

**Advanced Design of Steel Structures**  
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**Lecture - 33**  
**Critical buckling load - Numerical examples**

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The slide contains the following handwritten text:

- Lecture 33
- Advanced Steel design
- Numerical examples to determine critical buckling load
- Mat lab program

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Friends, welcome to the 33rd lecture of the course Advanced Steel Design. In this lecture we are going to do some numerical examples to determine Critical Buckling load we are going to use MATLAB programs for doing this.

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Basis - Basic assumption - to obtain the buckling load of a system is

- Buckling loads are estimated
- structures only transfer axial forces
- In case, they encounter transverse loads, then these loads will cause additional moments
  - alter the  $k$  significantly
  - (P-M) interaction to estimate the buckling loads
- The deformation is sufficiently small compared to its initial condition
- Hence, critical buckling load can be estimated using linear theory



So, let us first see the basis of doing this analysis. Friends, one of the basic assumptions which we made to obtain the buckling load of a structural system is the deformation is sufficiently small compared to its initial condition that is it undergoes small deformation. And hence critical buckling load can be estimated using linear theory. Furthermore, friends it is also important that buckling loads are estimated under the assumption that structures only transfer axial forces.

In case they encounter transverse loads then these loads will cause additional moments which will alter the stiffness significantly. Therefore, in such situation one must use continuous P-M interaction curve to estimate the buckling load we must remember this. So, we are under the assumption the structure will undergo only or will encounter only axial forces.

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fundamental logic to obtain the buckling load

- neglect the axial deformation  
Hence, joint loads related to the un-restrained joint displacements are set to zero

UR	restr
1 ( $\theta_1$ )	1 ( $\delta_5$ )
1 ( $\theta_2$ )	1 ( $\delta_6$ )
-	-

$$\begin{aligned} UR &= 2 (\theta_1, \theta_2) \\ R &= 6 (\theta_3, \delta_4, \dots) \end{aligned}$$

Now, let us see what the basic fundamental logic is to obtain the buckling load. Friends, we already said we are taking a basic module we are neglecting the axial deformations. Therefore, joint loads related to the unrestrained joint displacements are set to 0. You may be wondering how to identify an unrestrained joint displacement we will give some examples.

Let us say I take here three span continuous beam like this let me mark the unrestrained displacements for all the type of supports first. So, if it is a roller support, the roller can undergo rotation, but roller cannot undergo vertical displacement. So, I will make a table unrestrained and restrained let me call this as  $\theta$  and this as  $\delta v$ . So, I showed a table unrestrained as 1 which is  $\theta$  and restrained as 1 which is  $\delta v$ .

If I have a hinged support this can undergo rotation, but again vertical displacement is restrained. So, one rotation unrestrained one vertical displacement restrained. If you have a fixed support rotation as well as restraints alright, so this 0 this is 2 1 is  $\theta_1$  is  $\delta v$ . So, these are unrestrained displacements of varieties of supports learning this we can now mark the unrestrained displacements in green.

So, as  $\theta_1$ ,  $\theta_2$  and restrained degrees of freedom in red  $\theta_3$ ,  $\delta_4$ ,  $\delta_5$ ,  $\delta_6$ ,  $\theta_7$ ,  $\delta_8$  by neglecting axial deformation. I can say this beam has got unrestrained joint displacements as 2 which is  $\theta_1$  and  $\theta_2$  restrained displacements are 1, 2, 3, 4, 5, 6 which is  $\theta_3$ ,  $\delta_4$  and so, on. So, what we are now saying is unrestrained joint displacements are set to 0.

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Matrix math calcs,  $\{J_L\} = \{0\}$  ✓

$[K] = \begin{bmatrix} k_{uu} & k_{ur} \\ k_{ru} & k_{rr} \end{bmatrix}$

$J_L = \begin{Bmatrix} J_{Lu} \\ J_{Lr} \end{Bmatrix}$

$\Delta_u = \begin{Bmatrix} \Delta_u \\ \Delta_r \end{Bmatrix}$

$\{J_{Lu}\} = [k_{uu}] \{ \Delta_u \} - (1)$

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So, mathematically the joint load unrestrained should be set to 0. We know for a given matrix  $k$  if we partition this matrix as unrestrained and restrained unrestrained and restrained. So, this is  $k_{uu}$ , this is  $k_{ur}$ , this is  $k_{ru}$ , this is  $k_{rr}$  we also know the joint load can also be partitioned as  $J_{Lu}$  and  $J_{Lr}$   $u$  stands for unrestrained and the displacements can also be partitioned unrestrained displacements and restrained displacements restrained displacements anyway will be equal to 0.

We have an equation now force is stiffness multiplied by displacement is it not. So, we already said this should be set to 0. So, look at this equation now.

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Now,

either  $\{ \Delta_u \}$  must be zero

or  $[k_{uu}]$  should be set to zero

$\{ \Delta_u \} \neq 0$  ; Non-trivial solution called by

$|k_{uu}| \equiv 0 \quad (2)$

The Eqn - is Buckle Anal is called characteristic determinant

- Expand this determinant, to obtain the buckling factor

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This equation left hand side is 0 which implies the fact. Now this implies a fact that either  $\delta u$  must be 0 or  $k_{uu}$  should be set to 0. Now friends you please see  $\delta u$  cannot be 0 is under it undergoes displacement is it not? Because unrestrained joint displacements are not zero. So, the nontrivial solution could be determinant of this  $k_{uu}$  matrix should be set to 0. The equation 2 in stability analysis is called not stability I will say buckling analysis is called characteristic determinant. Now when you expand the determinant to obtain the buckling failure condition.

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If the axial load is lesser than the critical buckling load,  
 if  $(P_a < P_{cr})$ ,  
 then, the joint displacements of unrestrained joints  
 will be zero.  
 Hence  $[k_{uu}]$  will be positive  
 - Stable condition  
 on the other hand,  $P_a > P_{cr}$ , this will refer to unstable condition  
 $[k_{uu}]$  will be negative

Friends, we should also remember if the axial load is lesser than the critical buckling load that is if axial load is lesser than critical buckling load then the joint displacements of unrestrained joints will be 0. Because for buckling failure you must impose this displacement hence determinant  $k_{uu}$  will be positive which corresponds to a stable condition.

On the other hand, if the applied load exceeds buckling load critical buckling load this will refer to unstable condition because we already said buckling is a mode of consequence of instability. So, in this case  $k_{uu}$  determinant will be negative that is an important point which we have to remember.

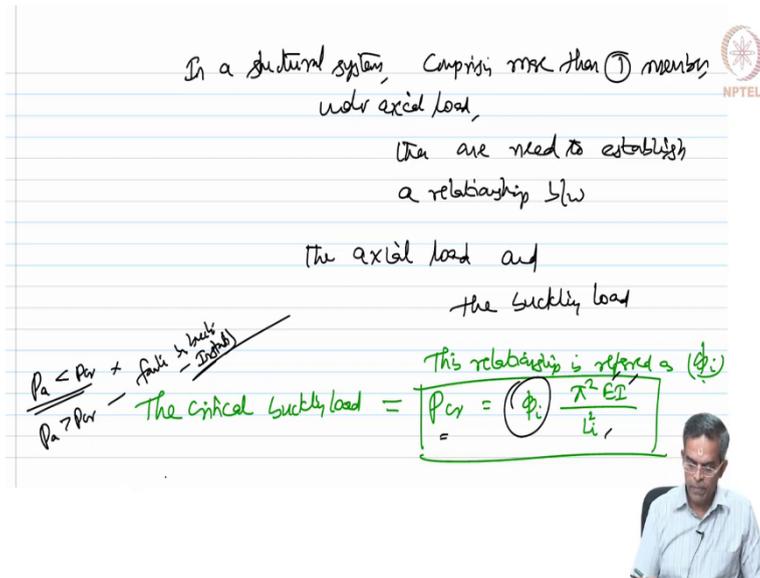
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In a structural system, comprising more than one member under axial load, one needs to establish a relationship between the axial load and the buckling load.

This relationship is referred to as  $\phi_i$ .

