Advanced Design of Steel Structures
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Lecture - 30
Stability functions - 2

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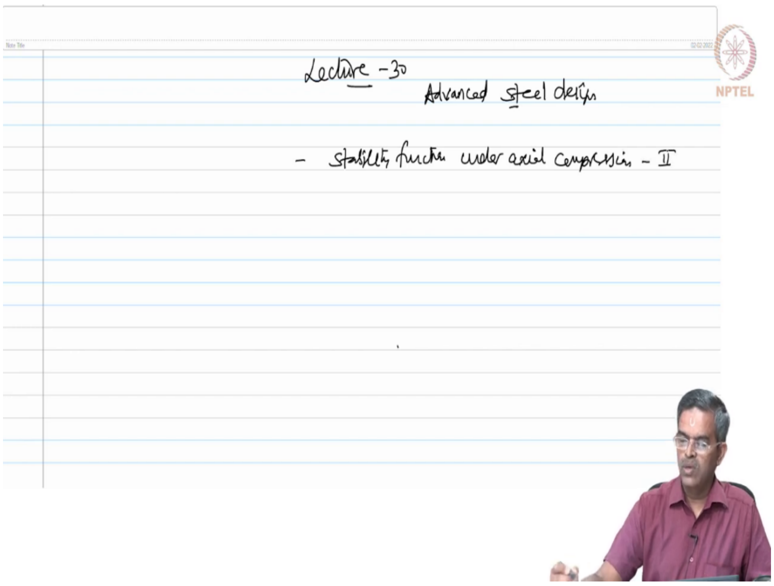

Friends, welcome to the lecture 30 on Advanced Steel Design. We will continue to discuss Stability Functions under axial compression, we will call selection number II on the same perspective.

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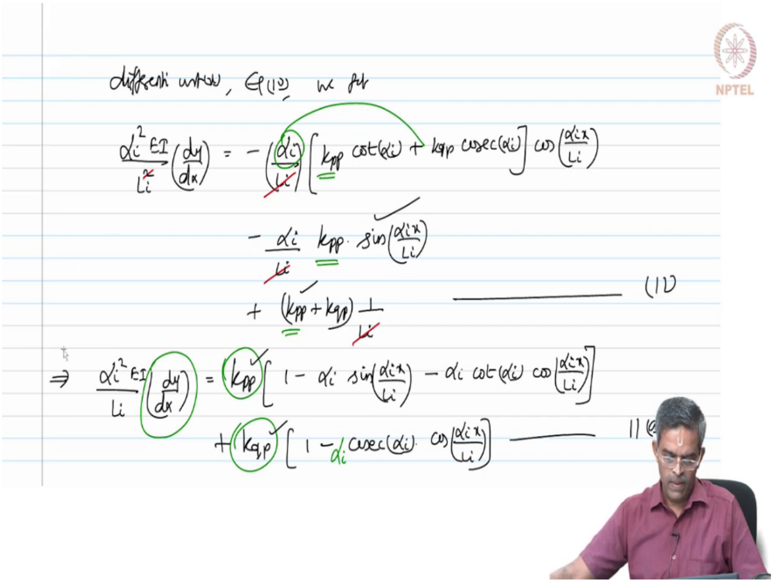

$$\frac{\alpha_i^2 EI}{L_i^2}\left(\frac{dy}{dx}\right) =- \left(\frac{\alpha_i}{L_i}\right)\left[k_{pp}\cot cot\left(\alpha_i\right) + k_{qp}\csc csc\left(\alpha_i\right)\right]\cos cos\left(\frac{\alpha_i x}{L_i}\right) - \left(\frac{\alpha_i}{L_i}\right)k_{pp}\sin sin\left(\frac{\alpha_i x}{L_i}\right) + \left(k_{pp} +\right.$$

$$\frac{\alpha_i^2 EI}{L_i}\left(\frac{dy}{dx}\right) = k_{pp}\left[1 - \alpha_i \sin sin\left(\frac{\alpha_i x}{L_i}\right) - \alpha_i \cot cot\left(\alpha_i\right)\cos cos\left(\frac{\alpha_i x}{L_i}\right)\right] + k_{qp}\left[1 - \alpha_i \csc csc\left(\alpha_i\right)\cos cos\left(\right.\right.$$

