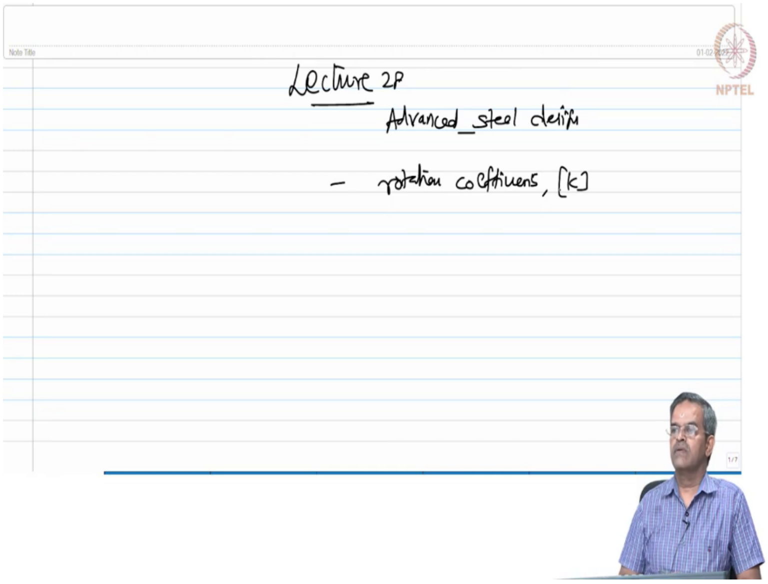

Welcome to the 28 Lecture on Advanced Steel Design course, we are in the process of deriving the stability functions. Now in this lecture we will continue to learn the rotation coefficients and then the stiffness matrix of a standard beam.

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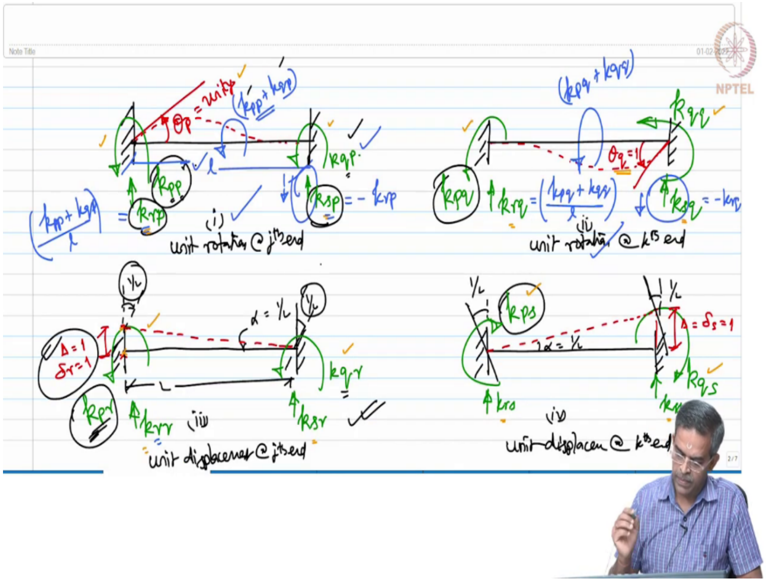

Let us rewind back this is what we discussed in the last lecture, we learnt how to obtain k_{rp} and k_{sp} in terms of p p and q p with reference to figure 1 and figure 2.

$$k_{rp} = \left(\frac{k_{pp}+k_{qp}}{l}\right)$$

$$k_{sp} =- k_{rp}$$

$$k_{rq} = \left(\frac{k_{pq}+k_{qq}}{l}\right)$$

$$k_{sq} =- k_{rq}$$

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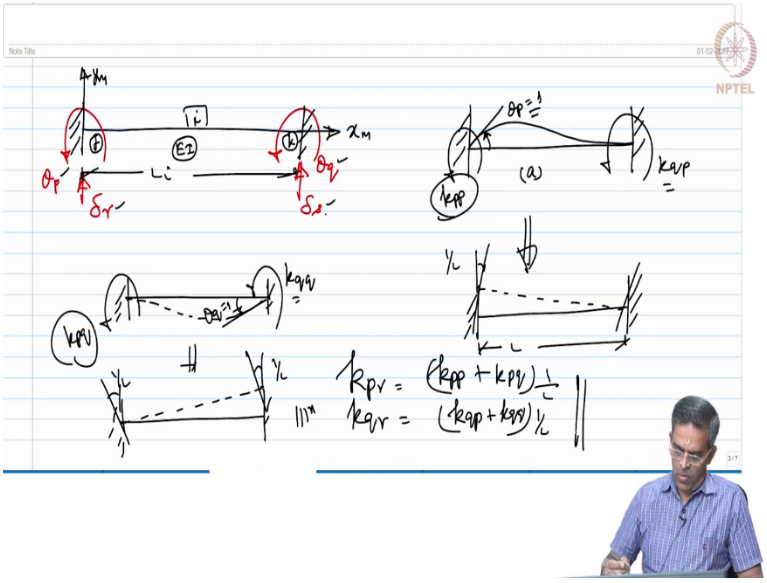