The critical buckling load =  $P_{cr} = \phi_i \frac{\pi^2 EI}{L_i^2}$

$\frac{P_a < P_{cr}}{P_a > P_{cr}}$  \* Failure by buckling (Instability)



Furthermore friends, in a structural system comprising more than one member under axial load then one needs to establish a relationship between the axial load and the buckling load. This relationship is referred to as  $\phi_i$  which we have used in the derivation. Therefore, the critical buckling load can be given by the relationship  $P_{cr} = \phi_i \pi^2 EI / L_i^2$ .

So, if you know  $\phi_i$  if you know  $\phi_i$  know  $E I$  know cross sectional moment of inertia  $I$  know the length of the member  $L_i$  I can find the critical buckling load and if  $P_a$  is lesser than  $P_{cr}$  applied load we do not have to bother if  $P_a$  exceeds  $P_{cr}$  we can say it is failing by buckling which is causing instability. So, with this background let us start doing a numerical example of estimating the critical load we will see this example 1.

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**Example 1**

Unrestrained dof (displacements) = 2 ( $\theta_1, \delta_2$ )

Restrained displacement = 4 ( $\theta_3, \delta_4, \delta_5, \delta_6$ )

Neglect axial def,  
= 3 ( $\theta_3, \delta_4, \delta_5$ )

choose j<sup>th</sup> end @ A  
k<sup>th</sup> end @ B

Labels (1, 3, 2, 4)

translations @ j<sup>th</sup> end along the y  
translations @ k<sup>th</sup> end along the y

rotations @ j<sup>th</sup> end  
rotations @ k<sup>th</sup> end

SC, ITM  
L, E, I

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So, on the screen you see a column subjected to axial compressive load the degrees of freedom are marked. So, unrestrained degrees are marked in green that is displacements. Displacements include both rotation and translation remember that are marked in green color and restrained displacements are marked in red color. So, now we can see very well here this problem has got 2 unrestrained degrees displacements that is  $\theta_1$  and  $\delta_2$  and this has got 6 well 4 restrained degrees namely  $\theta_3, \delta_4, \delta_5$  and 6.

Suppose if we neglect axial deformation then restrained degree will be 3 which will be  $\theta_3, \delta_4$  and  $\delta_5$  because this will also become  $\delta \phi$  this will also become  $\delta \phi$  I am neglecting axial deformation. So, now we have chosen j<sup>th</sup> end at here which is marked as A and this is k<sup>th</sup> end which is B at A and k<sup>th</sup> end at B. So, this is my x<sub>m</sub> and y<sub>m</sub>. Now I can write the stiffness matrix for stability functions for this problem we can recollect that equation which we wrote.

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So, the stability function which will be EI times of let us mark the labels of this problem. So, rotation at j. So, label is going to be rotation at j now rotation at k then translation along positive m positive y at j translation along positive y at k. So, 1, 3, 2, 4 are the labels. How do you do that? I repeat again rotation at j th end rotation at k th end translation at j th end along positive y translation at k th end along positive y that is standard know see look at the fixed beam this is x m this is y m  $\theta$  p  $\theta$  q  $\delta$  r and  $\delta$  s is it not.

So, p corresponds to rotation at j th end q corresponds rotation at k th end r corresponds to translation at j th end along positive y and s is at the k th end, so same logic here. So, let me write down this matrix. We already know this is r by l this is C r by l this is 1 plus c of r by l is minus of this in the next column I just interchange this. So, this is r by l this is c by l and sum of these two this is r c by l.

So, 1 plus c r by l minus 1 plus c r by l and the third column sum of these two by l. So, 1 plus c r by l sum of these two 1 plus c r by l then sum of these two by l, but I have to use my l square right this is l square by l square again. So, this is l square this is l square this is l square because it is sum of these two by l is it not? Again, this is l square sum of these two by l. So, now, this value k 3 3 will be sum of these 2 read by l. So, let me write that.

So, twice of 1 plus c r by l cube and I also say t translation function is it not. So, minus 2 r t 1 plus c by l cube. Fourth column is minus of third. So, minus of 1 plus c r by l square minus of 1 plus c r by l square minus of 2 r t 1 plus c l cube 2 r t 1 plus c l cube. So, this is what my



matrix is. Let me enter the labels we already marked the labels 1 3 2 4. So, 1 3 2 4 now friends please note this problem has two unrestrained degrees is it not.

So, therefore, I must partition this matrix at 2 by 2 to get my kuu. So, let me partition this matrix at 2 by 2 and this becomes my k uu is it not. So, I can now write kuu as r by l, c r by l sorry 1 2 sorry. So, not partitioning I think we can pick up the first and second. So, I must pick up k 1 1 k 1 2 and then k 2 1 and k 2. So, let me write kuu as k11 k12 1 plus c r by l square then k 2 1 1 plus c r by l square then k 2 2 which is 2 times of r t 1 plus c by l cube am I right. So, I have kuu now. So, let me copy this and put it here. I have kuu.

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$$k_{uu} = \begin{bmatrix} \textcircled{1} & \textcircled{2} \\ \textcircled{3} & \textcircled{4} \end{bmatrix} = \begin{bmatrix} r/l & (1+c)r/l \\ (1+c)r/l & 2rt(1+c)/l^3 \end{bmatrix}$$

setting  $|k_{uu}| = 0$

$$\left(\frac{r}{l}\right) \left(\frac{2rt(1+c)}{l^3}\right) - \left(\frac{(1+c)r}{l^2}\right)^2 = 0$$

simplify

$$\cancel{r} \frac{2t(1+c)}{l^2} = \frac{(1+c)\cancel{r}(1+c)\cancel{r}}{l^2}$$

$$\begin{matrix} t \\ c \\ r \end{matrix} \parallel \text{one stability function}$$

$$2t = 1+c$$

$$\boxed{(2t - c) = 1} \quad (1)$$

Let me that is kuu matrix as we just now discussed I must set determinant of this to 0. So, setting determinant of kuu to 0. So, if I do that. So, it is going to be r by l into 2 r t 1 plus c by l cube minus 1 plus c r by l square the whole square is it not? Should be set to 0. Simplifying you can simplify this very easily because you know 1 4 1 4 here 1 plus c into r 1 plus c into r here in the left-hand side. So, if I say r by l square of 2 r t 1 plus c is equal to 1 plus c into r 1 plus c into r by l 4 4 goes away 1 plus c goes away r goes away and one more r goes away.

So, I can now say 2 t is equal to 1 plus c or 2 t minus c will be 1 this is my control equation. These are all stability functions t c and r are stability functions friends which we derived in the previous lectures under axial compression axial tension we have these values. So, the equation 1 on the screen is called characteristic equation which is 2 t minus c is 1 is called characteristic equation.

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Eqn:  $2t - c = 1$  ✓ ✓  
 ↳ called characteristic Eqn

for  $\phi_i$  we have stability charts ( $r, c, t$ ) values  
 Choose  $\phi_i$  values, which satisfy the characteristic Eqn

Matlab program  
 for  $\phi_c = 0.25$ ,  $r = 3.6598$   
 $t = 0.7857$   
 $c = 0.5708$

$P_{cr} = \phi_i \frac{\pi^2 EI}{L_i^2}$

So, now, for different values of  $\phi_i$  we have the stability charts is it not which indicates  $r$ ,  $c$  and  $t$  values. So, what I should do I should go to the table choose  $\phi_i$  value which satisfies the characteristic equation. I will show you typically how the stability chart looks like give me a minute.

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
-10	11.18638	0.111817	4.967776											
-9.9	11.13816	0.112439	4.942899											
-9.8	11.08971	0.11307	4.917897											
-9.7	11.04104	0.113712	4.892768											
-9.6	10.99213	0.114365	4.867511											
-9.5	10.943	0.115028	4.842123											
-9.4	10.89362	0.115702	4.816602											
-9.3	10.84401	0.116388	4.790947											
-9.2	10.79415	0.117086	4.765154											
-9.1	10.74404	0.117795	4.739223											
-9	10.69368	0.118517	4.71315											
-8.9	10.64307	0.119251	4.686933											
-8.8	10.59219	0.119999	4.660571											
-8.7	10.54106	0.12076	4.63406											
-8.6	10.48966	0.121535	4.607398											
-8.5	10.43798	0.122324	4.580583											
-8.4	10.38604	0.123128	4.553612											
-8.3	10.33381	0.123947	4.526482											
-8.2	10.2813	0.124781	4.499191											
-8.1	10.2285	0.125632	4.471735											
-8	10.1754	0.126499	4.444112											

Friends, I will just enlarge this it is a typical stability chart friends for different values of  $\phi_{min}$  indicates tension and positive indicates compression.

