

Advanced Design of Steel Structures
Dr. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture - 35
Stability of shells

(Refer Slide Time: 00:17)

Lecture 35

- Learn to check the stability of cylindrical shells/tubes
- Stiffened cylinders
 - Common structural members - offshore structures
- classified as
 - ring-stiffened
 - stringer-stiffened
 - ring-stringer stiffened

cylinders (also known as orthogonally stiffened cylinders)

Friends, welcome to the 35th lecture on the course on Advanced Steel Design. In this course we are going to learn more about checking the Stability of cylindrical shells or tubes let us say or tubular elements. Now the stiffened cylinders are one of the major structural components of special kind of structures. They are quite common in offshore compliance structures. So, we are now talking about stiffened cylinders.

They are common structural members in offshore structures. They can be classified as ring stiffened stringer stiffened and ring stringer stiffened cylinders. So, the whole batch is also called as orthogonally stiffened cylinders. They are also known as orthogonally stiffened cylinders.

(Refer Slide Time: 02:37)

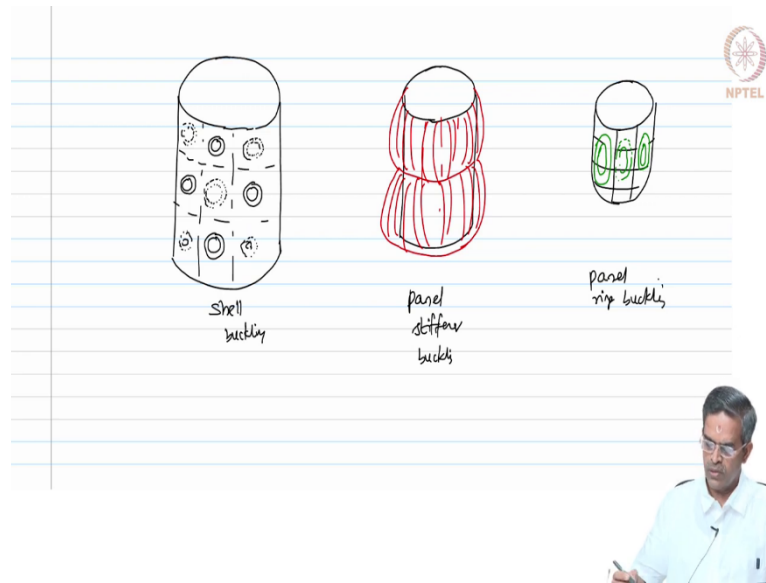
The slide contains handwritten notes on a lined background. At the top, the word "Stringers" is written. Below it, there are several bullet points: "- longitudinal stiffeners", "- externally vs internally", and "⊗ equal spacing". Further down, there are notes about cross-sections: "- c/s flat bar", "L", and "T sect". A bracket labeled "welding process" points to "geometric distortion" and "residual stresses". To the right of these, it says "orthogonally stiffened cylinders" and "generally fail by buckling". At the bottom, there is a list of buckling modes: "- different modes: shell buckling, panel stiffener buckling, panel ring buckling, column buckling". The NPTEL logo is visible in the top right corner of the slide area.

Now, let us quickly understand what stringers are. Stringers are longitudinal stiffeners which are either attached externally or internally to the cylinder at equal spacing. The stiffness can be of cross section there can be a flat bar they can be angles they can be T sections and so on.

They are generally integral welded to the shell, and they help in resisting the lateral loads. Now the structure is fabricated from hot or cold form plates essentially the welding will be a butt welding and therefore, we need to check whether the stiffened cylinders are stable for a given configuration.

The geometric distortion and residual stresses are some of the common problems which you heard in steel structures which commonly occur under welding process. So, we can say that welding process initiates geometric distortion and residual stresses. So, upon the types of failure orthogonally stiffened cylinders generally fail by buckling. This buckling can be in different modes. They can be shell buckling, panel stiffener buckling, panel ring buckling and column buckling.