In the last lecture, we derived equation 11 (a) which is expressed in terms of k_{pp} and k_{qp}. So, here on the left-hand side you will see there is a term representing the slope $\left(\frac{dy}{dx}\right)$ is the slope of the v, is it not which is given as a function of k_{pp} and k_{qp} which is given by the equation 11 (a).

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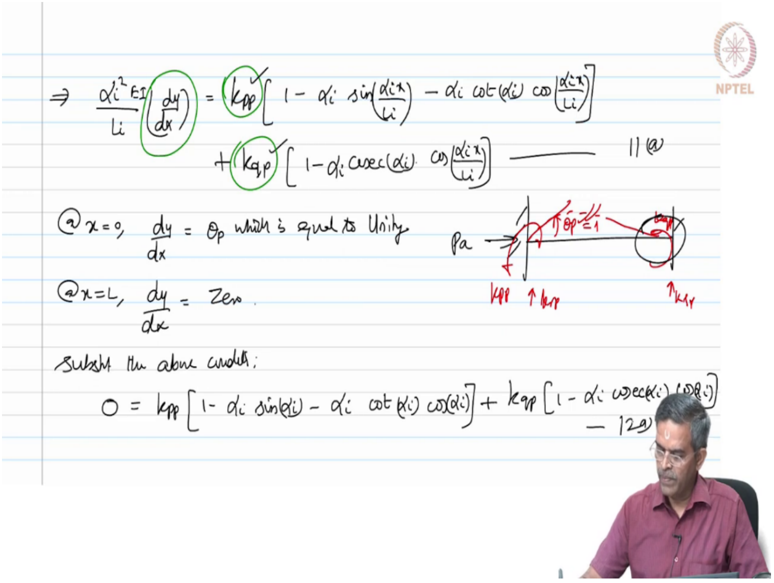

$$\frac{\alpha_i^2 EI}{L_i}\left(\frac{dy}{dx}\right) = k_{pp}\left[1 - \alpha_i \sin sin \left(\frac{\alpha_i x}{L_i}\right) - \alpha_i \cot cot \left(\alpha_i\right) \cos cos \left(\frac{\alpha_i x}{L_i}\right)\right] + k_{qp}\left[1 - \alpha_i \csc csc \left(\alpha_i\right) \cos cos \left(\right.\right.$$

We will copy this equation to the next screen and we anyway replace this with black color. So, we will do this, 11 (a).

@x=0, $\frac{dy}{dx} = \theta_p$ *which is equal to unity.*

@x=L, $\frac{dy}{dx}$ = Zero.

Substitute the above condition,

$$0 = k_{pp}\left[1 - \alpha_i \sin sin \left(\alpha_i\right) - \alpha_i \cot cot \left(\alpha_i\right) \cos cos \left(\alpha_i\right)\right] + k_{qp}\left[1 - \alpha_i \csc csc \left(\alpha_i\right) \cos cos \left(\alpha_i\right)\right]$$

Now, we know at x equal 0, $\frac{dy}{dx} = \theta_p$ because that is how we gave the rotation. This was the beam under axial compression P_a and we gave a rotation which is unity and this was θ_p we gave.

That is how we generated k_{pp} and we say k_{qp} δ_{rp} or k_{rp} and k_{sp}, that is how we did, is it not. So, we are given θ_p which is equal to of course, unity, but we have θ_p. So, let us say θ_p

which is equal to it whereas, at x is equal to L the slope is 0, see here. @x=L, $\frac{dy}{dx}$ = Zero. So, let us substitute this condition and see what happens; substituting the above conditions.