Let us focus on figure 3 now, friends the standard beam looks like this. So, if I give a unit rotation of θ_p as unity this invokes a moment which is k_{pp} and k_{qp}. If this is true then suppose if I have a beam where the rotation is 1 by L_i where this is L. So, this is figure a lets have one more figure and the k^{th} here.

$$k_{pr} = \left(\frac{k_{pp}+k_{pq}}{L}\right)$$

$$k_{qr} = \left(\frac{k_{qp}+k_{qq}}{L}\right)$$

If I give a rotation here then this gives me a moment of k_{qq} and k_{pq}. So, this is equivalent to a member where I give unit displacement here and the member rotates by 1 by L. Now considering these two figures and comparing this with figure 3 and 4 let us say k_{pr} is a moment which is invoked by giving unit displacement at the r^{th} end. But can I write k_{pr} as an effect of these two sum for unit rotation.

So, I am writing k_{pr} as k_{pp} plus k_{pq} into 1 by L why 1 by L k_{pp} and k_{pq} are the moments for unit rotation whereas, k_{pr} is the moment for 1 by L rotation. So, I am multiplying this with 1 by L. So, I can write k_{pr} is it not similarly can I also write k_{qr} similarly k_{qr} can also be a sum of k_{qp} plus k_{qq} by L, because again these are moments because of unit rotation, but I want moment because of o1 by L.

So, therefore, I can multiply this with 1 by L. So therefore, friends once in this figure I know k_{pr} and k_{qr} let me copy this I will put it here.

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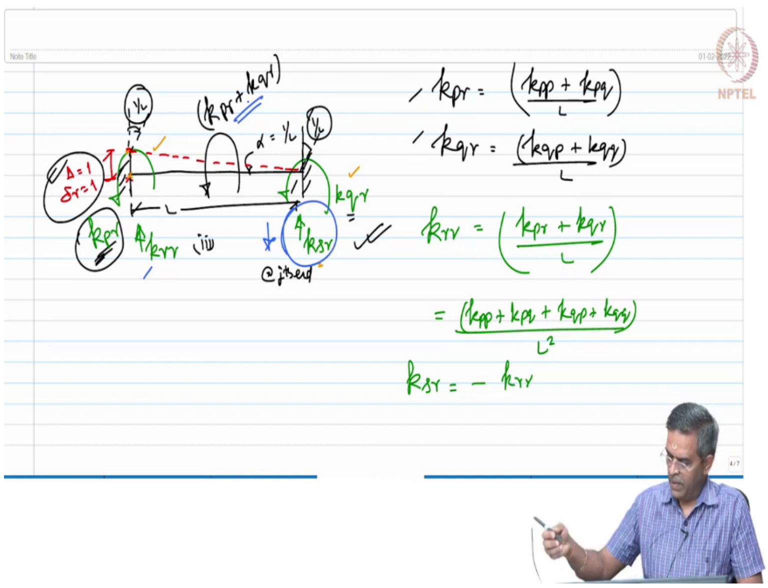

$$k_{pr} = \left(\frac{k_{pp}+k_{pq}}{L}\right)$$

$$k_{qr} = \left(\frac{k_{qp}+k_{qq}}{L}\right)$$

$$k_{rr} = \left(\frac{k_{pr}+k_{qr}}{L}\right)$$

$$k_{rr} = \left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$$

$$k_{sr} =- k_{rr}$$

So, I already know k_{pr} as k_{pp} plus k_{pq} by L. see here, I also know k_{qr} as k_{qp} plus k_{qq} by L. So now, I can write the net moment here is k_{pr} plus k_{qr} which is here. So, now I can say k_{rr} is actually k_{pr} plus k_{qr} by L because there is a couple, can you now write k_{rr} as $\left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$. So now, can I also write k_{sr} as minus of k_{rr} because that is a couple. So, this will be acting downward this will act up this becomes a couple the counter act this moment.