(Refer Slide Time: 05:53)

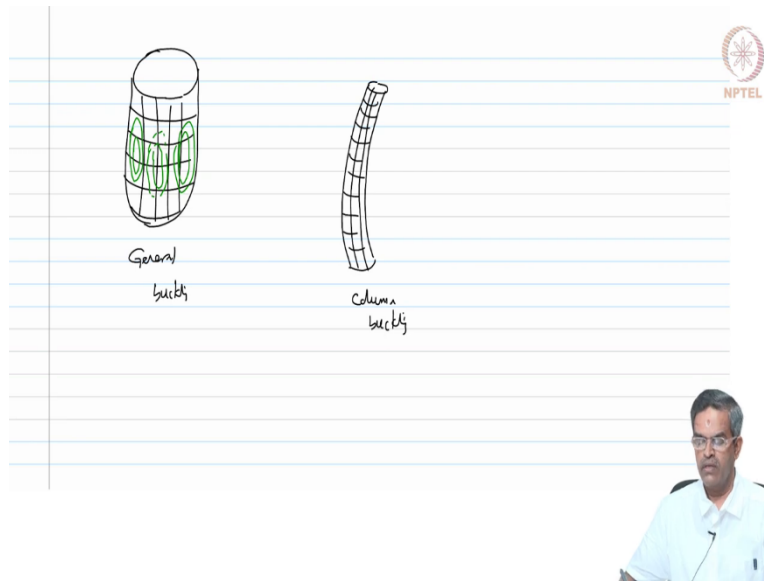


Now, let us try to draw them and show how do they look like. Let us say this is my cylinder. Now when there are different layers along the length and along the circumference some of them will have solid concentrations of stresses some of them will have negative. I am showing the positive ones as firm lines stress concentration factors and the negative ones as dotted lines.

So, this form of failure is called shell buckling. They generally form a pair; they generally occur alternatively they generally occur alternatively. The second one is called panel stiffener buckling. Let us take a shell again cylinder. Now what happens in this case is the panels will start buckling.

So, I should say and again there will be on different layers the compression tension. So, this is panel stiffener buckling. Panel ring buckling is a different phenomenon, which occurs in this form. Again, there will be alternate tension compression inundation and solid bulging. So, this will be panel ring buckling.

(Refer Slide Time: 09:06)



Of course, general buckling looks like this. So, there will be tension and compression stress concentrations or along the length of the member. So, this is called general buckling whereas, a column buckling phenomena will be like what we have seen in a column. So, these are different buckling modes of failure of orthogonally stiffened cylinders. We will do a problem in relationship with an international code and try to understand how code classifies stability of this circular cylinders.

(Refer Slide Time: 10:29)

Exercise Check the stability of a ring-stiffened thin cylinder under axial comp stress of 30 N/mm^2 against shell buckling.

Data given

dia of the cylinder	: 5.0m
Thickness	: 20mm
Length	: 8m
spacing of the ring stiffeners	: 800mm
σ_y	: 433 N/mm^2

So, we will do a simple exercise. We will say check the stability of a ring stiffened circular cylinder under axial compressive stress of 30 newton per mm square against shell buckling. We will just look only for one mode of failure. So, the data given is like this diameter of the cylinder is 5 meters, thickness of the cylinder 30 millimeters, length of the cylinder 8 meters, spacing of the ring stiffness is 800 mm center and take σ_y as 433 newton per mm square.

(Refer Slide Time: 12:17)

Solution

a) Stability requirements (DNV-RP-C202)

acc to sec 3.1 of the code

The stability requirements of a circular cylinder under axial σ is given by

$$\sigma_d \leq f_{ksd} \quad (1)$$

σ_d : design shell buckling strength

f_{ks} : characteristic buckling strength

$$f_{ksd} = \frac{f_{ks}}{r_m}$$

So, let us do this. So, let us say the solution. First let us try to find out the stability requirements, then we will use DNV recommended practice C 202 code. So, according to section 3.1 of the code when the moment I say code we are referring to this of the code. The stability requirements of a circular cylinder under axial stress is given by the following equation σ_d should be less than or equal to f_{ksd} we call this equation number 1.

