Advanced Design of Steel Structures
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Lecture - 32
Buckling and stability

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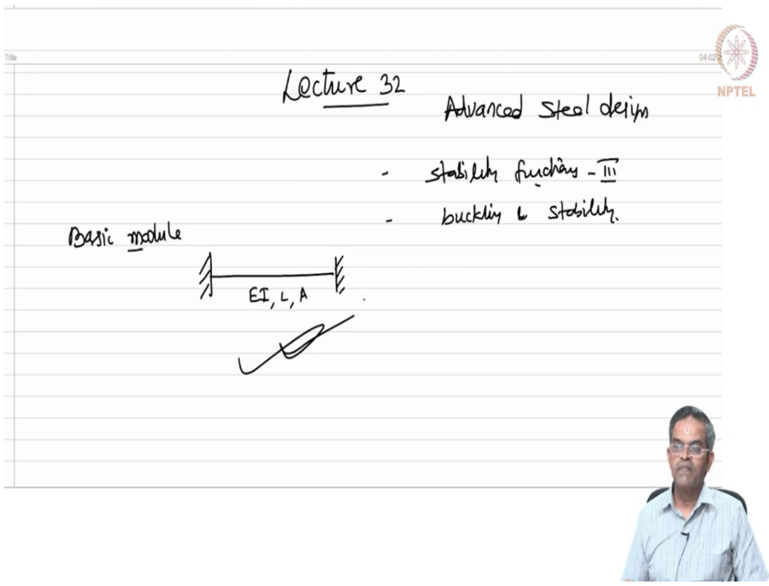

Friends, welcome to the 32nd lecture on Advanced Steel Design. We are discussing about derivation of stability functions in this lecture we will continue discuss that with some more interesting details about, difference between Buckling and stability.

So, friends while discussing the stability functions we already said we will compare a standard fixed beam which is used in stiffness method of matrix analysis. So, the basic module which is used to analyze a statically indeterminate beam or a frame or a structure is a fixed beam with some prismatic cross section and A. So, we have used this as a standard nomenclature.

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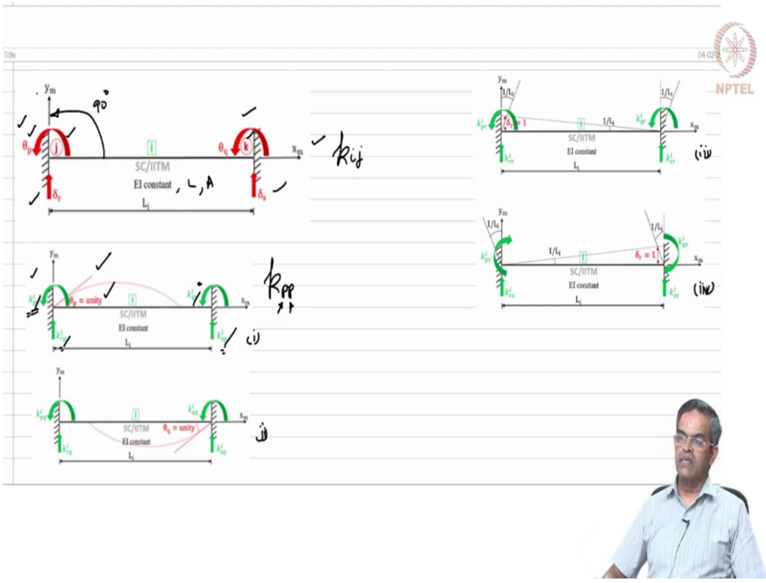

And we said that I can give unit rotation at different ends and displacements and try to derive the stiffness matrix. So, the standard fixed beam has got four degrees of freedom kinematically that is theta p theta q and del r and del s.

The member is prismatic with its EI, L and A. So now, this is a jth end of the member and kth end of the member. So, from j running towards k is xm and ym is anti clockwise 90 to xm this is a standard convention which we have said. Now I want to derive the stiffness coefficient for this beam which is statically indeterminate it is a fixed beam we know that. So, we know kij the stiffness coefficient is a force in ith degree of freedom.

For unit displacement in jth degree of freedom keeping all other degrees of freedom restrained we know this. So, what we did is we gave unit displacement rotation in pth degree, and we marked these forces as kpp kqp krp and ksp is it not. So, the in kpp for example, here the first subscript stands for the place or the degree of freedom we are measuring the force which is p and the second subscript refers to the degree of freedom where you are giving unit rotation.

So, theta p is unity therefore, its p you will find in figure one the second subscript of all you know p you know is p. So, unit rotation at jth end unit rotation at kth end unit displacement at jth end and unit displacement at kth end. So, standard convention we derive the stiffness matrix for this.