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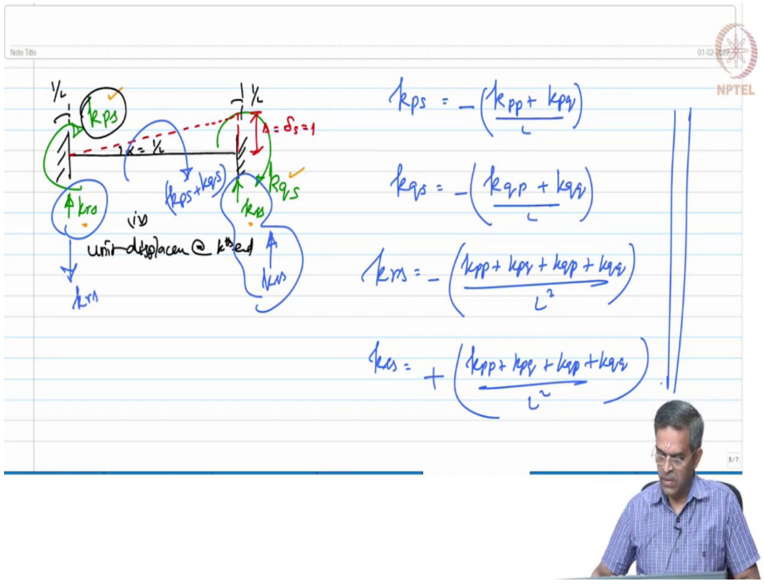

$$k_{ps} = -\left(\frac{k_{pp} + k_{pq}}{L}\right)$$

$$k_{qs} = -\left(\frac{k_{qp} + k_{qq}}{L}\right)$$

$$k_{rs} = -\left(\frac{k_{pp} + k_{pq} + k_{qp} + k_{qq}}{L^2}\right)$$

$$k_{ss} = +\left(\frac{k_{pp} + k_{pq} + k_{qp} + k_{qq}}{L^2}\right)$$

Similarly, let us do this for the other end let us copy this figure and put it here and then work on this similarly same order I am just trying to do for the other end. You know k_{ps} will be actually k_{pp} plus k_{pq} by L. k_{qs} will be k_{qp} plus k_{qq} by L, but there is a small change here originally k_{pp} look at here originally k_{pp} and k_{pq} were anticlockwise, but I am here looking for a clockwise point.

So, shall I say these two are minus is it not they are opposite to the original one is it not. So, if I know the net moment is k_{ps} plus k_{qs} then this will have a moment which is a couple like this. So, this is going to be k_{rs} going to be k_{ss}. So, I can now write k_{rs} as $\left(\frac{k_{pp} + k_{pq} + k_{qp} + k_{qq}}{L^2}\right)$, but with the minus sign. Whereas, k_{ss} will be $\left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$, but with the plus sign because this sign matches with this. So, now I have all the coefficients let me transfer these coefficients to the matrix.

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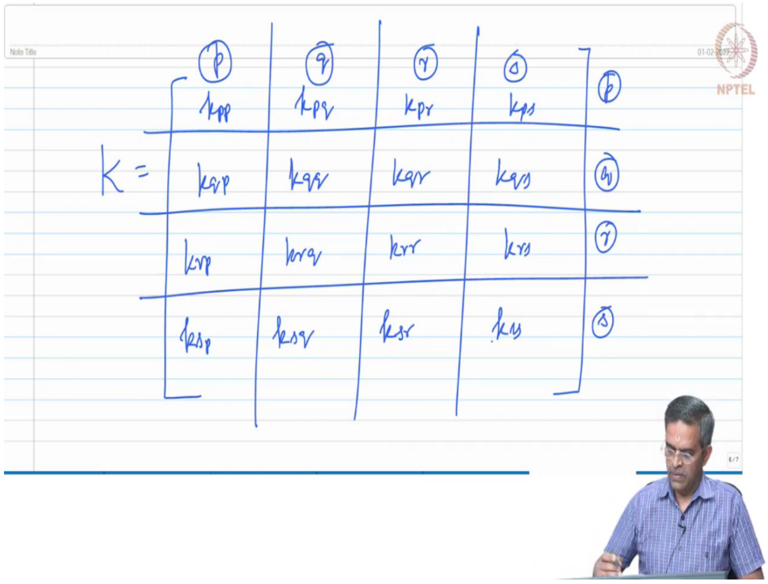

$$[K] = \left[k_{pp}\ k_{pq}\ k_{pr}\ k_{ps}\ k_{qp}\ k_{qq}\ k_{qr}\ k_{qs}\ k_{rp}\ k_{rq}\ k_{rr}\ k_{rs}\ k_{sp}\ k_{sq}\ k_{sr}\ k_{ss}\right]$$

So, what is the original matrix k the original matrix k is if this is p if this is q , r ,and s this is p, q, r and s are the labels. So, this is k_{pp}, k_{pq}, k_{pr}, k_{ps}, k_{qp}, k_{qq}, k_{qr}, k_{qs}, k_{rp}, k_{rq}, k_{rr}, k_{rs}, k_{sp}, k_{sq}, k_{sr}, k_{ss}. So now, I have all the values with me please look at the notes they have all the values.

