Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 31
Stability functions - 3

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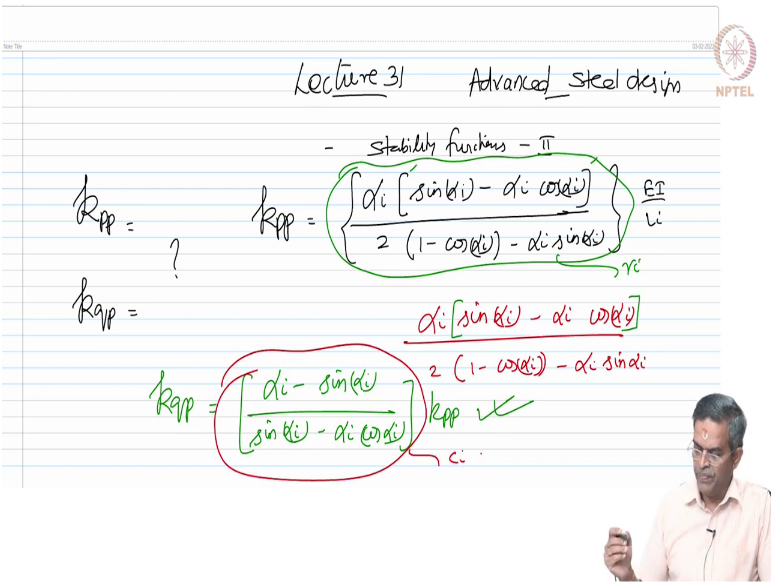

Friends, welcome to the lecture 31 on Advanced Steel Design course, we are going to discuss Stability functions. In the last lecture there is a small correction I apologize for that, let us rewrite that equation for k_{pp} and k_{qp}, please turn back your notes I am rewriting this equation there is a small correction. So, k_{pp} is given by

$$k_{pp} = \left[\frac{\alpha_i(\sin(\alpha_i) - \alpha_i\cos(\alpha_i))}{2(1-\cos(\alpha_i)) - \alpha_i\sin(\alpha_i)}\right] \frac{EI}{L_i}$$

So, the error what we had is that this was missing. So, it is written as αi sin αi - αi cos αi, divided by twice of 1 - cos αi - αi sin α this was written.

So, and of course, k_{qp} there is no change in that,

$$k_{qp} = \left[\frac{\alpha_i - \sin(\alpha_i)}{\sin(\alpha_i) - \alpha_i\cos(\alpha_i)}\right] k_{pp}$$

there is no change in this. The error was I missed out this bracket when we did the derivation, I mean you could have observed it, but; however, does not matter.

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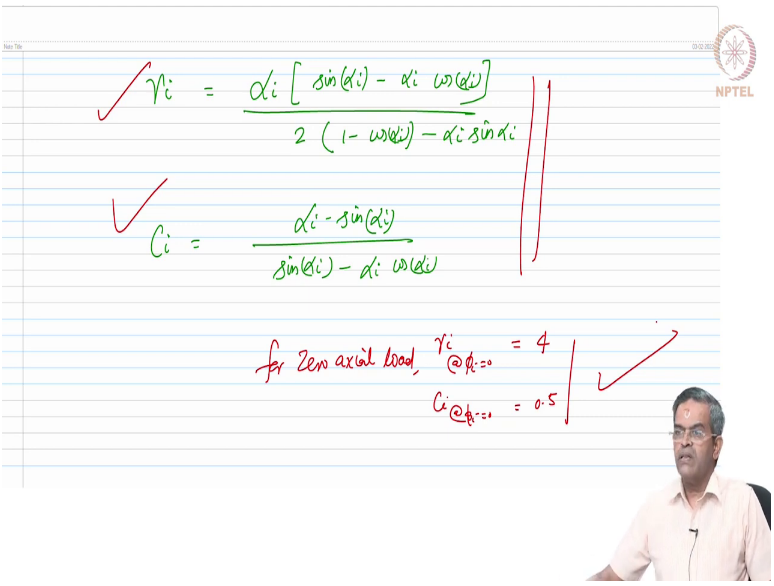

So, then subsequently we said the rotation coefficients can be expressed as r_i and c_i. So, ri again becomes

$$r_i = \frac{\alpha_i(\sin(\alpha_i) - \alpha_i\cos(\alpha_i))}{2(1 - \cos(\alpha_i)) - \alpha_i\sin(\alpha_i)}$$

And of course, ci is

$$c_i = \frac{\alpha_i - \sin(\alpha_i)}{\sin(\alpha_i) - \alpha_i\cos(\alpha_i)}$$

We applied this for zero axial load is a special case and we derived it using L'Hospital's rule. And we got for zero axial load, we got r_i at ϕ_i 0 special case is 4 and ci at ϕ i 0 special case is 0.5 I think we derived it.

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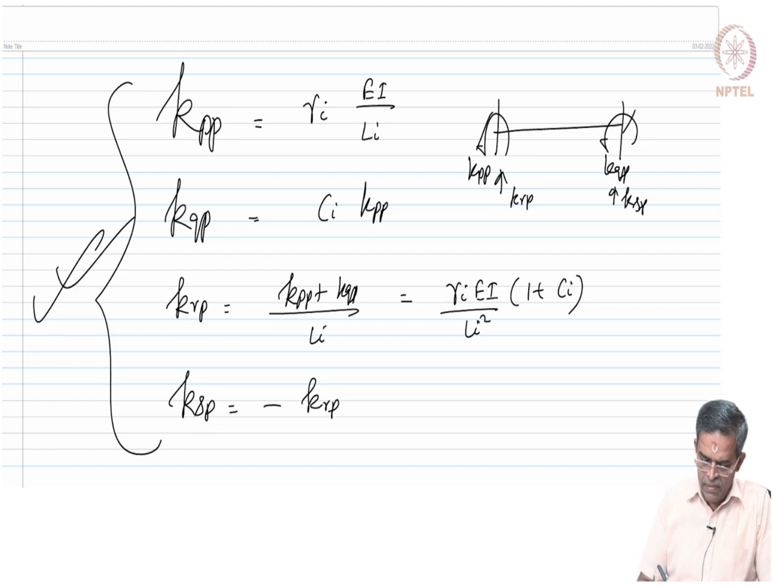

Furthermore, friends we made a very interesting and comprehensive statement saying that kpp is ri of EI by Li kqp is ci of kpp and krp is kpp + kqp by Li which becomes ri EI by Li square of 1 + ci. And we already know ksp is - of krp, this is actually the standard case where this is kpp, this is kqp, this is krp, and this is ksp, and these are the values what we have derived. Similarly, we extended this concept for unit rotation at the kth end.