Where, σ_d is known as design shell buckling strength ok where f_{ksd} is called characteristic buckling strength. This is given by f_{ks} by r_m where f_{ks} is the characteristic buckling strength ok not ksd . So, now, the question is how you find f_{ks} .

(Refer Slide Time: 14:31)



(b) characteristic buckling stress (f_{ks})

$$f_{ks} = \frac{f_y}{\sqrt{1 + \bar{\lambda}^4}} \quad (2)$$

$$\bar{\lambda}^2 = \frac{f_y}{\sigma_d} \left[\frac{\sigma_{ao}}{f_{ea}} + \frac{\sigma_{mo}}{f_{em}} + \frac{\sigma_{ho}}{f_{eh}} + \frac{\tau}{f_{ec}} \right] \quad (3)$$

$$\sigma_d = \sqrt{(\sigma_a + \sigma_m)^2 - (\sigma_a + \sigma_m)\sigma_h + \sigma_h^2 + 3\tau^2} \quad (4)$$

design axial stress, $\sigma_a = 30 \text{ N/mm}^2$ (given)
 design bending stress, $\sigma_m = 0$

Let us go for characteristic buckling stress f_{ks} . So, we know f_{ks} is given by f_y by root of $1 + \lambda^4$ we call equation number 2 where, λ^2 is given by f_y by σ_d of σ_{ao} by $f_{ea} + \sigma_{mo}$ by $f_{em} + \sigma_{ho}$ by $f_{eh} + \tau$ by f_{ec} equation number 3. And σ_d the design strength is given by the square root of squares of $\sigma_a + \sigma_m$ square - $\sigma_a + \sigma_m$ of σ_h + σ_h square + 3 tau square we call equation number 4.

They are available in DNV RP C 202 code. Let us substitute the values we know the design axial stress which is applied on the problem σ_a is 30 which is given in the problem ok and the design bending stress σ_m we should take it as 0 because we are saying it is under pure axial stress.

(Refer Slide Time: 16:38)

design circumferential stress, $\sigma_h = 0$
design shear stress, $\tau = 0$.

sub in Eq(4) we get $\sigma_d = 30 \text{ MPa}$

Characteristic buckling strength of circular cylinder for check under shell buckling is given by:

sec. 3.4.2, (PIS) of the code

$$f_E = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{r}\right)^3 \quad (5)$$
$$C, \text{ reduced buckling coeff} = \psi \sqrt{1 + \left(\frac{r\epsilon}{\psi}\right)^2} \quad (6)$$

So, then the design circumferential hoop stress σ_h is also 0 in our problem. The design shear stress τ is also set to 0. Now, I substitute this in equation 4 ok by substituting in equation 4 we get σ_d as 30 mega pascals. The characteristic buckling strength of circular cylinder under shell buckling is given by this following equation of circular cylinder for check under shell buckling is given by section 3.4.2 of page 15 of the code.

I am producing the equation again here f_E is given by C times of π^2 square E by $12(1-\nu^2)$ square of square t by r the whole square we will call it equation number 5. Whereas, in the above equation C is called reduced buckling coefficient is given by ψ times of root of $1 + \left(\frac{r\epsilon}{\psi}\right)^2$. So, equation number 6.

