Advanced Design of Steel Structures
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Lecture - 34
Stability problems - numerical examples

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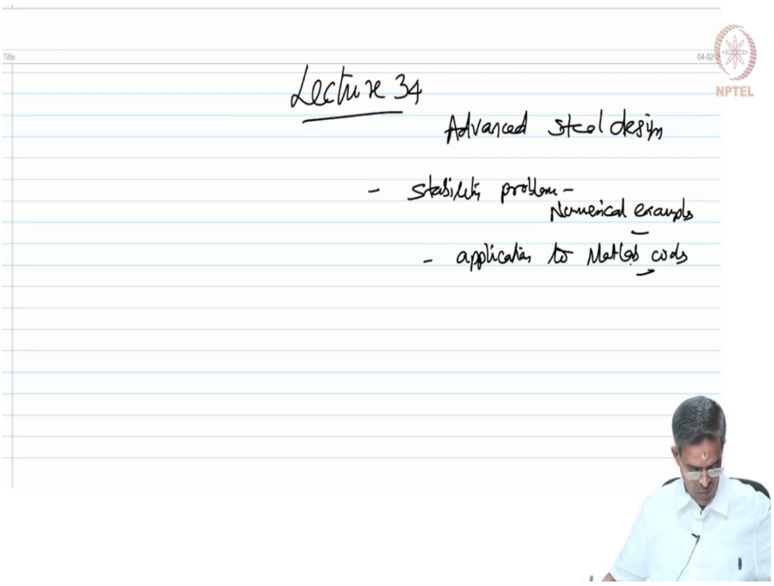

Friends welcome to the lecture 34 of the course Advanced Steel Design. In this lecture we are going to learn more about the stability problems numerical example. We will deal with more numerical examples in this lecture and learn couple of more with application to MATLAB programs.

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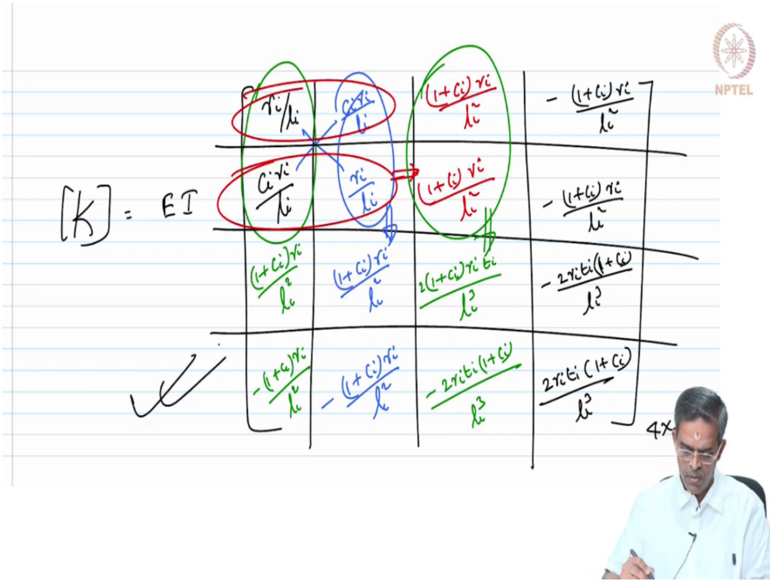

Before that let us revise quickly, that my stiffness matrix of the member is given by; the stiffness matrix is given by the basic equation which we should remember like that of a beam, which is a 4 by 4 matrix, which is ri by li, this is ci ri by li and these two by li will be this.

So, 1 + ci of ri by li square and negative of this. The 2nd column is swapping of this and sum of these two by li and the last one is negative of the previous one. The 3rd column is sum of these two by li is also sum of these two by li and the 4th one is the next one is sum of these two by li again. So, I should say 1 + ci ri twice, but introduce a new coefficient ti ok and this is minus of that. The 4th one of course, is negative of the 3rd one this is what we should, remember which is similar to what we have in the beam implication problem.

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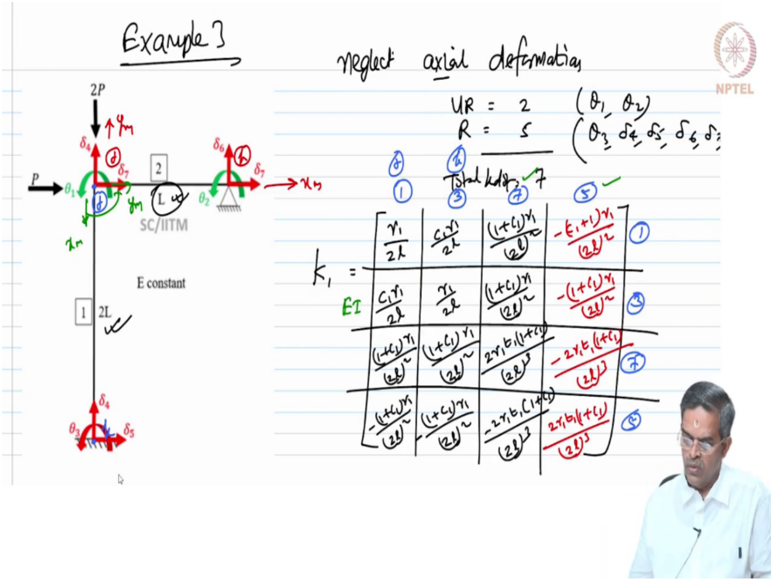

Having said this let us go to the example 3, as you see on the screen here. So, this example is about an orthogonal frame has got two members. One member is having lengths 2L the other one is L. We have marked the degrees of freedom as you see here.

So, the degrees of freedom are marked as theta L and theta two unrestrained ok and remaining all are restrained, we are neglecting axial deformation. So, restrained degrees, so it has got unrestrained degrees 2, which are theta 1 and theta 2 and restrained degrees are let us say theta 3, del 4, del 5, del 6 and del 7 which is about 5. So, the total kinematic degrees of freedom is 7. So, each matrix will be 4 by 4 and we can try to find out this very easily.

So, now for the member k a b or k 1 ok, for the member k 1 I should say r 1 ok by 2l, 1 + c 1 c 1 r 1 by 2l. So, 1 + c 1 of r 1 by 2l square sorry by li square. So, that is 2l the whole square ok so, the negative of this 1 + c 1 r 1 by 2l the whole square. The 2nd column will be c 1 r 1 by l, r 1 by l, 1 + c 1 r 1 by 2l the whole square - of 1 + c 1 r 1 by 2l the whole square with a negative sign.

The 3rd column will be 1 + c 1 r 1 by 2l or 2l the whole square. So, 1 + c 1 r 1 by 2l the whole square the 3rd one is 2 r 1 t 1 1 + c 1 by l cube. So, I should say 2l the whole cube and negative of this. The 4th column of course, negative of the 3rd column. So, we have this, let us mark the labels. We will take the j th end here and k th end here for this member. So, we should say this is 1, 3, 7 and 5; 1, 3, 7 and 5.

So, rotation at j th end, rotation at k th here, translation along positive y if I say this is my x m, anti-clockwise 90 is my y m. So, this becomes my degree of 7 and this is my label 5. So, k 1 for the member ab is established of course, we have an EI multiplier outside. So, we can do this. The next one is for the 2nd member k 2.

