Advanced Design of Steel Structures
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Lecture - 03
Failure theories -1

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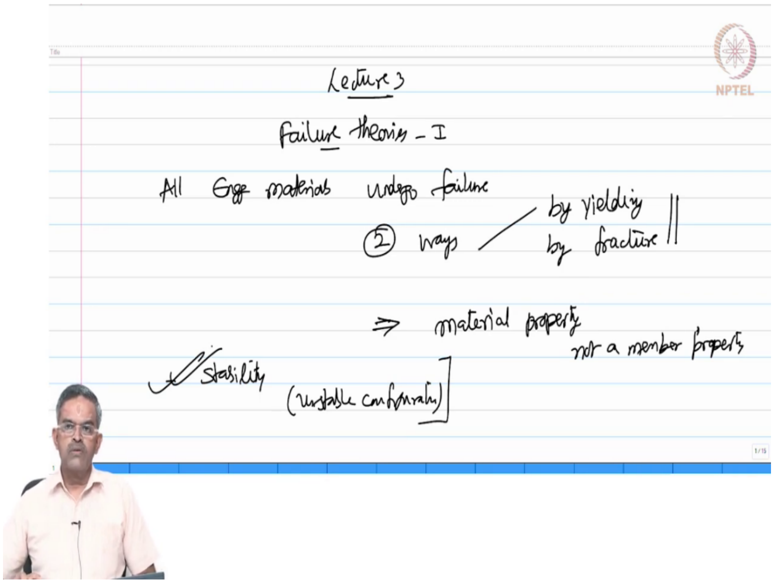

Friends, welcome to the 3rd lecture in which we are going to learn more on Failure theories. So, I will put this as Failure theories – I, this will have about series of lectures, where we are going to talk about the failure theories in detail. We know all engineering materials undergo failure, this failure can be in 2 ways, it can be either by yielding or by fracture. When we talk about failure, we are very closely associating this as a material property; it is not a member property.

Then one may ask me a question, if I wish to associate failure as a member property, what can I call that as in the present context of form dominant design, the answer could be if you want to associate failure, with respect to the geometric form, but not the material then I can say this can be related to stability or I can say it is a unstable configuration.

So, when we talk about failure of a geometric form, then stability is checked. When we talk about material failure related to the material form, it can occur broadly in two ways, one is yielding other is fracture. So, friends let us quickly see the difference between these two, they are definitely not same and as we all know that.

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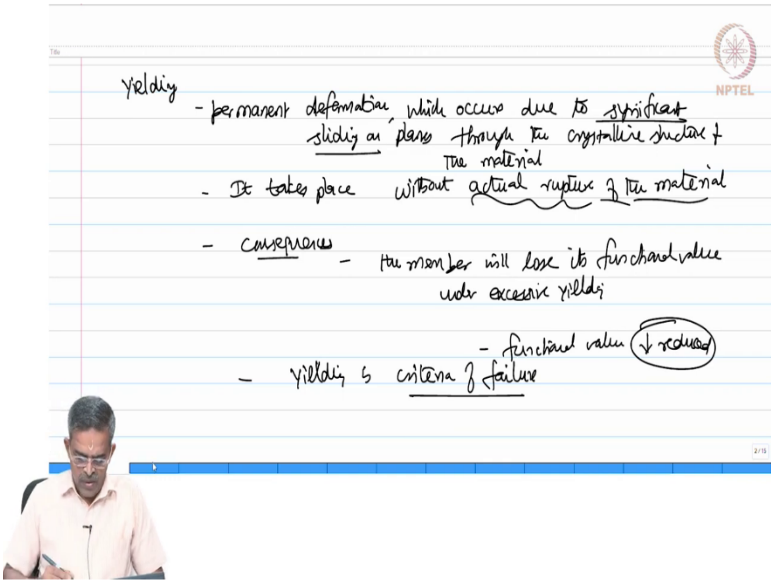

Yielding is a permanent deformation, which occurs due to significant sliding on planes through the crystalline structure of the material. I am defining this in the material perspective friends. When does it take place? It takes place without actual rupture of the material. What will be the consequence of yielding? One of the serious consequences of yielding is the member will lose its functional value under excessive yield.

Instead of saying we will lose we will say the functional value will get reduced ok, let us put it like this. Since the functional value of the member is deteriorated we can always say yielding is a criteria of failure though yielding actually is not actual rupture of the material, the material does not get ruptured, but still it is a failure criteria. Because it has a very severe consequence of degrading the functional value of the member therefore, yielding is considered as a criteria failure.

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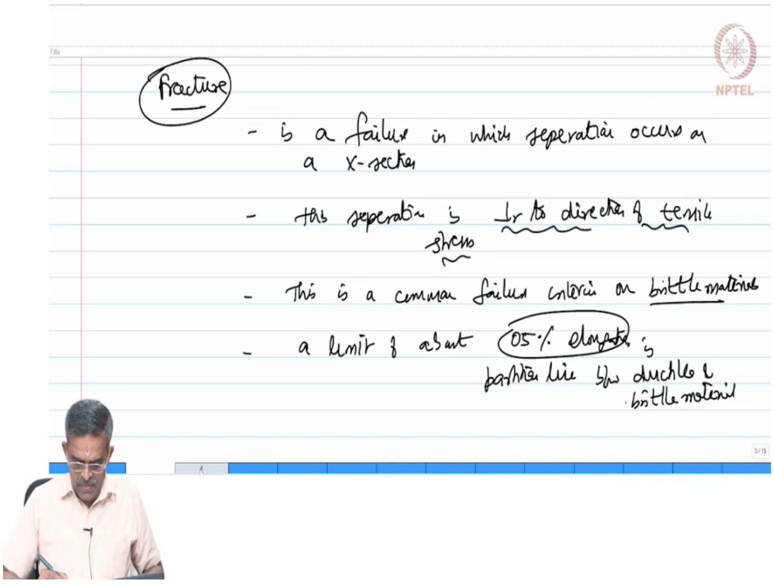

Alternatively what is fracture? Fracture is a failure in which separation occurs on a cross section, this separation is perpendicular to the direction of tensile stresses. Friends please note: fracture is also associated to the nature the direction of the stress. This is a very common failure criteria on brittle materials.