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A	B	C	D
-7.9	10.12201	0.127383	4.416319
-7.8	10.06832	0.128285	4.388353
-7.7	10.01432	0.129205	4.36021
-7.6	9.960003	0.130143	4.331887
-7.5	9.905371	0.131101	4.303381
-7.4	9.850415	0.132079	4.274689
-7.3	9.79513	0.133078	4.245806
-7.2	9.739509	0.134098	4.216729
-7.1	9.683548	0.13514	4.187455
-7	9.627239	0.136206	4.157978
-6.9	9.570578	0.137294	4.128296
-6.8	9.513556	0.138408	4.098404
-6.7	9.456167	0.139546	4.068297
-6.6	9.398406	0.140711	4.037972
-6.5	9.340264	0.141904	4.007423
-6.4	9.281734	0.143124	3.976646
-6.3	9.222808	0.144374	3.945635
-6.2	9.16348	0.145655	3.914385
-6.1	9.10374	0.146967	3.882894
-6	9.043582	0.148313	3.851152
-5.9	8.982995	0.149692	3.819156

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A	B	C	D
-0.7	4.848334	0.373502	1.518736
-0.6	4.735092	0.387231	1.450758
-0.5	4.619442	0.402114	1.380949
-0.4	4.501251	0.418298	1.309192
-0.3	4.380376	0.435956	1.235363
-0.2	4.256658	0.455289	1.159324
-0.1	4.129929	0.476541	1.080925
0	4	0.5	1
0.01	3.986823	0.50248	0.991762
0.02	3.973613	0.504986	0.983496
0.03	3.960368	0.507519	0.975203
0.04	3.947088	0.510078	0.966883
0.05	3.933773	0.512664	0.958534
0.06	3.920423	0.515278	0.950158
0.07	3.907038	0.51792	0.941753
0.08	3.893617	0.52059	0.93332
0.09	3.880161	0.523289	0.924858
0.1	3.866668	0.526017	0.916368
0.11	3.853139	0.528775	0.907848
0.12	3.839574	0.531563	0.899299
0.13	3.825972	0.534382	0.890721

It goes to 0 the 0 value is here then it goes positive.

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	A	B	C	D
0.14	3.812333	0.537232	0.882113	
0.15	3.798657	0.540114	0.873474	
0.16	3.784943	0.543028	0.864806	
0.17	3.771192	0.545975	0.856108	
0.18	3.757402	0.548955	0.847378	
0.19	3.743575	0.551969	0.838618	
0.2	3.729709	0.555018	0.829827	
0.21	3.715804	0.558101	0.821005	
0.22	3.70186	0.56122	0.812151	
0.23	3.687877	0.564375	0.803266	
0.24	3.673855	0.567567	0.794348	
0.25	3.659792	0.570796	0.785398	
0.26	3.64569	0.574064	0.776416	
0.27	3.631547	0.57737	0.767401	
0.28	3.617364	0.580715	0.758353	
0.29	3.60314	0.584101	0.749271	
0.3	3.588875	0.587527	0.740157	
0.31	3.574568	0.590995	0.731008	
0.32	3.56022	0.594505	0.721826	
0.33	3.545829	0.598058	0.712609	
0.34	3.531396	0.601655	0.703358	

So, positive compression. So, it is varying from minus 10  $\phi$  value to plus 4 we can see that plus 4.

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	A	B	C	D
1.61	1.199998	2.492711	-0.89563	
1.62	1.175871	2.553394	-0.91329	
1.63	1.151609	2.617018	-0.93109	
1.64	1.127212	2.683801	-0.949	
1.65	1.102677	2.753983	-0.96704	
1.66	1.078004	2.827828	-0.9852	
1.67	1.05319	2.90563	-1.0035	
1.68	1.028234	2.987712	-1.02192	
1.69	1.003134	3.074437	-1.04047	
1.7	0.977888	3.166206	-1.05915	
1.71	0.952495	3.263472	-1.07797	
1.72	0.926952	3.36674	-1.09693	
1.73	0.901259	3.476582	-1.11602	
1.74	0.875412	3.593641	-1.13525	
1.75	0.849411	3.718652	-1.15463	
1.76	0.823253	3.852448	-1.17414	
1.77	0.796937	3.995986	-1.19381	
1.78	0.770459	4.150366	-1.21361	
1.79	0.743819	4.316861	-1.23357	
1.8	0.717015	4.496949	-1.25368	
1.81	0.690043	4.692361	-1.27394	

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3.49	-13.3907	-1.08594	-13.9661												
3.5	-13.719	-1.08237	-14.284												
3.51	-14.0601	-1.0789	-14.6148												
3.52	-14.4149	-1.07551	-14.9591												
3.53	-14.7842	-1.07221	-15.318												
3.54	-15.1689	-1.06899	-15.6922												
3.55	-15.5702	-1.06586	-16.0829												
3.56	-15.989	-1.06282	-16.4912												
3.57	-16.4267	-1.05985	-16.9183												
3.58	-16.8845	-1.05697	-17.3655												
3.59	-17.364	-1.05417	-17.8343												
3.6	-17.8668	-1.05145	-18.3264												
3.61	-18.3946	-1.04881	-18.8435												
3.62	-18.9494	-1.04624	-19.3875												
3.63	-19.5335	-1.04375	-19.9608												
3.64	-20.1492	-1.04134	-20.5657												
3.65	-20.7993	-1.039	-21.2049												
3.66	-21.4868	-1.03674	-21.8814												
3.67	-22.215	-1.03455	-22.5987												
3.68	-22.9879	-1.03243	-23.3605												
3.69	-23.8096	-1.03038	-24.1713												

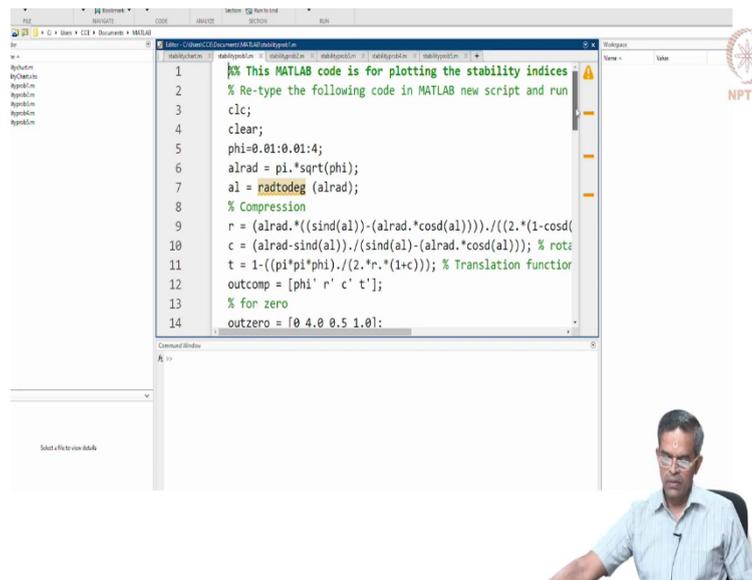
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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
3.24	-7.87795	-1.20919	-8.70195												
3.25	-8.032	-1.20279	-8.84642												
3.26	-8.18989	-1.19654	-8.99469												
3.27	-8.35175	-1.19042	-9.14691												
3.28	-8.51776	-1.18444	-9.30325												
3.29	-8.68809	-1.17859	-9.46388												
3.3	-8.86293	-1.17287	-9.62899												
3.31	-9.04246	-1.16728	-9.79876												
3.32	-9.2269	-1.16181	-9.97341												
3.33	-9.41646	-1.15647	-10.1532												
3.34	-9.61138	-1.15125	-10.3382												
3.35	-9.8119	-1.14614	-10.5289												
3.36	-10.0183	-1.14115	-10.7253												
3.37	-10.2308	-1.13628	-10.9279												
3.38	-10.4497	-1.13151	-11.1369												
3.39	-10.6755	-1.12686	-11.3526												
3.4	-10.9082	-1.12231	-11.5754												
3.41	-11.1485	-1.11787	-11.8055												
3.42	-11.3965	-1.11353	-12.0435												
3.43	-11.6528	-1.1093	-12.2897												
3.44	-11.9178	-1.10516	-12.5445												

So, the first column refers to  $r$   $t$  and  $c$ . So, these are the values. So, what I should do for this control equation; for this control equation to be satisfied I must choose a  $\phi$  value it is very difficult know it is very difficult to undergo and scan all entire table to satisfy this equation. So, to solve this problem we use the MATLAB program. So, I will show you. So, we are now using a MATLAB program, the program is available in the textbook referred in this course you can download the program easily MATLAB support is also extended for NPTEL students through IIT Madras link.