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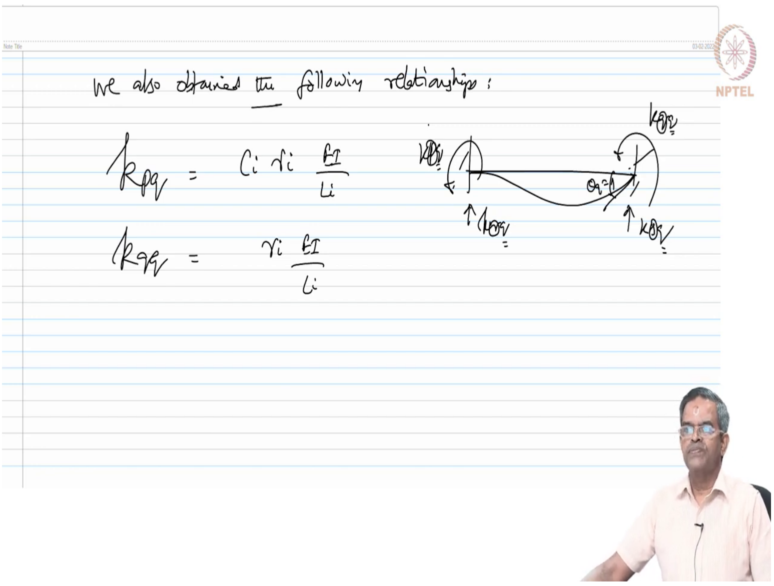

Then we also got we also obtained the following relationships, we got kpq as ci, ri, EI by Li and kqq as ri EI by Li and so on right. So, this is simple I am giving you know rotation here, this was kqq, this is kpq, this is krq, and this is ksq, the second subscript stands for the degree of freedom where we have given unit rotation or displacement. The first subscript stands for the degree of freedom where you are measuring the force.

I mean there is a standard logic we have in the stiffness derivation; we follow the same thing here there is even not a millimetre change in the whole concept of derivation. That is the idea why we presented the derivation of the fixed beam first, then we started extending that same algorithm and logic for deriving the stability functions for Euler's load, that is the idea we have here. Having said this; now, let us start focusing on deriving rotation functions under axial tensile load, let us go ahead.

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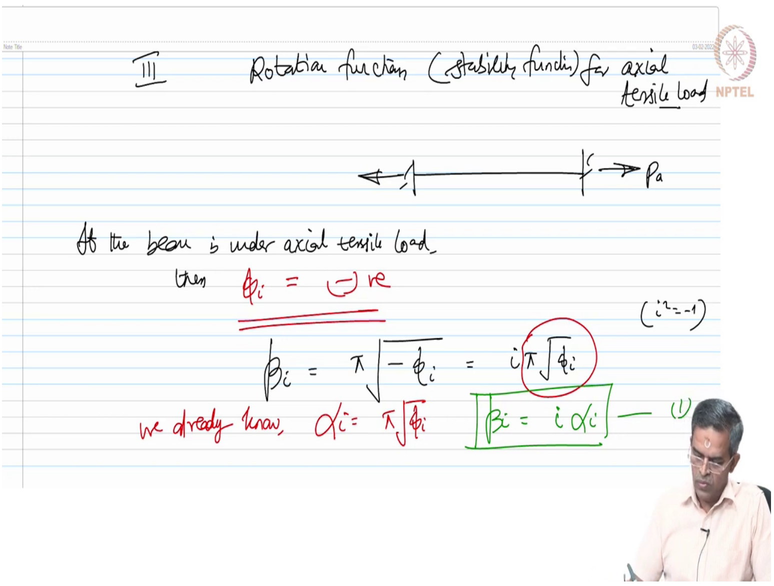

Now, we are going to do the 3rd part where I am deriving the rotation functions, they are actually stability functions for axial tensile load. If a beam is subjected to axial tension, then; obviously, ϕ i becomes negative is not that I have a beam, I am subjecting this axial tensile load of P a, I am looking for the stability of this. So, if the beam is under axial tension, then we can say my ϕ i is negative.

So, therefore, I now derive or describe a new variable β i which is now going to be πtimes of root of - ϕ i, which I can write as i π ϕ I, because you know i square is - 1. We already know friends this π ϕ i is $\alpha$is it not we already know αi is π square root of ϕ i, we have used it in the earlier derivation as well. So, can i now say β i as i αi; let us have this with us, we call the equation number 1, we also know we also know that sin βi, because there is an imaginary function here right.

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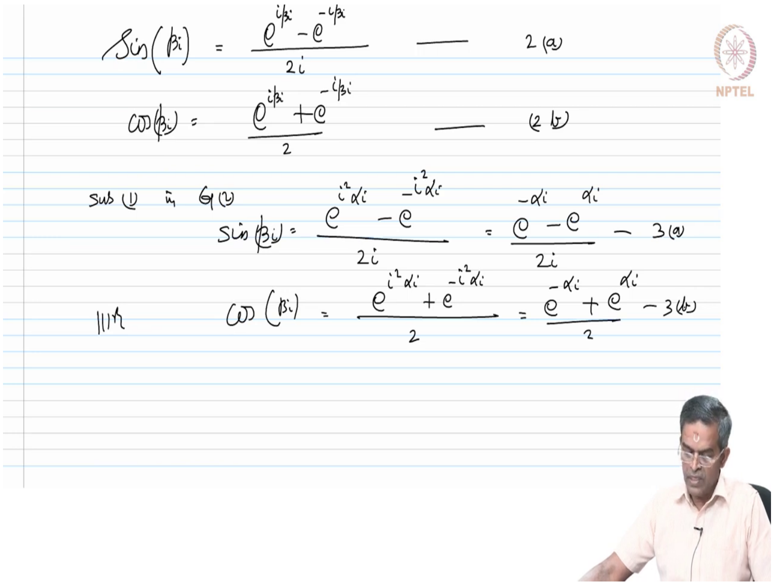

So, we should say this is equal to e i β i - e - i β i by 2 i, and cos β i is taken as e i β i + e - i β i by 2, we call equation number 2 a and 2 b. Let us substitute this 2 a and 2 b in this equation 1, we know that β i's αi. So, let us substitute substituting 1 in equations 2, what do we get? We get the following sin β i yes e i square αi - e - i square αi by 2i, which makes this as e - αi - e αi by 2i we call this as equation 3 a now.