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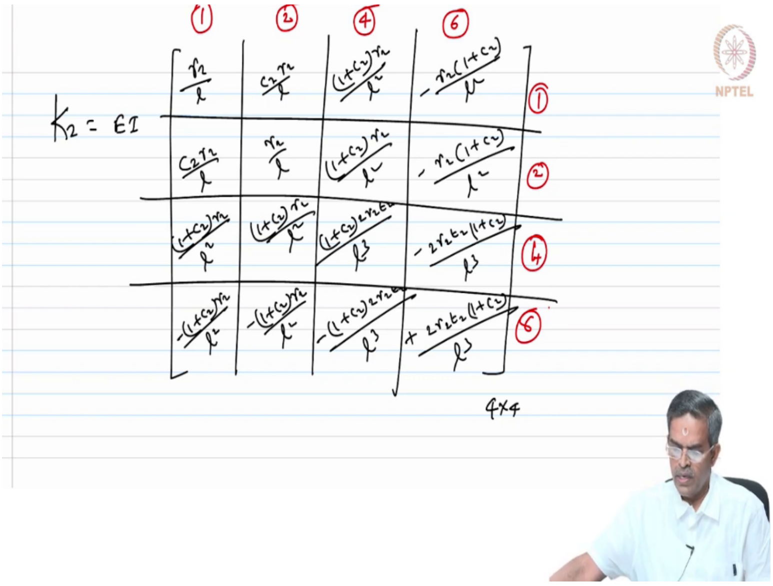

So, let us do it for the member k 2 which is again EI times of again a 4 by 4 matrix. So, this is going to be r 2 by l, because you see the length of the member 2 is only l. The length of the member 2 is only l. So, r 2 by l, then c 2 r 2 by l, 1 + c 2 of r 2 by l square.

So, let us mark the labels, look at this figure I am calling this as my j th end and this as my k th end, this becomes my x m and this becomes my y m for the member 2. So, I should say the labels are going to be 1, 2, 4 and 6. Let us mark them 1, 2, 4 and 6. Now, we can assemble these two matrices and try to get k u u, the k u u will be partitioned at 2 by 2.

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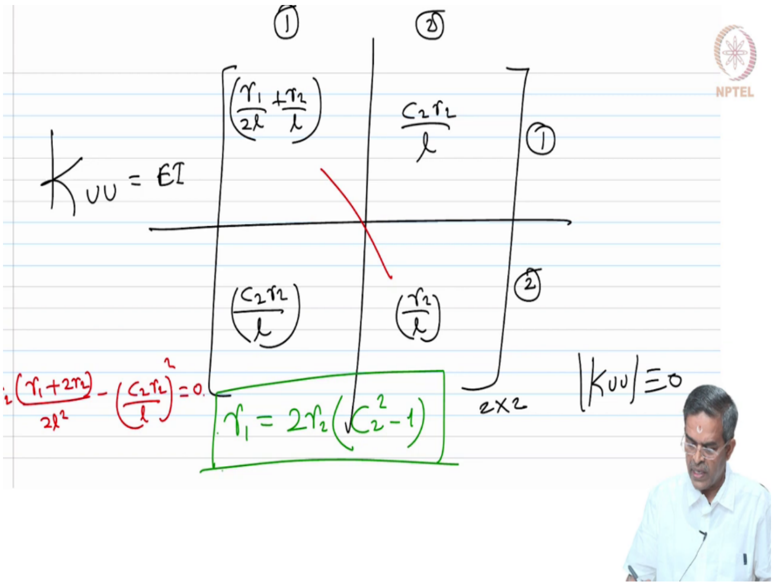

So, now I am getting try to get k u u because I have set determinant to 0 of this. So, that is k u u which is 2 by 2. Let us do this to be 2 by 2. Let us try to get this. So, we should look into this figure 1 1 is I have here, so r 1 by 2 l. So, let we take, r 1 by 2l, then do we have 1 1 here. So, we have r 2 by l as well.

This is 1 one ok let us say 1 2 which is not in the 1st matrix, but 1 2 can be in the 2nd matrix. So, 1 2 is c 2 r 2 by l, c 2 r 2 by l. Then let us say 2 1, 2 1 is here, 2 1 is the 2nd matrix here c 2 r 2 by l. So, c 2 r 2 by l and 2 2 is here which is r 2 by l ok right. So, I want to set this determinant to 0 to get my characteristic equation. So, if I do that I am writing it here. So, r 1 + 2 r 2 by 2l square - c 2 r 2 by l the whole square should be 0.

So, this gives me the expression r 1 + 2 r 2 by 2l square into r 2 ok, into r 2 because we have to product this r 1 by r 1 + 2 r 2 by 2l square into r 2 - this is set to 0, which gives me the characteristic equation as r 1 equals 2 r 2 times of C 2 square - 1, this is what we call as my characteristic equation, ok right. Now, I want to also see what my axial load is coming on the section.

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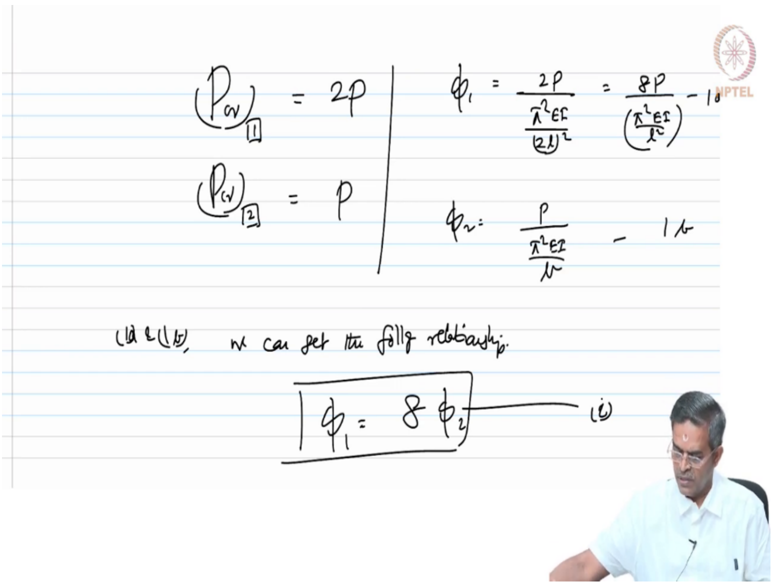

So, if I say P critical of the 1st member ok look at this figure. For the 1st member this is 2P, for the 2nd member this is P, is it not. So, 1st member this is 2P and for the 2nd member ok, let us I think use square brackets ok which will be P. So, by this logic ϕ1 should be 2P, because ϕ is the ratio between the applied load and the Euler's critical load which is pi square EI by l square.

So, in this case it is 2l the whole square ok, which will become 8P by pi square EI by l square, am I right? Similarly, for ϕ2 which will be P by pi square EI by l square. So, by this logic comparing equation 1 a and 1 b we can get the following relationship, which will beϕ1 is 8 times of ϕ2: equation 2. So, we got relationship we got k uu determinant as well.