You can also undo a free tutorial student MATLAB program is given to you and it does not require a sophisticated laptop or an desktop to run a MATLAB software. A normal laptop should be able to run the problem what I am discussing about MATLAB has got many facilities which requires a high-end computational facility. However, the problem what I am discussing now can be run in an ordinary conventional laptop do not worry about that. So, now, I will show you the MATLAB program.

(Refer Slide Time: 32:35)



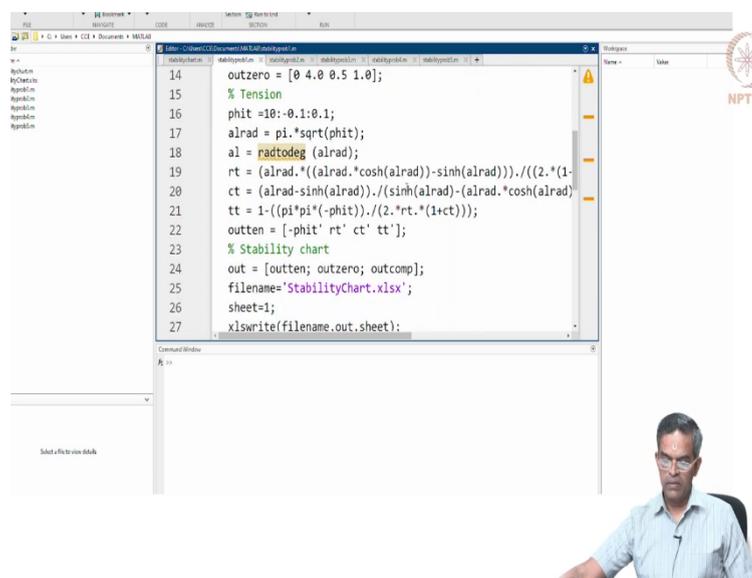
```

1  %% This MATLAB code is for plotting the stability indices
2  %% Re-type the following code in MATLAB new script and run
3  clear;
4  phi=0.01:0.01:4;
5  alrad = pi.*sqrt(phi);
6  al = radtodeg (alrad);
7  %% Compression
8  r = (alrad.*(sind(al)-(alrad.*cosd(al))))./(2.*(1-cosd(
9  c = (alrad-sind(al))./(sind(al)-(alrad.*cosd(al))); % rote
10 t = 1-((pi*pi*phi)./(2.*r.*(1+c))); % Translation function
11 outcomp = [phi' r' c' t'];
12 %% for zero
13 outzero = [0 4.0 0.5 1.0];

```

So, friends what you see on the screen is the MATLAB program.

(Refer Slide Time: 32:45)



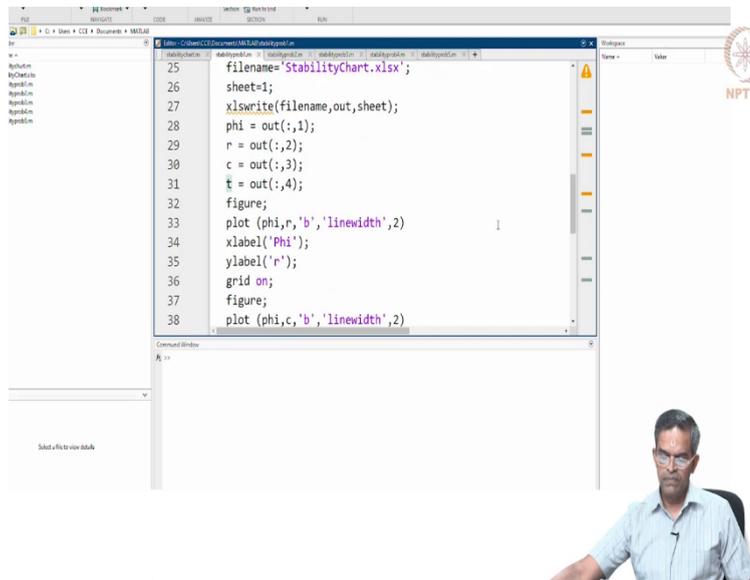
```

14 outzero = [0 4.0 0.5 1.0];
15 %% Tension
16 phit =10:0.1:0.1;
17 alrad = pi.*sqrt(phit);
18 al = radtodeg (alrad);
19 rt = (alrad.*(alrad.*cosh(alrad)-sinh(alrad))))./(2.*(1-
20 ct = (alrad-sinh(alrad))./(sinh(alrad)-(alrad.*cosh(alrad)
21 tt = 1-((pi*pi*(-phit))./(2.*rt.*(1+ct)));
22 outten = [-phit' rt' ct' tt'];
23 %% Stability chart
24 out = [outten; outzero; outcomp];
25 filename='Stabilitychart.xlsx';
26 sheet=1;
27 xlswrite(filename,out,sheet);

```

So, this is a program written for compression you can say from 0.01 to 4 we saw the value just now on the table and for tension it plots up to 10, tension is minus. There is minus right and then for 0 also we have plotted.

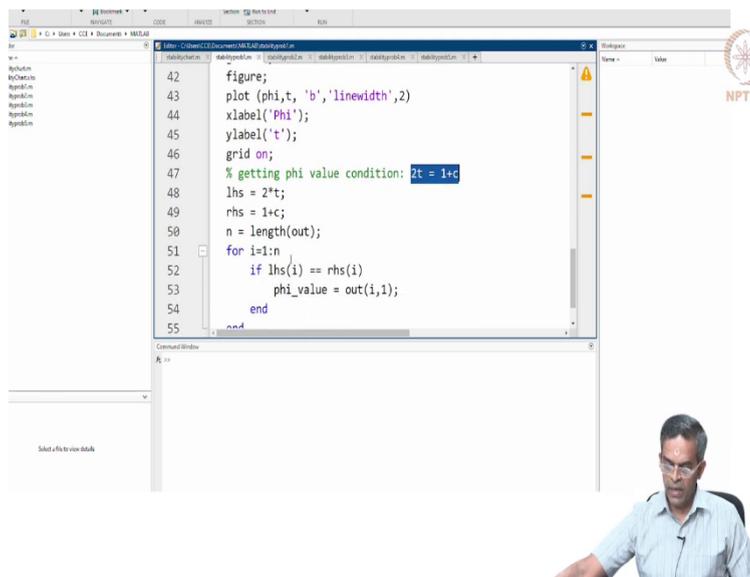
(Refer Slide Time: 33:08)



```
25 filename='StabilityChart.xlsx';
26 sheet=1;
27 xlswrite(filename,out,sheet);
28 phi = out(:,1);
29 r = out(:,2);
30 c = out(:,3);
31 t = out(:,4);
32 figure;
33 plot(phi,r,'b','linewidth',2)
34 xlabel('Phi');
35 ylabel('r');
36 grid on;
37 figure;
38 plot(phi,c,'b','linewidth',2)
```

It is a program it gives you a stability functions r c and t this is an equation what we already have derived you know this. So, we can easily, and chart is also prepared.