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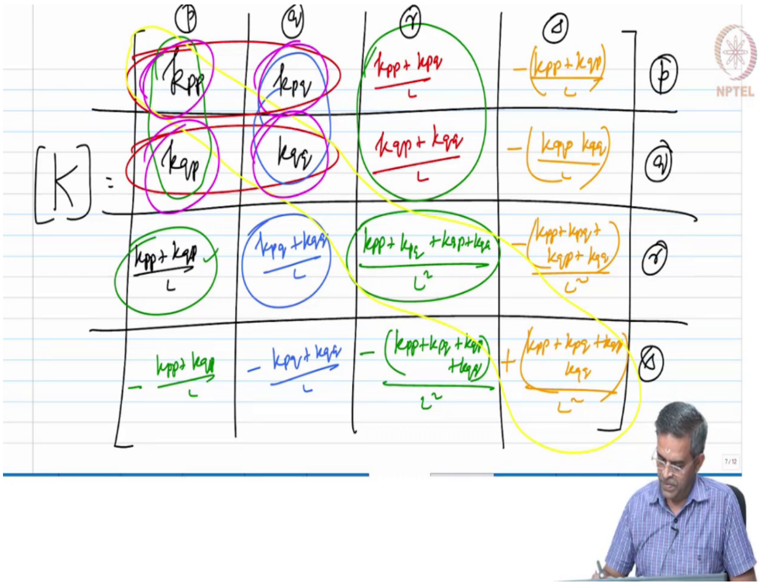

Let me write down these coefficients here I will tell you very simple way to remember this. It is a very simple way the k matrix can be easily remembered please watch here. So, it is easy for you to recollect. So, it s a 4 by 4 matrix please watch here p,q,r,s & p,q,r and s are the labels. this is k_{pp} and this is k_{qp} this is k_{pq}, this is k_{qq} and this value is k_{pp} plus k_{qp} by L it is nothing but sum of these two by L and this is minus of this.

$$for\ matrix\ [K] = \Big[k_{pp}\ k_{pq}\ \frac{k_{pp}+k_{pq}}{L}\ -\left(\frac{k_{pp}+k_{qp}}{L}\right) k_{qp}\ k_{qq}\ \frac{k_{qp}+k_{qq}}{L}\ -\left(\frac{k_{qp}+k_{qq}}{L}\right) \frac{k_{pp}+k_{pq}}{L}\ \frac{k_{pq}+k_{qq}}{L}\ \frac{k_{pp}+k_{pq}+k_{qp}+k}{L^2}$$

Similarly, sum of these two by L will be here this is minus of this. The third column is very easy sum of these two by L second element sum of these two by L, third element sum of these two further by L. So, that becomes k_{pp} $k_{qp}k_{pq}$. k_{qp} *plus* k_{qq} by L square is it not and this will be $-\left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$.

Now, once you written the 3rd column the 4th column is very easy the 4th column is minus of 3rd column. So, minus of k_{pp} k_{qp} by L minus of $k_{qp}k_{qq}$ by L. $-\left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$. then $\left(\frac{k_{pp}+k_{pq}+k_{qp}+k_{qq}}{L^2}\right)$. Friends you can always note that the elements along leading diagonal are positive they are positive they can never be negative. So, please check that when I do the derivation and understand very carefully. So, in this matrix these 4 values 1 2 3 and 4 these are called rotation coefficients.

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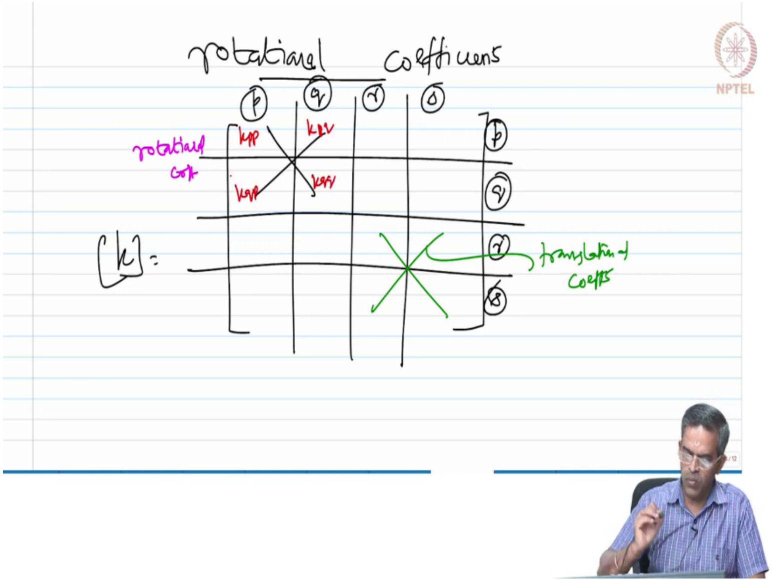

Let us say this is my matrix this is p q r and s are the degrees of freedom, these are called rotational coefficients and these 4 are called translational coefficients. Friends carefully look at the derivation back look here in this equation if we know this value and this value or these 4. I can fill up the remaining matrix very fast. So, I need to know only the rotational coefficients if you know the rotational coefficients I can fill up this matrix very fast.