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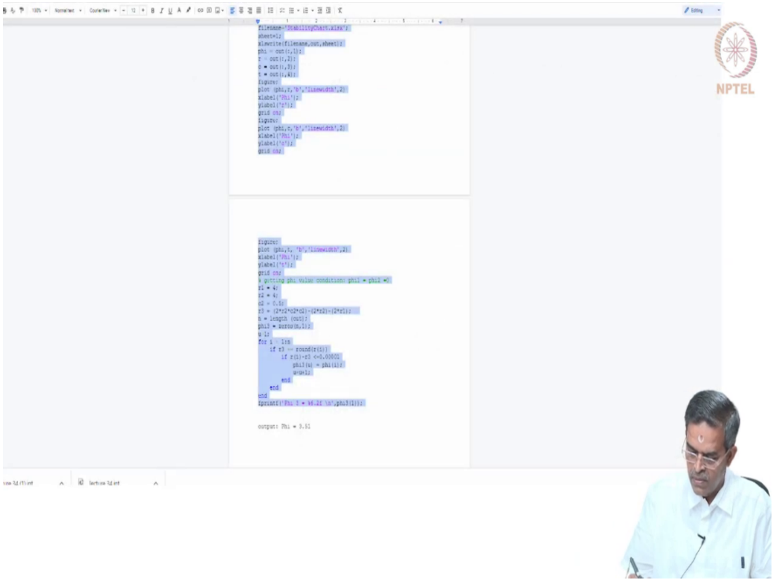

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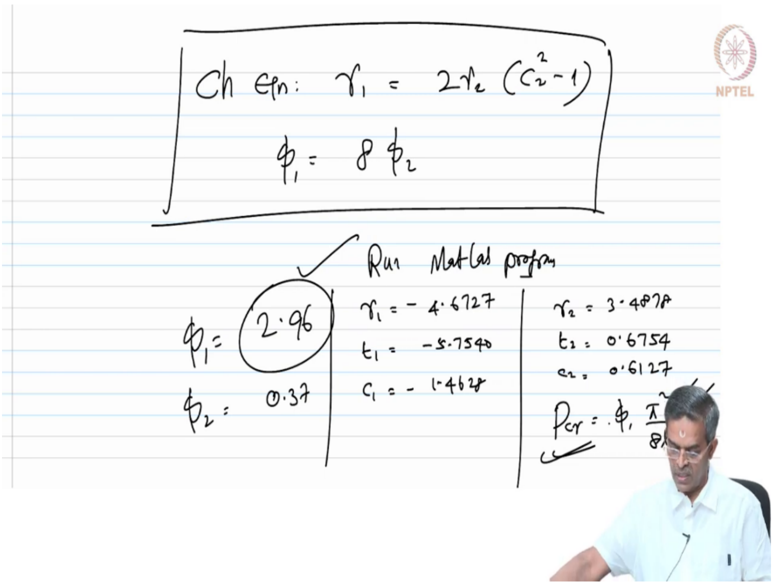

So, I am going to now satisfy these two conditions. What are the conditions? So, I have to satisfy the characteristic equation which is given by r 1 is equal to 2 r 2 c 2 square - 1 the other condition is I must satisfyϕ1 is 8ϕ2, is what I have to satisfy. So, let us run the MATLAB program to do this. Let us now run the MATLAB program and try to obtain the values ok, let us do that.

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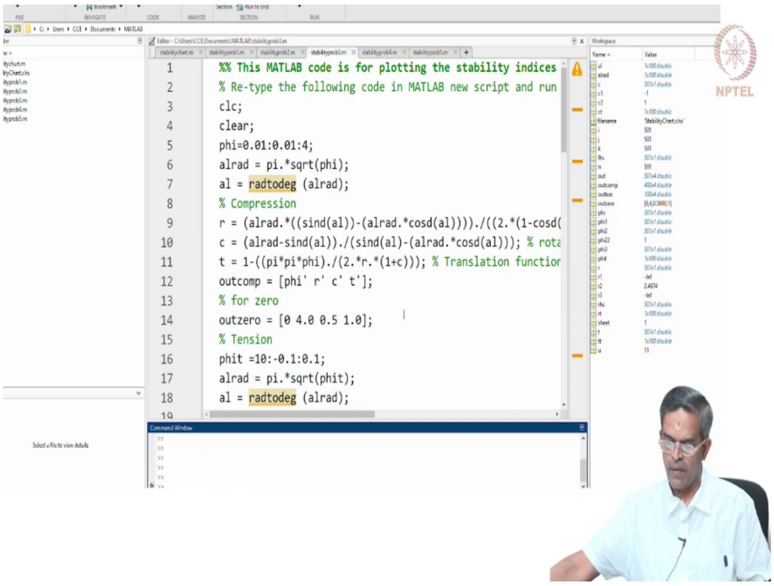

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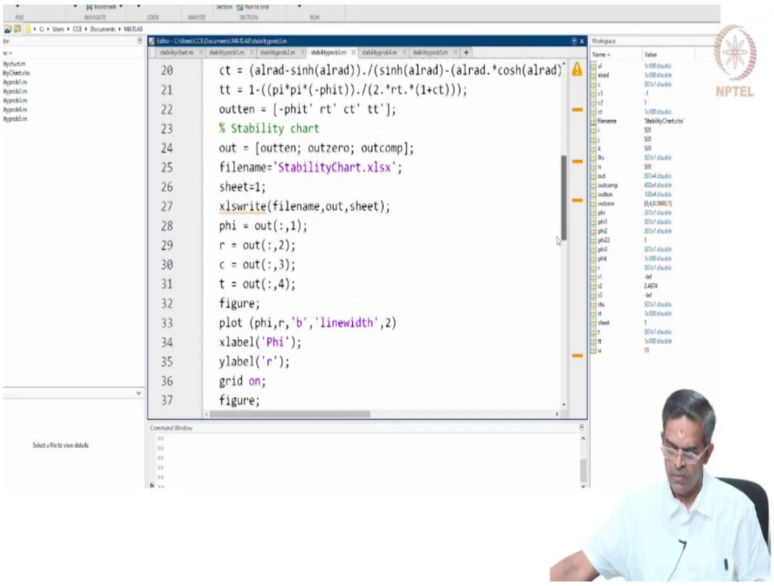

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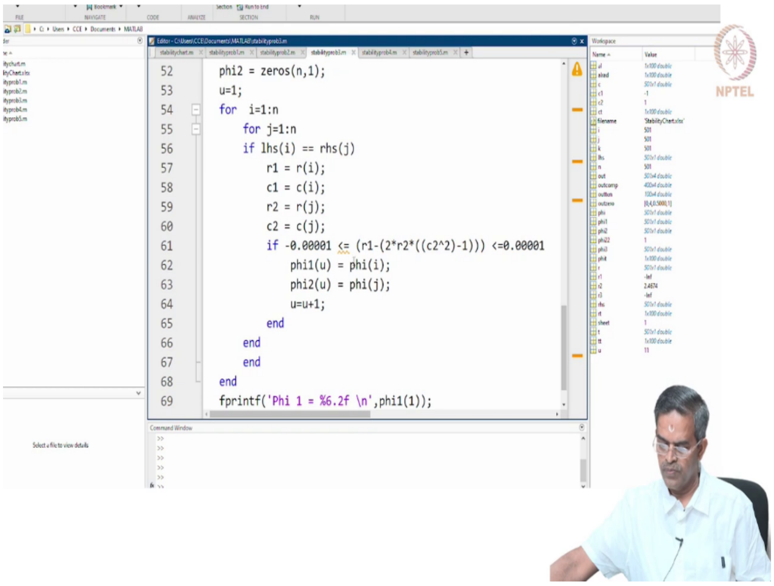

So, friends this is the program which you are going to run. You can see here the control equation is this r1 square ok r, let us say r1 is equal to or r 1 - 2 r2 times of c2 square - 1 should be set to close to 0. We simply say exactly 0, we may not be able to pick up the correct value of r1 r2 and c2 and c1 from the chart so, closely.