So, let us say slope is 0. So, that is going to happen at x is equal to L. So, let us write that. $0 = k_{pp}\left[1 - \alpha_i \sin sin \left(\alpha_i\right) - \alpha_i \cot cot \left(\alpha_i\right) \cos cos \left(\alpha_i\right)\right] + k_{qp}\left[1 - \alpha_i \csc csc \left(\alpha_i\right) \cos cos \left(\alpha_i\right)\right]$, Equation 12a.

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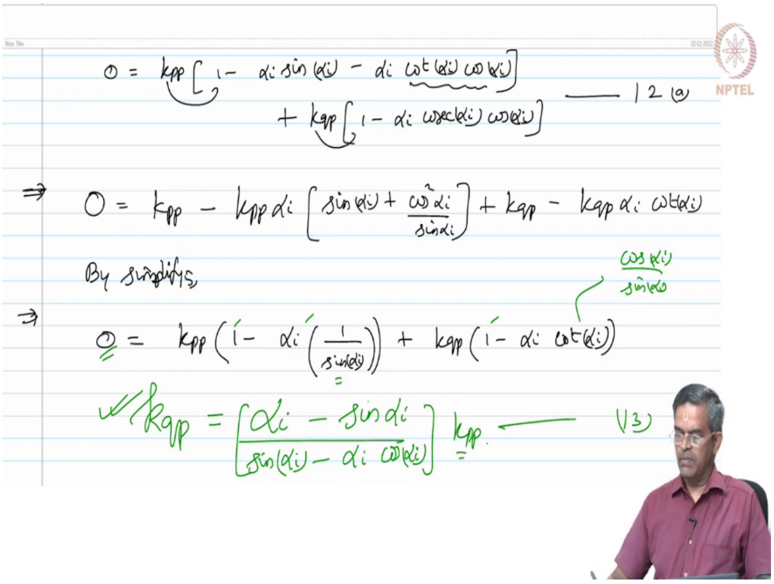

$$0 = k_{pp} - k_{pp}\alpha_i\left[\sin sin \left(\alpha_i\right) + \cos cos \left(\alpha_i\right)\frac{coscos \left(\alpha_i\right)}{sinsin \left(\alpha_i\right)}\right] + k_{qp} - k_{qp}\alpha_i \cot cot \left(\alpha_i\right)$$

This was the equation, 12a. By simplifying,

$$0 = k_{pp}\left(1 - \alpha_i\left(\frac{1}{sinsin \left(\alpha_i\right)}\right)\right) + k_{qp}(1 - \alpha_i \cot cot \left(\alpha_i\right))$$

$$k_{qp} = \left[\frac{\alpha_i - sinsin\, \alpha_i}{sinsin \left(\alpha_i\right) - \alpha_i coscos \left(\alpha_i\right)}\right]k_{pp}$$

This was equation number 13; is very simple because you know this is expressed as cos by sin. There is sin here, take a common denominator, multiply that with 0, the denominator goes away.

So, rearranging you will get k_{qp}. So, k_{qp} is given by equation 13, that is one of the coefficient we have, rotation coefficient.

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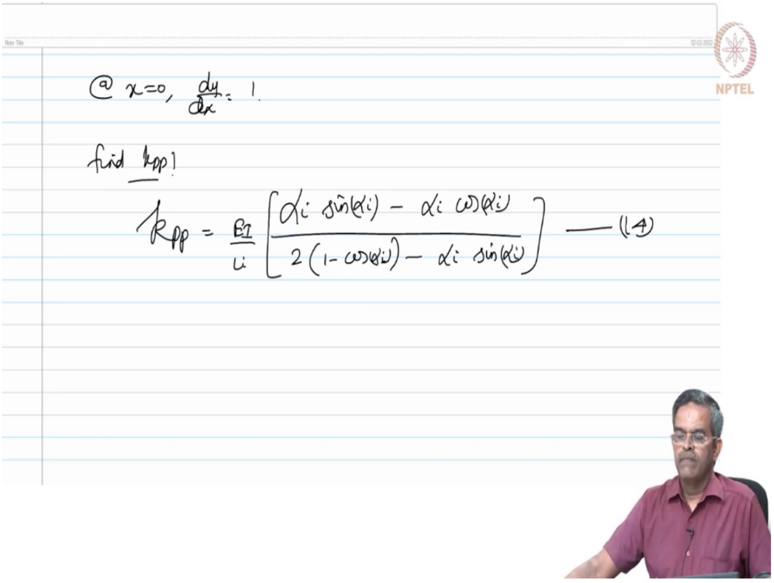