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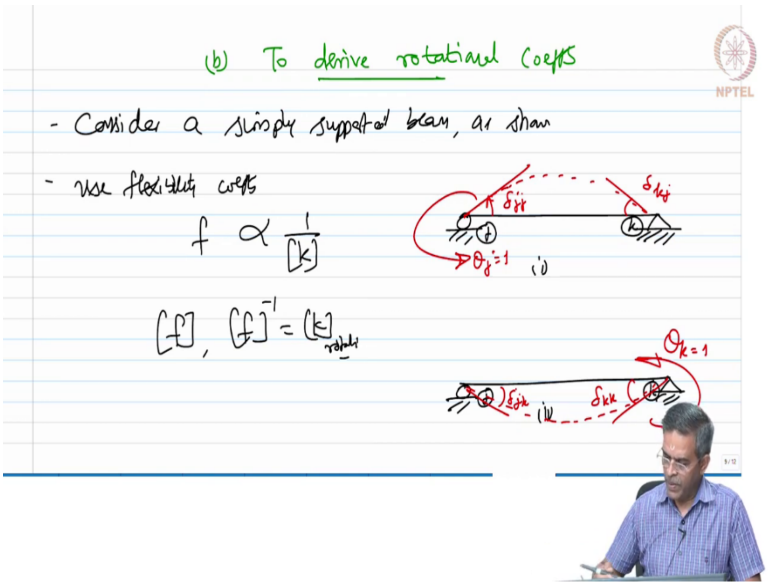

So, our job is to now derive the rotational coefficients. So, the rotational coefficients will be derived based on the flexibility method. So, we want to consider a simply supported beam as shown in the figure. So, I will draw the figure.

$$f \,\alpha\, \frac{1}{[k]}$$

Let us consider simply supported beam, we will consider 2 figures in one figure we will give unit rotation at j^{th} end this is my j^{th} end this is my k^{th} end is it not, we will give unit rotation. So, I am going to now use flexibility coefficients, I know flexibility and stiffness are inversely proportional we know that. So, I will find the flexibility coefficient matrix and invert it to get the stiffness matrix which is rotational matrix to do that I want to give.

Let us say unit rotation, so let me give unit notation here. So, let us say I give θ_j is unit. So, I draw a tangent to this I call this as δ_{jj} and I call this as δ_{kj}, where these are flexibility coefficients. Similarly I give unit rotation at the k^{th} end and I call this as δ_{kk} and I call this as δ_{jk}. So now, imagine we will compare this.

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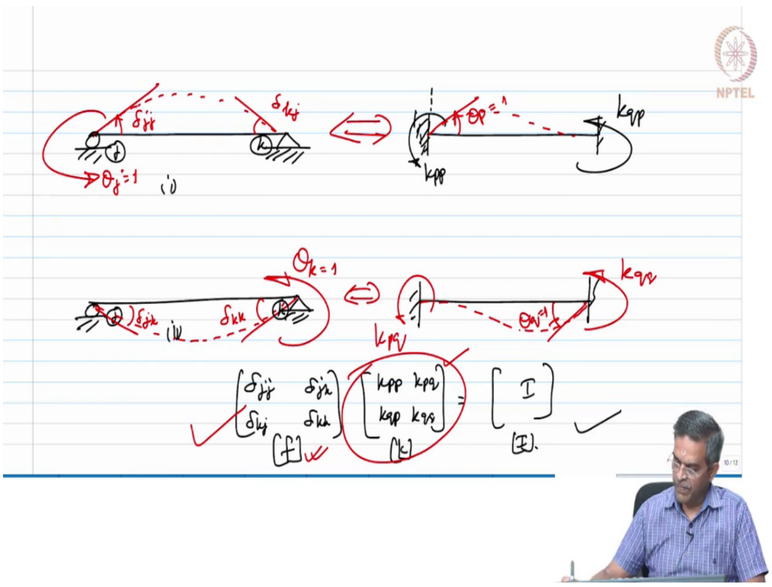

I will copy this I will compare this with our standard member which is initially normal and this is k_{pp} we know that and this was k_{qp} and this was θ_p which is unity. I am comparing these two figures, similarly an equivalent figure for this would be comparison of we can even draw this straight θ_q is unity and this was k_{qq} and this was k_{pq} and these are equivalent.

$$\left[\delta_{jj}\ \delta_{jk}\ \delta_{kj}\ \delta_{kk}\right]\left[k_{pp}\ k_{pq}\ k_{qp}\ k_{qq}\right] = [I]$$

$$[f]\ [k] = [I]$$

Now comparing these two figures I can write the control equation like this $\delta_{jj}\delta_{jk}\delta_{kj}\delta_{kk}$ which are flexibility coefficients multiplied by $k_{pp}\ k_{pq}\ k_{qp}\ k_{qq}$, will give me identity matrix.