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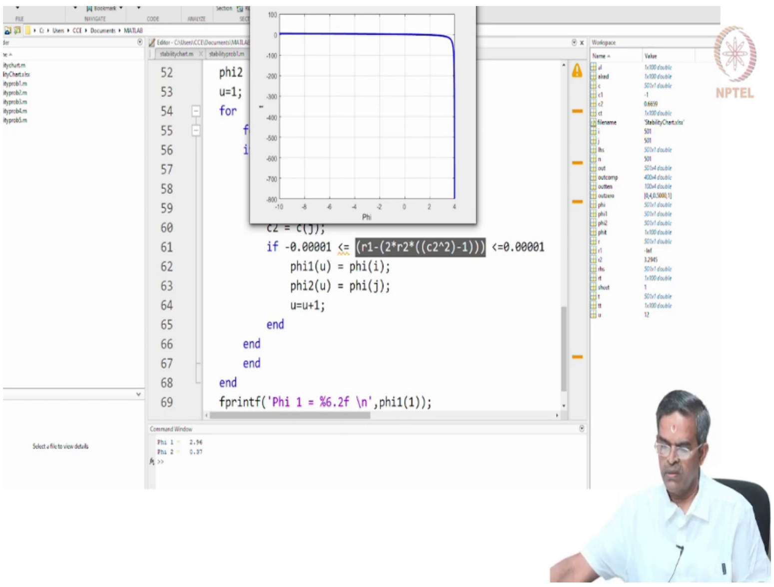

So, let us closely pick up this and let us run this program. So, I get these charts, I think we already have this charts with us, we know right. So, we know these charts of r i and t. So, the answers are ϕ1 is 2.96 and ϕ2 is 0.37. So, I will go back here. So, I get ϕ1 as ϕ1 as 2.96 and ϕ2 as 0.37.

So, for these values if you look at the chart, I get r 1 as - 4.6727, I get t 1 as - 5.7540 and I get c 1 as - 1.4628 and I get further r 2 as 3.4878 from the chart, t 2 as 0.6754 and c 2 as 0.6127. So, now, I can interpret P critical as ϕ1 times of pi square EI by 8 l square ok, 8 l square. So, we will be able to get the critical load because I know ϕ1. If I know if I know EI and length, I can get critical value (Refer Time: 20:29) critical ok, it is very simple and straightforward.

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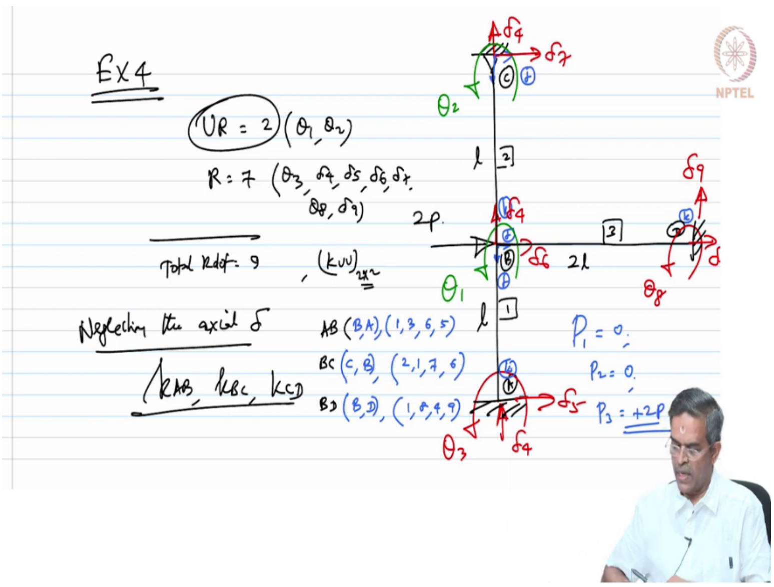

Let us go to example 4; we will go to example 4. So, let us draw this figure here, one end is fixed other end is hinged and this is also fixed. Let us call this as A, B, C and D, this is of two length l, this is l and this is l and this is member 1 and this is member 2 and this is member 3. This is subjected to in horizontal load of 2P here of the whole problem.

Let us know mark the unrestrained degrees of freedom. So, I get rotation here theta 1, I am marking them in green rotation here theta 2, these are my two unrestrained degrees of freedom. Let us mark restrained degrees of freedom theta 3 del 4 if I say del 4 and I neglect axial deformation, I have del 4 here and I have del 4 here also. And this is del 5 ok, this is del 6 therefore, this is also del 6 because I am neglecting axial deformation ok and this is del 7 and this is theta 8 and this is del 9.

So, I have 9 degrees of freedom, the unrestrained degrees of freedom are 2, which is theta 1 and theta 2. The restrained degrees of freedom r theta 3, del 4, 5, 6, 7, theta 8 and then 9, so which is 7. So, the total kinematic degrees of freedom are 9 and k uu will be of size 2 by 2, because there are 2 unrestrained degrees and remember we are anyway neglecting the axial deformation.

So, I can easily write k AB, I can write k BC, I can write k CD ok, assemble them and k k u that is what I am going to do now. So, let us do that exercise, very clearly it is simple and we can do that.

So, let us look at the labels for the member AB, let us look at what are the labels, we will take the j th end here ok we will take the j th end here and mark it; j th end here and k th end here. This is my j th end and this is my k th end. So, let us mark a table quickly, j th end is at B, and k th end is at A and the labels are rotation at j rotation at k deformation or displacement along positive y at j and positive y at k ok, let us do it for the member BC.

So, for the member BC, we will take the j th end here. So, j th end here and k th end here. So, I should say the j th end is at C and k th end is at B and the labels are rotation at the j th end rotation at the k th end displacement along positive y at the j th end and at the k th end. Let us do for the member say BD ok, for the member BD. So, for the member BD which is horizontal the length is of course, 2l we should remember that and we will take the j th end here and the k th end here for this member.

So, the j th end is at B, k th end is at D therefore, the labels are 1 rotation, then 8, then 4 and 9. These are the labels; I do not think we have any difficulty in writing the stiffness matrices. For all the 3 members let us quickly do that.

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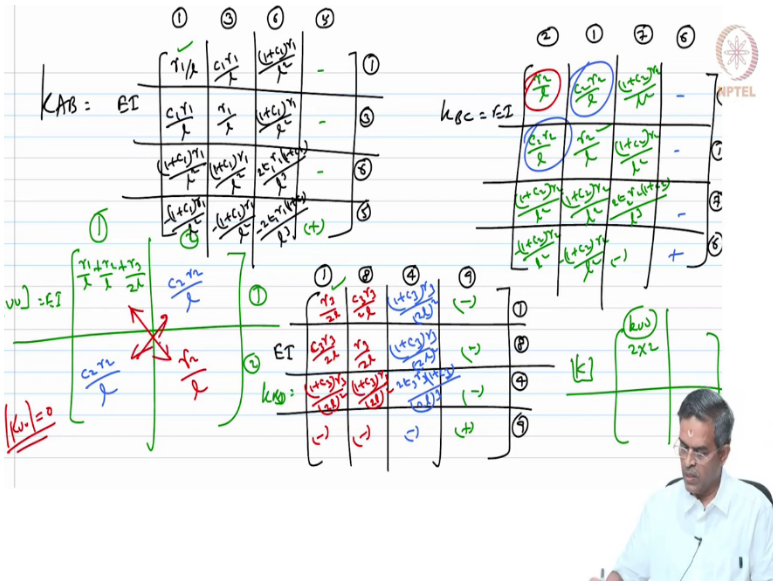

So, I should say k AB is EI times of, similarly k BC is EI times of, similarly k BD EI times of remember this is of 2 length l. So, be careful about the length. We also know the labels let us write the labels 1, 3, 6, 5, that is for the member AB, for the member BC, BC 2, 1, 7, 6; 2, 1, 7, 6 and for the member BD we know the labels are 1, 8, 4, 9, is it not; 1, 8, 4, 9.