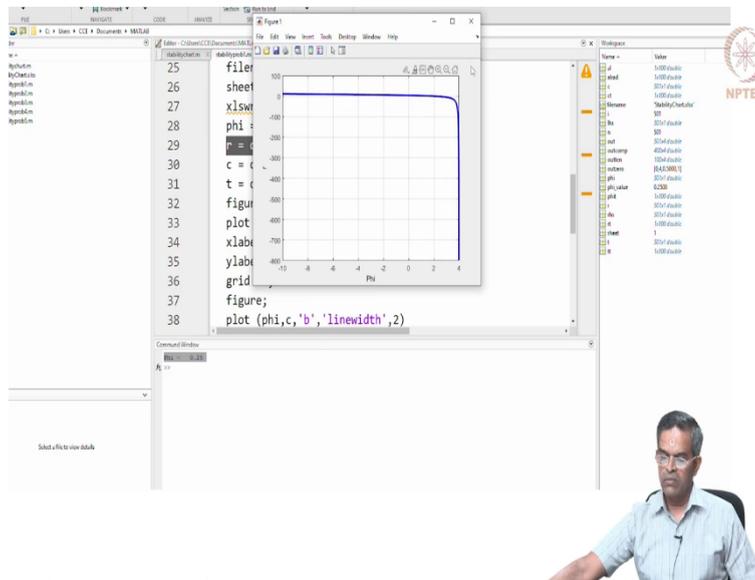
(Refer Slide Time: 33:23)



```
42 figure;
43 plot(phi,t,'b','linewidth',2)
44 xlabel('Phi');
45 ylabel('t');
46 grid on;
47 % getting phi value condition: 2t = 1+c
48 lhs = 2*t;
49 rhs = 1+c;
50 n = length(out);
51 for i=1:n
52     if lhs(i) == rhs(i)
53         phi_value = out(i,1);
54     end
55 end
```

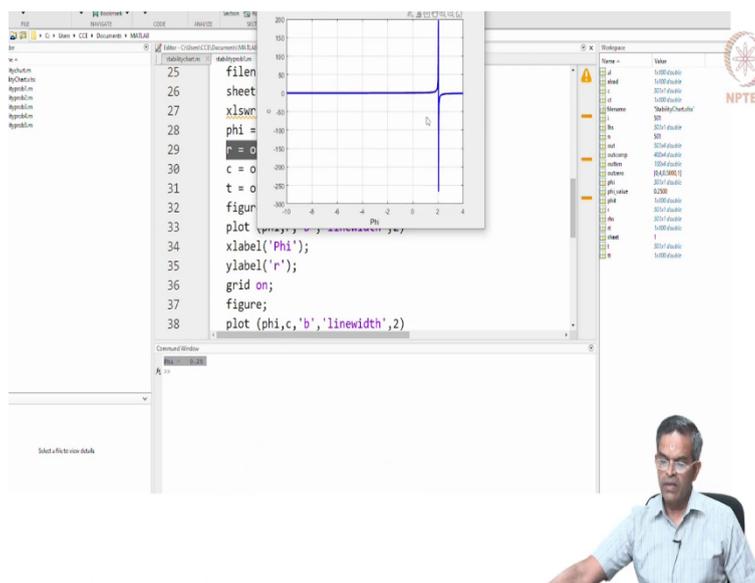
Then further friends we have a control equation which is the characteristic equation for this problem  $2t$  minus  $c$  is 1. So, a program is written to equate and find a specific  $\phi$  value from the output file which is generated from here.

(Refer Slide Time: 33:57)



Satisfy this equation. So, this program is now run. So, it gives me the  $\phi$  value please see on the screen 0.25, it also shows me the chart for your learning this is my  $r$  value variation, and this is my  $c$  variation with respect to  $\phi$  and this is variation of  $t$  with respect to  $\phi$  it is a chart.

(Refer Slide Time: 34:13)





So, it gives me after searching in the table or the chart gives me a  $\phi$  value as 0.25. So, now, I got  $\phi$  value as 0.25 from the MATLAB. So, I will go to the screen now. So, I have got  $\phi$  value as 0.25 now how do we get the buckling load that is the question is it not. So, now, for  $\phi$  value of 0.25, from the chart or table you can refer  $r$  will be 3.6598,  $t$  will be 0.7854 and  $c$  is 0.5708 which satisfies the characteristic equation.

So, now, what I do, I substitute  $P_c r$  as  $\phi$  of  $\pi$  square  $E I$  by  $L$  i square where  $\phi$  is 0.25 if you know the material constant if you know the second moment of area or moment of inertia of the cross section if I know the length of the member, I can find the Euler's critical load. So, the problem is very simple stability functions help you to obtain the Euler's critical load which is going to help you to estimate the buckling load.

So, program is given, derivations are clear, application is being done and I think it is very easy for us to practice a greater number of problems is it not let us do one more problem.

(Refer Slide Time: 36:23)

Ex 2  
 $0.707P = P_{BC}$   
 $P_{AB} = 0.707P$

$(P_{AB})_{AB} = 0.707P$  (Comp)  
 $(P_{BC})_{BC} = 0.707P$  (Comp)

UR = 2 ( $\theta_1, \theta_2$ )  
R = 5 ( $\delta_3, \delta_4, \delta_5, \theta_6, \delta_7$ )  
⑦

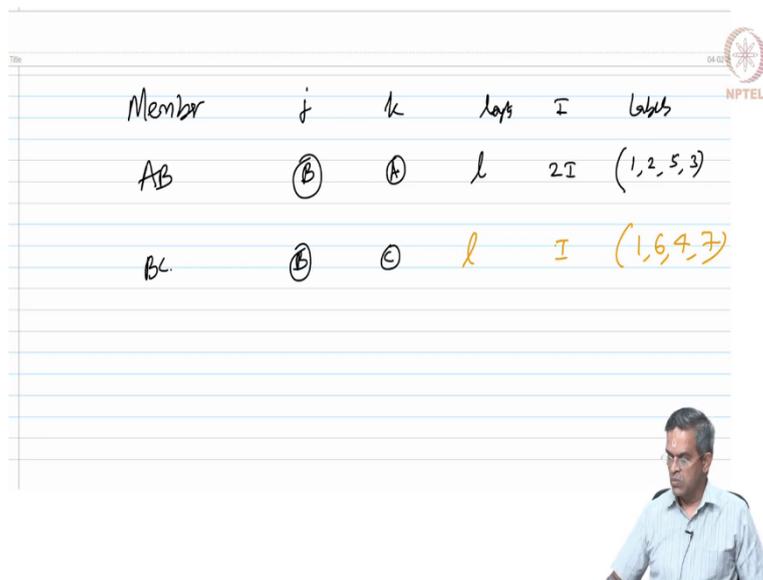
neglect axial  $\delta$

So, example 2 we will do one more problem. So, this is an example 2 where one member is having  $2I$  other member has moment of inertia as  $I$ . A B and C are the nodes. As the length of the member is same, I can interpret this angle has 45 degrees right. Now I can also find at node B; at node B if I have a load  $P$  applied here, I can find the component of this load which I am going to call as  $P_{BC}$  and the component of this load which I am going to call as  $P_{AB}$  or  $P_{BA}$  as  $0.707P$ .

Now the angle is 45 degree right this angle is 90 and this angle is 45. So, now, I can say the axial load on the member AB is  $0.707 P$  compressive the axial load on the member BC is  $0.707 P$  compressive. Now, there are two members sharing this load  $P$ . So, we must carefully form the characteristic equation now let us mark the unrestrained degrees one can see here on the screen  $\theta_1$  and  $\theta_2$  are the rotations which are unrestrained remaining all are restrained and look at  $\delta_4$  same both places indicating we are neglecting axial deformation.

Similarly,  $\delta_5$  and  $\delta_6$  neglect axial deformation along the member BC. So, I can say this member has unrestrained degrees 2 which are  $\theta_1$  and  $\theta_2$  restrained degrees as 1 2 3 1 3 4 5 6 and 7. So, 5 which are  $\delta_3, \delta_4, \delta_5, \theta_6$  and  $\delta_7$ . So, the total kinematic degree of freedom is 7 neglecting axial deformation restrained is 5 and unrestrained is 2 as usual I must now derive the stiffness matrix using rotational stability functions for the member AB and for the member BC.