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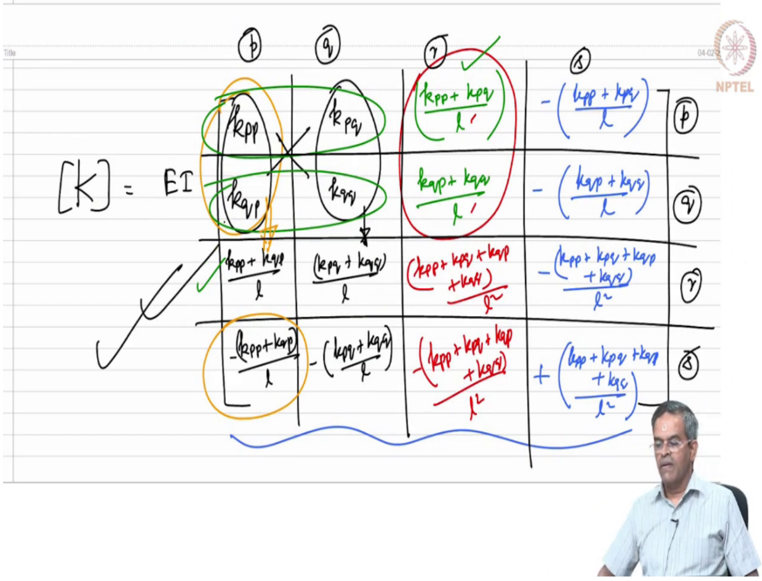

And we said the stiffness matrix for a fixed beam neglecting axial deformation is given by this. So, 4 by 4 matrix.

So, standard derivation you can refer to my book on analysis of structures using MATLAB written by for CRC. This book is a very popular book and this gives you the MATLAB program for doing analysis of indeterminate structures using stiffness method please refer to this book and MATLAB programs can be downloaded free. I can use this book as an important reference for a teaching and research as well.

So, this book refers the derivation of this in detail, but let us say this is my p q r and s labels and we know this is kpp this is kqp and this is sum of these two. So, I should say is kpp + kqp by l and the fourth one is negative of this. The second column is just the cross of this. So, this becomes kpq and this is kqq and sum of these two by l will be here, so kpq + kqq by l. Now fourth one is negative of this.

Coming to the third column of this matrix sum of these two by l will be here incidentally friends this value and this value are same because stiffness matrix is symmetric ok, but do not go by that there are possibilities where the stiffness matrix can become asymmetric as well, so let us not bother about that.

So, sum of these two by l is here which is kqp + kqq by l. Then sum of these two by l further is here which is kpp + kpq + kqp + kqq by l square, because there is already an l here I am dividing it further by l and the fourth one is negative of this. The fourth column is very straightforward negative of the third column.

This is standard derivation we have not done this derivation in this course, but it is a standard derivation available in all standard reference books which are meant for analysis of indeterminate structures using stiffness method.

So, I am going to borrow the same algorithm and we explain that in the previous lectures I am just summarizing it now. So, that stability functions can be easily remembered friends that is very important, because generally stability is an area or is a topic of discussion which is having a lot of confusion. People do not feel very comfortable to teach in the PG course or definitely not in UG course UG level and in research it is handled with most complicated manner.

So, the fundamental reason for this is the stability functions and the approach of stability problem is not understood properly. So, let us deprive of this particular feeling get out of this and try to remember this from the basic understanding of a simple statically indeterminate structure which we already know. So, I am following the same algorithm how we did the derivation for this. So, what we did is instead of giving the rotations of this.

We started finding out this using flexibility method and we also wrote this equation for us in the previous lecture.

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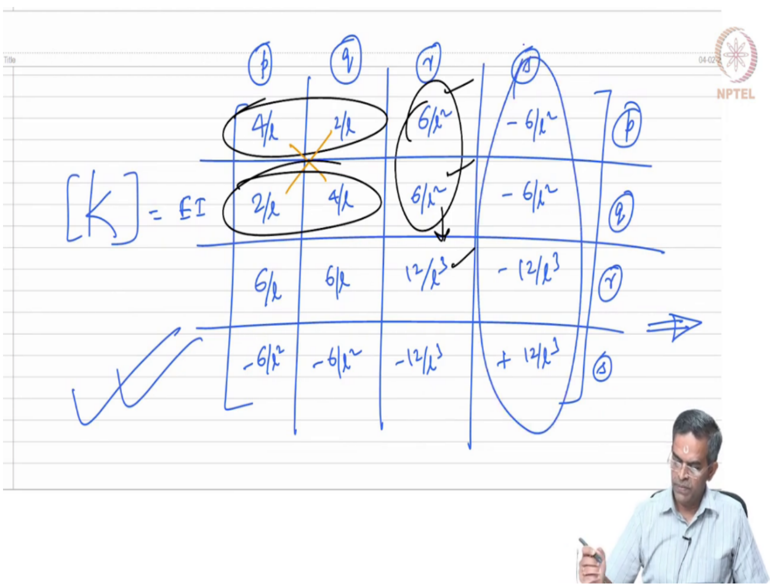

And we said that my K matrix for a simple fixed beam neglecting axial deformation is given by this. Let us have the labels as p q r and s this is q ok, let us say we know this is 4 2 6 by l 4 by l 4 by l 6 by l - 6 by l square. So, 2 by l 4 by l 6 by l - 6 by l square, 6 by l square 6 by l square 12 by l cube - 12 by l cube, - 6 by l square - 6 by l square - 12 by l cube + 12 by l.