$$@x = 0, \frac{dy}{dx} = 1$$

$$k_{pp} = \frac{EI}{L_i}\left[\frac{\alpha_i \sin\sin\left(\alpha_i\right) - \alpha_i \cos\cos\left(\alpha_i\right)}{2(1 - \cos\cos\left(\alpha_i\right)) - \alpha_i \sin\sin\left(\alpha_i\right)}\right]$$

Similarly, next condition at x is equal to 0, the slope is 1. So, substitute this condition in the original equation and you will find now k_{pp}. Call this is equation number 14. So, now, I can express the rotation functions in terms of stiffness coefficients. Let us do that.

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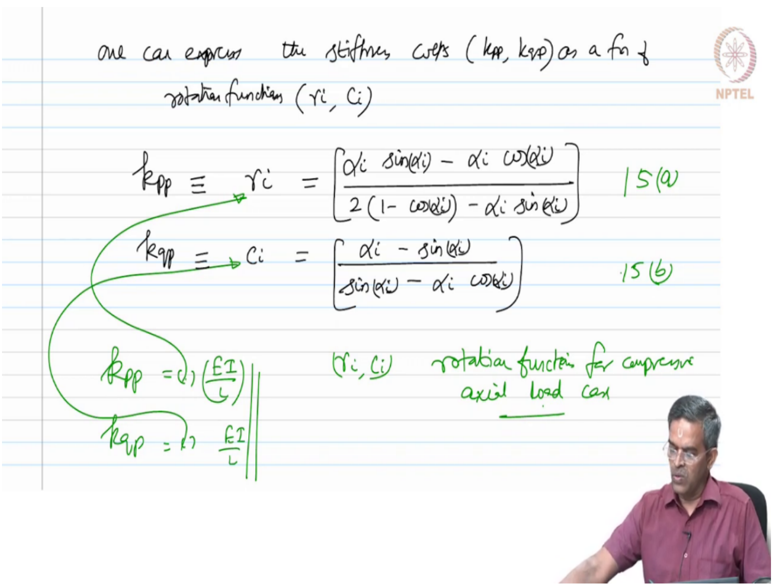

$$k_{pp} = r_i = \left[\frac{\alpha_i \mathrm{sin}sin\left(\alpha_i\right) - \alpha_i \mathrm{cos}cos\left(\alpha_i\right)}{2(1 - \mathrm{cos}cos\left(\alpha_i\right)) - \alpha_i \mathrm{sin}sin\left(\alpha_i\right)}\right]$$

$$k_{qp} = c_i = \left[\frac{\alpha_i - \mathrm{sin}sin\left(\alpha_i\right)}{\mathrm{sin}sin\left(\alpha_i\right) - \alpha_i \mathrm{cos}cos\left(\alpha_i\right)}\right]$$

One can express the stiffness coefficients, that is k_{pp} and k_{qp}. These are rotation coefficients; as a function of rotation functions r_i and c_i.

Friends, if you remember k_{pp} this was some function of EI by L and k_{qp} is also some function of EI by L. Here also you look at this equation k_{pp} is a function of EI by L and k_{qp} is a function of k_{pp} which is again EI by L. So, I am just trying to express this, only these k_{pp} as r_i and k_{qp} as c_i. We will call this equation as 15 (a) and 15 (b). These are called stability functions, that is rotation functions for stability.