In practice a limit of about 5 percent elongation is the partition line between the ductile and the brittle materials. What does it mean is, if the elongation exceeds 5 percent beyond the elastic limit elongation, then the material can be said as a ductile material. Otherwise, the material is designated as a brittle material; this 5 percent elongation is just a thumb rule.

Friends, all material do not obey this partition line and I am not talking about only steel here, I am talking about materials in general. So, fracture is also a failure, yielding is also a failure, though yielding does not show any pronounced rupture of the material, but it degrades the functional value of the member therefore, it is considered as a failure criteria, what are theories saying about this ok.

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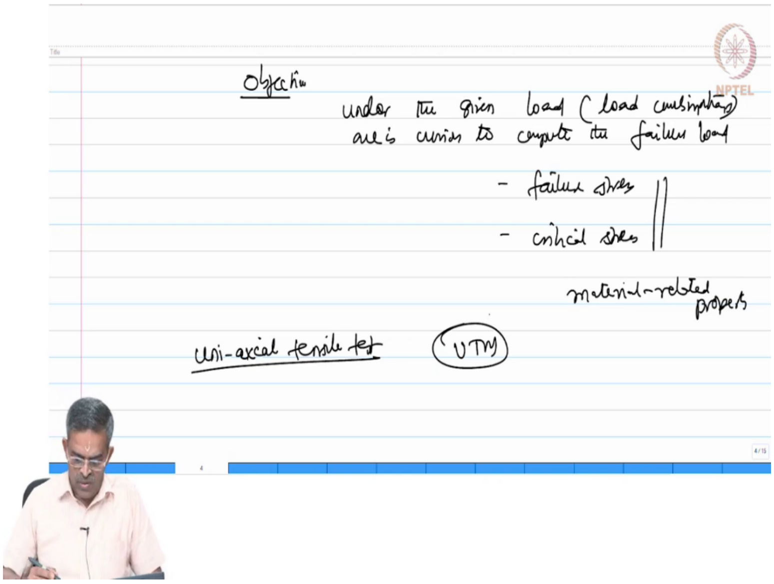

Now, when we talk about estimating this failure low so, what is our objective? Under the given load or to be very precise load combinations, one is interested to know the failure. In fact, to be a very precise one is center to know the failure stress, but it is addressed as critical stress.

So, we all agree that it is a material related property, generally in engineering perspective you have learnt about estimating this failure loads in your undergraduate level itself. We conduct something called uniaxial tensile test, is it not we are conducted this in an UTM, we used to conduct this test. So now, let us pay attention more to this to explain the failure theory.

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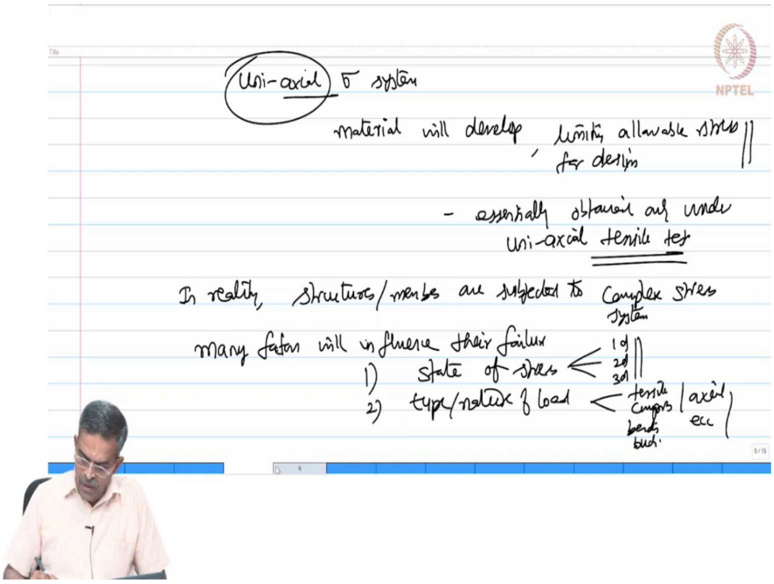

When we talk about an uniaxial stress, system the material will develop, limiting allowable stress for design. All codes look into the stress strain curve of the material, fix certain limits and say that is my design stress, is it not. Take for example, IS 800, IS 456 the steel and concrete design codes, they look into the stress strain curve of the material and fix up the value as a design stress.

And they are all based exactly on uniaxial stress system ok, which is essentially up time please note this is essentially up time only under uniaxial tensile test ok. It is always assume that the material obeys the same property both in tension and compression, with an ideology many material do not obey this ideology, please understand that ok.

So, in reality, structures or members are subjected to complex stress system. So, this uniaxial concept is actually ideal. In reality this does not work members are under complex stress systems. Therefore, many factors will influence their failure. What are those factors? State of stress that is it uniaxial, bi-axial or triaxial is it a 1-dimensional stress state, 2-dimensional or 3-dimensional stress state.

So, state of stress will govern the failure criteria, the second could be type and nature of load. If the load is tensile compressive, if it is axial or if it is eccentric will it cause bending and buckling?

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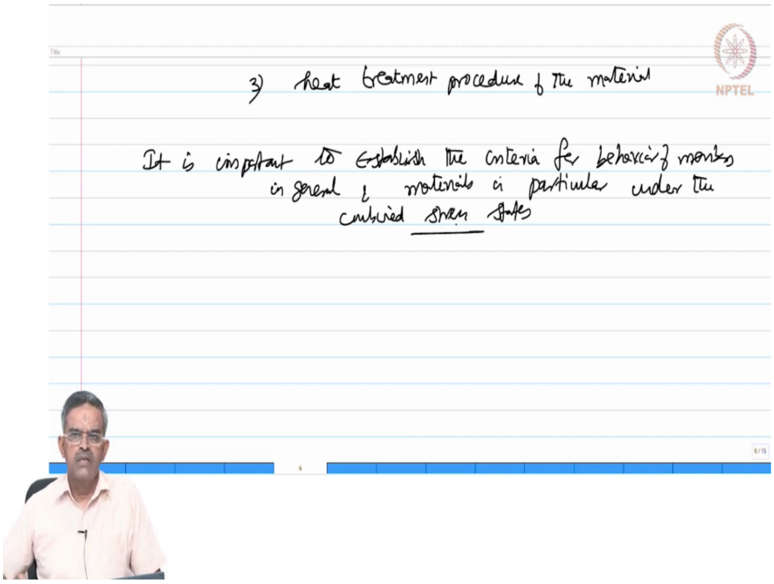

The third could be heat treatment procedure of the material, which is used for fabricating that structure. Therefore friends, since many factors govern the failure criteria in reality it is important to establish the criteria for behavior of the members in general and behavior of the materials in particular under the combined stress states; is it not?