I am following the same logic as we have explained here to this. We have used the same form I want you to remember this form and I will correlate this form to the stability functions now that is what we are going to do. So, that if you remember this form you can always remember the stability functions form that is what it is.

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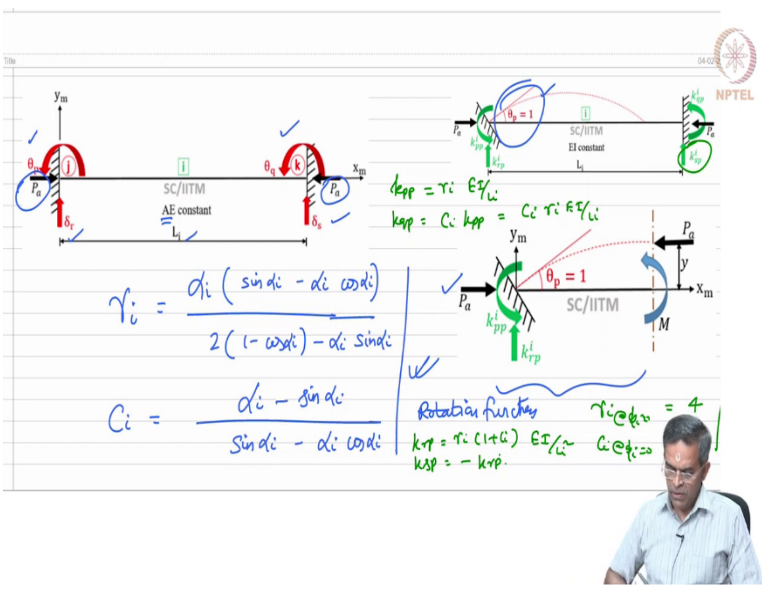

So, what we did is we took the same fixed beam we took the same fixed beam of theta p theta q del r and del s of A E L I etcetera, but we added one new concept here we said i am applying an axial force; because I am studying stability, we applied axial force right.

So, we gave unit rotation at jth end then we wrote the free body the equation for the free body diagram at the displaced position under the presence of axial load we started with axial compressive load. And we derived we derived the rotational functions that is what we did we derived rotation function, using second order ordinary differential equation from the three here flexure and we found out the rotation coefficients from the solution.

And we said r i is α i times of sin α i - α i cos α i by 2 times of 1 - cos α i - α i sin α. This was the r i and c i we derived as α i - sin α i by sin α i - α i cos α we derived this they are called as rotation functions.

Why rotation functions we derive this by giving unit rotation we derived this. We also proved at axial 4 0 at axial load 0 we showed that I write it here r i at phi i 0 axial load 0 is 4 and c i at phi i 0 is 0.5 is it not we use l hospitals rule then we derived this.

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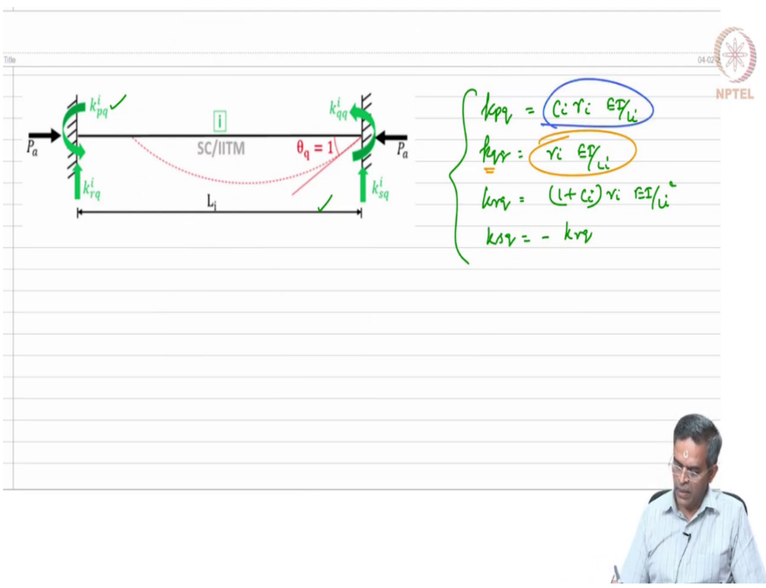

Then subsequently friends if we recollect, if we recollect, we rotation at the kth end and derived the coefficients simultaneously.