They are called as r_i and c_i are called rotation functions for compressive axial load case. I can take a special case of axial load zero and see what happens to these functions. Let us take a special case of axial load zero and see what happens.

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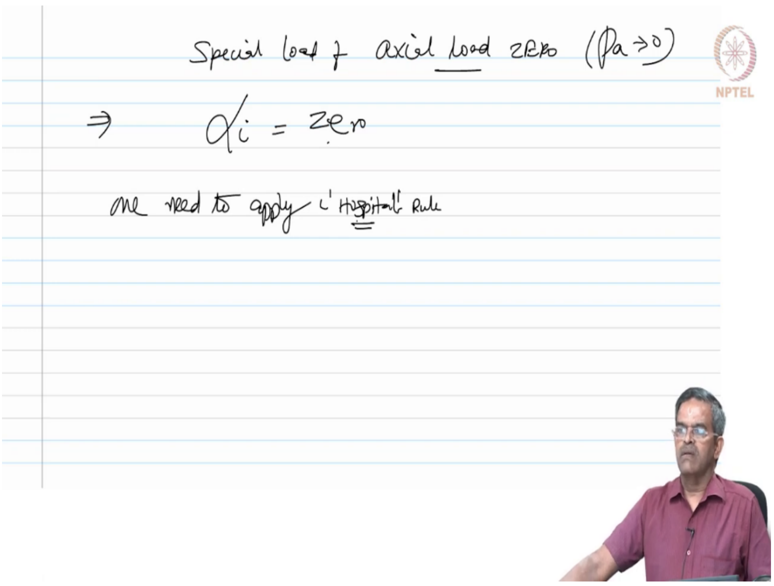

Special case of axial load zero, that is P_a is set to 0. So, what does it mean? Immediately, it implies a fact that α_i will become zero. So, when α_i becomes zero, look at the rotation coefficients which we just now derived. There is a possibility of 0 by 0. So, one need to apply L' Hospital's rule, let us explain the L' Hospital's rule very briefly for our learning, though we have learnt it in mathematics, but still, it is important.

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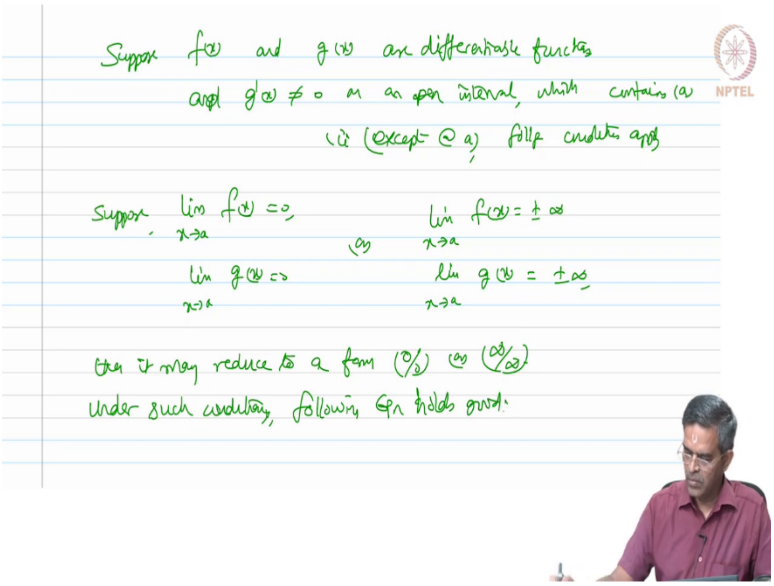

Suppose, f(x) and g (x) are differential functions and g dash of x is not equal to 0 on an open interval which contains that is except at a following conditions apply. Suppose, limit x tends to a, f(x)= 0, limit x tends to a, g(x) is 0, or limit x tends to a, f(x) is plus or minus infinity, limit x tends to a g(x) is plus or minus infinity. Then, it may reduce to a form 0 by 0 or infinity by infinity, under such conditions following equation is good.