Similarly, cos β i will be e i square αi + e - i square αi by 2 which now become e - αi + e αi by 2 which is 3 b. So, the equations 2 a and 2 b got refined to 3 a and 3 b by substituting equation 1 into them. I mean there is no confusion in this it is a straight away clear derivation, I hope you understand and follow the steps clearly.

Now, what I do is, we want to give rotation constants at the jth here let us work out the rotation constants we already have them with us we just now wrote them also right. So, we have rotation constants, we have rotation constants ri and ci; so, let us copy this copy these two equations write it here we already have.

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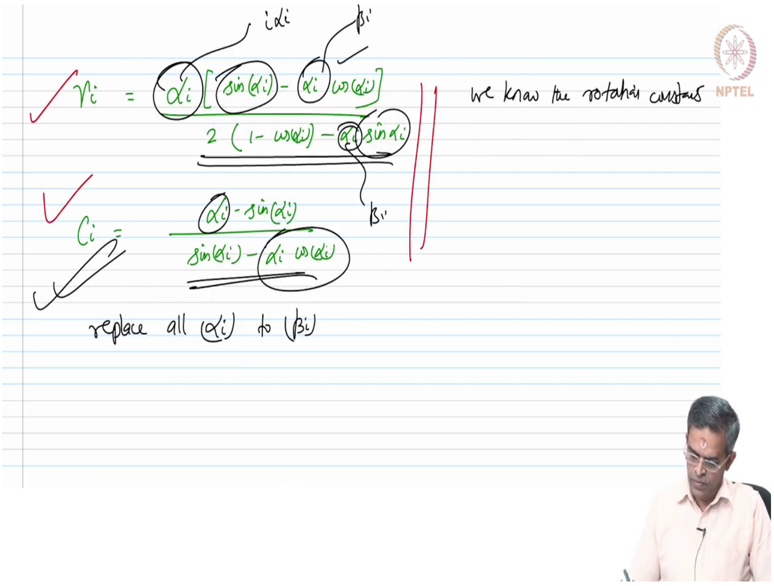

So, we know the rotation constants which are useful in stability functions . So, now, what I am going to do is, I am going to replace them as β i instead of αi am going to replace in β i right; so, I am going to do that. So, α β i will be i αi you can see that β i will be i αi; so, I am going to just replace all αs into β.

So, replace all αi to β i that is what I am trying to do. So, for example, this will become i β i mean i αi and so on. And this will become sin β i; so, i must get this equation extended there, I mean this equation extended there and so on is not that. So, αi to β i I keep on doing this right; so, let us do that please do this I will do it parallelly here.

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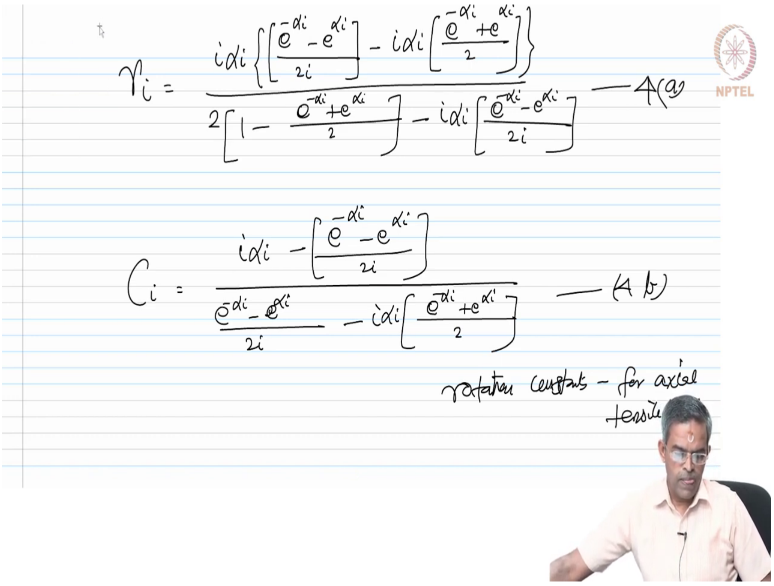

So, my ri will now become i αi e to the power of - αi - e αi by 2 i, this is 1 - you can see here as the first term second term here - i αi, because i have replaced as β i here i αi of. So, I should have cos β i which is here, which is here, this is here. So, let us do that, e to the power of - αi + e to the power of αi by 2 am I right, let me close this bracket divided by twice of you know divided by twice of this.

So, let us do that 2 of 1 - e to the power of - αi + e to the power of αi by 2 - see this is again β i which i say yes, i αi of then this is sin I have the value which is e to the power of - αi - e αi by 2 i. Similarly, let us write down for ci also we have ci also here; so, again the same story i αi see here my αi becomes β i.

So, - sin e to the power of - αi - αi by 2i divided by this one, e to the power of - αi - e αi by 2 i - this value i αi of cos. So, e to the power of - αi + e αi by 2, we call this as equation numbers 4 a and 4 b, 4 a and 4 b which are now the rotation constants, for axial tensile load 4 a and 4 b are rotation constants for stability function for axial tensile load.

Now, I can simplify these two equations further I can simplify, because there are I can just show you a simple manipulation. Let us say if you do this, I will copy this equation to the next sheet then I will do that I will copy this equation to the next sheet then I will do this.

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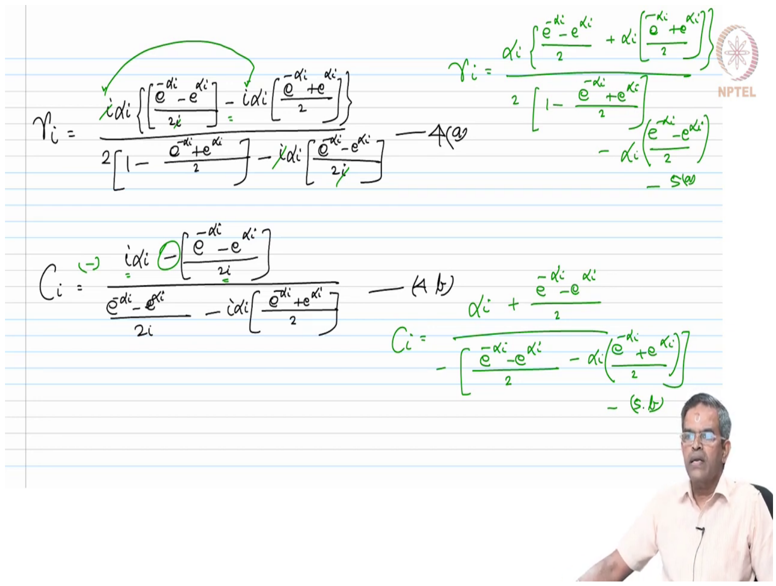

Let us do this a simplification how let us say this i and this i gets cancelled and this i with this becomes - 1 and that with this becomes +; so, these are the changes what I am going to expect when I write. So, I am writing this new ri as, αi times of e - αi - e αi by 2 + αi times of e - αi + e αi by 2, am I right divided by 2 times of 1 - e - αi + e αi by 2 -, this i again goes away.