This is 1 into 1 match, 1 is to 1 match given identity matrix. So, this my flexibility matrix, this my stiffness matrix, my anti matrix. Now my question is to find these values of flexibility matrix, if I know this, I can invert this matrix to get my rotational coefficient matrix you see here this is what I want this is what I want this is k_{pp} this is k_{pq} this is k_{qp} this k_{qq} this is what I want this is here.

So, to get this I will first find the flexibility matrix and invert it I will get this matrix.

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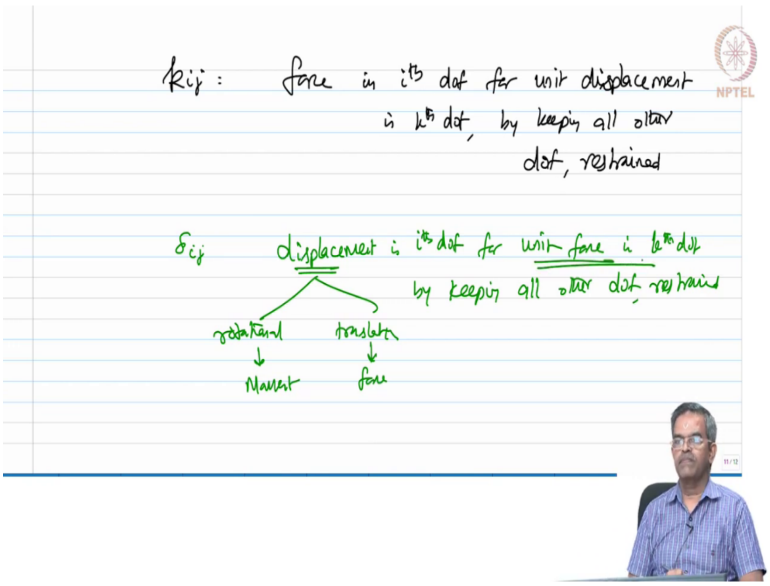

To get that flexibility is inverse of stiffness let us define what is stiffness first k_{ij} is force in i^{th} degree for unit displacement in the k^{th} degree by keeping all other degrees of freedom restrained. Now I want to write the flexibility definition, flexibility coefficient will be displacement in i^{th} degree for unit force in k^{th} degree by keeping all other degrees of freedom restrained is it not just a reverse of this.

So, displacements can be of 2 rotational translational, if I give a rotational displacement I will get a moment if I give a translational displacement I get a force. So now, I must give unit rotation at i^{th} degree I mean I have to find the rotation i^{th} degree by giving unit moment at the k^{th} degree, that is what I am going to do now let us do that.

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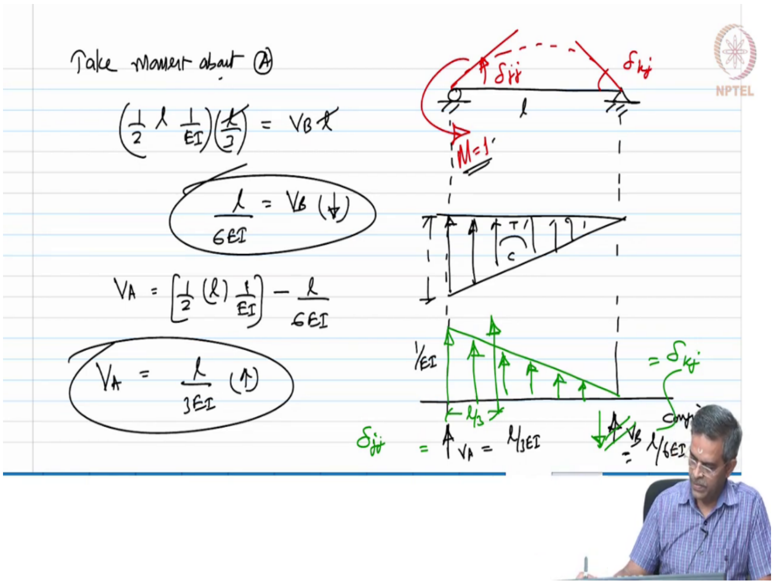

So, let me draw the figure of the simply supported beam again is having length L this my displaced configuration.

$$\left(\frac{1}{2} l \frac{1}{EI}\right)\left(\frac{l}{3}\right) = V_B$$

$$\frac{l}{6EI} = V_B$$

$$V_A = \left(\frac{1}{2}(l)\frac{1}{EI}\right) - \frac{l}{6EI}$$

$$V_A = \frac{l}{3EI}$$

I want to give unit moment and the displacement cost is rotation which is δ_{jj} and here this is δ_{kj}. So, let me draw the bending moment diagram of this. So, I have given unit moment here and this is 0. So, by this logic tension will be the top and compression will be the bottom is it not I given moment like anti clockwise here. So, let me apply this as an M by EA diagram to my beam. So, this is my conjugate beam I am loading this beam with my M by EA diagram this is my M by EA diagram. This is how this diagram is done.