And we found out that for this case kpp was found to be r_i times of EI by L_i kqp was found to be c i times of kpp which is c_i times of r_i of E I by L_i. And we said k_{rp} is r_i $(1 + c_i)$ EI by L_i square and k_{sp} which is here was $-k_{rp}$. We derived this is it not then we gave unit rotation to the kth end derive the stiffness matrix in the same.

Then we said k_{pq} which is here is, now c_i r_i EI by L_i whereas, kqq is ri E I by Li and krq is 1 + ci of ri E I by Li square and ksq is - of krq we got this. Then we also did for tensile load we also did for tensile load, and we expressed beta as a function of α.

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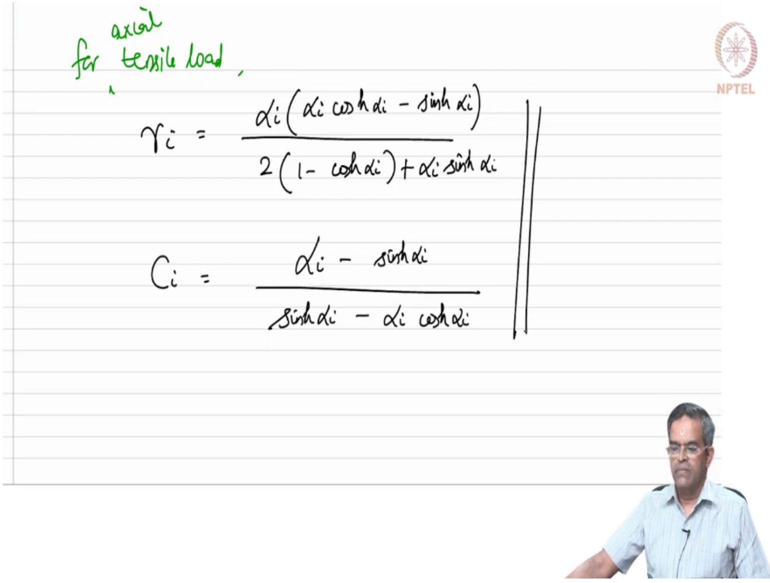

And we worked out we should say axial tensile load we got r i as αi of α i coshαi - sinhαi by twice of 1 - coshαi + αi sinhαi. And ci was found to be αi - sinhαi by sinhαi - αi coshα.

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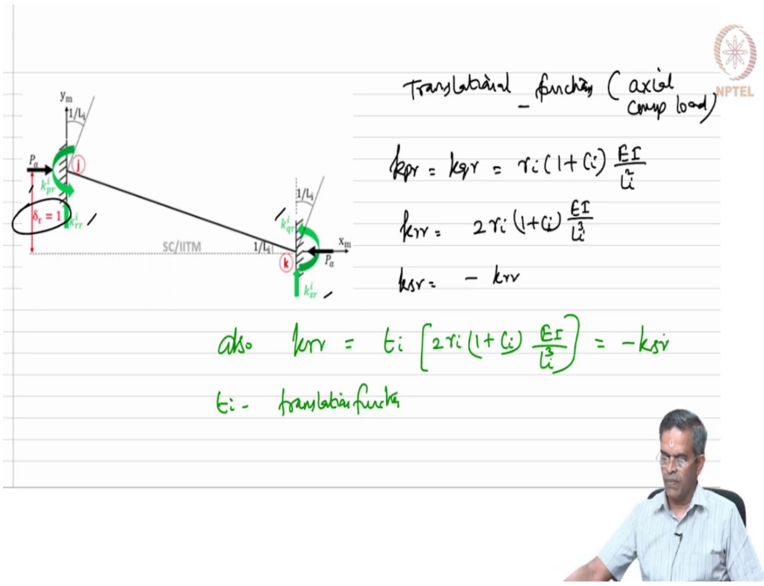

Then we started working on translational coefficients, so we worked on translational coefficients functions let us say under axial compressive load. So, we gave unit displacement at r we also did same way in s as well and these forces generated kpr kqr krr and ksr were determined for this free body diagram is it not. We found that kpr is same as kqr which is same as ri into 1 + ci of EI by Li^2.

And krr is 2 r i 1 + c i of E I by L i q and ksr is - of kr this is what we derived am I right. We wanted to express this as a function of translational constant, so we said also let krr be expressed as t i times of 2 ri 1 + ci E I by Liq where ti is called as translation function we also say this is - of ksr.