So, I can straight away say αi times of, let me write it here - αi times of e - αi - αi by 2 am I right, that is my ri. Let us write down the function for ci; so, I can now take a constant and say αi + e to the power of - αi - αi by 2; so, I make changes to +.

So, what happens is if you multiply this i with this i get - here there is already a - here, I am putting that - the denominator there is a reason. See this mathematical simplification is very important, because this is how the standard expression will be reached as you see in the literature; so, let us carefully look at this.

So, I am putting the - here then I am saying this equation is I think incomplete it should be there is a 2i here there is a 2i here, I think I have not copied that - of e to the power of - αi - αi by 2, I am removing this i . When you do that, this becomes - i gets cancelled, I am taking i constant out right - αi times of e - α + e α by 2 that is my ci call this as equation number 5 a and 5 b friends.

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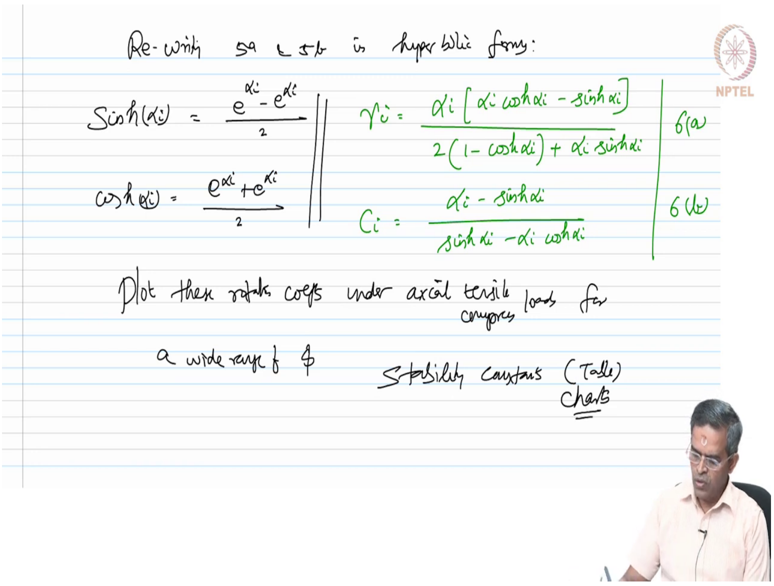

Now, I can rewrite this in hyperbolic forms, now rewriting 5 a and 5 b in hyperbolic forms. What is an hyperbolic form? We know that sin hyperbolic αi is e αi - αi by 2, cos hyperbolic αi is e αi + αi by 2. So, I am using this relationship, I am writing ri and ci like this.

So, ri now becomes αi times of αi cos hyperbolic αi - sin hyperbolic αi by twice of 1 - cos hyperbolic αi + αi sin hyperbolic α, you can look at this simplification. And ci now becomes αi - sin hyperbolic αi by sin hyperbolic αi - αi cos hyperbolic α. So, we call this is equation number 6a and 6b, these are expression of rotation coefficients in terms of hyperbolic functions.

Now, interestingly friends we can plot this rotation coefficients, we can plot these rotation coefficients under axial tensile and axial compressive loads for a wide range of ϕ is a variable ϕ is a variable. These typical tables are available in the literature of stability books, that is called stability tables I can say stability constants they are also called as stability charts.

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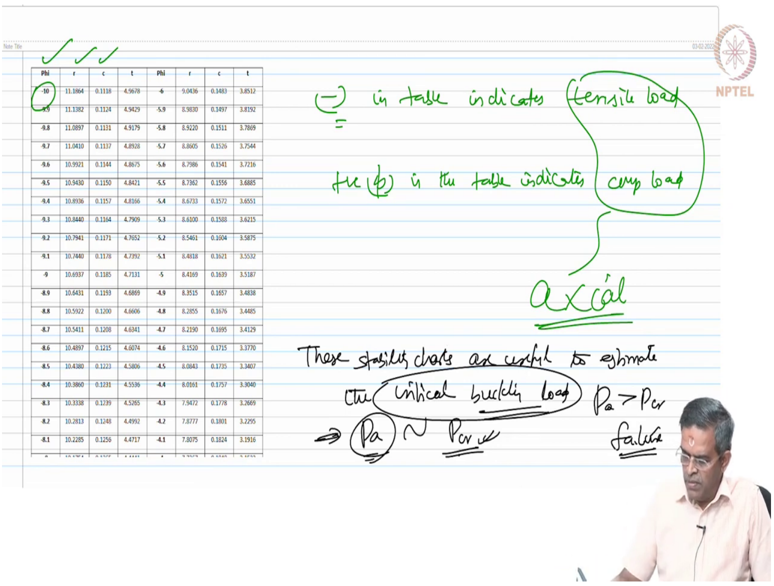

I will show you a typical chart here; so, for different values of ϕ r and c are plotted, I mean given the table for closer intervals. You can also do that I will also show you a figure and then I show you a MATLAB program, which can plots this, but let us say it is a continuous table which starts from a specific value. So, negative in the table indicates tensile load, positive ϕ in the table that is this value; if it is negative tensile, positive in the table indicates compressive load.

Please understand both of these loads are axial, they are axial loads we are talking about stability functions. Now where are they used these charts are useful to estimate the critical buckling load you may wonder, why I am talking about buckling load estimate very interesting question, buckling load will help you to estimate Pa. Now, Pa will be compared with buckling load and if Pa exceeds buckling load then it is a failure is not that.

So, to ascertain whether the structure will have a stability failure under the given load you must know, what is the capacity of the structure to sustain its stability? So, that is available from Pcr we already discussed that in the stability theory earlier Euler's criteria. So, now, I can estimate this ϕ I mean Pa b tensile or compressive from this table are charts compare it with critical load, then I will know critical load will help you to estimate from the charts axial load is the one which is applied to the system.

We can now compare and identify whether the structure will have destabilized mode under this Pa. So, remember stability starts will not help you to give Pa, Pa is a known input, stability charge will help you to calculate critical buckling load; so, we have done that; so, now, interestingly we can also plot these charts.