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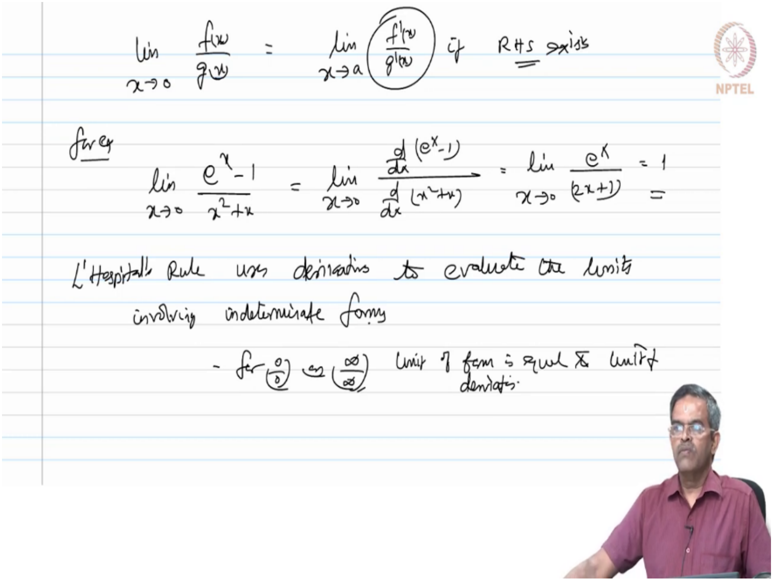

Limit x tends to 0, f(x) by g(x) is equal to limit x tends to a f dash of x g dash of x, if the right hand side of the equation exists. Let us say for example, limit x tends to 0 e power x minus 1 by x square plus x is our 0 by 0 form, but the denominator derivative is it not 0, is it not it exists. So, I can say is equal to limit x tends to 0 d by dx of the numerator d by dx of the denominator which is going to be limit x tends to 0 e power x by 2 x plus 1. Now, the answer will be 1, for which the condition is the right-hand side should exist.

So, L' Hospital rule uses derivatives to evaluate the limits involving indeterminate forms. It states that for indeterminate functions, where the unity tends to form 0 by 0 or infinity by infinity, the limit of that form is equal to the limited derivative itself.

For 0 by 0 form or this form, limit of the form is equal to limit of the derivative. L' Hospital rule can be applied any number of times until the function does not reduce to a condition back again to 0 by 0 or infinity by infinity. So, we have a problem which is to be used in this specific case. So, let us consider the rotation functions 15 (a) and 15 (b).

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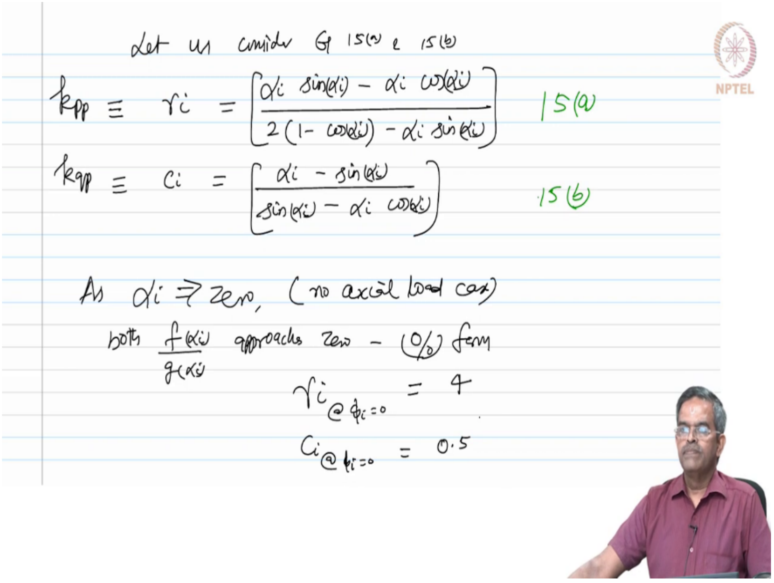