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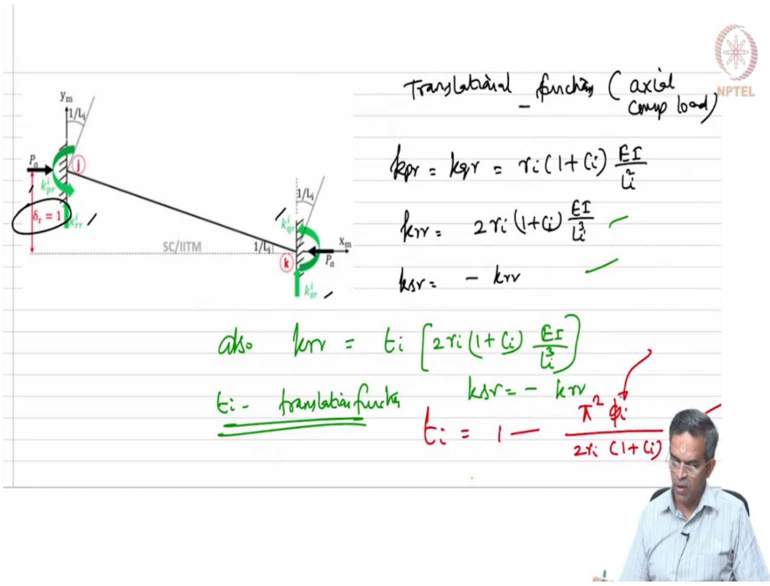

So, we should write this way ksr is - of kr we follow the same logic here. Where t i is a translation function which was given by t i is equal to 1 - pi square phi i 2 r i 1 + c i. Remember phi is a multiplier or the ratio between the axial load and the Euler's buckling load that is a ratio, I think we already know this equation how phi is connecting p and p e because we derived this.

After deriving this we summarize these constants ok, we summarize them and we wrote it in a matrix form which is quite convenient and this resembles the same I am writing the matrix form here.

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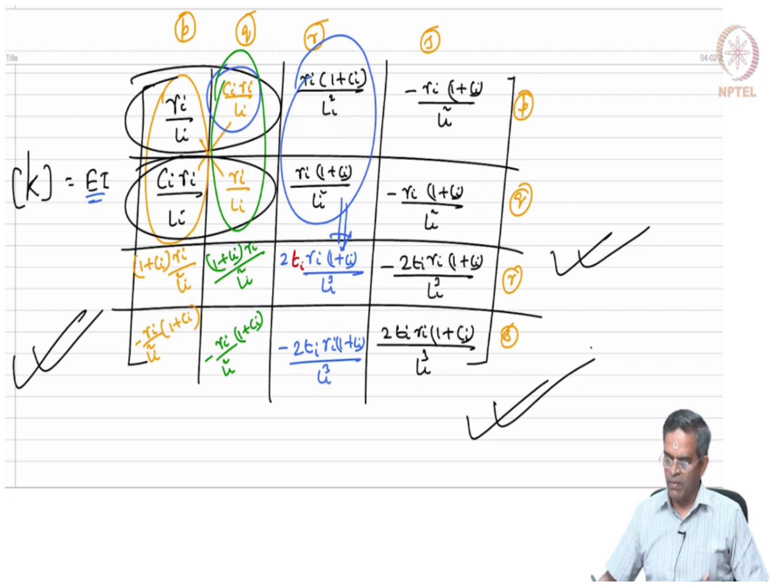

The stiffness matrix for stability function k E I times ofi i r i by L i c i r i by L i let us.

Then I am using the same logic see here how we wrote this sum of these two by l was this and this is negative of this is it not that is what we did I am using the same algorithm here. Some of these two by l that is 1 + c i of r i by L i square this is - of r i by L i square 1 + c i am using the same logic. If you look at this is crossed, is it not see here this is crossed.

So, I will cross it here I will cross this here. So, I am writing this as c i r i by L i this is simply r i by L i. In fact, that is what you will get here see here, kpp kqp this is the first column the second column will be kqq is r i E I by L i, I am doing the same thing this is p this is q r and s.

So, I am looking for let us say kpq I am looking for kpqkpq let us see here kpq is c i r i E I by L i c i r i E I by L i. So, I am doing the same thing I am just so, I am making you to easily remember this equation that is what I am trying to do. Similar to what we wrote in the beam analysis. So, once we do this some of these two will be here by l.

So, that is 1 + c i of r i by L i square then - of this r i by L i square 1 + c. Now if you look at the third value here sum of these two by L i. So, r i times of 1 + c i by L i square sum of these two by L i.

So, that is r i 1 + c i times of by L i square this is what we did in the beam also is it not see here. Sum of these two by L is this, sum of these two by L is this ok, sum of these two by L is this same way here. Sum of these two by L will be this which will be 2 times of r i 1 + c i by L i cube.

Since it is a translation function, I put t i here let me do it in different colour I am putting t i here. So, therefore, this will be - two t i of r i of 1 + c i L i cube. The fourth column as you see here is negative of the third column see here, I am doing the same thing here negative of the third column. So, - of r i 1 + c i by L i square - of r i 1 + c i by L i square - of 2 t i r i 1 + c i by L i cube + of 2 t i r i 1 + c i by L i cube.