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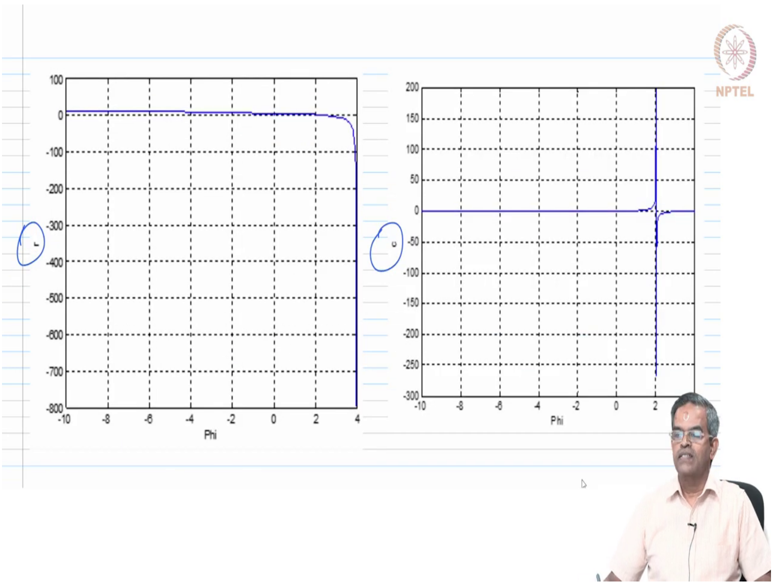

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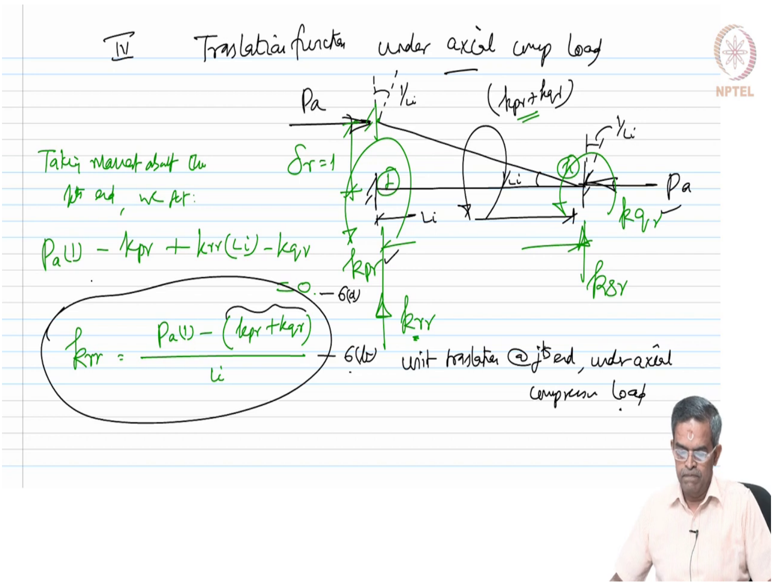

But before that let us also do the translation coefficients, let us also do the translation coefficients. So, let us do the translation coefficients, let us say under axial compressive load. So, how do you do a translation coefficient or translation function? Initially I will have a beam both ends fixed; I will give unit translation to this end the beam will move here, this will be unit translation I call this as del r unity.

Now, the beam will take this position; so, if this angle is 1 by Li this is a Li, we will agree upon this fact that this angle is also 1 by Li, this angle is also 1 by Li there is no dispute in this . Let us now mark the moments and shear causing this; so, this will be k pr, this will be kqr, and this shear will be krr and this shear will be ksr is it not.

Now, let us subject this to an axial load compressive Pa; so, what we have done? We have given unit translation at jth end under axial compressive load that is what you have done right that is the fuel. Now, we will take moments about the kth end, taking moment about the kth end, where is the kth end? This is my jth end, this is my kth the end of the beam right we get.

Let us see what do we get Pa into 1 that is clockwise - k pr antic clockwise + krr into Li clockwise - kqr should be 0 sigma m . So, from this I can write krr as P a into 1 - of kpr + kqr by Li, what is this actually very clear this is the unbalanced moment in the section right. So, let us remember this we call this equation number I think 5 a 5 b let us call the 6 let us call the 6 a and 6 b.

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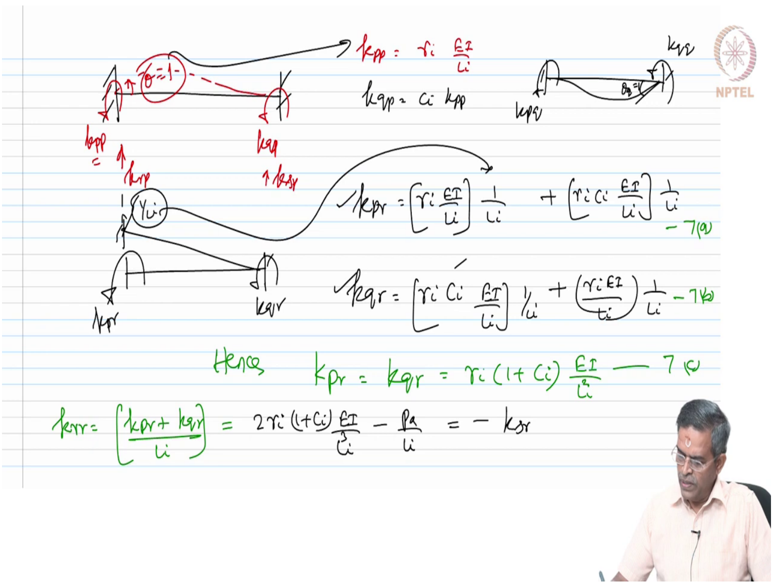

Now, when have a member with unit rotation and this was kpp and this is kqp this we know, and this was krp, and this was ksp, I think we know this . We also know that k pp that kpp is actually ri times of EI by Li and so on we remember that is it not. We wrote it in the beginning itself see k p a pp is ri times of this this is ca times of kpp and so on we already wrote that I am going to make use of this relationship now.

Now, friends if this rotation is unity this is kpp, if this rotation is not unity if this rotation is 1 by Li what would be this value. Let us say I know this is going to be k pr and this is going to be kqr, this is only at the jth end I am talking about. So, can I write k pr as ri times of EI by Li of 1 by Li, because this is for unity this value for 1 by Li this value, is not it? This is only for the jth end.

Can I write kqr as ri ci EI by Li times of 1 by Li, where kqr we already have here kqr or kqp is c i times of kpp; so, I am using that ci times of kpp I am using that.