(Refer Slide Time: 19:07)

Use Table 3.2 of the Code

$$\rho = 1.0$$

$$\xi = 0.702 \sqrt{z_1}$$

$$z_1 = \frac{l^2}{rt} \sqrt{1-\nu^2}$$

$$\rho = 0.5 \left[1 + \frac{r}{150t} \right]^{-0.5}$$

We know the spacing of the stiffener, $l = 800 \text{ mm}$

Cylinder radius, $r = 2500 \text{ mm}$

Cylinder thickness, $t = 30 \text{ mm}$

Poisson's ratio, $\nu = 0.3$

Subst. in Eq (7) we get

Now, what we do? We use table 3.2 of the code and we say psi is unity. And zeta is 0.702 z 1 where z 1 is l square by r t root of 1 - mu square we call this equation number 7 a this as equation number 7 b and this further ok this as 7 b and rho is given by 0.5 times of 1 + r by 150 t this is r ok whereas, this is nu - 0.5. We call it as 7 C.

Now we know the spacing the stiffener the spacing of the stiffener given is l which is 800 mm. We also know the cylinder radius which r is 2500 mm we also know the cylinder thickness which is t is 30 mm we also know the Poisson's ratio which is nu which is 0.3.

(Refer Slide Time: 21:11)

$$z_1 = 7.765$$

$$\xi = 5.451$$

$$\rho = 0.401$$

from Eq (6), reduced buckling coeff, $C = 2.404$

The characteristic buckling stress is given by

$$f_E = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{l} \right)^2$$

$$= 2.404 \frac{\pi^2 (2.1 \times 10^5)}{12(1-0.3^2)} \left(\frac{30}{800} \right)^2$$

$$= 641.84 \text{ N/mm}^2$$

Now substituting this in equation 7 we get Z 1 as 7.765 zeta as 5.451 and rho as 0.401 now from equation 6. Let us say equation 6 we get now the reduced strain is it not equation 6 gives me the reduced buckling coefficient is given by 2.404. You can see here in this equation we know all the values you can substitute and get C.

Now the characteristic buckling strength is given by $f_c E$ which is equal to C times of π^2 square $E I^2 / (1 - \nu)$ square t^3 by l the whole square you know that is the equation we have. Let us substitute 2.404 π^2 square 2.5 10^5 we are talking about t ok 12 $1 - \nu$ we have taken this as 0.3 the Poisson's ratio. So, $1 - 0.3$ square of square t^3 by l the whole square am I right. So, this value comes to about 641.64 newton per mm square.

(Refer Slide Time: 23:08)

(ii) check for stability

$$\bar{\lambda}^2 = \frac{f_y}{\sigma_d} \left(\frac{\sigma_{a0}}{f_{Ea}} \right) = 0.675$$

sub this $\bar{\lambda}$ in $f_{k s} = \frac{f_y}{\sqrt{1 + \lambda^4}} = 297.514 \text{ N/mm}^2$

As per sec 31, P13 of the code,
 $\gamma_m = 0.85 + 0.60 \bar{\lambda} \quad (0.5 \leq \bar{\lambda} \leq 10)$
 Hence, $\gamma_m = 1.343$

So, now let us check for the stability from equation 3. What is equation 3? Let us see this equation here from equation 3 we know this value can be now obtained for substitution for the equation 3 lambda bar square is given by f_y by σ_d times of σ_{a0} by f_{Ea} right remaining all are set to be 0s is it not which is coming to be 0.675; substituting this in equation 2. What is equation 2?

See here equation 2 will give you the characteristic buckling strength ok $f_{k s}$ is given by f_y by root of $1 + \lambda^4$ right. So, let us do that which is going to be 297.514 newton per mm square. So, now, using section 3.1 page 13 of the code ok, we get γ_m as $0.85 + 0.60$ lambda bar. If lambda bar is between 0.5 and 10 have a lambda bar 5 lambda bar is 0.675 therefore, we get γ_m as 1.343.

(Refer Slide Time: 25:44)

Now, $f_{ksd} = \frac{f_{ks}}{\gamma_m} = \frac{297.514}{1.343} = 221.54 \text{ N/mm}^2$

Since $\sigma_d = 30 \text{ N/mm}^2 < f_{ksd}$,

Hence, the circular cylinder is safe against shell buckling.