But so far, we always estimate the permissible loads based on uniaxial tensile test, then one may ask me a question, what are those challenges which are over sighted when we use the test results from uniaxial tensile test?

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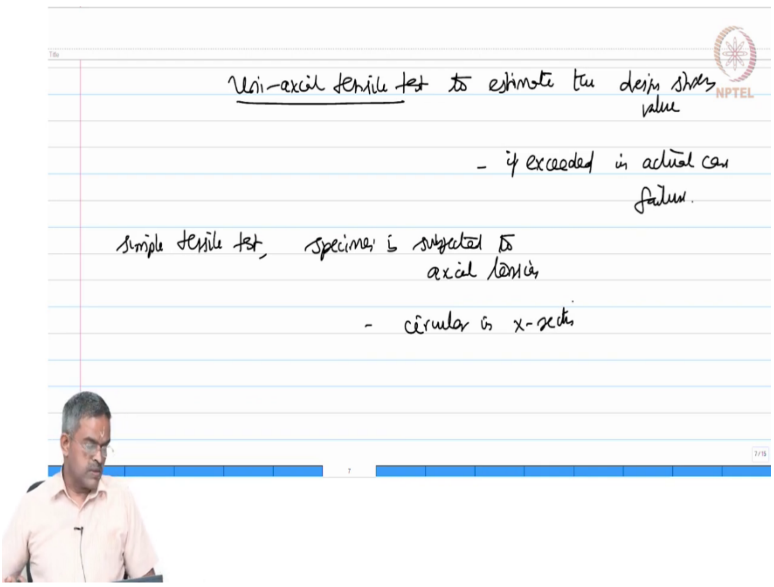

So, if you use an uniaxial tensile test, to estimate the design stress value and if this design stress value is exceeded in the actual case then we call this as a structural failure that is right. So, all the time we try to fix up the threshold limit of the stress of the material based upon uniaxial tensile test.

And we compare the actual stresses coming on the cross section with this value obtained from the uniaxial tensile test and then we declare whether structure is failed or satisfactory performed. That is the design process and procedure what we have been following for eras, but there are some difficulties associated with uniaxial tensile test. So, what is an uniaxial tensile test? It is a simple tensile test where the specimen is subjected to axial tension. Usually the specimen is circular in cross section, is usual practice ok circular in cross section.

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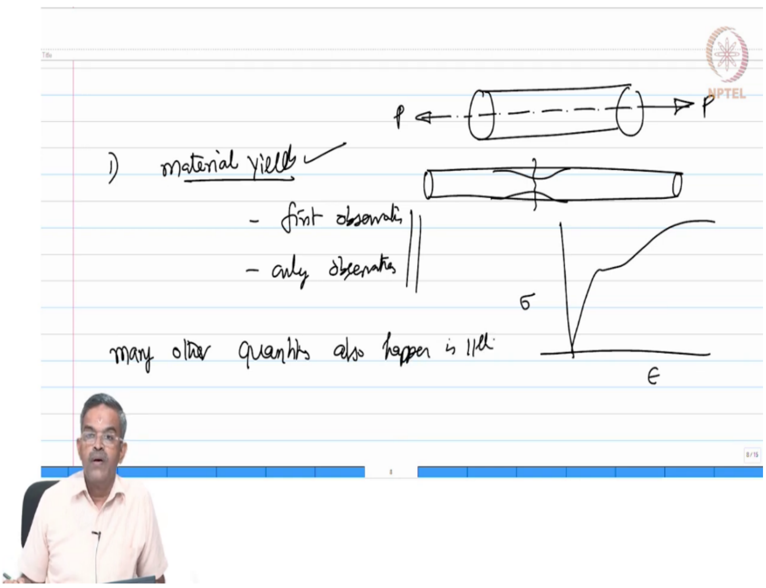

So, for example, when you take a circular bar, subject it to axial pull p the bar elongates and the diameter gets shortened and there is a neck formation and the member fails. This is a usual observation what we have seen and learned, in simple tension test in undergrad levels. So, if this material happens to be steel and try to plot the stress strain curve of this a typical stress strain curve of steel will look like this.

So, what we first observe is the material yields that is the first observation what we make, unfortunately friends that is the only observation we make. And based on this observation we fix everything elastic limit, proportionality, upper and lower yield point’s, ultimate strength, breaking strength, everything we fix looking only at this value. But there are many things which are happening in parallel to this, what are they?

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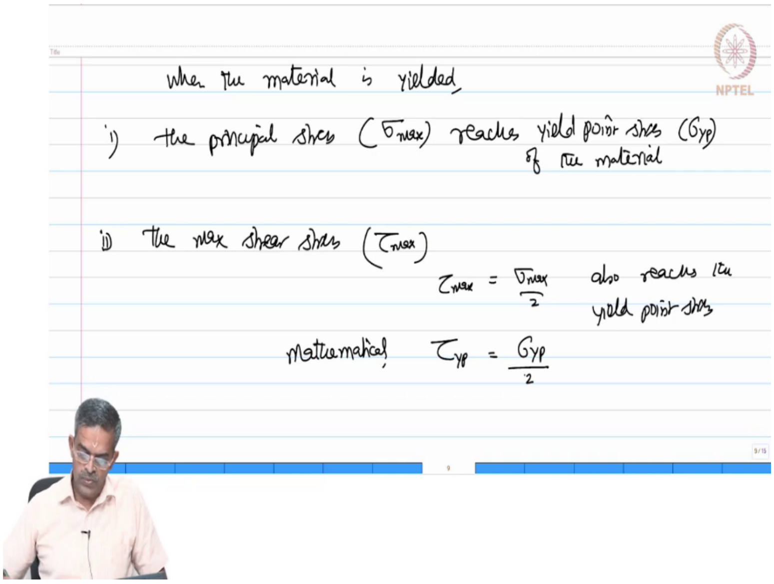

When the material is yielded, the principal stress which is addressed as σ max reaches yield point, stress which is σy, let us put it as σyp of the material. Secondly, the maximum shear stress which is τ max is usually taken as σ max by 2 also reaches the yield point stress. Mathematically, the yield point stress in shear is yield points in axial tension by 2. So, this happens simultaneously.