Friends it is very easy to remember this matrix if you remember this matrix. So, I have conveniently mapped for your convenience of at least remembering a standard beam derivation which is a fixed beam, to a stability function which is much more complicated in derivation see. The derivation is complicated, but you can remember this equation very well without any mistake.

And whereas, this r i and c i values are already given we already derived them you can remember them right.

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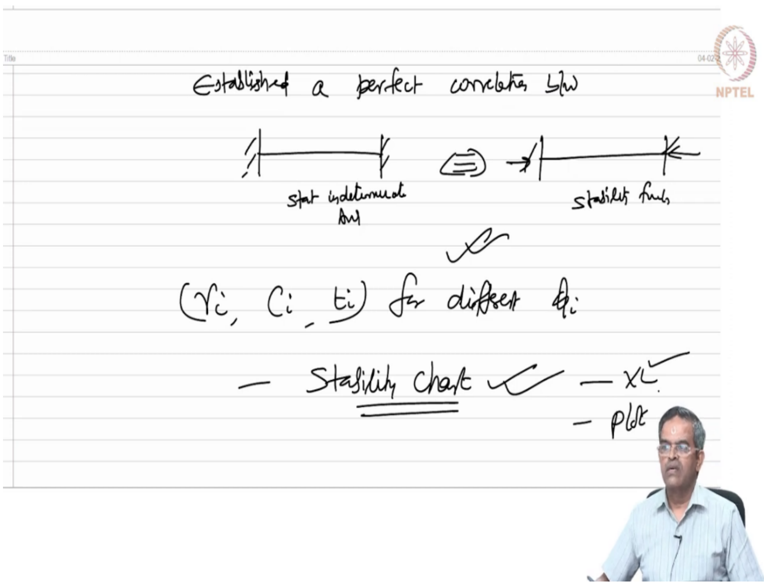

Now having said this, now we have established a perfect correlation between a standard fixed beam used for static indeterminate analysis with that of a problem with axial load to derive stability functions there is a perfect correlation is it not. So, this has got to be clearly understood and one can also plot this r i c i and t i values for different phi this is called stability chart which I showed you in the last lecture.

So, we have a program available in the textbook written by me which is referred in this course it has got a MATLAB program the MATLAB program will help you to generate this chart in excel sheet. Can also plot which I am going to use it for solving the problems in this coming lectures. Now the question comes friends people like to know, what is the difference between buckling and stability.

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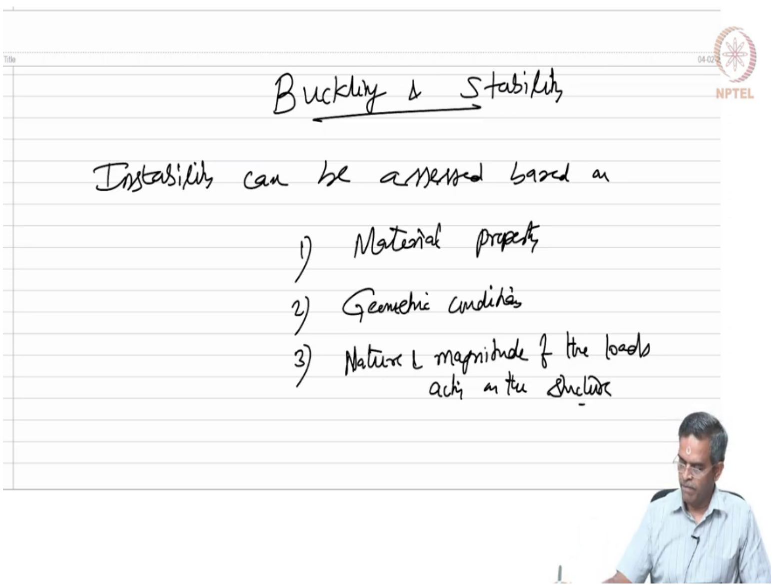

Friends, instability can be assessed based on the following 1 material property, 2 geometric conditions, 3 nature and magnitude of the loads acting on the structure the standard thing.

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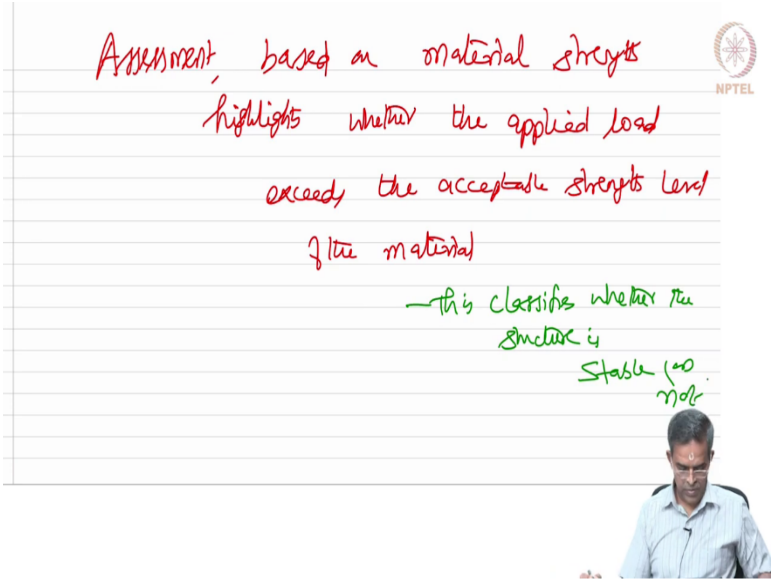