(Refer Slide Time: 39:44)



Member	j	k	Length	I	Labels
AB	B	A	$l$	$2I$	(1, 2, 5, 3)
BC	B	C	$l$	$I$	(1, 6, 4, 7)

So, for the member AB I will make a table for the member j and k and I and labels. Let us say for the member AB I am going to say the jth end is at B and kth end is at A. So, that is here. So, this is my  $x_m$  and this is my  $y_m$  for the member AB length of the member AB is  $l$  and moment of inertia is  $2I$  is it not let me mark the labels for the member AB. So, rotation at j rotation at k displacement along positive y at j and at k is it not. So, rotation is 1 rotation is then 2 then 5 and 3. So, the label should be strictly speaking 1, 2, 5 and 3. So, let us say the labels are 1, 2, 5 and 3.

Now, for the member BC for the member BC we will take the j th end at B and this at C. So, the j th end is here. So, this becomes my x m and y m for the member BC the labels are going to be 1, 6, 4 and 7 is it not. So, labels are 1, 6, 4 and 7 right the length of the member is again l, but the moment of inertia is only I. So, I do not think there is any confusion for you people to write this table. Once this table is done let us now write the matrix for the member AB and the matrix for the member BC 4 by 4.

(Refer Slide Time: 41:54)

The handwritten matrix  $[K]$  is:

$$[K] = \begin{bmatrix} \frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{6EI}{l^2} & \frac{12EI}{l^3} \\ \frac{6EI}{l^2} & \frac{4EI}{l} & \frac{6EI}{l^2} & \frac{6EI}{l^2} \\ \frac{6EI}{l^2} & \frac{6EI}{l^2} & \frac{4EI}{l} & \frac{6EI}{l^2} \\ \frac{12EI}{l^3} & \frac{6EI}{l^2} & \frac{6EI}{l^2} & \frac{12EI}{l^3} \end{bmatrix}$$

The matrix is partitioned into four 2x2 quadrants labeled 1, 2, 3, and 4. The top-left quadrant (1) contains  $\frac{12EI}{l^3}$  and  $\frac{6EI}{l^2}$ . The top-right quadrant (2) contains  $\frac{6EI}{l^2}$  and  $\frac{4EI}{l}$ . The bottom-left quadrant (3) contains  $\frac{6EI}{l^2}$  and  $\frac{6EI}{l^2}$ . The bottom-right quadrant (4) contains  $\frac{4EI}{l}$  and  $\frac{6EI}{l^2}$ .

Below the matrix is a 4x4 grid for  $[K]_{BC}$ :

$$[K]_{BC} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

So, E into 2 I for the member AB right. So, I should say it is r 1 by l, c 1 r 1 by l, 1 plus c 1 of r 1 by l square minus 1 plus c 1 r 1 by l square. So, then I cross this is c 1 r 1 by l, this is r 1 by l, this is 1 plus c 1 of r 1 by l, 1 plus c 1 of r 1 by l. Then we add these two and divide by l. So, 1 plus c 1 r 1 by l square, 1 plus c 1 r 1 by l square that is adding these two by l then adding these two by l, but t along with that.

So, that is going to be 2 t 1 r 1 1 plus c 1 by l cube right minus 2 t 1 r 1, 1 plus c 1 by l cube the fourth column is negative of 3. So, minus of 1 plus c 1 r 1 by l square minus of 1 plus c 1 r 1 by l square minus of 2 t 1 r 1 1 plus c 1 by l cube plus of 2 t 1 r 1 1 plus c 1 by l cube right. Let us mark the labels for the member AB is 1, 2, 5, 3, 2, 5 and 3 similarly here 1, 2, 5 and 3.

(Refer Slide Time: 44:35)

Handwritten matrix for stiffness matrix  $k_{BC}$  with columns labeled 1, 6, 4, 7 and rows labeled 1, 6, 4, 7. The matrix is:

$$k_{BC} = \begin{bmatrix} r_2/l & c_2 r_2/l & (1+c_2)r_2/l^2 & -(1+c_2)r_2/l^2 \\ c_1 r_2/l & r_2/l & (1+c_1)r_2/l^2 & -(1+c_1)r_2/l^2 \\ (1+c_1)r_2/l^2 & (1+c_2)r_2/l^2 & 2b_2 r_2(1+c_2)/l^3 & -2b_2 r_2(1+c_2)/l^3 \\ -r_2/l^2(1+c_1) & -(1+c_2)r_2/l^2 & -2b_2 r_2(1+c_1)/l^3 & 2b_2 r_2(1+c_1)/l^3 \end{bmatrix}$$

Then for the member BC let me do it to the next page.  $k_{BC}$  is  $2 \times 2$  by  $1 \times 1$  that is the length of the member  $c \times 2$  by  $1 \times 1$  plus  $c \times 2$  by  $1$  minus  $1$  plus  $c$  then it is crossed, so  $c \times 2$  by  $1 \times 2$  by  $1 \times 1$  plus  $c \times 2$  by  $1$  minus of. Then sum of these two by  $1$  this is  $1$  square right there will be  $1$  square here the previous case also  $1$  square that is right. It  $1$  square here then sum of these two by  $1$  then sum of these two by  $1$  again. So,  $2 \times 2 \times 2 \times 1$  plus  $c \times 2$  by  $1$  cube minus of  $2 \times 2 \times 2 \times 1$  plus  $c \times 2$  by  $1$  cube the fourth column is minus of this. We also know the labels of this is 1, 6, 4, 7. So, let us enter the labels 1, 6, 4, 7.

(Refer Slide Time: 46:57)

Handwritten matrix for stiffness matrix  $k_{UU}$  with columns labeled 1, 2 and rows labeled 1, 2. The matrix is:

$$k_{UU} = \begin{bmatrix} \frac{2r_1+r_2}{l} & \frac{2c_1 r_1}{l} \\ \frac{2c_1 r_1}{l} & \frac{2r_1}{l} \end{bmatrix}$$

Below the matrix, the determinant equation is written:

$$\left[ \left( \frac{2r_1+r_2}{l} \right) \frac{2r_1}{l} \right] - \left( \frac{2c_1 r_1}{l} \right)^2 = 0.$$

To the right,  $|k_{UU}| = 0$ .

Now, friends, let us form k uu. Let us form k uu how many unrestrained degrees are there? We have got two unrestrained degrees in this problem see here 1 and 2. So, I must get E I common 1 and 2. So, I must find k 1 1. Let us look at this k 1 1 is this I am just indicating with maybe the pink color k 1 1 is this from here. So, k 1 1 plus the other one is k 1. So, now, please note there is a two multiplier here. So, 2 r 1 by l 1 2 r 1 by l l 1 and is same. So, then for the second member it is r 2 by l then let us go to 1 2 from here 1 2 is this value.

So, 2 c 1 r 1 by l right 2 c 1 r 1 by l why this 2 has come? This 2 has come because of this 2 has come because of this. Now, I must get 2 1 go here 2 1 is this value. So, 2 c 1 r 1 by l correct then let us go to 2 2. 2 2 is this value we also have 2 2 here, we also have 2 2 here no we do not have it only this. So, 2 2s r 1 by l there is a 2 here. So, I should say 2 r 1 by l friends we have got kuu matrix now.

Now, I must set this determinant of k uu to 0. So, this diagonal multiply minus this diagonal and set it to 0. So, I should say 2 r 1 plus r 2 by l into 2 r 1 by l minus 2 c 1 r 1 by l the whole square should be 0 am I right.