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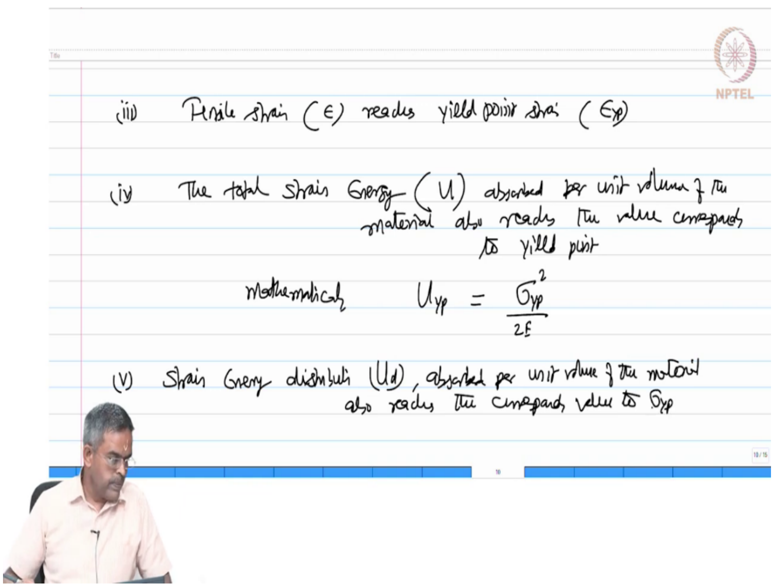

Thirdly, the tensile strain ε reaches yield point strain which is called as ε_{yp}. 4th, the total strain energy which is termed as u absorbed by the material per unit volume of the material also reaches the value corresponding to yield point; mathematically, u_{yp} is equal to σ_{yp}^2 by 2E. 5th, the strain energy distribution u_d absorbed per unit volume of the material also reaches the corresponding value to yield value; mathematically, u_d at yield point reaches 1 + μ by 3E of σ_{yp}^2.

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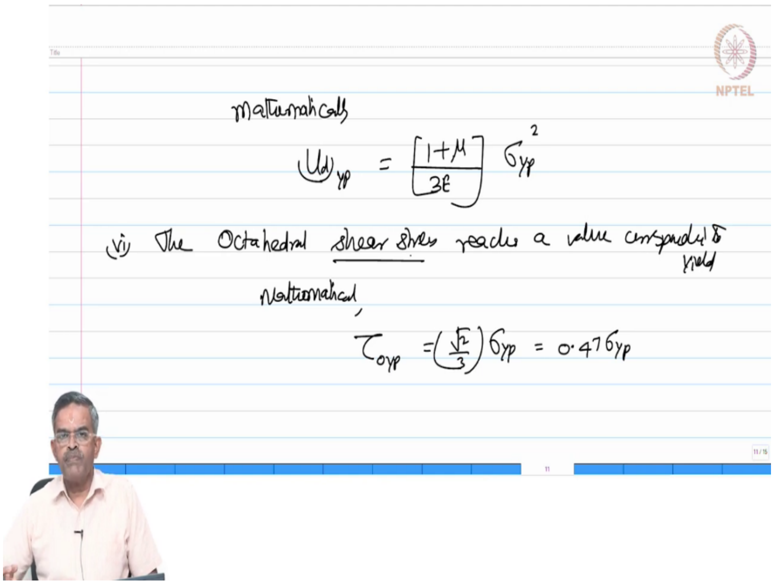

6, the octahedral shear stress reaches a value corresponding to yield mathematically octahedral stress at yield point is equal to √2 by 3 of σ_{yp}, which is about 0.47 σ_{yp}.

So, friends many quantities like shear stress, strain energy at distortion, tensile strain maximum shear stress, principal shear stress also reach the corresponding values, but they are all not considered when we try to fix up the threshold value of comparing the actual stress with the design stress limit for any distinct procedure. One may ask me a question, how these parallel values obtained during uniaxial tensile test govern the failure.

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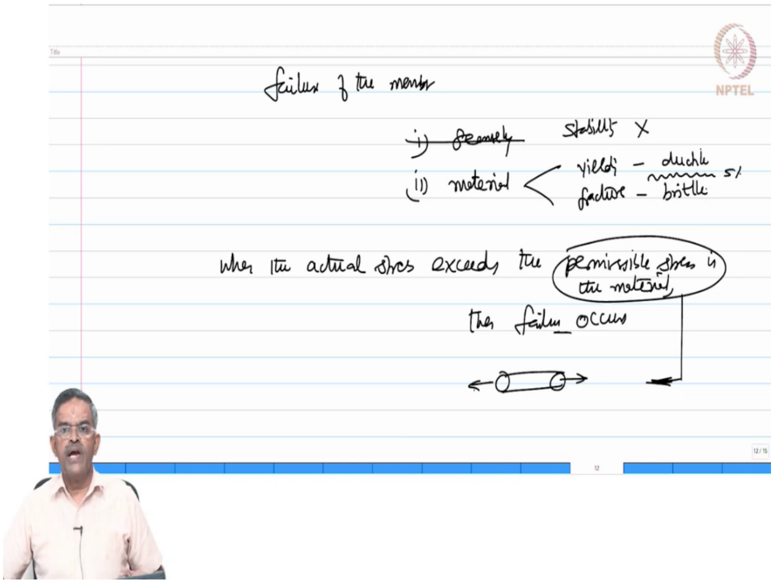

Now, let us redefine the failure again. We have agreed upon a concept that failure of a member is related to 2, 1 is by its geometry 2 by its material. The geometric failure is related to stability and let us say we ensure that the geometric failure does not happen. So, let us now try to define the failure only through the material perspective and this can have two things, one is yielding, other is fracture which we just now discussed.