Therefore, friends having said this assessment based on material strength highlights whether the applied load exceeds the acceptable strength level of the material this classifies, or this assessment classifies whether the structure is stable or not.

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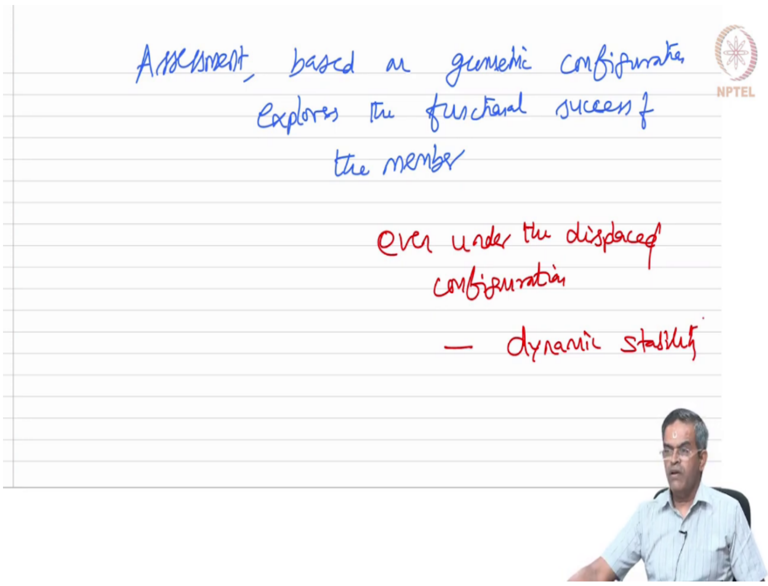

Now, assessment based on geometric configuration explores the functional success of the member, it explores a success even under that is a very important condition here friends even under the displaced configuration this is what we say as dynamic stability.

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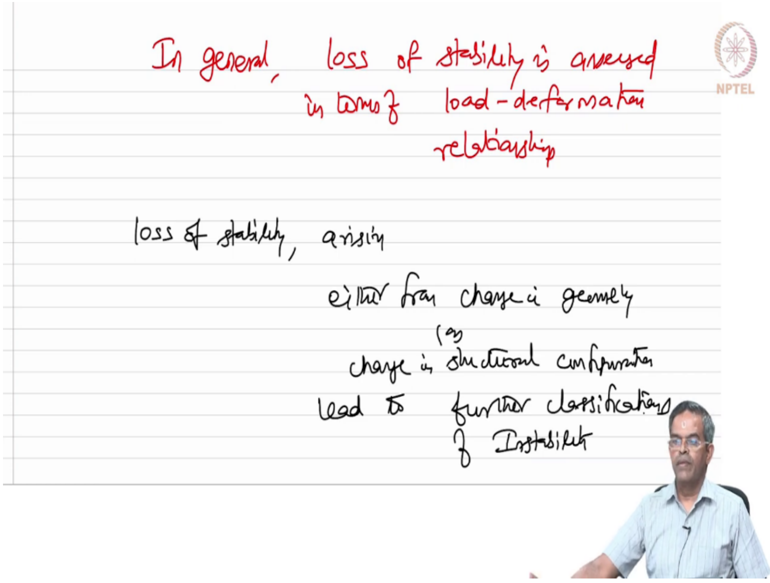

Therefore, friends in general in general loss of stability is assumed or is assessed in terms of load deformation relationship. So, loss of stability arising either from change in geometry or change in structural configuration, lead to further classifications of instability what are they.

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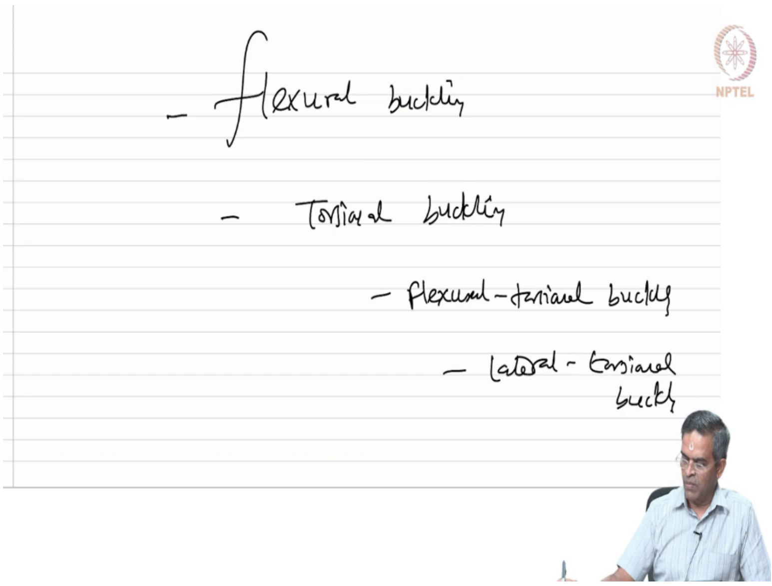