So, this is going to be 1 by EI why one because this is actually unity that is what is here right. So, this will have a CG somewhere here this will be at the distance l by 3. Now this will invoke reaction here and here let us see we call this as V_B this is V_A. Now let us take moment about A. So, half into base into height that gives the area of the triangle into the distance of the CG from a this is l by 3 should be equal to V_B into l provided V_B is now opposite.

So, I can now find V_B as so l, l goes away. So, it is going to be l by 6 EI am I right its acting downward. Now I can say V_A will be equal to half into base into height the total force minus l by 6 EI that is what my V_A is? So, this will give me V_A as l by 3 EI acting upward. So, this is l by 3 EI this is l by 6 EI is acting downward this acting upward. So, I have V_A and I have V_B.

Now, as per the conjugate beam effect this will be the displacements. So, I can straight away say they are going to be I can write here this is going to be δ_{jj} and this is going to be δ_{kj}. I can do the same way for the other end.

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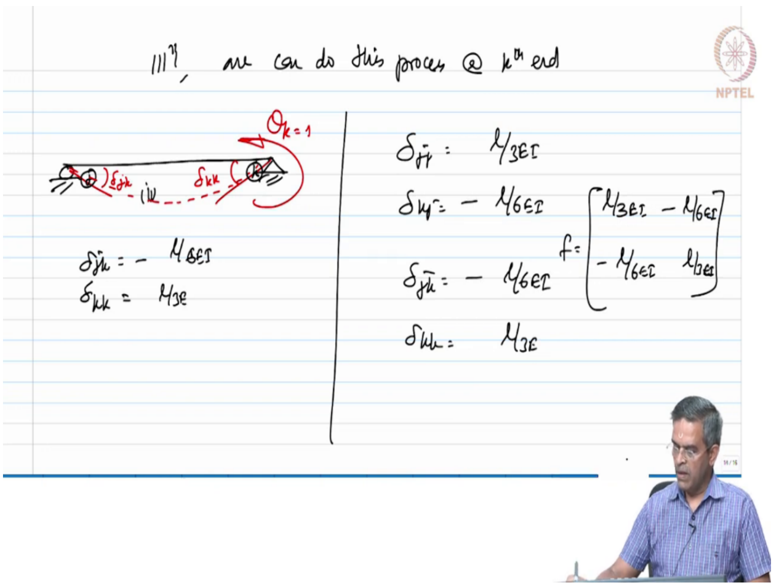

Similarly, one can do this process at k^{th} end. So, if you do that you will get the same equation.

$$\delta_{jj} = \frac{l}{3EI}$$

$$\delta_{kj} =- \frac{l}{6EI}$$

$$\delta_{jk} =- \frac{l}{6EI}$$

$$\delta_{kk} = \frac{l}{3EI}$$

$$f = \left[\frac{l}{3EI} \ \ -\frac{l}{6EI} \ \ -\frac{l}{6EI} \ \ \frac{l}{3EI}\right]$$

We do that you will find that we wanted to find $\delta_{kk}\delta_{jk}$. δ_{jk} will be minus 6 l by 6 EI and this will be l by 3 EI. So, we have now all the 4 values we have δ_{jj} as l by 3 EI δ_{kj} as l by 6 EI then δ_{jk} as minus l by 6 EI and δ_{kk} as l by 3 EI can write this in a matrix form as l by 3 EI minus l by 6 EI minus l by 6 EI and l by 3 EI this my flexibility matrix.

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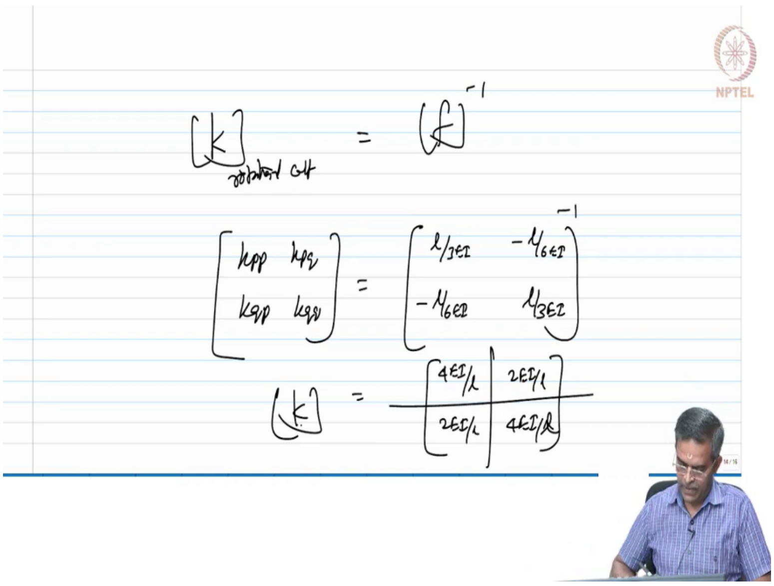