To be very clear friends let us have slightly an open minded understanding, yielding is related to ductile material and fracture can be related to brittle material. And there is a very fine mark between the ductile and brittle metal material in terms of elongation which is about 5 percent, though all material do not obey this fine patch. So, when the actual load or when the actual stress exceeds the permissible stress in the material then we say it is a failure. So, all the time failure is correlated to the permissible stress in the material and this permissible stress in general is obtained from a uniaxial tensile test, whatever may be the material.

But in uniaxial tensile test, when the material undergoes yielding 6 more parallel engineering values are obtained corresponding to this yielding value. Those engineering values are becoming the governing criteria for defining the failure, friends.

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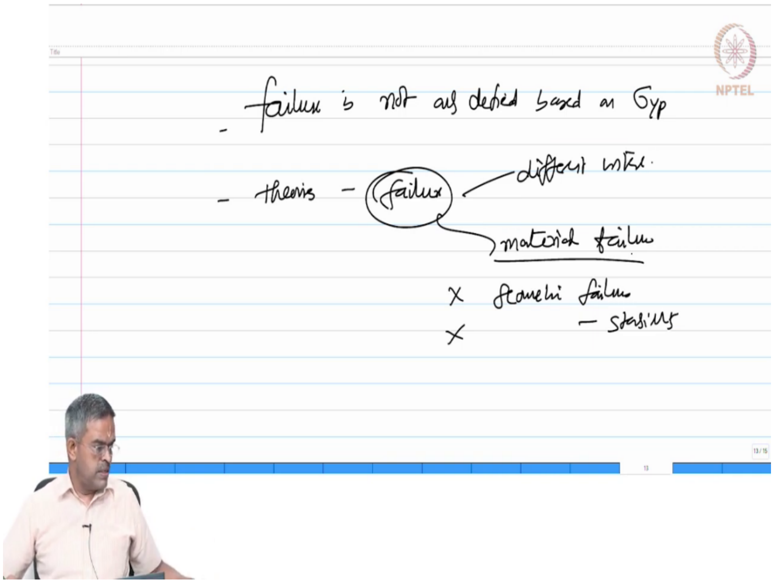

So, the failure is not only defined based on the yield value, there are many theories which classify the failure based on different criteria. Again, friends let me re-insist for our learning. We are talking only about the material failure.

We are not talking about the geometric failure which can be addressed otherwise by checking the stability conditions of the geometry. We are not discussing this now at this moment, we are focusing completely towards a failure criteria that arise only from the material. So, friends, let us see what all these theories addressing failure as, what are the governing factors which these theories talk about.

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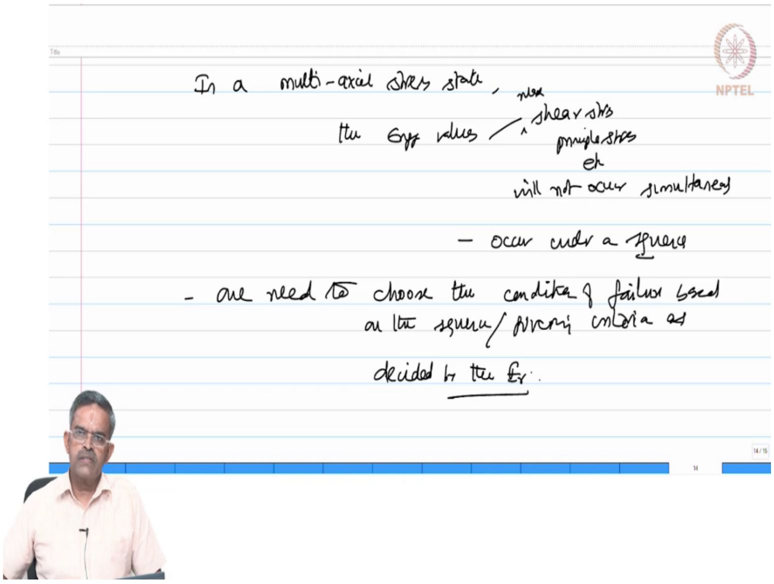

So, therefore, in a multi axial stress state the engineering values, like maximum shear stress, a principal stress etcetera will not occur simultaneously. So, they occur under a sequence and that sequence governs the failure criteria. So, therefore, one need to choose the conditions for failure based on the sequence or the governing criteria as decided by the engineer.

So, the decision cannot be purely based only on the yield stress value which is obtained from the uniaxial tensile test. So, engineers can always have a different governing criteria for design perspectives, based on this criteria based on the sequence of this failure an order of this criteria various theories define failure.

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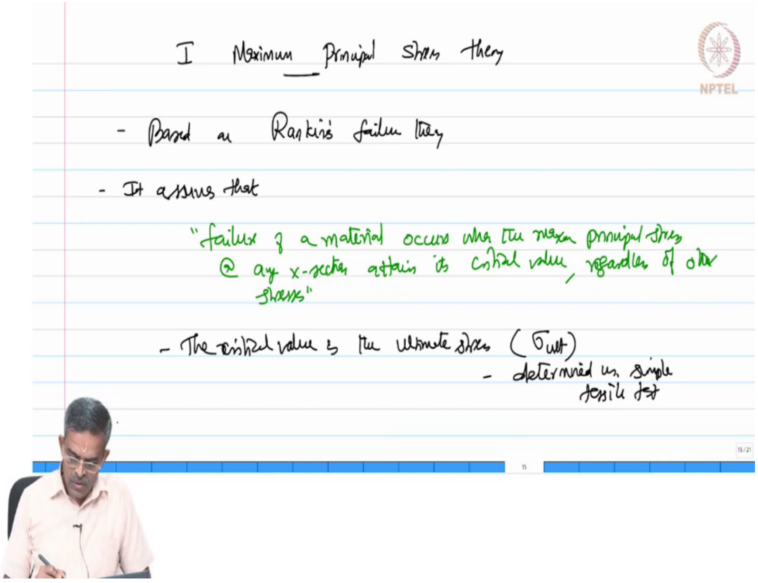

So, let us see the first one is a maximum principal stress theory, this theory is based on Rankine's failure theory, it assumes that failure of a material occurs when the maximum principal stress at any cross section attains its critical value regardless of other stresses.

Then what is the critical value? the critical value with which it is controlled is the ultimate stress, which I refer as $\sigma_{ultimate}$, this ultimate stress is determined using simple tensile test, that is what this theory says.