Once we get γ_m I can now find f_{ksd} which is nothing but f_{ks} by γ_m which is going to be 297.514 by 1.343 is it not which is going to be 221.54 newton per mm square. Now, since the design strength is 30 which is much lesser than f_{ksd} we can conclude that the circular cylinder is safe against shell buckling.

(Refer Slide Time: 26:54)

Ex 2 Calculate the characteristic buckling strength of the stiffened cylindrical shell with the following data

Data

- Diameter shell = 10 m
- Thickness = 40 mm

Stiffener

- flat bar: 300x40 mm as stiffener
- spacing between rings = 600 mm/c
- axial comp stress = 40 N/mm²
- circumferential comp stress = 70 N/mm²
- $\sigma_y = 430 \text{ N/mm}^2$

Let us do one more exercise we will say calculate the characteristic buckling strength of the stiffened cylindrical shell with the following data. Let us see what the data are given. So, diameter of the shell is 10 meters thickness of the shell is 40 millimeters. Let us say we are

using stiffness as follows. Using a flat bar of size 300 by 40 mm as stiffener ring frame spacing is 600 mm center center. Let us say the axial compressive stress is 40 mega pascal and circumferential compressive stress is 70.4 sorry is 70 mega pascal and yield strength of the material is 433.

(Refer Slide Time: 29:02)

Handwritten notes on a slide with equations for buckling stress and design stress. The slide includes the NPTEL logo in the top right corner. The text is as follows:

iv) characteristic buckling stress

$$f_{ks} = \frac{f_y}{\sqrt{1+\lambda^4}} \quad (10)$$

$$\lambda^2 = \frac{f_y}{\sigma_d} \left[\frac{\sigma_{ao}}{f_{E\sigma}} + \frac{\sigma_{mo}}{f_{Em}} + \frac{\sigma_{ho}}{f_{Eh}} + \frac{\tau_o}{f_{E\tau}} \right] \quad (11)$$

$$\sigma_d = \sqrt{(\sigma_a + \sigma_m)^2 - (\sigma_a + \sigma_m)\sigma_h + \sigma_h^2 + 3\tau^2} \quad (12)$$

design axial stress, $\sigma_a = -40 \text{ N/mm}^2$
 design bending stress, $\sigma_m = 0$

Let us solve this problem. So, 1 let us find the characteristic buckling stress I am using the same code rather the same class. f_{ks} is given by f_y by root of $1 + \lambda^4$ we call this equation number 10 and we know λ^2 is given by f_y by σ_d of σ_{ao} by $f_{E\sigma} + \sigma_{mo}$ by $f_{Em} + \sigma_{ho}$ by $f_{Eh} + \tau_o$ by $f_{E\tau}$ equation 11.

We already wrote this equation from the code and we also know σ_d is given by square root of $\sigma_a + \sigma_m$ the whole square - $\sigma_a + \sigma_m$ into $\sigma_h + \sigma_h$ square + 3 tau square equation number 12. We know the design axial stress σ_a is - 40 because it is compressive design bending stress σ_m is 0.

(Refer Slide Time: 30:57)

design circumferential $\sigma_h = 70 \text{ N/mm}^2$

design shear stress, $\tau = 0$

Sub in the above eq

$$\sigma_d = 60.828 \text{ N/mm}^2$$

in Elastic buckling stress

ref sec 3.3.2, p14 of the code

aspect ratio, (l/s) $l =$ c/c of the ring stiffness, 1000mm
distance b/w the longitudinal stiffness, $s = 600 \text{ mm}$

$$\text{aspect ratio} = \frac{1000}{600} = 1.667$$

Further, design circumferential stress is $\sigma_h = 70$. And design shear stress τ is 0. Substituting in the above equation we will get σ_d as 60.828 newton per mm square. Let us now compute the elastic buckling stress. So, we use section 3.3.2 on page 14 of the code. Let us talk about the aspect ratio l by s . Let us see what the value is where, l is the distance of the say spacing of the ring stiffness which is in this case 1000 mm.