And we know the 1st value here is r 1 by l, this is c 1 r 1 by l, 1 + c 1 of r 1 by l and - 1. This is again c 1 r 1 by l, r 1 by l, 1 + c 1 of r 1 by l - 1 + c 1 of r 1 by l. 1 + c 1 r 1 by l, 1 + c 1 r 1 by l, 2 times of t 1 r 1 1 + c 1 by l ok, these are all l square friends, I am sorry. This is l cube - 2 t 1 r 1 1 + c 1 by l cube and the 4th column is of course negative of the 3rd column.

Let us go to member BC. So, it is going to be r 2 by l, c 2 r 2 by l, 1 + c 2 r 2 by l square; so, c 2 r 2 by l r 2 by l 1 + c 2 r 2 by l square. Similarly, 1 + c 2 r 2 by l square, 2 times of t 2 r 2 1 + c 2 by l cube and - of this the 4th one of course, is negative of the 3rd column, I am not entering it ok it is very simple to remember.

Let us go to the member k 3. So, it is going to be, I will do it here r 3 by 2l, 1 c 3 r 3 by 2 l. So, 1 + r 3 of sorry, 1 + c 3 r 3 by 2l the whole square and negative of this, c 3 r 3 by l, r 3 by 2 l. So, 1 + c 3 of r 3 by 2l the whole square negative of this. So, this will be 1 + c 3 r 3 by 2l, 1 + c 3 r 3 by 2l this is going to be 2 times of t 3 r 3 1 + c 3 by 2l the whole cube. This is square is it not and this is negative of this. The last column of course, is negative of the 3rd column we can know this. So, now, if you assemble k and partition a 2 2 this is going to be k uu sub matrix.

So, now I should write k uu here, which is EI common of let me write this k BD somewhere here. So, 1 2 and 1 2, I am writing k uu. So, I should say r 1 by l that is 1, then I get r 2 by l then I also get r 3 by 2l ok, this one. Let us now do 1 2. So, 1 2 I can pick up from this which is c 2 r 2 by l, then I should say 2 1 2 1 I should pick up from here which is c 2 r 2 by l, then I should say 2 2, 2 2 I will pick up from here, which is r 2 by l, right. I should set this determinant to 0. So, let us multiply this and - of this let us do that. So, I can write that which will be, I will copy this matrix.

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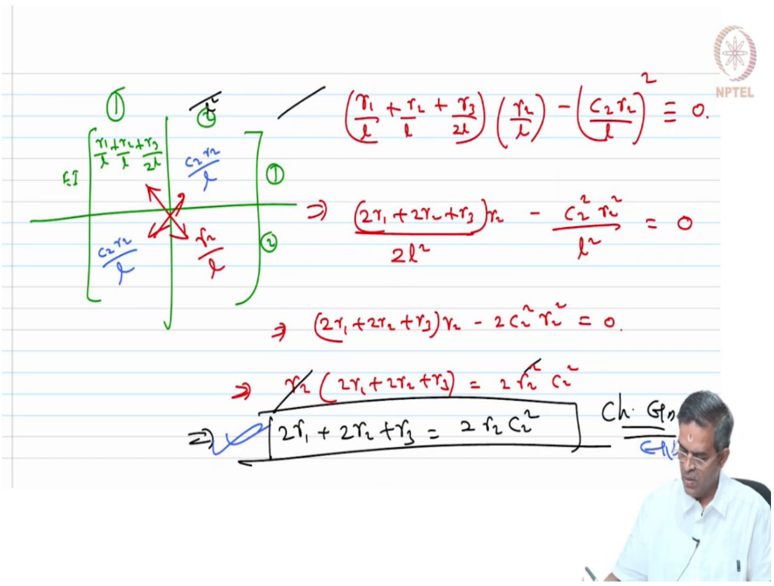

Let me put it here, let me rub this. So, I should say r 1 by l + r 2 by l + r 3 by 2l times of r 2 by l - c 2 r 2 by l the whole square should be set to 0, am I right. Which will be equal to 2 r 1 + 2 r 2 + r 3 by 2l square of r 2 - c 2 square r 2 square by l square is 0, which will become 2 r 1 + 2 r 2 + r 3 times of r 2 - 2 c 2 square r 2 square should be 0, am I right. r 2 times r 2 r 1 + 2 r 2 + r 3 is equal to 2 r 2 square c 2 square.

So, let us say r 2 goes away ok, r 2 goes away. We can say 2 r 1 + 2 r 2 + r 3 is equal to 2 r 2 c 2 square. So, that is my characteristic equation, the control equation to develop this. Furthermore, friends please see this figure, I should say the axial load on the member 1, the axial load on the member 1 is 0, the axial load for the member 2 is also 0, the axial load for member 3 is 2P compressive; am I right?

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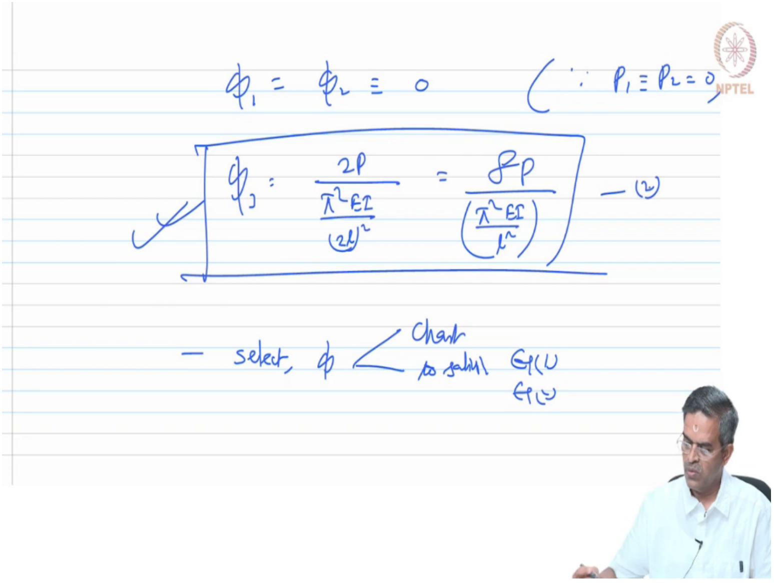

So, by this logic can I sayϕ1 andϕ2 will be 0? Because P1 and P2 are 0, am I right. So, let us sayϕ3 yes 2P times of pi square EI by 2l the whole square, which will be 8P by pi square EI by l square, let us keep this information with us to compute my critical load. So, I have got 2 control equations now, one is the characteristic equation, other is this relationship. I must now select ϕ in such a manner from the chart to satisfy equation 1 and equation 2, this is equation 2 sorry and equation 1 is this.