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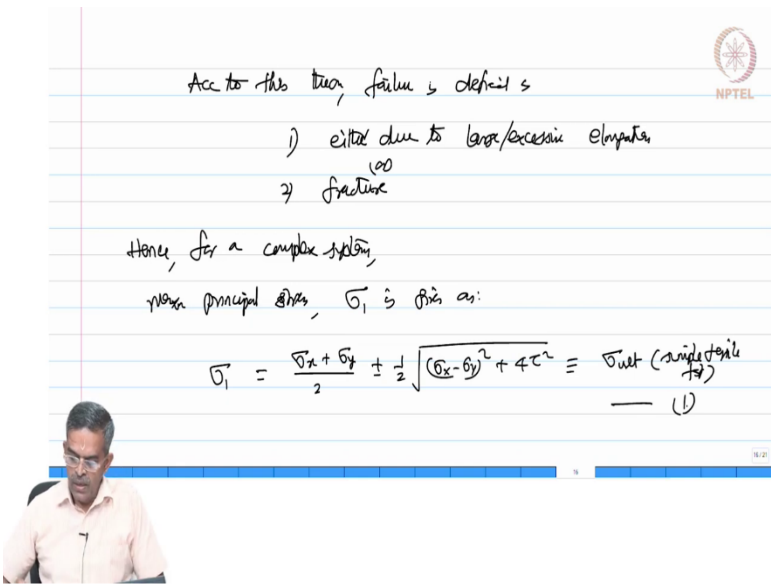

Therefore, according to this theory failure is defined as either due to large or excessive elongation, or it can be a fracture. Hence, for a system under complex loading, the maximum principal stress σ_1 is given by the following equation.

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y\right)^2 - 4\tau^2} = \sigma_{ult} \qquad \text{--------- (1)}$$

it is a classical equation, which we all know and we equate this to $\sigma_{ultimate}$ which is obtained from the simple tensile test.

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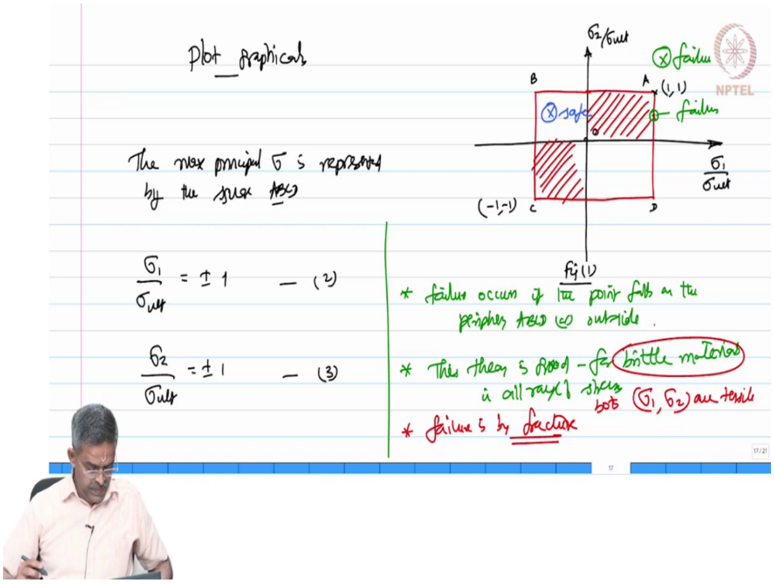

Let us plot this stress graphically, let us say this is my x and y axis. So, if this is my origin and this indicates $\sigma_1/\sigma_{ultimate}$ and this axis indicates $\sigma_2/\sigma_{ultimate}$, I like to draw the proportion.

So, in these points will be (1,1) and this will be (-1,-1). Let us name these points as A, B, C and D, this my origin O, this my failure envelope. So, any point lying outside the envelope is a failure point; any point lying inside the envelope is a safe point. So, try to find out the stress value, plot this ratio if it falls within this envelope then the member or the material is safe, it is beyond this envelope then it is a failure.

To be very clear, mathematically the maximum principal stress is represented by a square A, B, C, D as you see in figure 1. So, $\sigma_1/\sigma_{ultimate}$ is plus or minus 1, $\sigma_2/\sigma_{ultimate}$ is plus or minus 1. We call equation number 2 and equation number 3. So, friends based on this we can write some observations. Failure occurs if the point falls on the periphery A, B, C, D or outside.

So, even if it falls on the periphery, it is a failure. Interestingly experiments are conducted and those works show that this theory is good for brittle materials in all ranges of stresses provided both σ_1 and σ_2 are tensile. So, very clearly this theory says a failure is by fracture. So, in the 1st quadrant and in the 3rd quadrant the theory is in very good agreement for brittle materials.

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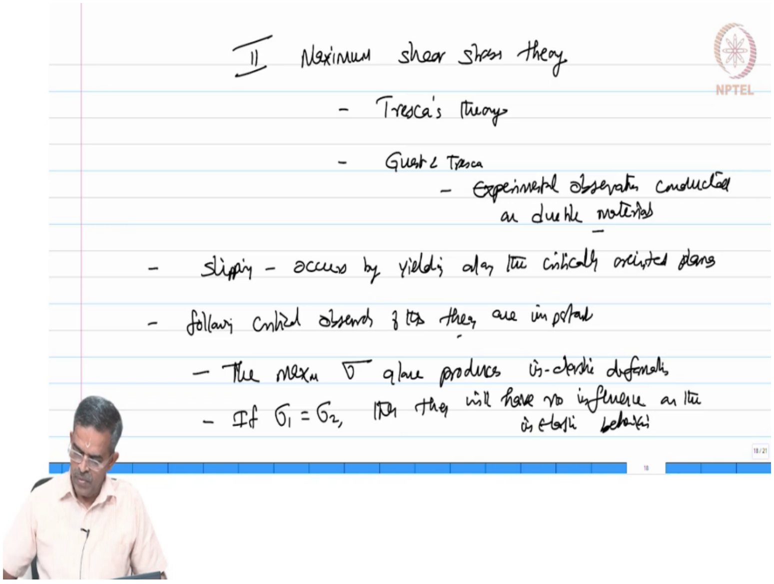

Let us talk about the next theory which is maximum shear stress theory. This theory is also called as Tresca's theory. This theory was suggested by Gust and Tresca based on the experimental observations conducted on ductile materials, they observe that slipping between the crystalline structure of the material occurs by yielding along the critically oriented planes. This theory is based on certain following observations;