So, you know kqp is ca times of kpp, I just multiplied this ci you can see that here, this is only for the jth end; so, I also have a similar contribution if I give unit rotation at the kth end and so on now. So, now, I can add that contribution here as this is just swapped ri ci EI by Li times of 1 by Li and this is swapped ri EI by Li of 1 by Li, we already derived this in the last lecture you can verify that. So, what we did is kpr and kqr we obtained, kpr and kqr we obtained; so, we have realized that kpr and kqr are equal.

So, we realize that hence kpr is kqr is equal to ri times of 1 + ci times of EI by Li square can I say that can I say that. So, we call this as equation we said 6 a and 6 b we call this as 7 a, 7 b, 7 c is that agreed. Now, it is very simple for me to find krr, can I say krr is sum of this by the span can I say that; so, I can say krr is k pr + kqr by Li, can I say that. So, substituting I will get this as 2 times of ri of 1 + ci of EI by Li cube can I say that.

Now, we should say this is also available to me here; so, comparing these two can I now say, because 6 b should be equal to this because I am going to equate both of them. So, can I say that krr is replaced as - Pa by Li, this factor is coming here. And we also know this will be negative of krr; so, can I say this as - ksr .

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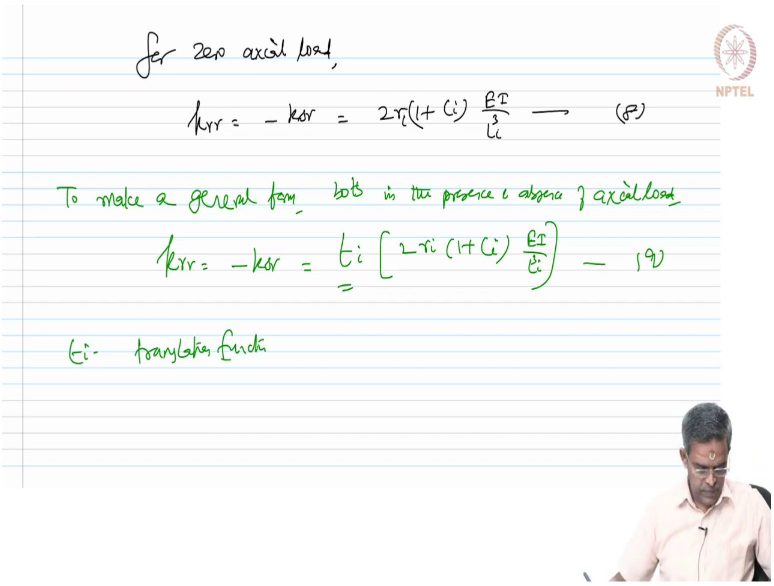

So, friends for zero axial load let me name this equation as 7 d, for 0 axial load krr is - ksr is 2 r 1 + c i 2 r i sorry. So, all for ith beam EI by Li cube can I say that, because Pa goes away that is what I will get equation number 8. Now, we want to make a general form, now to make a general form both in the presence and absence of axial load krr is - ksr is actually I am putting a new function ti of 2 ri 1 + ci of EI by Li cube. I am just introducing a new term this ti is called this equation 9 is called translation function, translation function.

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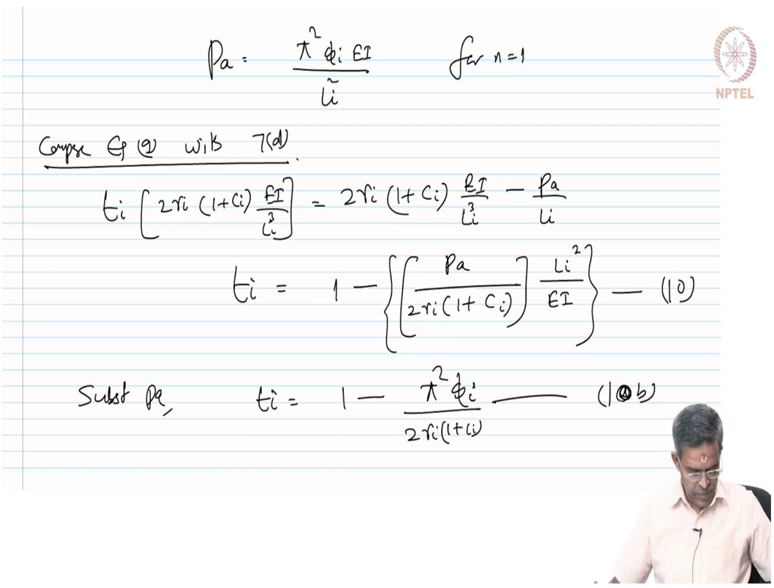

So, now they can say Pa is π square ϕ i EI by Li square for n equals 1 . Now, I want to equate the k rr that is equate 9 equation 9 to or with compare equation 9 with 7 d we will compare with 7 d . So, I can now say ti times of 2 ri 1 + ci EI by Li cube is equal to 2 ri 1 + ci EI by Li cube - pa by Li this is what I am writing. So, this gives me ti as 1 - pa by 2 ri of 1 + ci of Li square by EI equation number 10 substituting for pa, we get ti as 1 - π square ϕ i by 2 ri 1 + ci 10 b.

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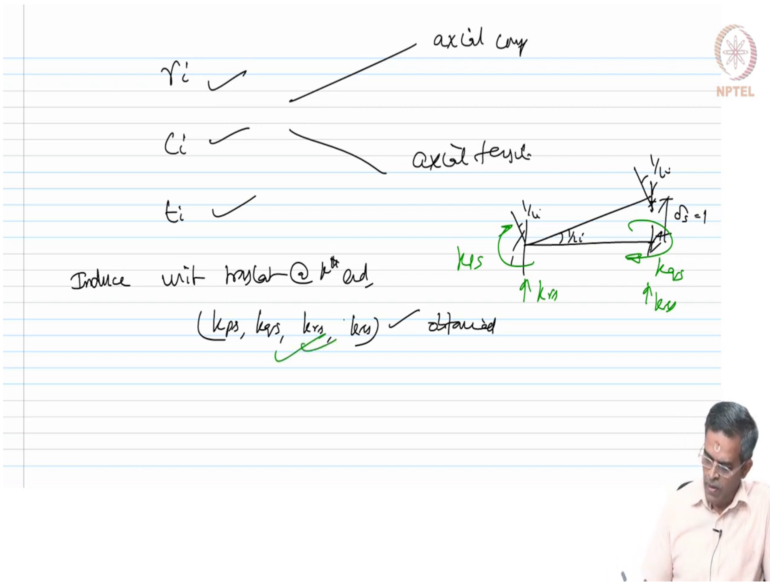

So, I have now ri, ci, ti equations with me for axial compression, axial tension am I right is it not. Similarly, I can induce unit translation at kth end and coefficients kps, kqs, krs, kss can be obtained. What I am trying to do is I am introducing translation here; this is 1 by Li this is del s which is unity. So, this is going to be 1 by Li 1 by Li and the moments created are kqs, kps, krs, and kss, this is what.