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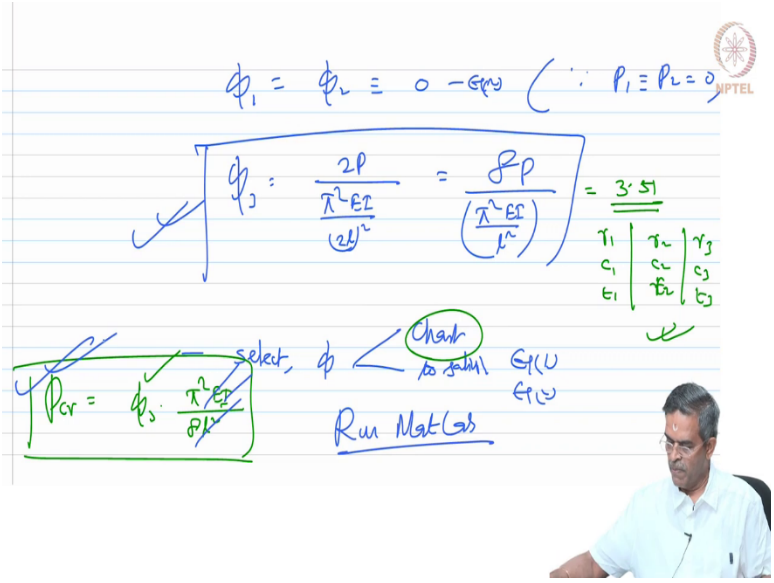

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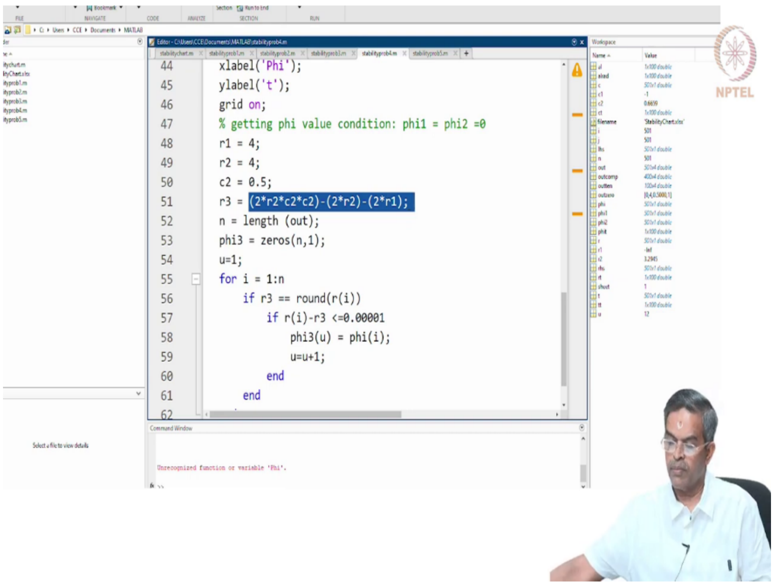

Let us run the MATLAB program and get the answer. So, now, I am running the MATLAB program. So, this is a program friends. So, the control equation is this.

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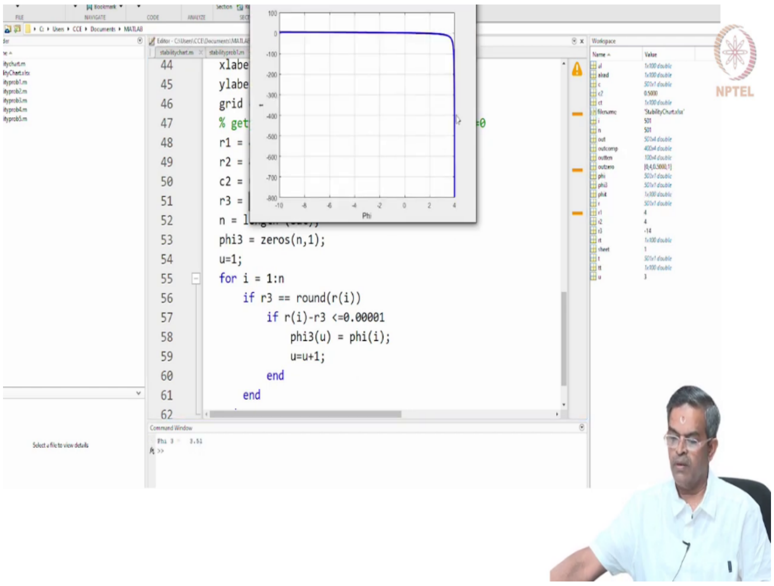

This is what we have. So, we can try to run the program and we get ϕ as 3.51 ok that isϕ3 ok 3.5. So, we should sayϕ3 was found to be 3.51 and the corresponding r 1 r 2 sorry c 1 and t 1 r 2 c 2 and t 2 r 3 c 3 and t 3 can be found out from the chart, I am not writing it here. So, we can do that.

So, if you have this value as 3.51. So, I can write P cr is equal toϕ3 of pi square EI by 8l square. So, this is known, if I know the cross section, if I know the length and if I know the E value I can find the Euler's critical, ok friends.

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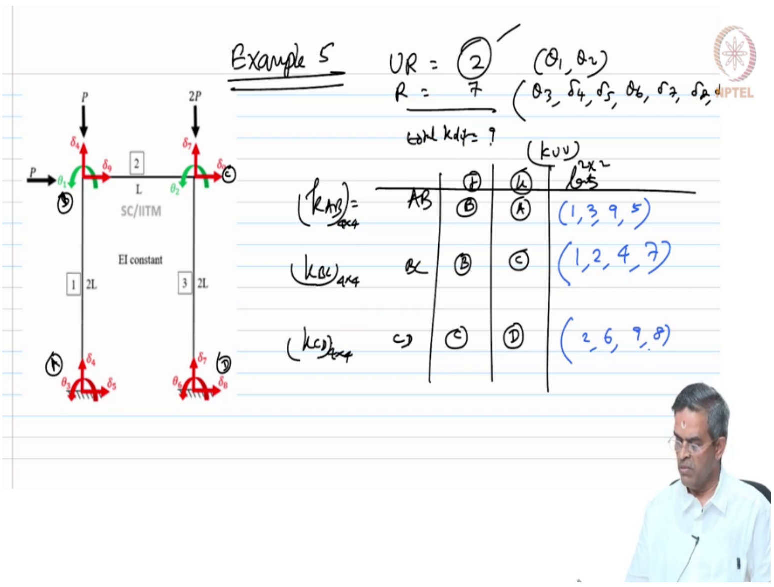

Let us do one more problem, example ϕ you can now see a single way single storey frame on the screen. So, I can quickly write some inferences, the unrestrained degrees of this problem are 2, which are theta 1 and theta 2 and the restrained degrees of this problem are 3, 4, 5, 6, 7, 8 and 9.

So, 7 the total kinematic degrees of freedom is 9. So, k uu will be of size 2 by 2 because unrestrained degree is only 2 and each k, that is k AB, let us say this is my A, this is B, C and D. So, k AB will be of size 4 by 4, k BC will be of size 4 by 4 and k CD will be of size 4 by 4. Let us write down quickly the labels for this ok, let us quickly write down the labels. So, let us make a table for the member AB, where is my j th end, where is my k th end ok and what are the labels.

We will keep for the member A B we will keep this as B and this as A. So, the labels are going to be quickly rotation at j, rotation at k, displacement at j and displacement at k. For the member BC we will keep the j th end at B and this at C. So, the labels are going to be 1, 2, 4 and 7 and for the member CD, we will keep the j th end at C, and this at D. Therefore, the labels are going to be 2, 6, 9 and 8. So, friends I think we can write easily the stiffness matrices of all the members, let us do that quickly.