1. The maximum stress alone produces in-elastic deformation

2. If σ_1 is equal to σ_2 then they will have no influence on the inelastic behaviour.

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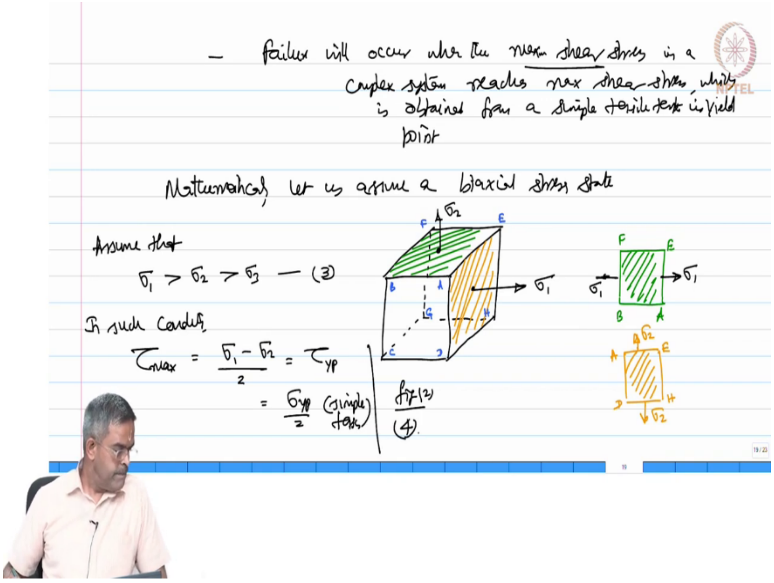

3. Failure will occur when the maximum shear stress, see the deviation. In a complex system reaches the maximum shear stress value, which is obtained from a simple tensile test in yield point. Mathematically, let us assume a bi-axial system or bi-axial stress state, the bi axial stress state is plotted. Let us hatch the surfaces, we call this stress σ1, we call this stress as σ2, let us say figure 2. Let us name the plane A, B, C, D, E, F, G, H.

Now, let us pick up the plane separately as seen here. Let us say I pick up a plane which is A, B, E, F, the green one let us pick up the green one. The green one says this is A, this B, this E and this is F and this is now subjected to let us say σ1. That is then draw the next plane which is A, E, H, D and this is now subjected to σ2.

We also assume that σ1 is greater than σ2 is greater than σ3. In such condition τ_{max} is actually equal to σ1 minus σ2 by 2. I equate this to the shear stress obtained from the yield point, usually this value is equal to the yield stress by 2, in simple tension. I call this equation number 4.

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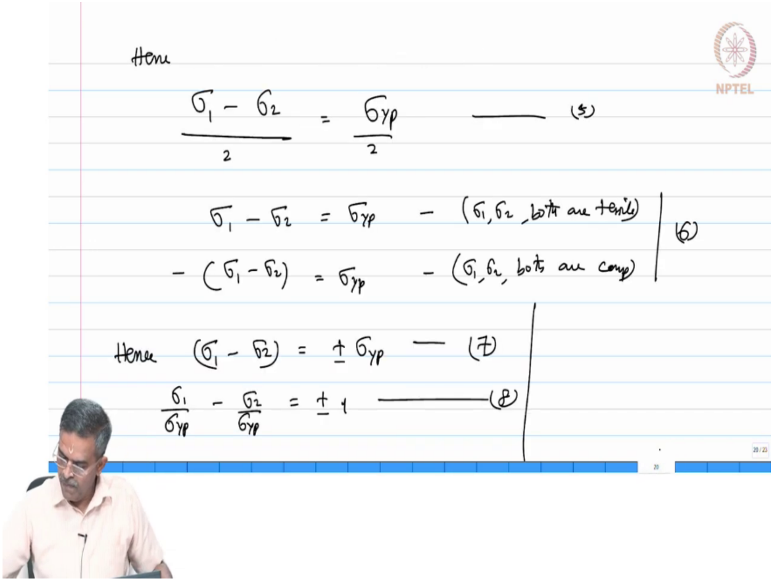

So, therefore,

$$\frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_{yp}}{2} \quad \text{--------- (5)}$$

$$\sigma_1 - \sigma_2 = \sigma_{yp}$$

$$-(\sigma_1 - \sigma_2) = \sigma_{yp}$$

$$\sigma_1 - \sigma_2 = \pm\sigma_{yp}$$

$$\frac{\sigma_1}{\sigma_{yp}} - \frac{\sigma_2}{\sigma_{yp}} = \pm 1$$

So, that gives me failure envelope based on this particular theory.

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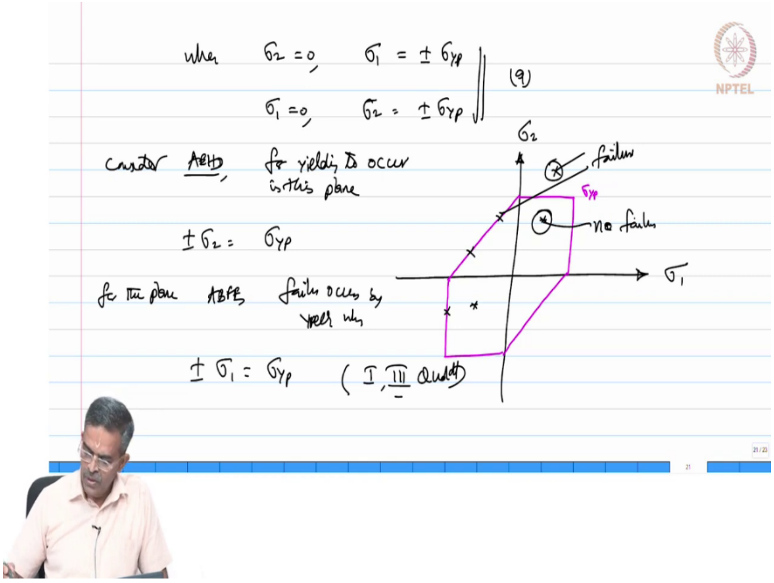

Interestingly friends, when σ2 is 0, σ1 will be plus or minus σ_{yp} when σ1 is 0, σ2 will be plus or minus σ_{yp}. So, this can be expressed graphically as below, say this is my σ1 and σ2 axis. Let us plot plus or minus 1 so, this is σyp, then plus or minus 1 in the third quadrant and the failure envelope simply joins this way.