Now, distance between the longitudinal stiffness. We call this as s which is 600 mm. So, the aspect ratio in our case is 1000 by 600 which is 1.667 which is greater than 1.

(Refer Slide Time: 33:02)

Then, the characteristic buckling stress of the cylindrical shell is checked for shell buckling

$$f_E = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{s}\right)^2 \quad (13)$$

reduced coeff, $C = \psi \sqrt{1 + \left(\frac{P}{P_c}\right)^2} \quad (14)$

Table 3.2, of the code, $\psi = 4$

$$\zeta = 0.702 z_s$$

$$z_s = \frac{s^2}{t^3} \sqrt{1 - \nu^2}^{-0.5} \quad (15)$$

$$P = 0.5 \left[1 + \frac{\gamma}{150E} \right]$$

When it is greater than 1, then the characteristic buckling strength of the cylindrical shell checked for shell buckling is given by $f E$ is C times of $\pi^2 E$ by $12(1 - \nu^2) t^3$ by 1 the whole square.

We call this equation number 13. Now the reduced coefficient C reduced buckling coefficient C is given by ψ times of square root of $1 + \rho \epsilon$ by ψ the whole square equation 14. So, from table 3.2 of the code ψ is 4. So, ζ will be $0.702 \zeta s$ is s square by $r t$ of root of $1 - \nu^2$ or ν square ρ is 0.5 square root sorry $1 + r$ by $150 t - 0.5$. We call this equation number 15.

(Refer Slide Time: 35:02)

spacing between longitudinal stiffeners, $s = 600 \text{ mm}$

cylinder radius, $r = 5000 \text{ mm}$

cylinder thickness, $t = 40 \text{ mm}$

Poisson's ratio, $\nu = 0.3$

Substituting, we get

$\zeta_1 = 1.638$

$\zeta = 1.15$

$\rho = 0.369$

Substituting in Eq (14), we get, $C = 4.022$

So, now we already know the spacing between the longitudinal stiffeners s is 600 mm, cylinder radius r is 5000 mm, cylinder thickness t as 40 mm and Poisson's ratio ν is 0.3. So, substituting these values we get ζ_1 as 1.638 ζ as 1.15 and ρ as 0.369. So, substituting in equation 14 we get, what is equation 14? This is equation 14 ok we get C we get C as 4.022.

(Refer Slide Time: 36:30)

The characteristic Buckling strength is given by


$$f_{Ea} = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{l}\right)^2$$

$$= 4.022 \frac{\pi^2 \times 2.1 \times 10^5}{12(1-0.3^2)} \left(\frac{40}{600}\right)^2$$

$$= 3392.78 \text{ N/mm}^2$$

(ii) Elastic buckling strength under circumferential stress

Table 3.2 $\psi = \left[1 + \left(\frac{s}{l}\right)^2\right]^{-1}$ — (16a)



Therefore, the characteristic buckling strength is given by f_{Ea} under axial a C times of π square E by 12 $1 - \mu$ square t by l the whole square. So, which will become 4.022 times of π square 2.1 10^5 12 $1 - 0.3$ square of square 40 by 600 the whole square which now becomes 3392.78. Now, the elastic buckling strength under circumferential stress is given by values from table 3.2. So, we say ψ is $1 + s$ by l the whole square ok and ok we call this equation number we call this equation number 16 a.