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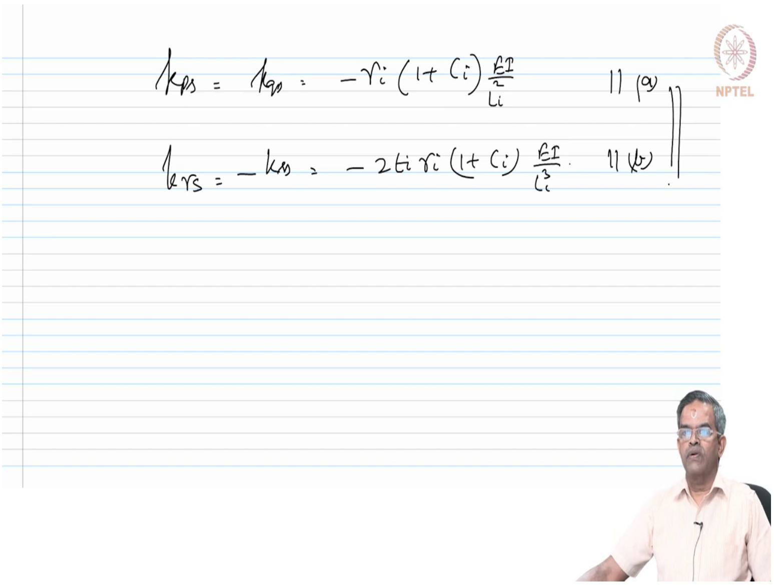

So, I can compute them I leave it this small exercise for you I am writing the equations for you. So, kps is given as equal to krs is - ri of 1 + ci EI by Li cube sorry Li square k rs. So, this is kps, kqs, krs is - of kss which is - 2 ti ri 1 + ci EI by Li cube, we call this equation number 11 a and b, 11 a, 11 friends please derive this get it satisfied before you proceed further.

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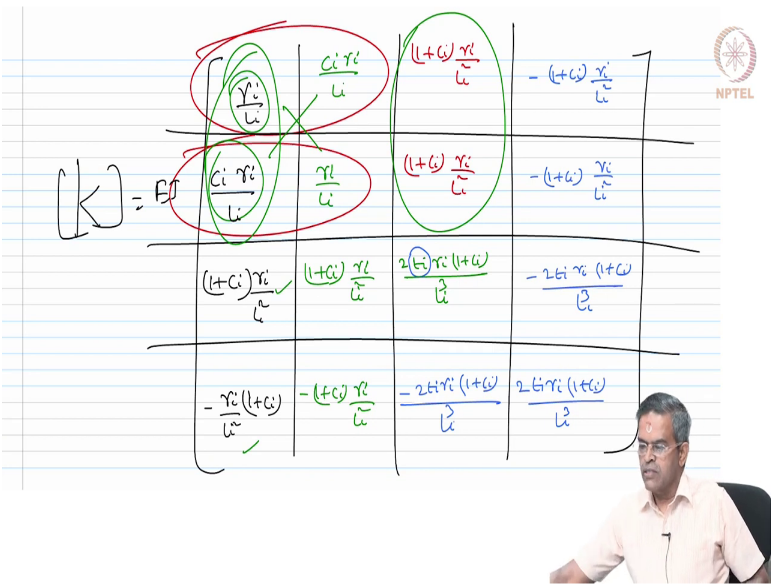

So, friends I can now write the stiffness matrix by taking EI constant as this is ri by Li this is ci ri by Li. This is 1 + ci of ri by Li square this is - of you can easily remember this, friends. If you remember this and this sum of these 2 by l will be this and - of this is this is as same as stiffness matrix of the steam beam member. Now, to get the second column swap this; so, this is ci ri by Li, this is simply ri by Li and this is same as that of the previous one previous column.

Now, to get this sum of these 2 by L we did the same thing in beams also is not that; so, 1 + ci into ri π Li square. Similarly sum of these 2 by L 1 + ci into ri by Li square to get this we have to add these two and divide by L. So, there I am going to do the transfer function 2 ti ri 1 + ci by Li cube, this ti is new here.

So, this is - 2 ti ri 1 + ci by Li cube, the fourth column is negative of the third column. So, we have got now the transfer functions which can be quickly plotted using MATLAB, I will show you the MATLAB program which can be used for this I will show you the MATLAB program.

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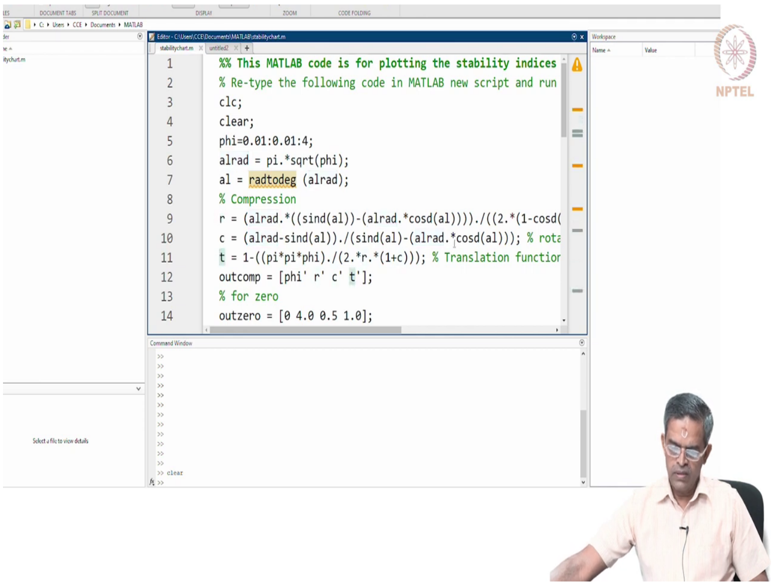

So, you can see the MATLAB program on the screen for plotting the stability indices. So, the ϕ is varied from this range and the degree is converted to radiance and the now r c t are computed from the equation.