They are; flexural buckling, torsional buckling, flexural-torsional buckling, lateral-torsional buckling. So, friends we are slowly connecting stability to buckling. So, very clearly loss of stability will lead to classifications of buckling.

Instability leads to classification of buckling that is what how they are connected to.

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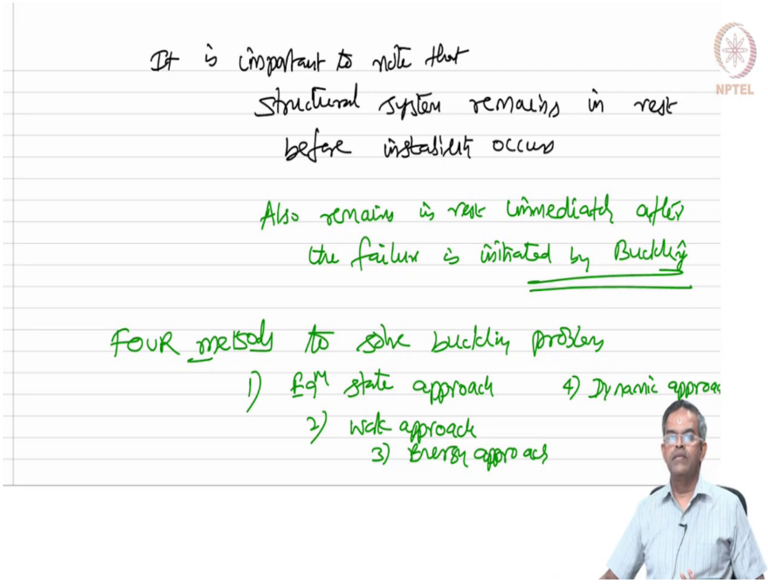

Furthermore, it is important to note that structural system remains in rest before instability occurs also remains in rest immediately after the failure is initiated by buckling. So, instability will initiate a failure and buckling is a failure instability is a geometric displacement, it is a geometric imbalance.

The consequence of instability is buckling. There are four methods to solve buckling problems; 1 equilibrium state approach, 2 work approach, 3 energy approach, 4 dynamic approach. So, we have connected bridged the understanding between stability and buckling is it not.

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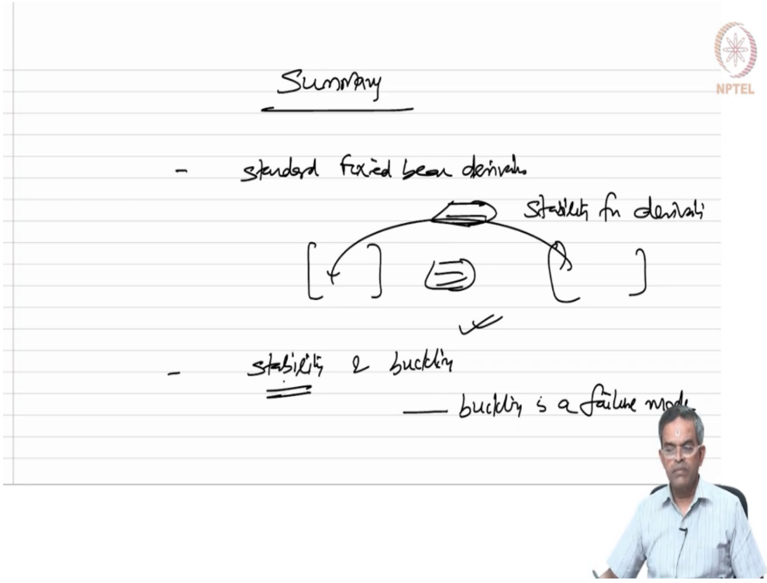

So, let us see what we learned from this lecture.

We have learnt how a standard fixed beam derivation is connected to stability function derivation. We have remembered the matrix with respect to the stability function and we realize that it is very easy, and it is having an one to one mapping. We have also learnt the difference between stability and buckling and we realize that buckling is a failure mode is consequence of instability.

So, in the next lecture we will take up some problems use MATLAB to find out or to check stability of a given structural system we will do some examples.

Thank you very much and have a good day bye.