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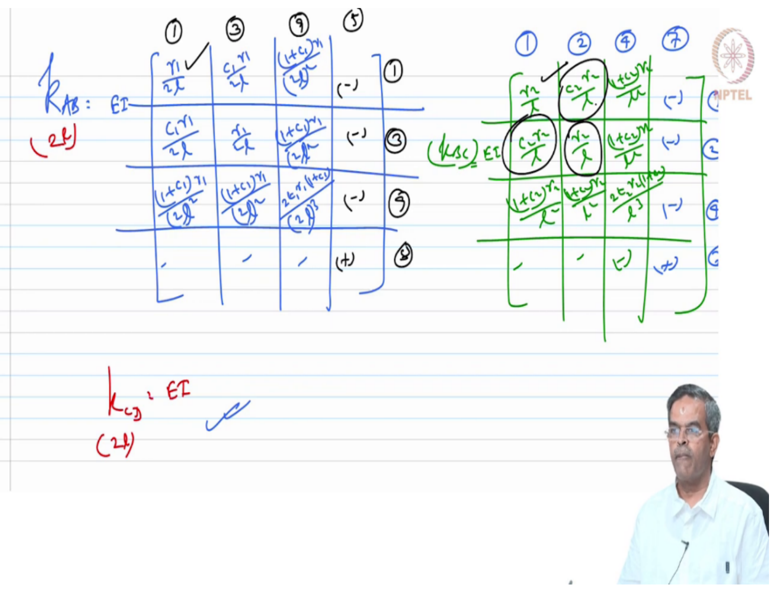

So, k AB will be E times of; so r 1 by l, 1 + c 1 sorry c 1 r 1 by l, then 1 + c 1 of r 1 by l - of this. Similarly, c 1 square so, c 1 r 1 by l, r 1 by l, 1 + c 1 of r 1 by l square and - of this. So, 1 + c 1 r 1 by l square, 1 + c 1 r 1 by l square, 2 times of t 1 r 1 1 + c 1 by l cube - of this, the

4th column is simply negative of the 3rd column and the labels are 1, 3, 9, 5; is it not 1, 3, 9, 5.

We can also write quickly for the member k BC, which is going to be E times of ok; r 2 by l, c 2 r 2 by l, 1 + c 2 of r 2 by l, - of this l square ok c 2 r 2 by l, r 2 by l, 1 + c 2 of r 2 by l square negative of this. So, 1 + c 2 r 2 by l square, 1 + c 2 r 2 by l square, 2 times of t 2 r 2 1 + c 2 by l cube - of this. And the 4th column will be negative of the 3rd column the labels are going to be 1, 2, 4 and 7 you can see here 1, 2, 4 and 7.

So, let us do it for k CD, which is E times of we should remember the length is 2l in this case is it not it is 2 no sorry, it is 2L in this case. So, we have to be careful ok even this also 2l, even this also 2 l. So, we should I think write 2l, 2l, 2l, the whole square ok right; k BC is just only l that is ok and k CD we can enter this, which will be I will do k CD in the next slide.

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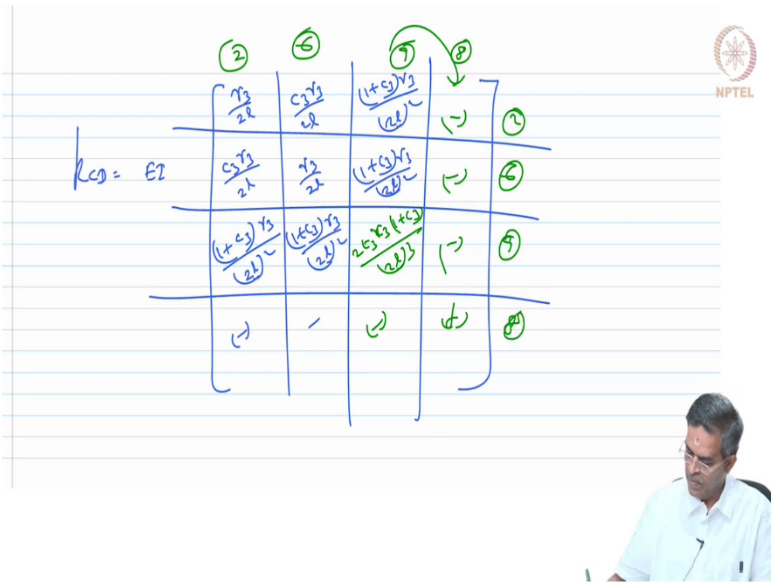

So, k CD EI times of which will be r 3 by 2l, c 3 r 3 by 2l 1 + c 3 of r 3 by 2l the whole square negative and this value will be 2 times of t 3 r 3 1 + c 3 by 2l the whole cube negative so, negative + of this column. So, the labels are going to be 2, 6, 9, 8 you can see here 2, 6, 9.

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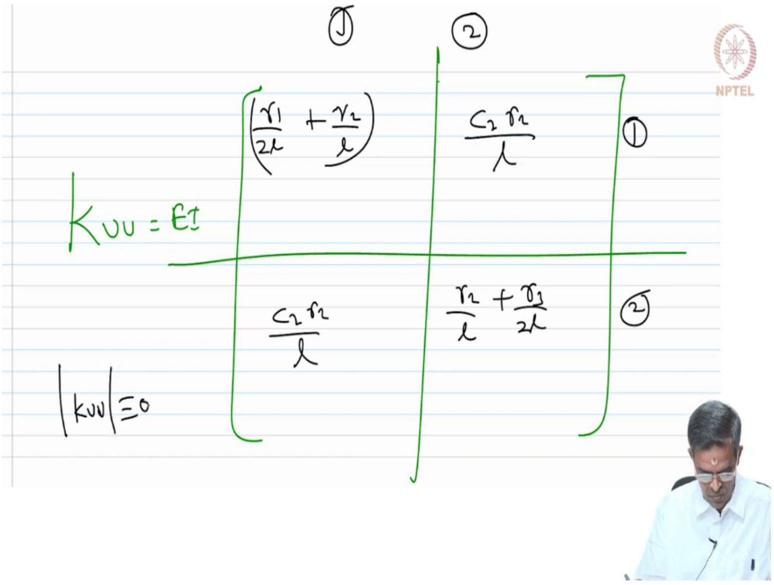

So, we have all the 3 matrices, I can assemble them and get k uu which will be E times of ok substituting we will get this as kuu. So, let us say for example, I should ok, should 1, 3, 9 and 5. So, I am looking for one more. So, I should get this and this. So, r 1 by 2l + r 2 by l so, r 1 by 2l + r 2 by l. 1, 2 and 1 and 2, let us do for 1 2.

So, 1 2 is here, c 2 r 2 by l, c 2 r 2 by l. So, let us say 2 1; 2 1 is here c 2 r 2 by l. So, c 2 r 2 by l let us go for 2 2, 2 2 are 2 places let us say r 2 by l ok r 2 by l + then we also have r 3 with 2l right. So, now, let us set this determinant to 0.