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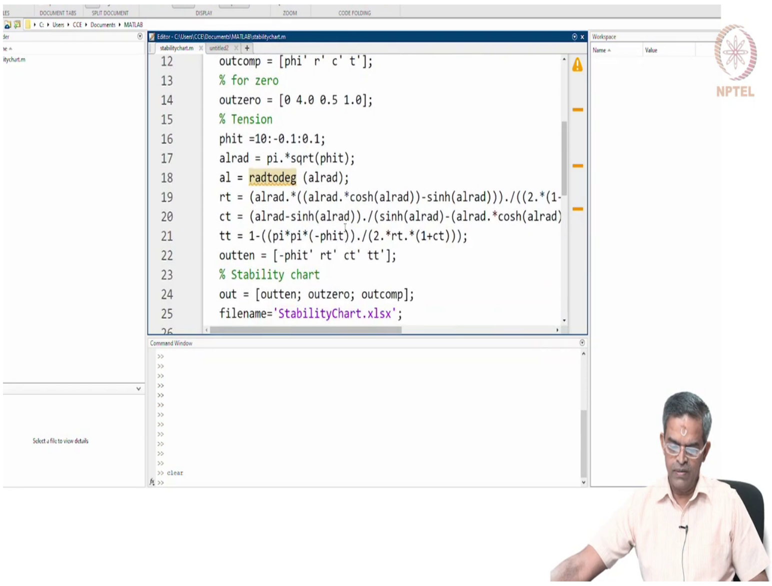

Then for 0 for tension and for compression for compression all the three are computed then the chart is prepared.

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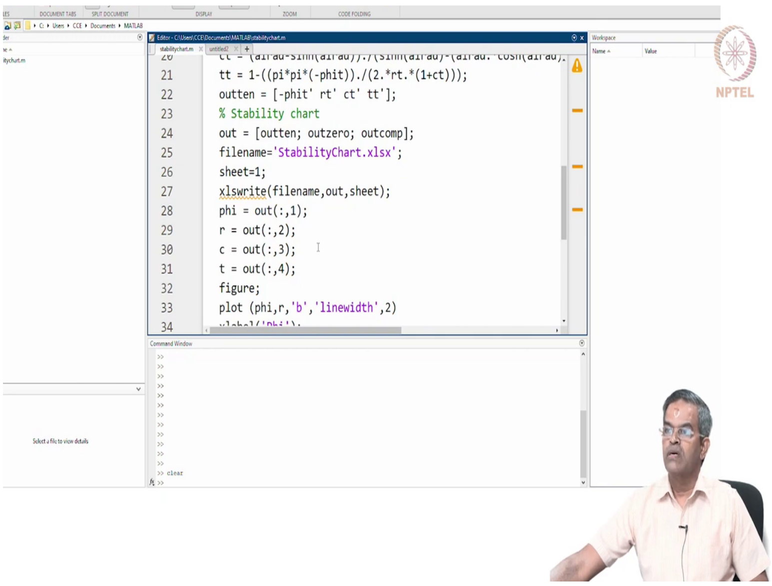

This program is available in a reference book download the program run in MATLAB, MATLAB supports you for NPTEL courses through IIT Madras.

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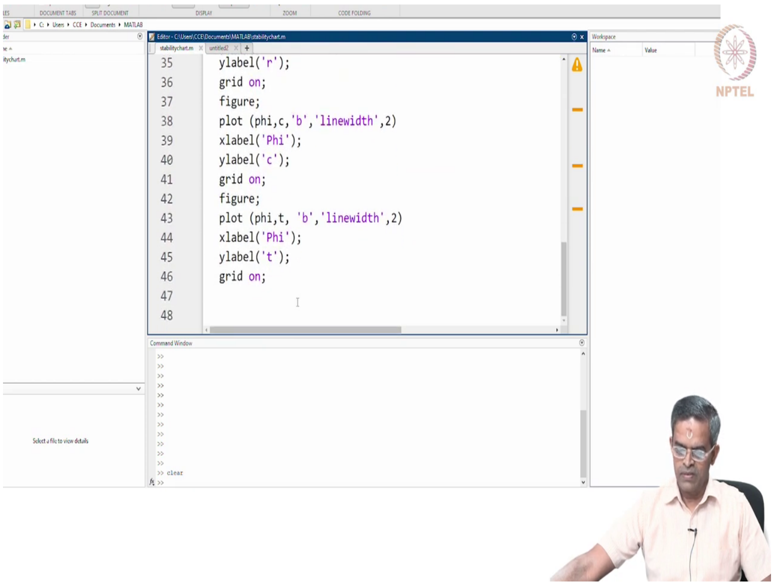

So, please download MATLAB software, use it for this particular course intensively this is what it is; so, this is the MATLAB program we have. So, if I do that my plot for the rotation function r c and t appears like this, I wish that you should plot this and see yourself how they are appeared please see that friends.

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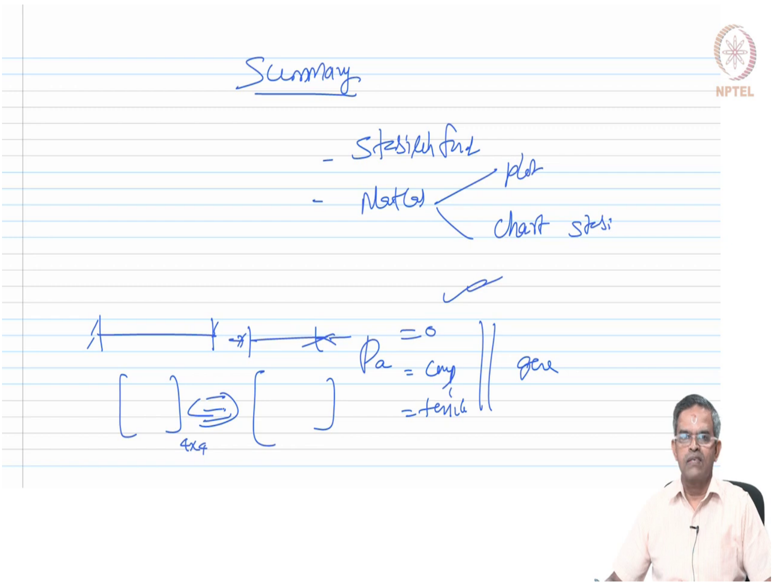

So, in this lecture we learnt the stability functions, we also use the MATLAB program to plot the stability function, we also prepared the chart for stability functions right. We learnt it for zero axial load, axial load compression, axial load tension we have developed a generic case.

And very interestingly this compares variable with the standard fixed beam derivation without axial load same algorithm.

So, this is also 4 by 4 and the stability function is also 4 by 4; so, there is an absolute similarity between these two in the derivation. So, that is the beauty of the whole exercise can easily remember this and try to teach this in the class in the same order. So, the stability derivations become very simple like a stiffness analysis of a fixed beam.

Thank you very much friends, have a good day bye.