$$k_{pp} = r_i = \left[\frac{\alpha_i \mathrm{sin}sin\left(\alpha_i\right) - \alpha_i \mathrm{cos}cos\left(\alpha_i\right)}{2(1 - \mathrm{cos}cos\left(\alpha_i\right)) - \alpha_i \mathrm{sin}sin\left(\alpha_i\right)}\right]$$

$$k_{qp} = c_i = \left[\frac{\alpha_i - \mathrm{sin}sin\left(\alpha_i\right)}{\mathrm{sin}sin\left(\alpha_i\right) - \alpha_i \mathrm{cos}cos\left(\alpha_i\right)}\right]$$

So, let us copy those functions here. Let us rub this. As α_i reduces to zero or approaches zero because we are considering no axial load case, is not that; both f(α_i) by g(α_i) approaches zero. So, this will turn to a 0 by 0 form, is it not. So, I can apply L' Hospital rule which will give me the function value as r_i will now become that is r_i at ϕ_i equal 0 will become 4 and c_i at ϕ_i becomes 0 is 0.5.

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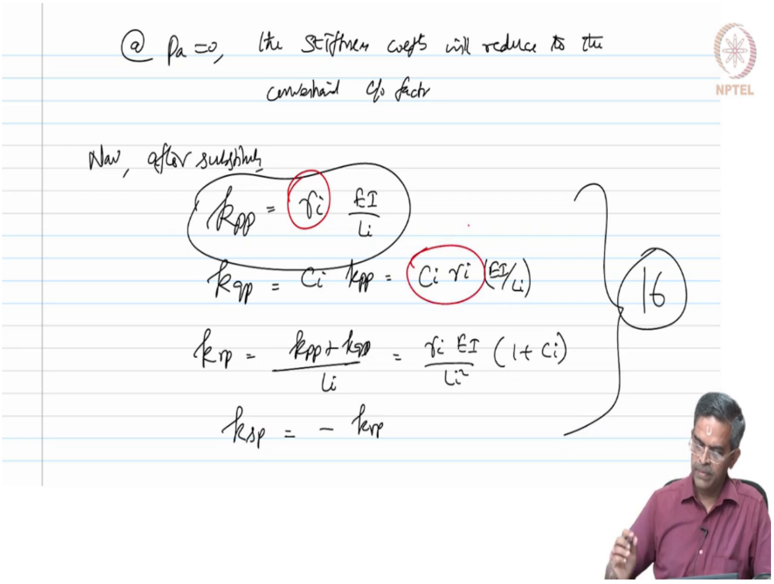

So, at P_a equals 0, the stiffness coefficients will reduce to the conventional carryover factors.

$$k_{pp} = r_i \frac{EI}{L_i}$$

$$k_{qp} = c_i k_{pp} = c_i r_i \left(\frac{EI}{L_i}\right)$$

$$k_{rp} = \frac{k_{pp} + k_{qp}}{L_i} = \frac{r_i EI}{L_i^2}\left(1 + c_i\right)$$

$$k_{sp} = - k_{rp}$$

Let us call this equation as equation 16. Similarly, friends by applying any rotation the kth end, another set of stiffness coefficients can be derived.