(Refer Slide Time: 38:16)

$$\xi = 1.04 \frac{s}{l} \sqrt{Z_s}$$

$$Z_s = \frac{s^2}{rE} \sqrt{1-\nu^2}$$

$\nu = 0.3$


s , the longitudinal stress = 600mm

cylinder radius, $r = 500$ mm

or t , $t = 4$ mm

$$\nu = 0.3$$

Eq (16a) $\psi = 1.05$; $\xi = 0.799$



Zeta is 1.04 s by l root z s ok and z l is s square by rt root of 1 - nu square. Now, we know rho is 0.6 s which is the distance between the longitudinal stiffness is 600 mm cylinder radius is 5000 millimeters, Poisson's cylinder thickness is 40 mm, Poisson's ratio is 0.3. So, substituting in equation 14 we get psi as 1.85 equation 14 is what we have here. Equation 16 a psi as 1.85 and zeta as 0.799 and these are all equation 16 b, 16 c and so on.

(Refer Slide Time: 39:58)

substituting in Eq. C = 1.911

Hence, the characteristic buckling strength:

$$f_{Eh} = C \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{R}\right)^2$$

$$= 1.911 \frac{\pi^2 \times 2.1 \times 10^5}{12(1-0.3^2)} \left(\frac{40}{600}\right)^2$$

$$= 1612.04 \text{ N/mm}^2$$

Eq. 11

$$\bar{\lambda}^2 = \frac{f_y}{\sigma_d} \left[\frac{\sigma_{ao}}{f_{Ea}} + \frac{\sigma_{ho}}{f_{Eh}} \right] = 0.393$$



$$\bar{\lambda} = 0.627$$

So, now substituting this in equation 12 let us go back to this and try to find out capital C as 1.91 one in equation 12 is what C for C here oh not equation 12 equation 14 sorry equation 14. So, therefore, the characteristic buckling strength is given by f Eh C times of pi square E E by 12 1 - nu square square t by l the whole square which will be 1.911 pi square 2.1 10 power 5 12 1 - 0.3 square square 40 by 600 the whole square which becomes 1612.04 newton per mm.

Now, lambda bar square is given by f y by sigma d see this equation, equation 11. So, substituting in equation 11 f y by d sigma ao by f Ea + sigma ho by f Eh which is 0.393 and therefore, lambda bar will be 0.627.

(Refer Slide Time: 42:01)

Hence, the characteristic buckling strength is given by.



$$f_{ks} = \frac{f_y}{\sqrt{1 + \lambda^4}} = \underline{\underline{402.996 \text{ MPa}}}$$


Hence the characteristic buckling strength is given by f_{ks} , which is f_y by root of $1 + \lambda^4$ which is 402.996 mega pascal.

(Refer Slide Time: 42:34)

Summary

- Example(s) - built up of stiffened cylinder
- Int code - Checks the stability as recommended by the code



So, friends in this lecture we learnt an example. In fact, of finding the buckling strength of stiffened cylinder in fact, we have used an international code and checked the stability as recommended by the code. So, that is the procedure how buckling strength or buckling stress estimates or practiced and applied in design as far as international codes are concerned.

So, friends in these like set of lectures on stability we learnt different types of stability. We learnt stability is a mode of failure and we have learnt the difference between stability and buckling and we have derived the stability functions for axial compression, 0 axial load and axial tension.

The rotational and translational functions of stability equations from the first principles and we really understood that the stiffness matrix formulated for a stability problem is more or less similar to that of a matrix generated for a simple analysis of statically indeterminate structure.

Provided you remember the order of the label and remember the stiffness matrix. Once you have this matrix we have formulated the characteristic equation where we set k_{uu} determinant to 0 for a non trivial solution and we obtained the characteristic equation. We used a stability charts or stability tables available in the literature to choose the ψ value sorry the ϕ value which is the ratio between the axial force and the Euler's critical load.

And we found out that to satisfy this equation and we determine this value. We also exposed to you the MATLAB programs useful to plot the stability functions and also subroutines to solve the problems to estimate the critical buckling loads. In the next lectures we will talk about unsymmetric bending, curved beams and crane hook design and analysis using standard equations and MATLAB applications.

Thank you very much and have a good day.