(Refer Slide Time: 50:14)

By simplifying we get  
The CE as

$$\Rightarrow 2r_1(1 - c_1^2) + r_2 = 0 \quad \text{--- (1)}$$

$P_{cr1} = P_{cr2} = P_{cr}(0.707)$        $\phi_1 = \frac{P(0.707)}{\pi^2 EI l^2}$

Also state that  $2\phi_1 = \phi_2$        $\phi_2 = \frac{P(0.707)}{\pi^2 EI l^2}$

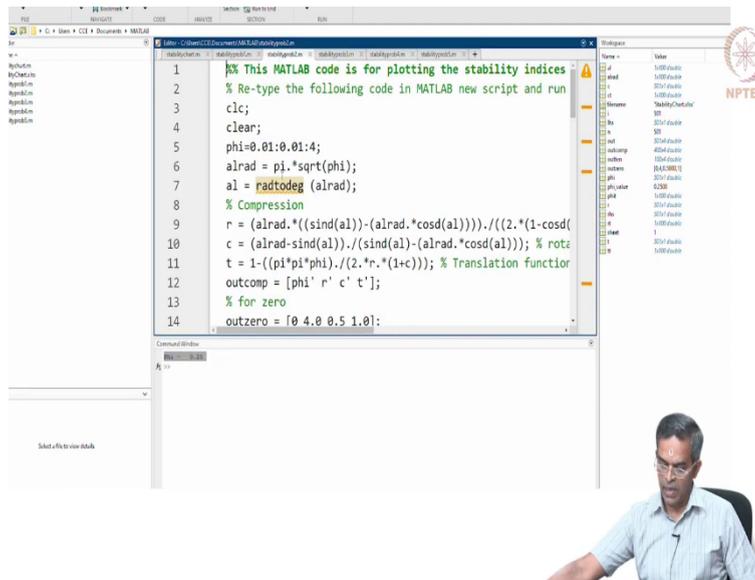




This is what I must get set it to 0 when I set this to 0 and expand and simplify we get the characteristic equation as 2 r 1 of 1 minus c 1 square plus r 2 is u this my characteristic equation. Further, we know P c r 1 is P c r 2 which is P c r of 0.707 is it not? Well, that is the load which is being applied therefore, friends we have 1 more relationship  $\phi_1$  will be P times of 0.707  $\pi$  square E to I by l square  $\phi_2$  will be P of 0.707 by  $\pi$  square E I by l square.

So, comparing these two equations we can now also say  $2\phi_1$  is  $\phi_2$  you can see easily from here  $2\phi_1$  will be  $5.2$  right. So, I have to satisfy two equations while selecting  $\phi$ . So, I must find  $\phi_1$  and  $\phi_2$  because  $\phi_1$  and  $\phi_2$  will be helpful to get P and the characteristics equation should be satisfied. So, I can refer to the chart or the table and try to get.

(Refer Slide Time: 52:19)



The image shows a MATLAB script in a script editor. The script is for plotting stability indices. It starts with a comment: "% This MATLAB code is for plotting the stability indices". The code includes the following lines:

```

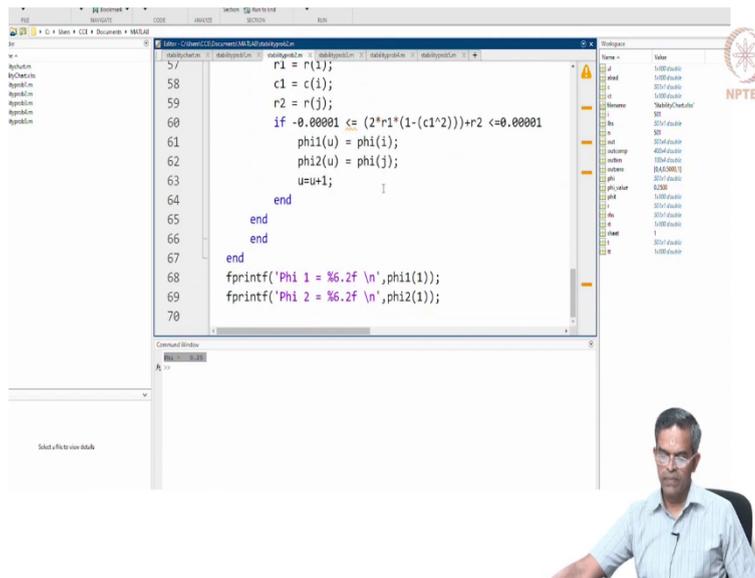
1  % Re-type the following code in MATLAB new script and run
2  c1;
3
4  clear;
5  phi=0.01:0.01:4;
6  alrad = pi.*sqrt(phi);
7  al = radtodeg (alrad);
8  % Compression
9  r = (alrad.*(sind(al))-(alrad.*cosd(al)))/(2.*(1-cosd(
10 c = (alrad-sind(al))/(sind(al)-(alrad.*cosd(al))); % rote
11 t = 1-((pi*pi*phi)/(2.*r.*(1+c))); % Translation function
12 outcomp = [phi' r' c' t'];
13 % for zero
14 outzero = [0 4.0 0.5 1.0];

```

The script is running in a MATLAB environment, and the Command Window is visible below the script editor. The NPTEL logo is visible in the top right corner of the MATLAB window.

So, I am going to use MATLAB again, so using MATLAB.

(Refer Slide Time: 52:25)



The image shows a MATLAB script in a script editor. The script is for plotting stability indices. It starts with a comment: "% This MATLAB code is for plotting the stability indices". The code includes the following lines:

```

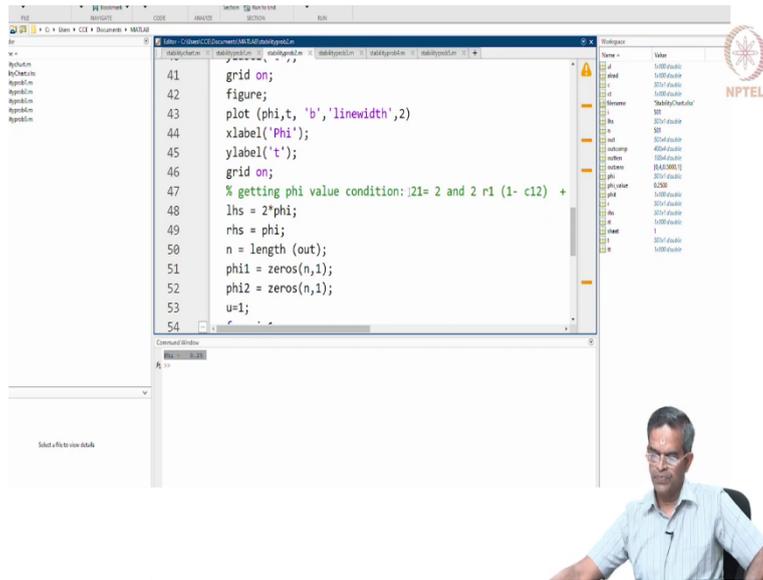
57  r1 = r(1);
58  c1 = c(1);
59  r2 = r(j);
60  if -0.00001 <= (2*r1*(1-(c1^2)))+r2 <=0.00001
61     phi1(u) = phi(i);
62     phi2(u) = phi(j);
63     u=u+1;
64   end
65 end
66 end
67 end
68 fprintf('Phi 1 = %6.2f \n',phi1(1));
69 fprintf('Phi 2 = %6.2f \n',phi2(1));
70

```

The script is running in a MATLAB environment, and the Command Window is visible below the script editor. The NPTEL logo is visible in the top right corner of the MATLAB window.

So, again the same plot, but I am going to show you the control equation.

(Refer Slide Time: 52:32)



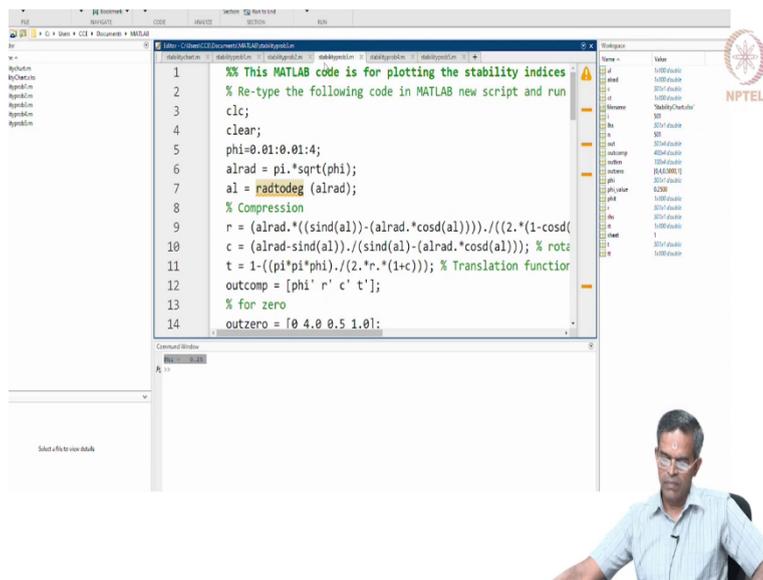
```
41 grid on;
42 figure;
43 plot(phi,t, 'b','linewidth',2)
44 xlabel('Phi');
45 ylabel('t');
46 grid on;
47 % getting phi value condition: |z1|= 2 and 2 r1 (1- c12) +
48 lhs = 2*phi;
49 rhs = phi;
50 n = length (out);
51 phi1 = zeros(n,1);
52 phi2 = zeros(n,1);
53 u=1;
54
```