So, this becomes my failure envelope. So, in this figure consider plane A, E, H, D, for yielding to occur in this plane plus or minus σ2 should be equal to σ_{yp}. similarly for the plane A, B, F, E failure occurs by yielding when plus or minus σ1 is σ_{yp}.

So, this will represent the first and third quadrants of the failure envelope. If there is a point which is lying in between here inside this envelope no failure, if it lies on the envelope or outside the envelope this indicate failure.

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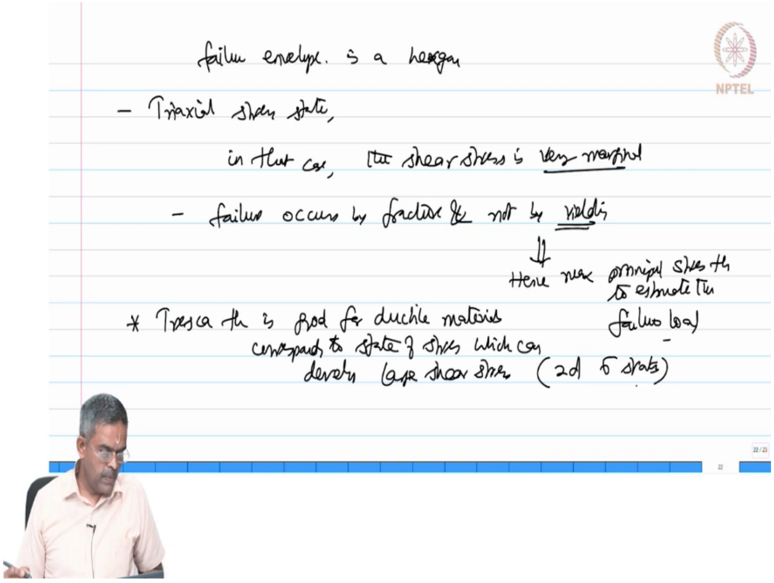

So, the failure envelope in this case is a hexagon. If you consider a triaxial stress state, then in that case the shear stress is very marginal in magnitude, in that case failure occurs by fracture and not by yielding. So, in that case you must apply maximum principal stress theory, to estimate the failure load. Friends, there are some observations and literature based on this theory. It says that Tresca's theory is good for ductile materials corresponding to state of stress which can develop large shear stress. So, in short it is very good for 2-dimensional stress states.

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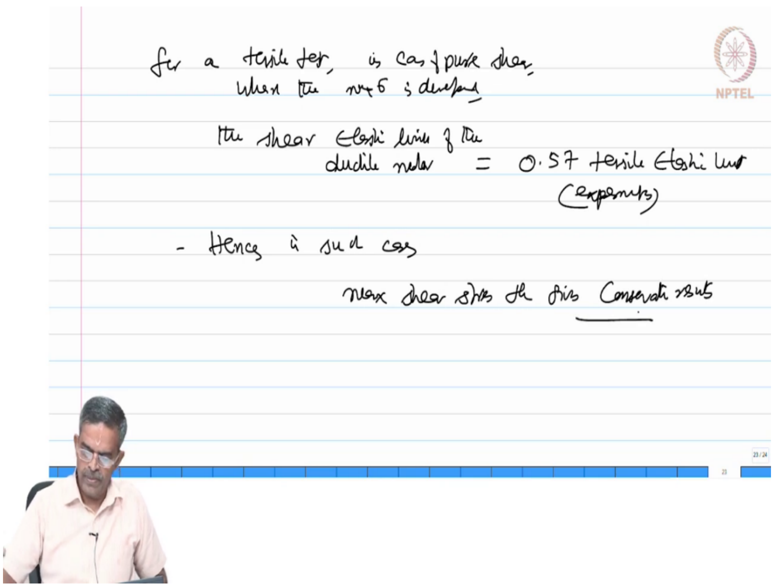

For a tensile test, in case of pure shear, where the maximum shear is developed the shear elastic limit of the ductile material is taken as about 0.57 of the tensile elastic limit. This is by experimental observations. Hence in such cases maximum shear stress theory gives conservative results.

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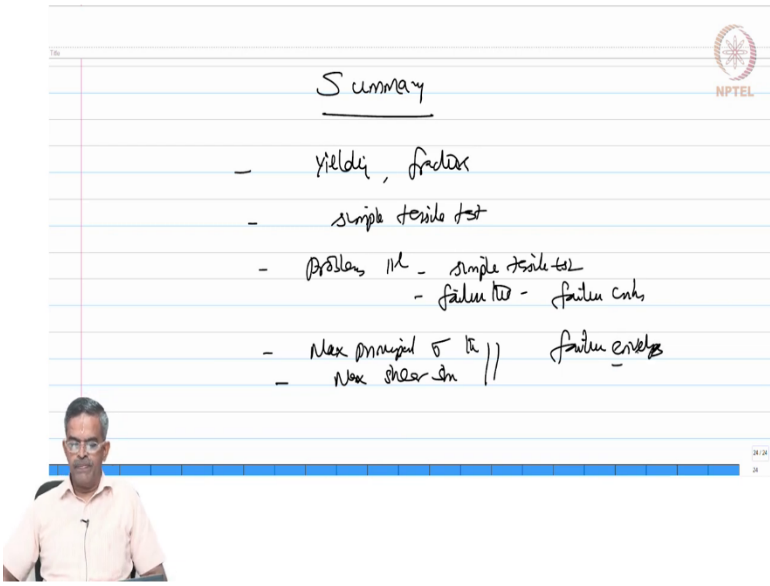

So, in this lecture friends, we have learnt what is the difference between yielding and fracture. Why simple tensile test cannot be a governing criteria for design, what are the problems associated in parallel with this simple tensile test and how the failure theories argue based on the failure criteria.

Then we have learnt the maximum principal stress theory, then maximum shear stress theory and we understood the failure envelope of both these theories in this lecture. I believe you will be able to follow this and have a good reading back on this with additional support material and any doubts accumulated you will post back to me in the discussion forum.

Thank you very much, have a good day, bye.