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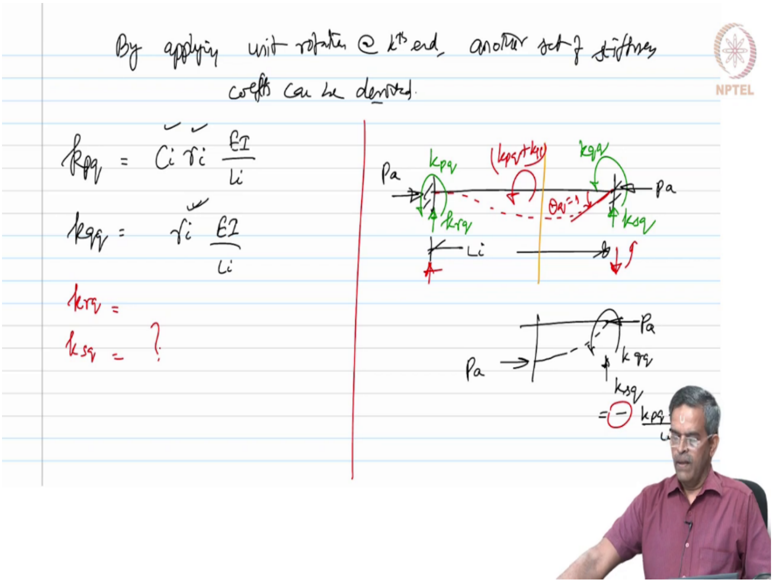

$$k_{pq} = c_i r_i \frac{EI}{L_i}$$

$$k_{qq} = r_i \frac{EI}{L_i}$$

Now, by applying unit rotation of the k^{th} end, another set of stiffness coefficients can be derived. Let us draw the figure and understand that I am trying to apply unit rotation at the k^{th} end. So, this is θ_q which is unity and we know the length of the beam is L_i and we have already applied the axial load P_a. And, this is k_{pq}, this is k_{qq} and this force of coefficient is k_{rq} and this is k_{sq}.

So, I can again cut a section here, draw the free body diagram. P_a is applied here and P_a is reciprocated here. And, there is a moment k_{qq} and there is force k_{sq} which is actually equal to $-\frac{k_{pq}+k_{rq}}{L_i}k_{qq}$. Why it is minus? I think you realize it there is a net moment of $k_{pq} + k_{qq}$. Here, the counteract that this will be the couple and this is opposite to k_{sq}. Therefore, there is a minus sign here.

So, you can follow the same logic and we can write the coefficients directly as here. I am leaving it for learning, you easily do that.

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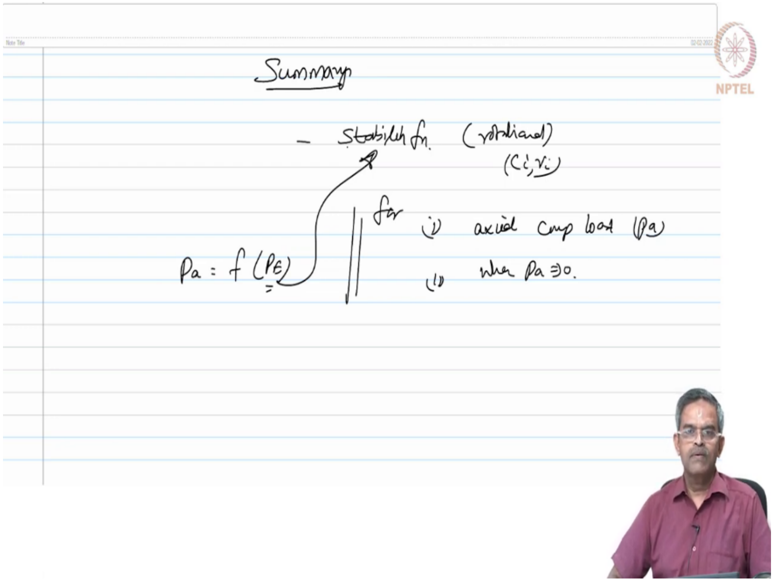

So, friends we have learnt the derivation of stability function, that is rotation coefficients which are c_i and r_i for two cases. Case i, under axial compressive load P_a, ii when P_a is tend to 0 and we realize that in both the cases P_a is expressed as a function of PE. Why? We are looking for stability functions. So, please look at the derivation back again and try to understand that is very simple.

It has got involvement of differential equation understanding, have a parallel reading on some text book on engineering mathematics. And, learn this and try to get a hold of the derivation. The next lecture we are going to discuss about the rotation function and stability functions of a beam under axial tensile load.

Thank you very much friends. Have a good day.