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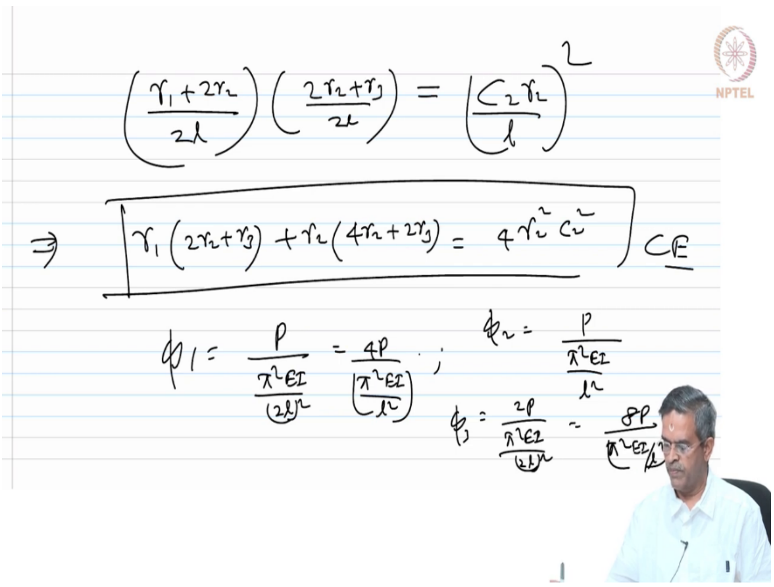

So, I can write that that is going to be r 1 + 2 r 2 by 2l see here, r 1 + 2 r 2 by 2l into 2 r 2 + r 3 by 2l should be equal to I can say should be equal to c 2 r 2 by l the whole square right, c 2 r 2 by l the whole square. So, this gives me rise to the characteristic equation which will be r 1 times of 2 r 2 + r 3 + r 2 times of 4 r 2 + 2 r 3 should be equal to 4 r 2 square c 2 square, that is my characteristic equation which I will use in MATLAB now and do it.

But further, let us also writeϕ1, see the figure, ϕ1 will be P, ϕ2 will be P whereas,ϕ3 will be 2P. So, let us do that. So, ϕ1 will be P by pi square EI by l square, which is 2l whole square, which will now become 4P by pi square EI by l square. Let us sayϕ2 is P by pi square EI by l square whereas, ϕ3 will be 2P by pi square EI by 2l the whole square which will become 8P by pi square EI by l square.

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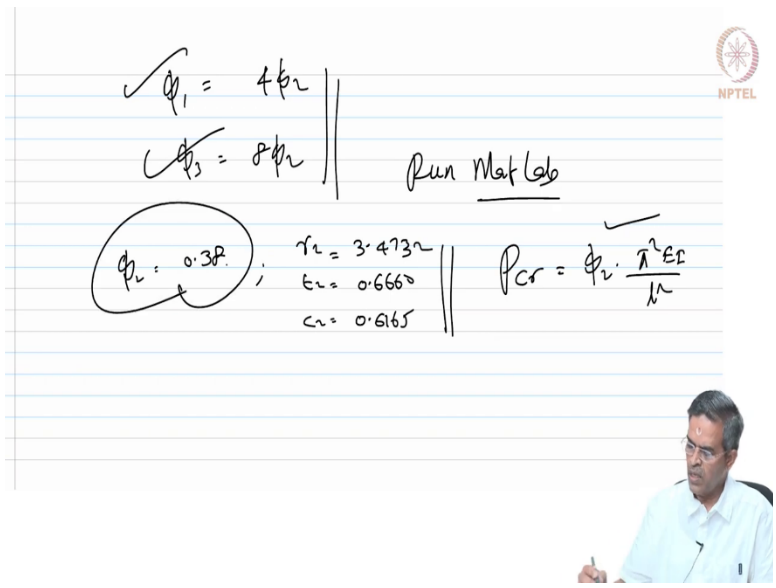

So, now I can write a relationship whereϕ1 is 4ϕ2 andϕ3 is 8ϕ2 ok, let us run the MATLAB program for the 5th problem.

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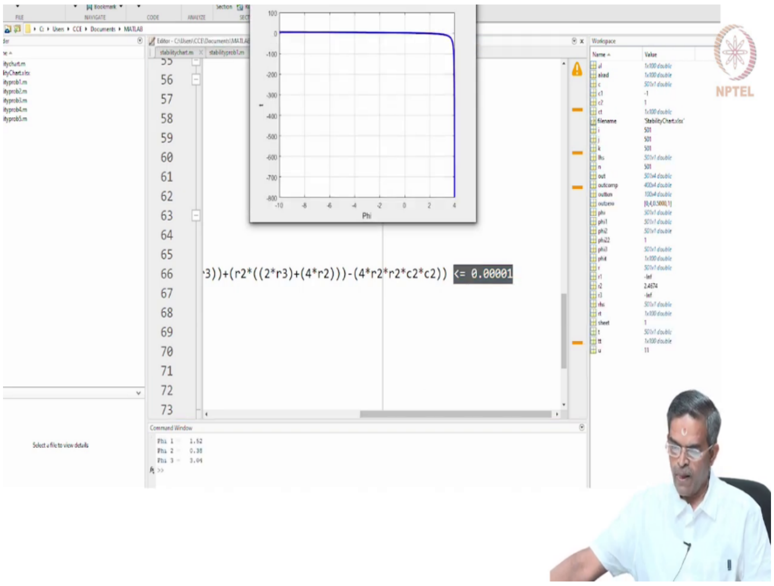

So, this is the 5th problem, we have the program here, we have the control equation which is here. Set to close to 0 0 ok, not 0, 00 let us run this program, I get these are the charts of r t and p, I get $\phi 1$, $\phi 2$, $\phi 3$ as you see on the screen. So, let us say $\phi 2$ is 0.38 and $\phi 1$ $\phi 3$ can be obtained. I will minimize this, I can now say $\phi 2$ is 0.38, I can find $\phi 1$ and $\phi 3$ does not matter. So, for this I get r 2 c 2 and t 2. I got r 2 as 3.4738 sorry 32, c 2 as point triple 60 and this is point. So, this is t 2 and this is c 2 which is 0.6165. So, now, I can say P cr is $\phi 2$ times of pi square EI by l square, I have $\phi 2$. If you know E, I and l I can find P c.

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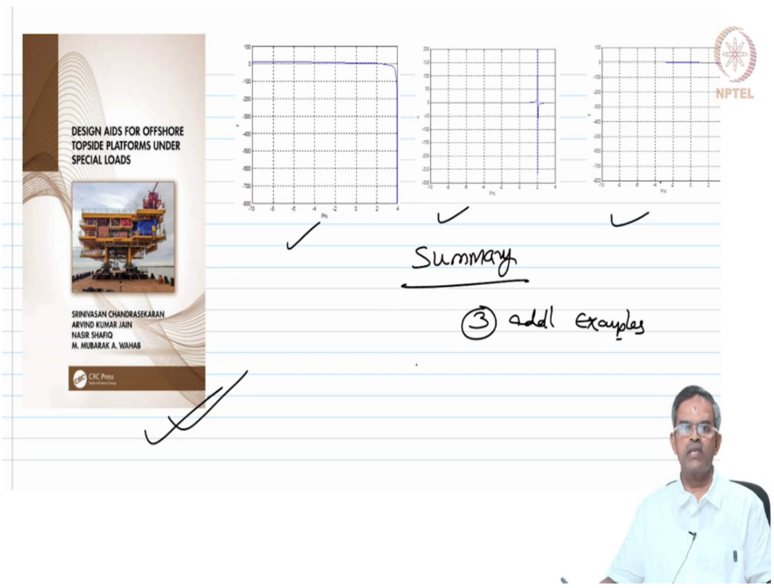

So, friends in this lecture we also learned 3 additional numerical examples to find Euler's critical law. This book is a very good reference which you must use for stability functions derivation understanding more problems and computer programs and MATLAB, you can download them, and this book is available in open access.

So, please see the book and try to use it for your library, recommend it to a library and use it thoroughly. The MATLAB programs are available in this book, you can copy paste them and run them these are the typical charts what you will see when you run the program. So, friends practice more examples in stability problems and if you have any difficulty, you can always discuss it in the forum, we will try to help you out.

Thank you very much and have a good day bye.