I want to find the stiffness matrix of rotational coefficients which will be inverse of this matrix. So, that will be k_{pp} k_{pq} k_{qp} k_{qq} which will be conduct inverse of this which is l by 3 EI minus l by 6 EI minus l by 6 EI and l by 3 EI inverse, you will find this matrix as 4 EI by l

2 EI by l 2 EI by l 4 EI by l. So, the rotational coefficients can be obtained like this let us go back to the original matrix.

$$[k]_{rotational\ dof} = [f]^{-1}$$

$$\left[k_{pp}\ k_{pq}\ k_{qp}\ k_{qq}\right] = \left[\frac{l}{3EI}\ -\frac{l}{6EI}\ -\frac{l}{6EI}\ \frac{l}{3EI}\right]^{-1}$$

$$[k] = \left[\frac{4EI}{l}\ \frac{2EI}{l}\ \frac{2EI}{l}\ \frac{4EI}{l}\right]$$

So, we are talking about the rotational coefficients which are these 4, if I know these fours k_{pp} $k_{pq}k_{qp}$ k_{qq} is what we have just found out. So, let me say if I know this I can write the remaining, let us see how? Let us copy this I want the full stiffness matrix, let us rub the remaining one let us see how do you fill up this we know this.

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$$[k] = \left[\frac{4EI}{l}\ \frac{2EI}{l}\ \frac{6EI}{l^2}\ -\frac{6EI}{l^2}\ \frac{2EI}{l}\ \frac{4EI}{l}\ \frac{6EI}{l^2}\ -\frac{6EI}{l^2}\ \frac{6EI}{l^2}\ \frac{6EI}{l^2}\ \frac{12EI}{l^3}\ -\frac{12EI}{l^3}\ -\frac{6EI}{l^2}\ -\frac{6EI}{l^2}\ -\frac{12EI}{l^3}\ +\right.$$

Let us rub this this is 4 EI by l and this is 2 EI by l and this is 2 EI by l and this is 4 EI by l just now we computed there see here just now we computed. Once you know this sum of these two by l is 6 EI by l square minus of this. Similarly sum of these two by l 6 EI by l square then minus of this, then sum of these two by l sum of these two by l then sum of these two by l, then minus of that the last column is negative of the third column. That is how we have the stiffness matrix of the beam element is ready.

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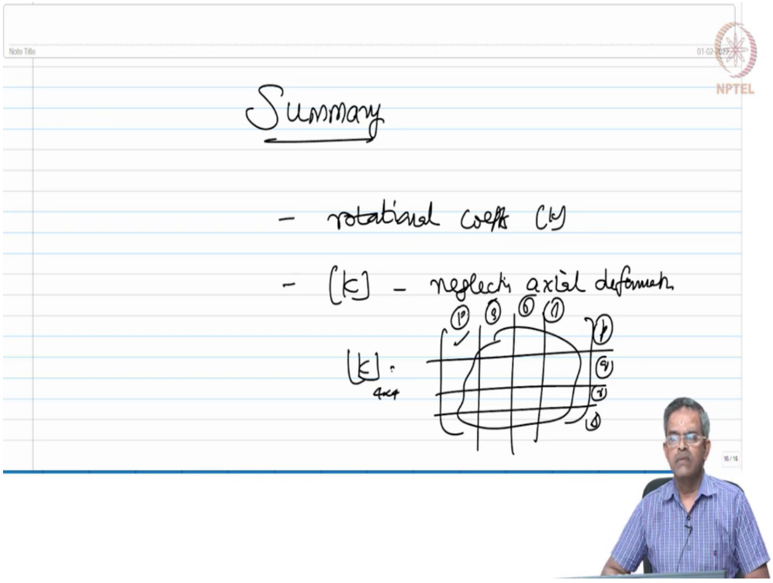

So, friends in this lecture we learnt how to derive the rotational coefficients of stiffness matrix. We also learnt how to formulate the complete stiffness matrix neglecting axial deformation and we learnt that k is 4 by 4 is very easy to write. If you know this label p q r and s if I know this value I can keep on writing the remaining all very easily based on this. So, in the next lecture we are going to continue discussing stability functions and axial compression. But based on the same derivation technique what we used for a fixed beam.

So, keep on revising what we discussed in the previous lectures that will be very helpful for you and the reference already slated for you. Please go through the additional references given in the website of this course and improve your additional learning skills in parallel. So, that the derivation becomes very clear for you.

Thank you very much and have a good day bye.