Command Window

Select a file to view details



(Refer Slide Time: 52:39)



```
1 %% This MATLAB code is for plotting the stability indices
2 % Re-type the following code in MATLAB new script and run
3 clc;
4 clear;
5 phi=0.01:0.01:4;
6 alrad = pi.*sqrt(phi);
7 al = radtodeg (alrad);
8 % Compression
9 r = (alrad.*(sind(al))-(alrad.*cosd(al)))/((2.*(1-cosd(
10 alrad-sind(al)))/(sind(al)-(alrad.*cosd(al))))); % rote
11 t = 1-((pi*pi*phi)/(2.*r.*(1+c))); % Translation funcion
12 outcomp = [phi' r' c' t'];
13 % for zero
14 outzero = [0 4.0 0.5 1.0];
```

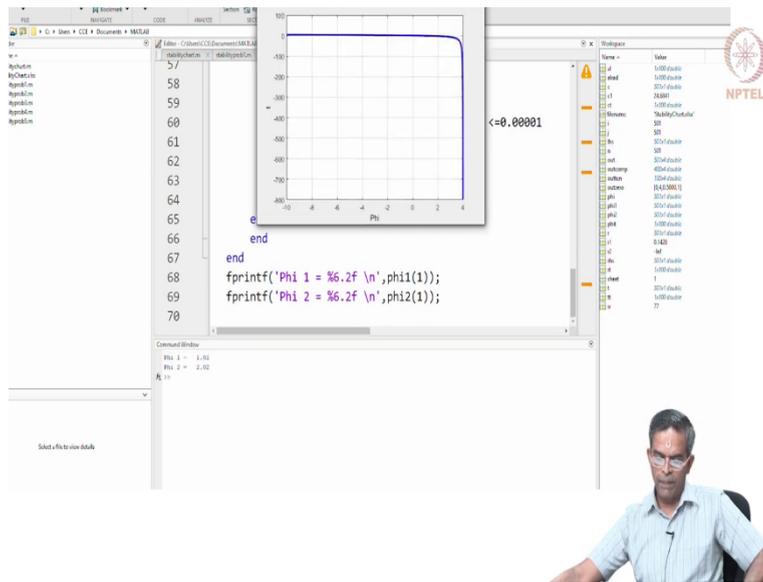
Command Window

Select a file to view details



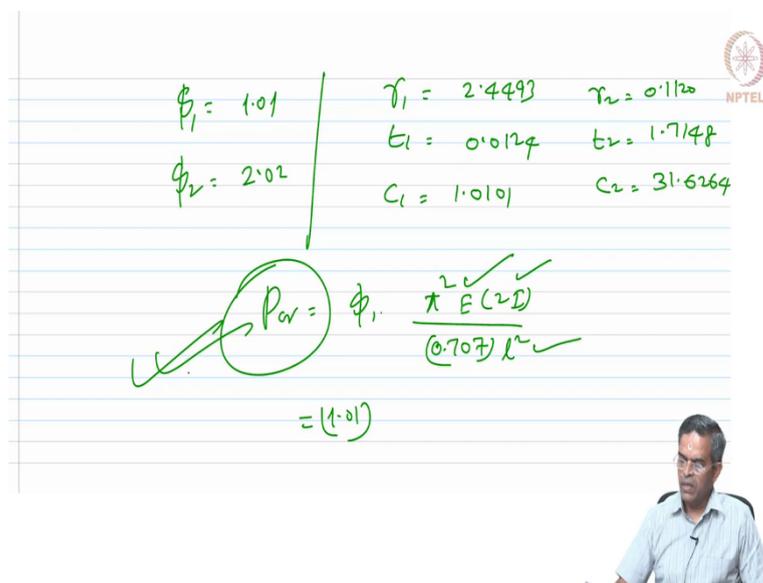
The control equation what we had is  $2 r 1 1$  minus  $e 1$  square plus  $r 2$  is 0 is it not this is the control equation here.

(Refer Slide Time: 53:07)



So, we have developed this control equation apply this and wrote an algorithm and then we run this program. So, forget about these plots, we get  $\phi_1$  as 1.01 and  $\phi_2$  as 2.02. So, I will write it here I will close this minimize this, these are charts being plotted I will minimize this.

(Refer Slide Time: 53:31)



So, now I can write  $\phi_1$  is 1.01 and  $\phi_2$  is 2.02 for these values we got  $r_1$  as 2.4493,  $t_1$  as 0.0124 and  $c_1$  as 1.0101 and  $r_2$  as 0.1120,  $t_2$  as 1.7148 and  $c_2$  as 31.6264. So, now I have got  $p$  critical is  $\phi_1$  times of  $\pi$  square  $E$  of  $2I$  by  $0.707$  of  $l$  square. So,  $\phi_1$  I already have, which is 1.01. So, if you know  $E$  if you know  $I$  if you know  $l$  I can find.



(Refer Slide Time: 54:55)

Summary

- Buckling/stability
- Set the charact eq to solve a buckling problem

$|k_{uu}| = 0$

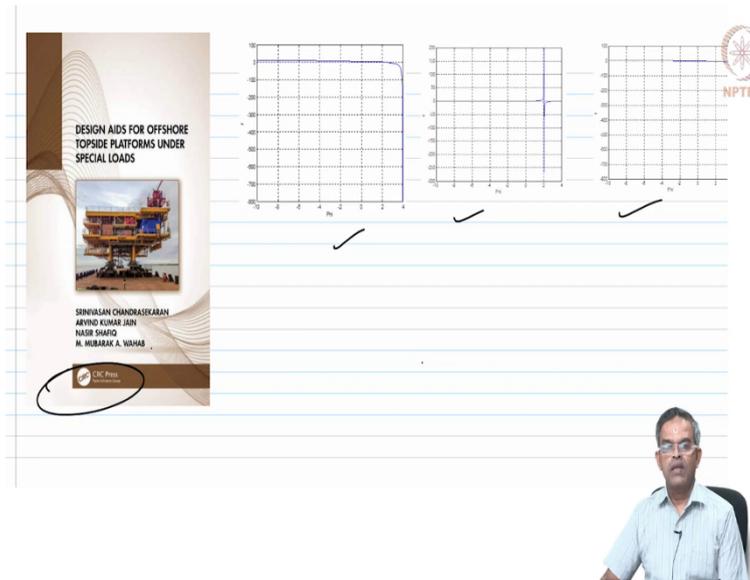
$\leftarrow$  Choose  $\phi_c$  - Char Table

p or

So, friends in this lecture we learnt the difference between buckling and stability analysis. We also learnt how to set the characteristic equation to solve a buckling problem for a frame subject to axial loads. So, we said that set the  $k_{uu}$  determinant to 0 for non-trivial solution and try to get the control equation and then choose the  $\phi$  values from the chart or the table and try to find the  $\phi$ .

Once I know the  $\phi$  value I can find  $p$  critical. So, I wish you should do more examples to solve more problems and try to learn more about this. So, friends stability analysis is not difficult it is easy please practice more problems and use the MATLAB program given.

(Refer Slide Time: 56:16)



And also this book will be very helpful for you which is being used as a reference. So, please refer to this book Design Aids for Offshore Structures Topside Platforms under Special Loads which has got the stability charts and the table and also the MATLAB program. This is a CRC press release this book is available in public domain please access to this book through your institute library subscribe this to a library recommend this to your resource scholars and friends and faculty colleagues and I hope you will find an extremely useful contributions from this book for your additional learning on this course.

Thank you very much and have a good day bye.